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Wickedness In Social Choice

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Abstract

In an article from 1973, Rittel and Webber distinguished between "tame" or "benign" problems on the one hand and "wicked" problems on the other. The authors argued that wicked problems occur in nearly all public policy issues. Since different groups adhere to different value-sets, solutions can only be expressed as better or worse. By no means can they be viewed as definitive or objective. In this paper we shall consider, from this very angle, the theory of social choice which is about the aggregation of individual preferences with the aim to derive a consistent social preference. We shall show that collective choice offers wicked problems of various types which differ in their degree of severity. We shall hereby concentrate on welfare functions and voting schemes of different kinds and shall discuss these in the light of various criteria such as Arrow's independence condition, Condorcet consistency, monotonicity, manipulability, and other properties.

Keywords:

Social choice, value judgments, wickedness, conditions of independence, consistency, monotonicity, non-manipulability

1. Introduction

In their article "Dilemmas in a General Theory of Planning", which proved to be very influential in the areas of urban planning and environmental policy, Horst Rittel and Melvin Webber (1973) distinguished between "tame" or "benign" problems and what they called "wicked" problems. According to their view, examples of the first type are problems of mathematics such as finding a solution to some equation or analyzing the structure of some unknown material by chemical methods. Examples of problems of the second kind include, according to the authors, nearly all public policy issues. "For wicked planning problems", so Rittel and Webber argue, "there are no true or false answers" (1973, p. 163). Due to special value-sets of the parties involved, proposed solutions can only be expressed "as 'good' or 'bad', or, more likely, as 'better or worse' or 'satisfying' or 'good enough' " (p. 163). By no means can these solutions be viewed as definitive or objective. Among the distinguishing properties that wicked problems have, there is one, according to the authors, which says that "there is no immediate and no ultimate test of a solution" (p. 163) to such problems. We take these quotations as points of orientation for our own reflections on "wickedness in social choice".

Value judgments are ubiquitous in parts of economics. They are particularly numerous in welfare economics, social choice theory and the analysis of inequality, deprivation and exploitation, but not only there. That part of economic policy which deals with public policy issues (such as, for example, the opening of a new elementary school in a multi-ethnic community or, alternatively, investments in old age health care) also involves, quite naturally, value judgments. These are very different from statements like "the growth rate of country *x* has, due to a world-wide recession, declined from 5% to 2% during the last two years". Value judgments are different from laws such as the law of gravity in physics or the law of demand in economics which says that demand for a particular commodity goes down when the price of this commodity goes up (and vice versa) though, in a strict sense, the law of demand is a claim that reflects an empirical regularity, and nothing more. As Prasanta Pattanaik argued in private communication, "value judgments do not yield to the true/false classification and can only be characterized as 'plausible', 'implausible', 'reasonable', 'unreasonable', etc." Pattanaik added that "people differ in their judgments regarding the plausibility or reasonableness of value judgments".

Amartya Sen made the distinction between basic and nonbasic value judgments (1970, chapt. 5). If a person sticks to a particular judgment under all imaginable circumstances, Sen calls this judgment basic. If this verdict only holds under certain circumstances, this judgment has to be seen as nonbasic in the person's value system. The equal treatment of all individuals concerned seems to be a basic value judgment, at least amongst civilised communities. It seems to us, however, that the line between basic and nonbasic is sometimes difficult to draw. The statement that situation *x* is preferable to situation *y* because under *x*, everyone is better off than under *y*, can with some legitimacy be called a basic value judgment. However, if under *x*, the degree of inequality is significantly higher than under *y*, one may start wondering about the general validity of the former assessment.

The demand that individuals truthfully report their preferences and abstain from any manipulative voting behaviour seems to represent another basic value judgment but as we will see later on, this view is not shared by everyone, under all circumstances. The statement that all kinds of individual preferences should be taken into consideration, even the most malicious or most evil, when deriving the general will of society, involves various aspects so that, again, the question of whether such a verdict is either basic or, rather, nonbasic is not easy to decide.

There are many distinguishing criteria that represent value judgments of a stronger or weaker form. The dichotomy "ordinal – cardinal" can, at first sight, perhaps be viewed more as a technical issue than as a deep value-laden problem. The cardinal concept of utility requires much more than the ordinal concept so that, according to Occam's razor, one should stay in the ordinal world of utility as long as one does not need the measurement of utility differences. General equilibrium theory in a situation of complete certainty is a case in point. However, as soon as aspects of risk enter the analysis, a purely ordinal approach may perhaps be viewed as unsatisfactory or deficient. In relation to the issue of distributive justice, John Harsanyi (1978) expressed a very strong value judgment when he asserted that the Bayesian theory of rational behaviour under risk, together with a Pareto optimality requirement, "entail *utilitarian ethics* as a matter of mathematical necessity" (p. 223, the italics are Harsanyi's).

