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Michael Friedman
Department of Philosophy
University of Illinois
Chicago, USA

POINCARÉ'S CONVENTIONALISM AND THE LOGICAL POSITIVISTS

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Abstract. The logical positivists adopted Poincare's doctrine of the conventionality of geometry and made it a key part of their philosophical interpretation of relativity theory. I argue, however, that the positivists deeply misunderstood Poincare's doctrine. For Poincare's own conception was based on the group-theoretical picture of geometry expressed in the Helmholtz-Lie solution of the "space problem", and also on a hierarchical picture of the sciences according to which geometry must be *presupposed* be any properly physical theory. But both of this pictures are entirely incompatible with the radically new conception of space and geometry articulated in the general theory of relativity. The logical positivists's attempt to combine Poincare's conventionalism with Einstein's new theory was therefore, in the end, simply incoherent. Underlying this problem, moreover, was a fundamental philosophical difference between Poincare's and the positivists concerning the status of synthetic a priori truths.

The great French mathematician Henri Poincaré is also well-known, in philosophical circles, as the father of geometrical conventionalism. In particular, the logical positivists appealed especially to Poincaré in articulating and defending their own conception of the conventionality of geometry. As a matter of fact, the logical positivists appealed both to Poincaré and to Einstein here, for they believed that Poincaré's philosophical insight had been realized in Einstein's physical theories. They then used both – Poincaré's insight and Einstein's theories – to support and to illustrate their conventionalism. They thus viewed the combination of Poincaré's geometrical conventionalism and Einstein's theory of relativity as a single unified whole.

How, then, do the logical positivists understand Poincaré's argument? They concentrate on the example Poincaré presents in the fourth chapter of *Science and Hypothesis*: the example, namely, of a world endowed with a peculiar temperature field. According to this example we can interpret the same empirical facts in two different ways. On the one hand, we can imagine, in the given circumstances, that we live in an infinite, non-Euclidean world – in a space of constant negative curvature. On the other hand, we can equally well imagine, in the same empirical circumstances, that we live in the interior of a finite, Euclidean sphere in which there also exists a special temperature field. This field affects all bodies in the same way and thereby produces a contraction, according to which all bodies – and, in particular, our measuring rods – become continuously smaller as they approach the limiting spherical surface. (Poincaré of course obtains the law of this contraction from his own model of Bolyai-Lobachevsky space.) We are thus here confronted with a case of observational equivalence; and so no empirical facts can force us to select either the Euclidean or the non-Euclidean description as the uniquely correct description. In this sense the choice of geometry is entirely free and therefore conventional.

Moritz Schlick, the founder of the Vienna Circle, presents just such an interpretation of Poincaré's argument in his 1915 article on the philosophical significance of the theory of relativity – which was the first article on relativity theory within the tradition of logical positivism:

> Henri Poincaré has shown with convincing clarity (although Gauss and Helmholtz still thought otherwise), that no experience can compel us to lay down a particular geometrical system, such as Euclid's, as a basis for depicting the physical regularities of the world. Entirely different systems can actually be chosen for this purpose, though in that case we also have at the same time to adopt other laws of nature. The complexity of non-Euclidean spaces can be compensated by a complexity of the physical hypotheses, and hence one can arrive at an explanation of the simple behavior that natural *bodies* actually display in experience. The reason this choice is always possible lies in the fact (already emphasized by Kant) that it is never space itself, but always the spatial behavior of bodies, that can become an object of experience, perception and measurement. We are always measuring,

> as it were, the mere product of two factors, namely the spatial properties of bodies and their physical properties in the narrower sense, and we can assume one of these two factors as we please, so long as we merely take care that the product agrees with experience, which can then be attained by a suitable choice of the other factor. (Schlick (1979), pp. 168-169)

This argument and relativity theory fit together especially well, according to Schlick, because relativity theory is also based on the idea that space and matter cannot be separated from one another.

