What do we mean when using the acronym 'BCS'? The Bardeen–Cooper–Schrieffer theory of superconductivity

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Alexander M Gabovich[1] and Vladimir I Kuznetsov[2]

[1] Institute of Physics, Nauka Ave 46, Kyiv 03680, Ukraine
[2] National University of Kyiv-Mohyla Academy, 2 Skovorody vul, Kyiv 03680, Ukraine

E-mail: alexander.gabovich@gmail.com and vladkuz8@gmail.com

Abstract
The history and use of the acronym 'BCS' (named after Bardeen, Cooper and Schrieffer) in the science of superconductivity is traced and analysed. It is shown that a number of different theories are labelled 'BCS'. The confusion in the application of the term 'BCS' is shown to be common because the term 'theory' itself is not precisely defined in physics. Recommendations are given to physics readers and students on how to distinguish between various theories referred to as 'BCS'. Contributions from scientists in different laboratories to the creation and formation of the superconductivity theory are indicated.

1. Introduction

The majority of physical science papers and monographs include the term 'theory'. There are two cases of its use. The first designates general theories that are accepted as basic ones in certain large areas of physical studies. For example, Newtonian mechanics is a theory that describes and explains a full range of macroscopic body movements. Quantum mechanics plays a similar role for a variety of microscopic phenomena. The mechanics of special relativity is a generalization of Newtonian mechanics for particle (body) velocities comparable with the velocity of light c.

In the second case, the term 'theory' labels particular theories of specific classes of like phenomena or even a unique phenomenon. For instance, atomic theory relates to atomic structures and their interaction with electromagnetic radiation, while quark theory describes the phenomenology and behaviour of quarks. In fact, particular theories use fundamental (general) theories as reliable and reasonable tools that should be adjustable to the entities under study.

Unfortunately, many physicists and philosophers of physics avoid explaining what they mean by a 'theory' or give a very general and uncertain description of a physical theory as such. Similarly, the authoritative encyclopedic physical sources do not contain entries specifically devoted to the notion of a 'physical theory' [1–3]. The required entries are absent in many recent encyclopedic sources on the philosophy of science and the philosophy of physics. See, for example, [4, 5]. When these entries are at hand, a reader meets the exposition of rather different interpretations of what theories are [6–8].

Our goal here is much more modest than to propose an elaborate and detailed reconstruction of what a 'physical theory' is. We imply a correct, detailed description rather than a precise definition. To this end, one can examine other sources [9–12].

We wish merely to identify and distinguish the main features of a theory that are intuitively clear to professional physicists, but are not specified in their textbooks and monographs. Since one example can give more than enough theorization, we shall restrict ourselves to the groundbreaking theory of superconductivity [13]. This theory was, and still is, an extremely important source of ramifications so our study will be of use to students engaged not only in condensed matter studies but also in statistical physics and elementary particle physics as well.

2. Preliminaries concerning the Bardeen–Cooper–Schrieffer theory

Formally speaking, BCS is short for Bardeen, Cooper and Schrieffer, three famous American scientists who created the beautiful, successful (semi-)microscopic theory of superconductivity published in 1957 [13] (see also an account made by one of the most prominent scientists in this research field [14]). Nevertheless, people usually write 'BCS' to refer to this theory or the paper expounding it. Incidentally, this theory, which is most often considered microscopic [15, 16], is 'only' semi-microscopic. Indeed, specific calculations made in the paper concerned involve the electron–electron attraction as a constant V and a quasiparticle density of states $N(0)$ (DOS) at the Fermi surface. Both are phenomenological quantities in essence in the framework of [13]. The product of V and $N(0)$ is considered in [13] as a dimensionless electron–phonon interaction constant λ that determines the strength of the interaction leading to the spin-singlet Cooper pairing [17] of charge carriers.