We think that the reader will agree that such a strong verdict reflects more than a purely technical debate.

As far as social welfare functions and other voting schemes are concerned, there was (is) a debate whether Kenneth Arrow's well-known condition of the independence of irrelevant alternatives should be satisfied or not. Closely related to this question is the debate between binary, mostly majority-based aggregation functions and positionalist methods such as the rank-order scheme proposed by de Borda. This debate is still very lively, with Condorcet-followers on one side and rank-order adherents on the other. How do these different methods fare? If one asks a question like this, one has to explain very quickly what the measuring-rod is for such a comparison. The list of criteria to be used for such a comparison is nowhere written down. It can be short or very long. Let us just give a few criteria that will be examined in more detail in this paper.

Apart from the independence condition just mentioned, one can ask whether a Condorcet winner, when he exists, is always picked by the different rules to be considered. One can further ask whether those rules which are widely discussed satisfy a partition consistency condition, sometimes called reinforcement, where the overall electorate is partitioned into subsets of voters. The question being posed is whether the choices among subsets and the choice over the entire set of voters bear a particular relationship, namely that the nonempty intersection of choices among subsets of voters is identical with the choice over the entire electorate.

Further questions come to one's mind. Are the various schemes responsive or monotonic in relation to changes in the underlying preferences of the individuals? Do they fall a prey to the no-show paradox? Can they be easily manipulated? We shall see that all these criteria, and there are many more, are such that the different aggregation functions that we shall examine satisfy or dissatisfy them to various degrees. We are far away from a case of vector dominance which if this dominance existed, would tell us that there is one "best" aggregation method or, perhaps more cautiously, a small set of best aggregation rules. Referring to Rittel and Webber again, there is, in our opinion, no definitive or objectively founded set of rules to provide judgment. Even a "better-or-worse" relationship can only be formulated partially.

The observations and remarks above already indicate the structure of this paper. We start in section 2 with a brief discussion of Arrow's famous impossibility theorem and a comparison with Sen's Pareto-extension rule which provides an escape from Arrow's negative result. We then propose in section 3 alternative aggregation rules that will, in the following sections, be confronted with various criteria that have been considered as important by many social choice theorists. Section 4 looks more closely at the fulfilment of Arrow's independence condition, section 5 focuses on partition consistency, section 6 examines the Condorcet winner and loser property, section 7 considers the aspect of monotonicity and responsiveness as well as the no-show phenomenon, section 8 discusses the problem of manipulability, while section 9 views the earlier findings in the light of Rittel and Webber's wickedness issue. Section 10 concludes.

2. The Arrow Problem: Impossibility vs. Possibility

The theory of social choice considers the problem of aggregating the preferences of the members of a given society in order to derive a social preference that accounts for the general will or the common good of this society. Economists argue that the common good finds its expression in a so-called *social welfare function* which represents a compromise among the divergent interests of those who belong to society. Unfortunately, Arrow (1951, 1963) showed in a path-breaking analysis that if the aggregation procedure is to satisfy a small number of rather plausible requirements or properties, a social welfare function of such type just does not exist.

Arrow's requirements were (1) that a social welfare function should generate a *social ordering*, in other words, a complete and transitive binary relationship. The latter means that every given alternative should be comparable to every other alternative, and this comparison should be done consistently, that is, both the strict preference relation P and the indifference or equivalence relation I should satisfy the transitivity property. Then, (2) a social welfare function as an expression of the general will of the populace should be able to deal with whatever kind of preferences the individual members of a given society possess (*unrestricted domain*).The next requirement (3), called *weak Pareto principle*, says that, if for any two alternatives x and y, let's say, all members of society agree that x, for example, is strictly preferred to y, then society should have exactly the same strict preference. Next, (4) information gathering for the aggregation procedure should be parsimonious, that is, if society has to make a decision between two alternatives, let us call them again x and y, the individual preference rankings with respect to x and y only, and not any other preferences, should be taken into account in order to distil the social ranking between these two alternatives (*independence of irrelevant alternatives*). Lastly, (5) there should not exist a particular person in society such that whenever this person has a strict preference for some alternative over another, society "automatically" has the same strict preference, for any two alternatives and any preference profile of the members of society (*non-dictatorship*). For Arrow, these five conditions were necessary requirements for a democratic decision procedure, perhaps not sufficient since there may be other demands as well. Arrow's result says that in the case of at least two persons and at least three alternatives, these five conditions cannot be simultaneously fulfilled by any social welfare function.