Approximately fifty years later (1966) we find Rudolf Carnap still presenting essentially the same argument in his *Introduction to the Philosophy of Science*:

> Suppose, Poincaré wrote, that physicists should discover that the structure of actual space deviated from Euclidean geometry. Physicists would then have to choose between two alternatives. They could either accept non-Euclidean geometry as a description of physical space, or they could preserve Euclidean geometry by adopting new laws stating that all solid bodies undergo certain contractions and expansions. As we have seen in earlier chapters, in order to measure accurately with a steel rod, we must make corrections that account for thermal expansions or contractions of the rod. In a similar way, said Poincaré, if observations suggested that space was non-Euclidean, physicists could retain Euclidean space by introducing into their theories new forces – forces that would, under specified conditions, expand or contract the solid bodies. (Carnap (1974) pp. 144-145)

Carnap then concludes this chapter on Poincaré's philosophy of geometry by remarking that we will see in the next two chapters on relativity theory how Poincaré's insight into the observational equivalence of Euclidean and non-Euclidean theories of space leads to a deeper understanding of the structure of space in relativity theory.

In my opinion, however, this conception of the relationship between Poincaré and Einstein rests on a remarkable – and in the end ironical – misunderstanding of history. The first point to notice is that the logical positivists' argument from observational equivalence is in no way a good argument for the conventionality of geometry – at least as this was understood by Poincaré himself. For the argument from observational equivalence has no *particular* relevance to physical geometry and can be applied equally well to *any* part of our physical theory. The argument shows only that geometry considered in isolation has no empirical consequences: such consequences are only possible if we also add further hypotheses about the behavior of bodies. But this point is completely general and is today well-known as the Duhem-Quine thesis: *all* individual physical hypotheses require further auxiliary hypotheses in order to generate empirical consequences.

Poincaré's own conception, by contrast, involves a very special status for physical geometry. He emphasizes in the Preface to *Science and Hypothesis*, for example, that his leading idea is that hypotheses of different kinds should be carefully distinguished from one another:

> We will also see that there are various kinds of hypotheses; that some are verifiable and, when once confirmed by experiment, become truths of great fertility; that others, without being able to lead us into error, become useful to us in fixing our ideas, and that the others, finally, are hypotheses in appearance only and reduce to definitions or conventions in disguise. (Poincaré (1913), p. 28)

Poincaré then enumerates the sciences where we are involved principally with the free activity of our own mind: arithmetic, the theory of mathematical magnitude, geometry, and the fundamental principles of mechanics. At the end of the series of sciences, however, comes something quite different: namely, experimental physics. Here we are certainly involved with more than our own free activity:

> Up to here [mechanics] nominalism triumphs, but we now arrive at the physical sciences properly speaking. Here the scene changes: we meet with hypotheses of another kind, and we recognize their great fertility. No doubt at first sight our theories appear fragile, and the history of science shows us how ephemeral they are; but they do not entirely perish, and from each of them something remains. It is this something that it is necessary to try to discover, because it is this, and this alone, that is the true reality. (Poincaré (1913), pp. 29-30)

The fourth part of *Science and Hypothesis* explicitly considers precisely these physical sciences properly speaking. There, under the heading "Nature," Poincaré discusses what he takes to be genuinely physical theories: e.g., optics and electrodynamics. Despite the obvious fact that the above-mentioned Duhemian argument applies equally well to these theories as well, Poincaré nevertheless considers them to be *non*-conventional. Hence, this Duhemian argument can certainly not – at least by itself – be Poincaré's own argument for the conventionality of geometry.

Poincaré's own argument involves two closely related ideas. The first is the already indicated idea that the sciences constitute a series or a hierarchy. This hierarchy begins with the purest a priori science – namely arithmetic – and continues through the above-mentioned sciences to empirical or experimental physics properly speaking. In the middle of this hierarchy – and thus in a very special place – we find geometry. The second idea, however, is the most interesting and important part of Poincaré's argument. For Poincaré himself is only able to argue for the conventionality of geometry by making essential use of the Helmholtz-Lie solution to the "problem of space." This specifically group-theoretical conception of the essence of geometry, that is, is absolutely decisive – and thus unavoidable – in Poincaré's own argument. In what follows I will consider these two ideas more closely.