Note that the microscopic background for the calculation of thermodynamic, electrodynamic and other properties is introduced in the second section of the BCS paper with reference to the basic works of Fröhlich [18], Bardeen [19] and Pines [20] (the latter cited prior to its publication). However, the corresponding deep microscopic understanding of the subject, although crucial to formulate the theory, is hidden in the subsequent elegant derivation of the energy-gap, $\Delta_{BCS}(T)$, equation, where T is temperature. The solution can be expressed in terms of $\delta_{BCS} \equiv \Delta_{BCS}(T)/\Delta_{BCS}(0)$ versus $t \equiv T/T_c$, where T_c is the mean-field critical temperature of the superconducting transition. Hence, the solution is given by the universal function $\delta_{BCS}(t)$, similar to those appropriate to other theories of the corresponding states, such as the Van der Waals theory of gases and liquids [21]. The universality and general applicability of the BCS theory result from incorporated elements of phenomenology. In this context, phenomenology is an advantage rather than a shortcoming.

The details indicated here testify that even in the case of a recent theory with a well-documented history and well-known sources, its very status (a true microscopic versus semi-microscopic or even phenomenological theory) may be uncertain. Below we show that further development of the BCS approach into a wide and important area of research and (we dare to say) industry made the use of the term 'BCS theory' even more vague and changeable. Many substantially different theories are labelled 'BCS', but they are mostly generalizations of the

original superconductivity theory [13] that go far beyond the prototype. The polysemy of the very significant physical term 'BCS theory' is detrimental to the teaching of superconductivity and leads to confusion among undergraduates and even professionals. Therefore, in section 4 we analyse use of the notation 'BCS' in various cases and show that it is ambiguous. We classify the different 'BCS'-labelled theories that exist to date.

From a methodological point of view, the situation is no surprise since the concept of 'theory' itself is not properly explicated in scientific and methodological literature. We shall briefly discuss the situation in section 3 and a closer analysis will be published elsewhere.

3. Compositional complexity of the physical theory

When a physical theory is studied or taught it should be treated from different viewpoints since too strong an emphasis on one of them will result in a one-sided vision. In particular, from the system viewpoint, a theory is a relatively independent system containing a certain set of elements and links between them. From the net viewpoint, a theory is included in a complicated network of existing theories and uses a lot of their fragments. From the historical viewpoint, both the system and the network may be considered as either fixed or varying in the course of the theory development. In fact, a variety of intermediate versions are available.

Subsystems and elements of the mature theory (both, hereafter, named components) have specific features. For instance, one can indicate rigour, completeness, universality, consistency and adequacy. The inclusion of different components in the theory forms the basis of the theory classification.

Thus, fundamental theories include general laws, principles, models, equations and problems that are all components of particular theories. As an example, quantum mechanics is fundamental to all theories elaborated on its basis. One can mention the daughter condensed matter theory and the theory of superconductivity. Fundamentality is relative so that a theory may be considered as fundamental with respect to certain theories and as non-fundamental with respect to others.

Phenomenological theories comprise schemes involving relations between observables and auxiliary entities based on the experimental data and not explained (at the current stage of available knowledge) from the fundamental principles. In the science of superconductivity one should mention three early successful phenomenological theories: Gorter–Casimir two-fluid [22], London [23] and Ginzburg–Landau [24, 25]. It is remarkable that after the creation of the BCS theory [13], coefficients of the Ginzburg–Landau expansion [24] were rendered concrete by Gor'kov [26] in terms of the BCS-related quantities.

The network viewpoint emphasizes the incorporation of a theory as a system into the relevant network of related theories, which constitutes a necessary condition of its emergence, existence and development. Leading experts in modern physics stress the significance of the theory as a multiply connected web [27, 28]. It means that, in principle, one should analyse the relations between theories while studying the validity and applicability of any specific one. In the context of BCS theory, it relies on the validity of the Landau Fermi-liquid theory [29], i.e. the insignificance of strong electron–electron correlations associated with the studies of Hubbard [30] and Mott [31]. It proves once more that any modern theory is built, at least partially, on the basis of already existing ideas and theories that are taken for granted, unquestioningly. On the other hand, as was indicated by Einstein: 'Unless one sins against logic one generally gets nowhere; or, one cannot build a house or construct a bridge without using a scaffold which is really not one of its basic parts'; it is impossible to construct a new theory using logic and previous theories only, without any further insight [32]. BCS theory involved three main new ideas that were not logical consequences from pre-existing physical theories:

Cooper pairing, retardation of the electron–phonon interaction, and the built-in adiabaticity of the electron–ion metallic plasma [17, 19, 33].