Consider the following aggregation mechanism: alternative x is socially at least as good as y if it is not the case that (a) everyone in society finds y at least as good as x and (b) there exists at least one person who strictly prefers y to x. Sen (1970) called this procedure a *Pareto extension rule* and showed that it satisfies all of Arrow's requirements except one. It does not fulfil the demand that the social relationship be *fully transitive. Quasi-transitivity*, i.e. transitivity of the strict part of the relation (strict preference P) is still satisfied, but transitivity of the indifference relation I may be violated.

Is this "the" solution to Arrow's impossibility result? Consider the following situation where the number above each ordering indicates the number of voters declaring such preferences:

Example 2.1	99	1
most preferred	x	y
second preference	y	z
least preferred	z	x

There are two immediate observations. According to Sen's Pareto-extension rule x and y and x and z are socially indifferent, whereas y is socially strictly preferred to z. This shows what we have just claimed: transitivity of the indifference relation I is not generally satisfied on the aggregate level. The second point is that a single person, the outsider, let's say, can, by pronouncing a strict preference over a particular pair, guarantee that there be at least social indifference over this pair. This has been termed *weak dictatorship* in social choice theory. Apparently, Sen's rule is such that opposite strict preferences within a given preference profile, independent of where they occur, "automatically" generate instances of social indifference or equivalence. In other words, whenever there is some opposition among the members of society, even if, in absolute numbers, it is very small, cases of social indifference may become ubiquitous. If Arrow's requirement of non-dictatorship is replaced by the stronger condition that there be no weak dictatorship and the requirement of full transitivity of the social relation is relaxed to quasi-transitivity, another impossibility result emerges. Is the mileage Sen's result has gained a great leap forward?

A very basic property that voting rules should fulfil is of the following kind: imagine that, with respect to a given preference profile of society, there is social indifference between two alternatives *x* and *y*, let's say. Assume now that some voter changes his or her individual preference from a strict preference for *y* over *x* to an indifference between them, or an indifference between *x* and *y* to a strict preference for *x* over *y*. It appears intuitive that such a change should at least not turn out to be negative for *x*. *Positive responsiveness*, a particular type of monotonicity, requires somewhat more: such a preference change should result in a strict social preference for *x* over *y*. Sen's Pareto-extension rule, however, does not satisfy this property for more than two voters, as can be seen from the two variants of the following situation. Voters 1 and 2 show a strict preference, though in opposite direction, and the third voter first is indifferent between *x* and *y*:

Example 2.2a	1	1	1
most preferred	x	y	
			xy
least preferred	y	x	

Example 2.2b	1	1	1
more preferred	x	y	x
less preferred	y	x	y

In both examples, the Pareto-extension rule generates social indifference between x and *y*; positive responsiveness would require *x* to be strictly preferred to *y* socially when going from 2.2a to 2.2b. So we arrive at another impossibility result, once we require positive responsiveness on top of transitivity of strict social preference together with Arrow's other conditions, given that there are more than two voters (for more details, see Andreu Mas-Colell and Hugo Sonnenschein (1972)).

We hope to have shown that impossibilities and possibilities are located very close to each other. Weakening one of Arrow's conditions, to wit the rationality requirement, to quasi-transitivity yields a possibility result, strengthening non-dictatorship to an exclusion of weak-dictatorship or, alternatively, requiring positive responsiveness again lead to impossibilities. As long as we stay close to Arrow's axiomatic structure, there does not seem to be an uncontested exit from his problem. Given this "fact", Michel Balinski and Rida Laraki (2007, 2010) argue that the framework of Arrow in terms of orderings should be replaced by a completely new concept, namely by evaluations or grades attributed to the alternatives. Their argument is that evaluations are much more discerning than orderings. However, as will be shown in this paper, this alternative approach creates major problems as well.

The following section introduces various aggregation functions all of which, except for one, have the purpose to offer, under the requirement of unrestricted domain, an escape from Arrow's general impossibility theorem.