The series or hierarchy of sciences begins, as we said, with arithmetic. For Poincaré arithmetic is of course not a branch of logic; for logic is a purely analytical science and thus purely tautological, whereas arithmetic is the first and foremost synthetic science – which therefore genuinely extends our knowledge. Arithmetic is synthetic, because it is based on our intuitive capacity to represent the (potentially) infinite repetition of one and the same operation. And this intuition is then the ground for the characteristically mathematical procedure of reasoning: namely, mathematical induction or reasoning by recurrence. Such reasoning by recurrence comprehends as it were an infinite number of syllogisms and is precisely for this reason in no way merely analytic. For no merely analytical procedure can possibly lead us from the finite to the infinite. Nevertheless, arithmetic is wholly a priori as well: mathematical induction forces itself upon us uniquely and necessarily, because it is precisely the expression of a unique power of our own mind. Therefore, arithmetic is neither an empirical science nor conventional.

The next lower level in the hierarchy of sciences is occupied by the theory of mathematical magnitude. Here Poincaré considers what we nowadays refer to as the system of real numbers. Poincaré, however, is not only interested in the purely formal properties of this system; on the contrary, he is interested above all in the psychological-empirical origin of our concept of this system. Specifically, he explains the origin of our concept of the system of real numbers in two steps. He first describes how the idea of the continuum arises: namely, through the repeated or iterative application of the principle of non-contradiction to just noticeable differences in Fechner's sense. But here we have only obtained the idea of an order-continuum, which does not yet contain metrical or measurable magnitudes. In order, then, to construct the latter, we must introduce a further element: namely, an addition operation. And, according to Poincaré, the introduction of such an addition operation is almost entirely arbitrary. It must of course satisfy certain conditions – the conditions for a continuous, additive semi-group. Nevertheless, we are according to Poincaré entirely free to introduce any addition operation whatsoever that satisfies the given formal conditions. Here, therefore, for the first time, we have a convention properly speaking – that is, a free stipulation.

So far we have considered only one dimensional continua. When we attempt to apply these ideas to multi-dimensional continua we reach the next level in the hierarchy of sciences: namely, the science of geometry. A multi-dimensional continuum becomes an object of geometry when one introduces a metric – the idea of measurability – into such a continuum. And, analogously to the case of one dimensional continua, we achieve this through the introduction of group-theoretical operations. In this case, however, the structure of the operations in question is much more interesting from a mathematical point of view. In the case of a three dimensional continuum, for example, instead of a continuous, additive semi-group of one dimension, we have a continuous group of free motions (in modern terminology a *Lie group*) of six dimensions. And, in my opinion, we can achieve a deeper understanding of Poincaré's own conception of the conventionality of geometry only through a more careful consideration of precisely these group-theoretical structures.

I will come back to this question in a moment. First, however, it is necessary briefly to consider the remaining two levels in the hierarchy of sciences. The next lower level after geometry is occupied by the science of mechanics. The laws of mechanics – for example, the Newtonian laws of motion – govern the fundamental concepts of time, motion, mass, and force; and these laws are also according to Poincaré conventional – at least for the most part. I understand him here to be arguing that the fundamental concepts of time, motion, mass, and force have no determinate empirical meaning *independently* of the laws of mechanics. Thus, for example, the laws of motion supply us with an implicit definition of the inertial frames of reference, without which no empirically applicable concepts of time or motion is possible; the concepts of mass and force are only empirically applicable on the basis of the second and third Newtonian laws of motion; and so on. The laws of mechanics do not therefore describe empirical facts governing independently given concepts. On the contrary, without these laws we would simply have no such concepts: no mechanical concepts, that is, of time, motion, mass, and force. In this sense the laws of mechanics are also free creations of our mind, which we must first inject, as it were, into nature.