In order that a physical theory properly describes certain experimental objects and their attributes, the corresponding information must be represented in the theory. Common and symbolic names as well as models of objects and their attributes are required data carriers. Given the theory, a theoretician analyses the phenomenon via available models. In the course of studies, controversies arise that cannot be solved in terms of the current models. Hence, new models are constructed, helping to solve the initial controversy and related problems. In BCS theory the initial model was augmented by the description of the Coulomb repulsion, which led to the appearance of the Coulomb pseudopotential concept and the Eliashberg modification of the BCS theory (see section 4).

Later on, the simple isotropic form of the pairing interaction and the order parameter structure were generalized to include anisotropy and gave birth to a variety of models [34, 35], which nevertheless preserved much from the starting point [13]. It should be emphasized that the continuous flow of experimental data, together with the great potential included in the BCS theory at the very beginning ensured the long and fruitful development of this theory and the concomitant models of superconductors.

4. The ever-fluctuating boundary between BCS and non-BCS superconductors. Problems with terminology

As indicated above, the BCS theory [13] was inspired by the idea of phonon-mediated dynamic attraction between two electrons leading to their subsequent pairing. Specifically, any *such* Cooper pair, being a loose quasi-boson entity, includes two fermionic quasiparticles with oppositely directed spins and momenta. In other words, the pair is spin-singlet and isotropic.

All paired electrons constitute a collective system with a unique coherent wave function [13, 36] so that boson properties of the loose Cooper pairs (their size is estimated by the Pippard coherence length $\xi_0 = \hbar v_F/\pi \Delta \approx 10^{-4}$ cm in pure metals [37] where v_F is the normal–electron–liquid Fermi velocity and $\hbar$ is Planck's constant) are only an approximation (see, e.g., [38]). Indeed, due to the inequality $\xi_0 \gg a$, where a is the crystal lattice constant, Cooper pairs overlap and are inseparable. The collective of pairs *does not* exist in a metal above T_c, i.e. in a normal state of this metal. Hence, a transition into the BCS superconducting state below T_c is *not* 'Bose–Einstein condensation (BEC) of Cooper pairs'. Due to these circumstances, the traditional term '*condensation* into the superconducting state' used in many textbooks (see, e.g., [39–41]) is at the least inaccurate.

In particular, we totally agree with the unification of the superconducting state and its Bose–Einstein-condensed counterpart, being a collective of true bosons (e.g., ^{4}He atoms), into one class of phenomena on the basis of the so-called off-diagonal long-range order [38, 42, 43]. However, the use 'for brevity' of the term 'quantum condensation' concerning superconducting transition [42] may be misleading for undergraduates, postgraduates and even professional experimentalists who are not involved in sophisticated considerations and calculations comprising the background of this.

Simultaneously to the work of the BCS team, a different, obviously non-BCS-like, model of superconductivity was elaborated [44], treating electron pairs as local entities (in the co-ordinate space) and named 'quasimolecules' by the authors. Their size is suggested to be comparable to or less than the inter-pair distance. In the framework of this model, the superconducting state was considered as a BEC state similar to that observed in the quantum liquid mentioned above, where boson atoms are partially condensed below the λ-point [15, 42, 43].

It is worthwhile indicating that, whatever the relationship between the characteristic lengths in the systems concerned, a superconducting or superfluid state with a broken gauge symmetry can be obtained by the same Bogoliubov method of the uv-transformation from initial quasiparticles to new elementary excitations of the diagonalized Hamiltonian [38, 42, 43, 45, 46]. This mathematically formal similarity does not mean that those low-T states are of similar spatial structure or common origin. It is important that the Bogoliubov method leads to the same energy gap and T_c equations as the BCS theory [13]. There is also a third approach in terms of the so-called anomalous Green's functions, which arrives at the BCS equations in a very elegant way [47]. This is a good example of the frequently encountered situation in physics where two (or several) approaches lead to identical results. It means that the theories in question, dissimilar at first glance, are actually different formulations of the same theory. In mechanics, one can recall Newton equations and Lagrange equations of the second kind as another example of the formulation diversity [48].