3. Aggregation Rules to Be Considered

We start out with the *simple majority rule*, perhaps the most widely applied rule of all aggregation schemes. This rule proceeds pairwise where for any x and y from set X, the set of feasible alternatives, the votes in favour of x and those in favour of y are counted and subtracted from each other. The winner is that alternative which receives a positive "net"-majority. Parity between any two contestants is possible and admissible as well. We know since Kenneth May's (1952) analysis that the simple majority rule is characterized by unrestricted domain, *anonymity* (an equal treatment of voters), *neutrality* (an equal treatment of alternatives by the voting mechanism – it is not allowable that x and y, for example, are processed differently than z and w, let's say), and positive responsiveness. Condorcet's paradox is perhaps the most illustrious example to demonstrate that the majority rule can yield a cyclical social relation, in other words does not satisfy Arrow's transitivity requirement under the condition of unrestricted domain.

Sen's Pareto-extension rule that leads to many instances of equivalence because of weak dictatorship which is inherent in this scheme, was already described in the previous section.

Charles Dodgson (1876), better known as Lewis Carroll, had an idea that is related to what Condorcet himself was considering when he became aware of the fact that the simple majority rule can generate preference cycles on the social level. *Dodgson's rule* demands that a so-called *Condorcet winner* (an alternative that is majority-wise at least as good as every other available alternative) be selected whenever it exists. If this does not hold, then for each alternative x from X, the number of binary preference reversals is calculated such that x becomes a Condorcet winner. A binary preference reversal or inversion is an interchange between two adjacent positions in some voter's preference ordering. That (those) alternative(s) is (are) eventually selected for which the number of binary reversals is minimal.

The *Copeland method* is based on pairwise contests as well, typically applying the simple majority rule. For each alternative x from X, a score s(x) is calculated which stands for the number of other alternatives that x beats or ties in pairwise comparison. The winner(s) according to the Copeland method consist(s) of those elements x in X for which s(x) is maximal.

While the Dodgson and Copeland methods are intimately related to the majority principle, the *Borda ranking rule* is a positionalist non-binary method which counts ranks that are assigned to the alternatives which are available for choice. If there are m alternatives, ranks $m-1$, $m-2$ down to 0 are assigned to positions from top to bottom within each voter's linear ordering, and the alternative(s) with the highest aggregate rank score is (are) chosen. Peyton Young (1974), amongst others, characterized the Borda rule. The four conditions that he used in order to define this method uniquely were neutrality, *partition consistency* (or *reinforcement*), *faithfulness* and *cancellation*. The first two properties were explained above. Faithfulness means that with respect to a collective decision "socially most preferred" and "individually most preferred" have the same meaning when society comprises just one voter. The cancellation property requires that, whenever there is a preference profile such that for all pairs of alternatives (a_i,a_j) from set *X*, let's say, the number of voters who prefer a_i to a_j equals the number of voters with the opposite preference, then all alternatives from this set are to be chosen. Without this last property, so-called *scoring functions* can be characterized (Young, 1975). Once the cancellation property is added, one obtains an equi-distanced voting scheme where the rank-difference between any two adjacently placed alternatives is the same everywhere within each individual's rank ordering.

One can distinguish between the *broad* and the *narrow* Borda method. When going from a superset *X* to any strict subset *X'*, the broad rule uses the scores assigned to the elements in superset *X* for any possible subset *X'*, so that an alternative that scored highest in the superset will always be picked in a subset, as long as it belongs to this subset. The narrow variant calculates new rank numbers for all elements in every subset *X'*; *m* now stands for the number of elements in each *X'*. It is well known (see, for example, Wulf Gaertner, 2009, pp. 107-08) that the broad Borda rule satisfies transitivity but violates Arrow's independence condition, while the narrow version has no problems with independence but can violate a set consistency requirement both when going from superset *X* to subset *X'* and when extending the choice from *X'* to *X*. Sen (1977) called these requirements *set contraction* and *set expansion consistency*, where, as has become clear from what we just stated, this notion of consistency refers to the various sets of alternatives to be considered and not to the set of voters as in the earlier concept used by Young.

All voting mechanisms that we wish to analyse in this paper have in common that they are based on the entire rankings of all voters concerned. This is why we shall leave out *approval voting* (Fishburn, 1978; Brams and Fishburn, 1983) which is a non-ranked voting scheme that specifies just two classes of equivalent objects, namely the group of alternatives that a voter finds acceptable and the group of objects a voter does not endorse. With this reservation in mind, the reader may find it strange that we consider the *plurality rule* in our analysis. One can argue that under the plurality rule, each voter just casts one vote for a single alternative with the consequence that the alternative(s) with the highest number of votes is (are) chosen. One can, however, also assert that the plurality rule only registers the top element in each voter's ranking. This formulation allows one to easily describe a modification of the plurality rule, namely *plurality with run-off*, which is a two-stage rule. In the first round, the top elements of all individuals' rankings are considered. If among these, no candidate receives a simple or (which is often the requirement in real-world elections) an absolute majority, then in a second round, only the two candidates with the highest number of first-round votes compete against each other, and the one with a majority of votes wins. For the second round, we obviously need information on lower-placed candidates in each voter's ranking.