Now, however, we have finally reached the empirical laws of nature properly speaking. For we have now injected precisely enough structure into nature in order to extract the genuinely empirical laws from nature. We do this, for example, by discovering particular force laws that realize the general concept of force defined by the laws of mechanics. Poincaré himself considers in this connection the Maxwell-Lorentz theory of the electromagnetic field and electrodynamic force especially, for this theory was of course of most interest in his time. But the point can perhaps be made even more clearly if we consider Newton's theory of universal gravitation. For Newton's *Principia* had already clearly shown how we can empirically discover the law of universal gravitation – on the *presupposition*, that is, of the Newtonian laws of motion and Euclidean geometry. Without these presuppositions, however, we would certainly not have been able to discover the law of gravitation. And the same example also shows clearly how every level in the hierarchy of sciences presupposes *all* of the preceding levels: we would have no laws of motion if we did not presuppose spatial geometry, no geometry if we did not presuppose the theory of mathematical magnitude, and of course no mathematics at all if we did not presuppose arithmetic.

I now return to a more detailed consideration of geometry. The metrical properties of physical space are based, as indicated above, on a Lie group of free motions; and the idea of such a group arises, according to Poincaré, from our experience of the motion of our own bodies. We thereby learn, in particular, to distinguish between changes in external objects and changes (that is, motions) of our own bodies. Then, through an idealization, we construct a separate concept of these latter changes (motions of our own bodies), and we represent this concept by means of a mathematical group. In this sense – that is, through an idealization – the idea of such a Lie group arises from our experience. At this point, however, a remarkable mathematical theorem comes into play: namely, the Helmholtz-Lie theorem. For, according to his theorem, there are three and only three possibilities for such a group: either it can represent Euclidean geometry (that is, it is a group of free motions of rigid bodies in a Euclidean space), or it can represent a geometry of constant negative curvature (hyperbolic or Bolyai-Lobachevsky space), or it can represent a geometry of constant positive curvature (elliptic, or, as it is sometimes called, Riemannian space). What is important here, for Poincaré, is that only the idea of such a Lie group can explain the origin of geometry, and, at the same time, this idea drastically restricts the possible forms of geometry.

Poincaré of course believes that the choice of any one of the three groups is conventional. Whereas experience suggests to us the general idea of a Lie group, it can in now way force us to select a specific group from among the three possibilities. Analogously to the case of the theory of mathematical magnitude we are here concerned basically with the selection of a standard measure or scale:

> This is the object of geometry: it is the study of a particular "group"; but the general concept of a group preexists in our mind, at least potentially. It imposes itself upon us – not as a form of our sensibility, but as a form of our understanding. However, from among all possible groups it is necessary to choose one that will be so to speak the *standard measure* [*étalon*] to which we relate the phenomena of nature. Our experience guides us in this choice but does not impose it upon us; it allows us recognize, not which is the truest geometry, but rather which is the most *convenient*. (Poincaré (1913), pp. 79-80)

> In our mind the latent idea of a certain number of groups preexists: those for which Lie has supplied the theory. Which shall we choose to be a kind of standard measure by which to compare the phenomena of nature? [...] Our experience has guided us by showing us which choice is best adapted to the properties of our own body. But there its role ends. (Poincaré (1913), p. 91)

But such a selection in this case is much more interesting from a mathematical point of view. In contrast to the case of one dimensional continua, a selection of the relevant group-theoretical operations here determines that the resulting system has one (and only one) of the three possible mathematical structures (Euclidean, constant negative curvature, or constant positive curvature). In this sense the mathematical laws here are completely determined by the selection of a particular scale.

Poincaré's conception becomes clearer when we contrast it with Helmholtz' earlier conception of geometry. For Helmholtz of course also proceeds from such group-theoretical considerations – that is, from the possibility of free motion – in attempting to justify a more *empiricist* conception of geometry; and, for precisely this reason, Helmholtz gives the title, "On the *Facts* which Lie at the Basis of Geometry," to his main contribution here. Where, then, lies the disagreement between Helmholtz and Poincaré? We should first remind ourselves that Helmholtz had first left Bolyai-Lobachevsky geometry completely out of consideration. His original idea was that there are only two possible geometries: namely, Euclidean geometry and elliptical (or spherical) geometry. From the fact that free motion in general is possible it follows that space must be either Euclidean or spherical. From the further fact that free motion is possible *to infinity* (so that an infinite straight line is possible) it then follows that space must be Euclidean. Now Helmholtz of course soon corrected this erroneous idea when he became acquainted with Bolyai-Lobachevsky geometry (through the work of Beltrami); but Poincaré, by contrast, clearly recognized from the very beginning that the most important and interesting choice is that between Euclidean and Bolyai-Lobachevsky geometry.