If one considers dirty superconductors (i.e. going beyond the original model [13] designed for pure metals) it is natural to expect that impurities modify the coherence length so that the actual ξ can be found from the equation [38]

$$\xi^{-1} = \xi_0^{-1} + l^{-1}, \tag{1}$$

where l is the mean free path. Hence, the coherence length may, in principle, become very small for alloys and compounds. Moreover, the quantity ξ_0 itself may become extremely small for superconductors with low charge carrier densities and large energy gaps (critical temperatures). It is really the case, e.g., for superconducting cuprates with high T_c where the observed $\xi \approx 10^{-7}$ cm are comparable with the lattice constants [49].

Such small coherence lengths in high T_c oxides imply a non-BCS scenario of superconductivity, namely a BEC one [15]. Double-charged bosons are often called bipolarons, i.e. they are regarded as bound pairs of polarons [50]. The latter are heavy electrons or holes 'dressed' by the polarized environment. As a consequence, the polaron effective mass m^* is much larger than its free-electron counterpart m so that $v_F = k_F/m^*$ is very small, resulting in small ξ_0 in agreement with its definition given above. Such a situation is reasonably recognized by the community as a non-BCS one and will not be discussed below.

One should bear in mind that a plethora of intermediate physical models can be realized [51] between two indicated extremes, namely, BCS and BEC states. In trapped Bose-atom gases, which can be considered to be a close analogue of a superconducting electron liquid, one can control the gradual transition from the BCS to the BEC state by changing the external magnetic field H [52].

Let us come back to the original BCS work [13]. It is a so-called weak-coupling theory when both Δ_{BCS} and T_c are considered small in comparison to other relevant physical parameters of the metal, in particular, the representative average phonon frequency ω_{av} (or a Debye frequency ω_D) and the Fermi energy $E_F \gg \omega_{av}$. The weak-coupling equation for $\Delta_{\mathrm{BCS}}(0)$ reads [13]

$$\Delta_{\mathrm{BCS}}(0) = 2\omega_{av} \times \exp(-1/\lambda). \tag{2}$$

In 1960 Eliashberg built [53, 54] a strong-coupling generalization of the BCS theory. Specifically, on the basis of the Fröhlich Hamiltonian [55] describing electron–phonon interaction, a system of two coupled equations for the energy, E,- and T-dependent gap function $\Delta_{\mathrm{El}}(E, T)$, and the so-called normal self-energy part $Z(E, T)$ (renormalization function) was obtained. The latter describes the effective electron mass m^* enhancement by the electron–phonon interaction [33, 56]. This renormalization originates from the same phenomenon that leads to the polaron formation discussed above. In essence, any electron-like (hole-like) quasiparticle in a metal is a polaron of a certain coupling strength so that the effective mass

m^* is slightly or substantially larger than the free-electron mass m. In rare-earth-based metals with narrow electron bands, the mass m^* of the (otherwise conventional) Landau Fermi-liquid can reach $(10^2–10^3)$ m. Such materials are called heavy-Fermion ones [57]. Many of their representatives are superconductors with peculiar unconventional properties but most probably involving Cooper pairs rather than bipolarons [58].

One should emphasize that the *energy*-dependent function $\Delta_{El}(E, T)$ is not identical to the original BCS gap $\Delta(T)$, even if $\Delta(T)$ is generalized to the anisotropic case and becomes a *momentum*-dependent ($\mathbf{k}$-dependent) function $\Delta(\mathbf{k}, T)$ [59]. Nevertheless, in the framework of the dielectric formalism [60] it can be shown [61] that (i) the true gap function $\Delta(E, \mathbf{k}, T)$ should be 'four-dimensional', i.e. depend on both energy and momentum; (ii) in the weak-coupling limit the Eliashberg function $\Delta_{El}(E, T)$ can be transformed into the BCS $\Delta_{BCS}(\mathbf{k}, T)$ in the anisotropic superconductor and into $\Delta_{BCS}(T)$ function in the isotropic case.