In the second place, however, Poincaré also clearly saw that the idea of the free motion of rigid bodies is itself an idealization: strictly speaking, there are in fact no rigid bodies in nature, for actual bodies are always subject to actual physical forces. It is therefore completely impossible simply to read off, as it were, geometry from the behavior of actual bodies, without first formulating theories about physical forces. (In my opinion, the point of the temperature field example is precisely to make *this* situation intuitively clear.) And it now follows that geometry cannot depend on the behavior of actual bodies. For, according to the above described hierarchy of sciences, the determination of particular physical forces presupposes the laws of motion, and the laws of motion in turn presuppose geometry itself: one must first set up a geometry before one can establish a particular theory of physical forces. We have no other choice, therefore, but to select one or another geometry on conventional grounds, which we can then use so to speak as a standard measure or scale for the testing and verification of properly empirical or physical theories of force. Moreover it is also remarkable (and we shall return to this point below) that relativity theory confirms Poincaré's conception more than it does Helmholtz'. For we here apply non-Euclidean geometry to nature, not through the mere observation of the behavior of rigid bodies, but rather through a fundamental revision of both the laws of motion and our physical theory of gravitation.

Nevertheless, relativity theory also shows that Poincaré's own conception of the role of geometry in physics is false in principle. For Poincaré's conception is entirely based, as we have seen, on an application of the Helmholtz-Lie theorem: geometry is conventional precisely because the general idea of a Lie group of free motions has three (and only three) possible geometrical realizations. Poincaré therefore presupposes throughout that the free motion of an ideal rigid body is possible and hence that space is homogeneous and isotropic: the only geometries that are possible on Poincaré's conception are the classical geometries of constant curvature. By contrast, in the general theory of relativity we use the much more general conception of geometry articulated in Riemann's theory of manifolds (not to be confused, of course, with the very particular case of constant positive curvature – which is sometimes also called Riemannian geometry). According to the general theory of relativity space (more precisely, the space-time continuum) is a manifold of *variable* curvature – and, in fact, a curvature that depends essentially on the distribution of matter.

Poincaré was not of course acquainted with the general theory of relativity. (He died in 1912.) He is nevertheless completely clear that his conception of geometry is not compatible with Riemann's theory of manifolds. And, for precisely this reason, he considers this more general theory to be purely analytical:

> If, therefore, one admits the possibility of motion, then one can invent no more than a finite (and even rather restricted) number of three dimensional geometries. However, this result appears to be contradicted by Riemann; for this scientist constructs an infinity of different geometries, and that to which his name is ordinarily given is only a special case. [...] This is perfectly exact, but most of these definitions [of different Riemannian metrics] are incompatible with the motion of an invariable figure – which one supposes to be possible in Lie's theorem. These Riemannian geometries, as interesting as they are in various respects, can therefore never be anything but purely analytic, and they would not be susceptible to demonstrations analogous to those of Euclid. (Poincaré (1913), p. 63)

The Riemannian theory is purely analytical, because it is not based on group-theoretical operations and therefore not on the possibility of repeating a given operation indefinitely:

> *Space is homogeneous and isotropic.* One may say that a motion that is produced once can be repeated a second time, a third time, and so on, without changing its properties. In the first chapter, where we studied the nature of mathematical reasoning, we have seen the importance that one should attribute to the possibility of repeating indefinitely the same operation. It is in virtue of this repetition that mathematical reasoning acquires its force; it is thanks to the law of homogeneity that it applies to the facts of geometry. (Poincaré (1913), p. 75)

Poincaré's conception is therefore entirely coherent. For the Riemannian manifolds of variable curvature contradict his explanation of the fact that geometry is a properly synthetic science.

Yet the general theory of relativity also contradicts Poincaré's conception in an even more fundamental way. This theory describes the motion of a body in a gravitational field as a geodesic (straightest possible curve) in a four dimensional manifold – that is, as a geodesic in a space-time continuum possessing a variable curvature depending explicitly on the distribution of matter. And this completely new formulation of the law of gravitation then also takes over the role previously played by the laws of motion. For the geodesics in space-time traversed by bodies in a gravitational field have here precisely the role previously played by the inertial motions. In other words, the law of gravitation takes over here the role of the law of inertia. It then follows, however, that one can no longer separate geometry from the laws of motion, and one can no longer separate the latter from the law of gravitation. On the contrary, in the general theory of relativity, geometry is simply identical to the theory of gravitation; this theory is in turn identical to the laws of motion or mechanics; and geometry is therefore also identical to mechanics.

In the general theory of relativity there can therefore by no question of a hierarchy of sciences in Poincaré's sense. Poincaré presents mathematical physics as a series of sciences in which every succeeding science presupposes all preceding sciences. General mechanics is presupposed by particular force laws and thus makes the latter possible; geometry is presupposed by general mechanics and thus makes both it and particular force laws possible; the theory of mathematical magnitude is presupposed by geometry; and arithmetic is presupposed by the theory of mathematical magnitude. In this way, Poincaré's conception of the sciences is actually quite similar to the Kantian conception. Yet Poincaré is writing at the end of the nineteenth century and can therefore not proceed from the idea that Euclidean geometry is the only possible geometry. In the context of the Helmholtz-Lie solution to the "problem of space" it then appears natural to suppose that we have a conventional choice among three (and only three) possibilities. And, precisely because geometry still appears to be the presupposition of all properly empirical sciences, this choice cannot itself be empirical. Thus, Poincaré's modernized Kantianism is particularly well adapted to the scientific situation of the late nineteenth century – such a modified Kantianism can no longer be maintained in the context of the radically new physics of the twentieth century, however.

In contrast to Poincaré, it is clear that the logical positivists, for their part, belong entirely to the twentieth century. And, in fact, Rudolf Carnap, Hans Reichenbach, and Moritz Schlick all attempted in their earliest writings philosophically to comprehend the theory of relativity. They even undertook the task of fundamentally reforming philosophy itself through precisely this attempt to comprehend Einstein's physical theories. Thus, for example, from the very beginning the logical positivists explicitly asserted that Einstein's new theories are completely incompatible with the Kantian conception of the synthetic a priori, so that this philosophical conception is simply now untenable. They also clearly recognized that Helmoltz' geometrical empiricism is untenable as well. For, in the general theory of relativity, we construct a non-Euclidean description of nature (as emphasized above), not by simply observing the behavior of rigid measuring rods, but rather by fundamentally revising both general mechanics and our theory of gravitational force. The logical positivists therefore sought for an intermediate position, as it were, lying *between* traditional Kantianism and traditional empiricism. And it seemed to them that precisely such an intermediate position is to be found in Poincaré's conception of convention.

We have seen, however, that Poincaré's own argument for geometrical conventionalism actually fails in the context of the general theory of relativity: neither his conception of a hierarchy of sciences nor his penetrating and insightful application of the Helmholtz-Lie theorem make sense in this new conceptual framework. The general theory of relativity essentially employs a geometry of variable curvature and also effects a holistic unification of previously separated sciences. For the logical positivists there was therefore no alternative but simply to ignore the characteristic elements of Poincaré's own argument and to concentrate instead solely on the example of the peculiar temperature field. In the absence of Poincaré's own conception of a hierarchy of sciences, however, it is clear that this example by itself can have no particular relevance to geometry. On the contrary, we thereby obtain (as emphasized at the very beginning) only a completely general holism, according to which every individual scientific hypothesis has empirical consequences only in connection with further auxiliary hypotheses. In other words, we thereby obtain only what is nowadays referred to as Duhemian or Duhem-Quine holism. And Quine himself, as is well-known, uses this Duhemian holism precisely to attack the conventionalism of the logical positivists: according to Quine there is of course no longer a difference in principle between facts on the one side and conventions on the other. It is therefore extremely problematic, at best, to base the thesis of the conventionality of geometry on Duhemian holism. As we have seen, what is most ironical here is the circumstance that just this holistic collapse of the conventional/factual distinction was already prefigured in the earlier encounter between Poincaré's geometrical conventionalism and the general theory of relativity.