In the general case, i.e. for the coupling of arbitrary strength, $\Delta_{El}(E, T)$ and $\Delta_{BCS}(\mathbf{k}, T)$ (or $\Delta_{BCS}(T)$) differ significantly both qualitatively and quantitatively. Nevertheless, the Eliashberg theory and solutions $\Delta_{El}(E, T)$ and T_c of Eliashberg equations are often labelled 'BCS' by community members. The main argument to do this is that both theories are based on the electron–phonon interaction. However, such a statement is invalid because any of these theories may be constructed assuming that the electron–electron interaction glue bosons are magnons [62, 63] (most probably inducing a spin-triplet superconducting state), excitons [64] or, say, plasmons [65] with the same significant differences between BCS and Eliashberg equations.

Incidentally, superconductivity based on the s-wave Cooper pairing but mediated by other bosons than phonons is sometimes called BCS superconductivity and sometimes not. Since the choice in different cases is based on varying criteria, it is impossible to regard the terminology used as valid or invalid. In each case one should understand which criterion is borne in mind. Otherwise, the term BCS becomes altogether meaningless.

Another question arises, which concerns Eliashberg generalization [53, 54] of the original BCS theory including its basic ingredients, such as the description of the electron–phonon interaction [13]. Specifically, many experts and authors of textbooks consider Eliashberg equations to be microscopic (see, e.g., [63, 66]). It is done quite similarly to the attribution of the microscopic status to the BCS gap equations [15, 16] indicated above. It is not accurate because the Fröhlich Hamiltonian [55], the starting point of the Eliashberg theory [53, 54], is not actually microscopic: it contains the phenomenological electron–phonon interaction constant F'. In Eliashberg equations the interaction is redefined as $\alpha^2(\omega)F(\omega)$, where $\alpha^2(\omega)$ is the Eliashberg electron–phonon function and $F(\omega)$ is the phonon DOS (see the excellent analysis of various important nuances in [67, 68]). The function $\alpha^2(\omega)$ is not only a phenomenological one, which is not at all bad. Difficult problems exist regarding how to derive the Fröhlich Hamiltonian [55] from the generic electron-ion Hamiltonian and whether the upper limit of the electron–phonon interaction constant exists [69].

The latter issue is important in studies of the crystal lattice instability, which is directly related to the problem of high-T_c superconductivity [70, 71]. Namely, if the electron–phonon coupling is strong enough, it often happens that ions are periodically displaced, accompanied by itinerant electrons so that the Fermi surface shrinks, at least partially. Such a scenario is especially easily realized when the parent Fermi surface has congruent (nested) sections, which is a visiting card of a quasi-one-dimensional or a quasi-two-dimensional electron spectrum [70, 72, 73].

Another shortcoming concerns the account of ubiquitous Coulomb forces. In the original BCS paper [13], the electron–electron Coulomb repulsion (being detrimental to superconductivity) was totally neglected, whereas in Eliashberg equations Coulomb

correlations are approximately taken into account and described by a single phenomenological constant [38, 67, 68]—a Coulomb pseudopotential

$$\mu^* = \mu[1 + \mu \log(E_F/\omega_{av})]^{-1}. \tag{3}$$

Here μ is an effective screened Coulomb interaction constant. Both μ and μ^* can be considered at most as adjustable parameters of unknown magnitude, although they are very significant and there are quite a number of Hubbard-model approaches to cuprates as well as Fe-based pnictides and chalcogenides [62, 63, 74, 75]. Meanwhile, all Hubbard-like theories assume that Coulomb-repulsion is the origin of high-T_c superconductivity.