It is therefore noteworthy that there was one logical positivist who, at least once in his life, correctly and explicitly recognized the incompatibility of Poincaré's conventionalism with the general theory of relativity. This was

Hans Reichenbach, in his first book, *The Theory of Relativity and A Priori Knowledge*, of 1920:

> It was from a mathematical standpoint asserted that geometry has only to do with conventional stipulations – with an empty schema containing no statements about reality but rather chosen only as the form of the latter, and which can with equal justification be replaced by a non-Euclidean schema.* Against these objections, however, the claim of the general theory of relativity presents a completely new idea. This theory makes the equally simple and clear assertion that the propositions of Euclidean geometry are just *false*. (Reichenbach (1965), pp. 3-4)
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> *Poincaré has represented this view. Cf. [*Science and Hypothesis*, Chap. III]. It is significant that for his proof of equivalence he excludes from the beginning *Riemannian* geometry, because it does not permit the displacement of a body without change of form. If he had guessed that precisely this geometry would be taken up by physics, he would never have been able to assert the arbitrariness of geometry.

Unfortunately, Reichenbach was soon convinced by Schlick that Poincaré's conception could still be valid in the context of the general theory of relativity. As is well-known, Reichenbach then occupies himself, in his later writings, precisely with the attempt to combine relativity theory with conventionalism. That this attempt must fail is implicit in the analysis of Poincaré's conventionalism I have presented.

Here, however, I will not pursue the story of Reichenbach's later conventionalism further. But I do want to emphasize how far the basic philosophical conception of the logical positivists deviates from that of Poincaré himself. For the empiricism of the logical positivists consists in precisely the circumstance that they completely reject the Kantian doctrine of synthetic a priori judgements. In their case the concept of convention is then a substitute for the synthetic a priori that is supposed to take over the function of the Kantian a priori in all domains of thought: they apply the concept of convention, not only to comprehend physical geometry, but also to explain pure mathematics and even logic. According to the logical positivists *all* a priori sciences rest in the end on conventional stipulations – and precisely in this way is Kantianism once and for all decisively overcome.

By contrast, Poincaré himself gives a central place to the synthetic a priori. In fact, as we have seen, his conception of arithmetic is extremely close to the original Kantian conception of arithmetic. First, arithmetic is based on our intuitive capacity for representing the indefinite repetition or iteration of one and the same operation, and therefore arithmetic for

Poincaré is not a merely analytic science. Second, arithmetic is also not conventional for Poincaré: mathematical induction forces itself necessarily upon us, and there are thus no alternatives here. Third, arithmetic occupies the apex or summit of a hierarchy of sciences: all other sciences – all other a priori sciences, in particular – presuppose arithmetic, because all others presuppose mathematical induction or reasoning by recurrence.

Now Poincaré's conception of geometry is also very similar to the Kantian conception of geometry. For Poincaré, as for Kant, geometry is synthetic, because it is based, like arithmetic, on the possibility of indefinitely repeating particular operations: namely, group-theoretical operations constituting a Lie group of free motions. Moreover, geometry is also viewed as the presupposition of all properly empirical physical theories: neither for Poincaré nor for Kant can geometry itself be either empirically confirmed or empirically disconfirmed. The difference, of course, is that Poincaré, in contrast to Kant, is acquainted with *alternative* geometries. Poincaré is acquainted, in particular, with the Helmholtz-Lie theorem, according to which geometry is constrained, but by no means *uniquely determined*, by the idea of a Lie group of free motions. It then follows for Poincaré, because three alternative possibilities are still left open, that we have here – in this very special situation – a conventional choice or free stipulation.