It is remarkable that shortly after the appearance of the BCS model, the leading experts in the field fully recognized its phenomenological status. To illustrate this we draw your attention to the words of Morel and Anderson [76]: 'We may recall that the BCS effective potential is instantaneous and displays a strongly oscillating behaviour in coordinate space (since its Fourier transform is sharply cut off in momentum space). Since the strength V of this effective interaction appears only as an adjustable parameter, then the BCS model is still adequate to describe most properties of superconductors . . . '. It is worthwhile including a more recent sceptical viewpoint [77] on the alleged microscopic nature of the BCS superconductivity [13]: 'Phonons as such are not explicitly present in this model; they exist only to set the energy scale for the cut-off of the interaction'. Nevertheless, *the same authors in the same book* call the theory [13] microscopic. Such is the power of tradition.

As for the cause of the Coulomb interaction suppression in equation (3), it is transparent [76, 78]: Coulomb interaction is smeared over the whole energy interval $0 < E < E_F$ above the Fermi surface while the electron–phonon attraction is concentrated near the Fermi surface in the shell $|E| < \omega_{av}$, where it overcomes the repulsion. This circumstance leads to the logarithm first found by Bogoliubov *et al* [79].

It is well known that many superconducting properties of existing materials are anisotropic. The actual superconducting order parameter $\Delta(\mathbf{k}, T)$ itself, as was pointed out above, is also anisotropic, in principle. The existence of two or more well-defined separate energy gaps is the extreme form of such anisotropy. Magnesium diboride with its high $T_c \approx 40$ K and an electron-phonon underlying mechanism is usually regarded as a true two-gap superconductor with coupled $\Delta_1(0)$ and $\Delta_2(0)$ and a single T_c [80] in accordance with well-developed theories [66]. It is no wonder that the ratios $\Delta_1(0)/k_B T_c$ and $\Delta_2(0)/k_B T_c$ differ substantially from the weak-coupling BCS universal value $\alpha_{\text{BCS}} \approx 1.764$. Here k_B is the Boltzmann constant. The same non-universality is observed, for other benchmark quantities, e.g., for the ratio $\delta C/C_N$ at T_c, where C_N is the normal state electron heat capacity and δC is a discontinuity at the superconducting phase transition. Nevertheless, this manifestly non-BCS behaviour is ignored when many specialists call MgB$_2$ and other similar materials 'two-gap BCS superconductors' (see, e.g., [66, 80]).

Of course, there are plenty of 'one-band' superconductors where the BCS corresponding-state universal laws are violated, which actually means that the famous model is oversimplified. Most probably, the deviations from the standard behaviour are mainly due to strong-coupling and crystal anisotropy effects. To make allowance for the deviations, the idea appeared to make the BCS theory even 'more phenomenological' [81] by introducing an extra scaling (fitting) parameter $\alpha_{\text{PNS}} = \Delta_{\text{PNS}}(0)/k_B T_c$ so that the actual (observed) energy gap equals

$$\Delta_{\text{PNS}}(T) = (\alpha_{\text{PNS}}/\alpha_{\text{BCS}})\Delta_{\text{BCS}}(T). \tag{4}$$

In the α-model [81] all the other reduced variables of the BCS theory are properly scaled on the basis of the ratio $\alpha/\alpha_{\text{BCS}}$ in order to fit the experimental data for specific superconductors. This is a rather useful and transparent phenomenological scheme that avoids cumbersome numerical work, the results of which still remain uncertain because condensed matter science

still cannot predict new superconductors and satisfactorily reproduce T_cs of the existing ones [70, 82–86]. The reader should decide on his (her) own whether the approach [81] is a version of the BCS one [13] or a new non-BCS theory, since the dependences $\delta_{BCS}(t)$ and $\delta_{PNS}(t)$ are identical, whereas all magnitudes of the effects are different.

For alleged two-gap superconductors the situation becomes so involved that the phenomenological two-gap model combined with the phenomenological α-model approach [81] was applied to the description of the electronic-specific-heat measurements in MgB_2 [87, 88] and $Ba(Fe_{0.925}Co_{0.075})_2As_2$ [88]. The authors of [88], contrary to all *ad hoc* approximations made in the framework of the α-approach, do not want to leave the BCS cradle, insisting that 'the validity of the obtained generalized BCS-like equations is verified by analysing different experiments on different compounds'. It seems that such extended interpretation is somewhat misleading, especially to newcomers.

Soon after publication [13] the Cooper-pairing idea was extended to cover non-zero angular momentum L [89] and spin-triplet [90] cases. The first successful applications dealt with the superfluidity of He^3 [38, 42, 43]. However, there is a large body of evidence that spin-triplet Cooper pairing also takes place in many superconducting metals, in particular, Sr_2RuO_4 [91]. We emphasize that spin-triplet superconductivity is associated with non-phonon boson-mediators. Hence, its connection to the original BCS argumentation is weak, although these types of superconductors are often referred to as BCS.

High-T_c oxides (cuprate oxides) are considered by the overwhelming majority of the 'superconducting' community to be a true example of $d_{x^2-y^2}$ anisotropic superconductors [63, 74, 92, 93], although there exists a conspicuous resistance supported by controversial experimental data [94–97]. Most experts stand for the spin-fluctuation mechanism of Cooper pairing in these ceramic compounds [63, 74]. Nevertheless, consensus has not been achieved, on this point at least, because there are scenarios of phonon-driven attraction with $L \neq 0$ [98]. Even if the dominant opinion is valid, it is still impossible to attribute the observed cuprate superconductivity to BCS or non-BCS type. That is why articles and books contain mutually exclusive characteristics of oxide superconductivity. One should be very careful with the terms 'BCS' or 'non-BCS' in this connection.

It should be emphasized that in the science of superconductivity there are at least two more widely used vague concepts that are detrimental to understanding, namely 'exotic' [49, 94] and 'unconventional' [63, 66, 98] superconductivity. The latter notion is even a source of polemics [91]: 'Therefore, in the present review, we define "unconventional superconductivity" as superconductivity due to non-phonon (non-BCS) mechanism . . . '. The real problem consists, however, in the undefined terms quoted above, justifying our current attempt to explain and classify BCS-labelled objects.

5. Conclusions

The totality of literature devoted to superconductivity demonstrates that all superconductors, excluding those that are claimed to be BEC systems, are denoted 'BCS' or 'generalized BCS' objects. This is merely the result of a 50-year tradition. Instead, 'superconductor with Cooper pairing' would be a much more appropriate term, since a Cooper pair might be spin-triplet or possess any angular momentum. Moreover, the term 'Cooper pairing' covers the cases of two- or multiple-gapness. Nevertheless, students are forced to consult the sources available with their vague use of fundamental concepts. Therefore, students should be aware of terminological difficulties and distinguish between different entities hidden under the same name. We hope this paper will be of help in the specific 'BCS' case.

One should draw another conclusion from the facts and opinions presented above. Namely, the theory of superconductivity does not exist yet (compare with the well-known opposite claim concerning high-T_c oxides [99]) and is phenomenological to a variable degree. This is a general feature of solid-state science, aggravated in the case of superconductivity by the exponential dependence of Δ and T_c on the interaction constant λ (see equation (2)). It is amusing that these unavoidable traits of phenomenology in condensed matter physics led to the appearance of the inadequate use of terms: 'a phenomenological microscopic model for the . . . ' [100].

As we have indicated above, more terms exist in superconducting-materials science that are as obscure as the term which is considered here in more detail. Unfortunately, such cases are not rare in modern physics. For instance, one can indicate the extremely important concept of 'photon' found earlier to be at the least very unclear [101–103]. Thus, specific branches of physics should be monitored from the methodological viewpoint to facilitate the mastering of subjects by students.

A complete analysis of the totality of superconductivity theories requires reliable and 'empirically' valid methodological reconstruction of the real and acting physical theories, which goes beyond the scope of the current study and will be published elsewhere. Factually, almost all available reconstructions are not well equipped to deal with the real nuts and bolts of particular superconductivity theories. As a consequence, they are not able to depict adequately the peculiarities of their historical development and the diversity of their interrelations.

Acknowledgments

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