Poincaré's basic philosophical conception thus by no means implies a general rejection of the synthetic a priori. On the contrary, without the synthetic a priori his argument simply makes no sense. Precisely because geometry – like arithmetic – is synthetic, but also – according to the Helmholtz-Lie theorem and in contradistinction to arithmetic – is not uniquely determined, it follows that geometry is conventional. For the logical positivists, by contrast, there can be no question of *this* kind of argument for geometrical conventionalism. Because arithmetic is no longer viewed as synthetic a priori in the Kantian sense, they, for their part, attach no particular importance to our intuitive capacity for representing the indefinite repetition of some or another operation. Moreover, because we now consider geometry first and foremost in the context of the Riemannian theory of manifolds, group theory and the Helmholtz-Lie theorem are no longer relevant in any case. And, finally, we now accept the general theory of relativity (indeed, as the very paradigm of a successful physical theory); and, according to this theory, there is no longer any possibility of conceiving geometry as the presupposition of properly empirical physics. As we have seen, we are in fact forced by this theory to subscribe to a holistic conception of the relationship between geometry and empirical physics. Before the development of the general theory of relativity theory we were of course free to adopt such a holistic conception if we wished – but after this development there is simply no alternative.

The main point of our earlier discussion, however, is that such a holism is much too weak to support a special, non-empirical status for geometry. Holism by itself is obviously also completely unable to explain the non-empirical status of arithmetic. If the logical positivists really wish to apply the concept of convention as an explanation of the status of the a priori in general, therefore, they clearly need to add some entirely new element that goes beyond mere holism. And this, in fact, is precisely what happens: When Rudolf Carnap then attempts to articulate a general conventionalistic conception of the a priori in *The Logical Syntax of Language* in 1934, holism plays only a very subsidiary role. Instead, everything depends on the new conception of *analyticity* he attempts to develop there.

Carnap considers purely formal languages or linguistic frameworks that can be chosen entirely arbitrarily. We can, for example, choose a language governed by the rules of classical (Frege-Russell) logic; but we can also, with equal justification, choose an entirely different type of language governed by the rules of intuitionistic logic. In fact, there can here be no question at all of either "justification" or "correctness," for the very concept of "correctness" itself only has meaning when we have antecendently specified a particular linguistic framework. Hence, the choice of one or another such framework can only be based on a convention, which we stipulate entirely freely on pragmatic grounds. What is most important, however, is the following: Relative to any particular formal language or linguistic framework there is a sharp distinction between the *logical* rules or *analytic* sentences of the framework and the *physical* rules or *synthetic* sentences of the framework. In particular, the former constitute the underlying logic of the framework which first makes questions of "correctness," "justification," and so on possible. Our conventional choice of a language – together with the characteristic logical rules of this language – then clarifies the special epistemological (and non-empirical) status of such rules.

Carnap does not therefore represent a general holism, according to which all sentences whatsoever have precisely the same status: instead, we are given a sharp distinction between logical and physical rules – analytic and synthetic sentences. Within a framework for classical mathematical physics, for example, (classical) logic, arithmetic, and the theory of the real numbers belong to the logical rules, whereas Maxwell's field equations belong to the physical rules. The former are therefore conventional in the context of this framework, whereas the latter are non-conventional and thus empirical. And what is the status of geometry here? From the present point of view Carnap's result is especially interesting and noteworthy. Within a framework for classical mathematical physics, in which space has constant curvature, geometry also belongs to the logical (or analytic) rules. Within a framework like that of the general theory of relativity, by contrast, in which space (more precisely, space-time) no longer has constant curvature but rather a curvature depending essentially on the distribution of matter – within such a framework geometry belongs rather to the physical (and therefore synthetic) rules! Carnap's result here thus agrees completely with our argument – and also with the conception defended by Reichenbach in his *first* book (last quotation). In the context of classical mathematical physics Poincaré is perfectly correct: physical geometry belongs to the a priori part of our theoretical framework and hence to the conventional part. In the context of the general theory of relativity, however, Poincaré is incorrect: in this context physical geometry belongs rather to the empirical part of our theoretical framework and hence to the non-conventional part.

Carnap's conception in *Logical Syntax* is thus in a much better position to establish a meaningful version of conventionalism than is a purely general Duhemian holism. Unfortunately, however, Carnap's conception has its own fatal difficulties – difficulties that have only become clear in the course of the Quinean criticism of the concept of analyticity. But this story I must definitely leave for another occasion.

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