Balinski and Laraki (2007, 2010) proposed a scoring rule, called *majority judgment*, where voters are supposed to assign ranks or grades to options on a qualitative scale to which all judges or voters agreed. This scale can stretch from, let's say, "excellent" to "fail" and has to be interpreted ordinally. Equi-distance of the various ranks is not required. Consequently, this method cannot add scores as in the Borda case but focuses, for each alternative, on its *median grade* and then picks the candidate with the highest median evaluation; in the case of an even number of grades, the lower of the two median grades is to be chosen.

In some sense, the Borda rule which assigns integer values from best (*m-1*) to worst (*0*) to the alternative options that are arranged linearly, is a very special voting scheme. One can imagine instead that there is a set *E* of non-negative discrete numbers, ascending or descending but equi-distanced, or a closed interval I with the requirement that all options over which a collective decision has to be taken be assigned to these discrete numbers or to the closed interval. Based on such a cardinal scale, each individual does this assignment according to his or her own scoring function. Similar to the Balinski-Laraki rule, the scores usually have a qualitative meaning, from best to worst or from left to right or from near to far, let's say, but a multitude of other denominations is possible as well. There are two major differences to the Borda scheme: one is the explicit cardinal character of this rule, the other more important one is that each voter is completely free how to assign the given candidates. What is essential also in this scheme is that the scale is commonly binding for all voters. A certain voter can then decide, for example, to allocate all candidates very close to each other, or assign some of them close to the top of the scale and all the others close to the bottom, or do the allocation in any other way. Such a scoring rule is additive like the Borda method. It aggregates the scores of each alternative option and finally chooses, as social outcome, the alternative(s) with the highest sum of scores. This rule is called *range voting* (Warren Smith (2000); for an axiomatic characterization see Gaertner and Yongsheng Xu (2012) as well as Marcus Pivato (2014)). Range voting satisfies all of Young's conditions that characterize the Borda count except for the cancellation property.

One last word. We stated some paragraphs above that all voting schemes that we shall consider are based on the entire rankings of all voters concerned. Balinski and Laraki (2007) would heavily protest as far as their own proposal is concerned. They assert that "a judge's message is simply a message, nothing more. It depends on the judge's preferences, but it is not and cannot be his or her preferences" (p. 8720). On the same page, they state that "a measure or grade is a message that has strictly nothing to do with a utility. A judge may dislike a wine and yet give it a high grade because of its merits…". If a judge assigns grade *a* to *x*, grade *b* to *y* and grade *c* to *z*, with the grades increasing from *a*, then, all things considered, this judge apparently ranks *z* above *y* and *y* above *x*. In this sense, there exists a rank ordering in the judge's mind that led to a particular assignment of grades.

4. Independence of Irrelevant Alternatives

We begin our investigation of fundamental properties with Arrow's independence condition. At the outset of section 3, we briefly discussed May's (1952) analysis of the majority rule where, amongst other conditions, a neutrality requirement was used to characterize this rule. Consider the following example for three individuals and three alternatives *x*, *y*, and *z*:

Example 4.1	1	1	1
most preferred	x	y	x
second preference	y	z	z
least preferred	z	x	y

According to simple majority voting *x* becomes a Condorcet winner. This result depends solely on *x*'s majority relation with respect to *y* and with respect to *z*. The majority relation between *y* and *z* has no influence on *x*'s status. Given the above profile, it just determines that alternative *y* is majority-wise ranked second. Clearly, simple majority rule satisfies Arrow's independence of irrelevant alternatives.

We know from Sen's (1970) analysis that the Pareto-extension rule satisfies the Arrovian independence condition.

The Copeland method does not satisfy independence as can be seen from the following example with five alternatives and five voters:

Example 4.2

1	1	1	1	1
x	x	y	w	v
y	v	v	v	x
z	w	z	x	z
v	y	w	z	w
w	z	x	y	y

We obtain s(*v*) = 4, s(*x*) = 3, and s(*w*) = s(*y*) = s(*z*) = 1, with alternatives *w*, *y*, and *z* forming a bottom cycle. The social preference according to Copeland is that *v* is preferred to *x* and both are preferred to *w*, *y*, and *z*. Let us now assume that the second voter changes his or her preference ranking to *x w y v z*. Then the Copeland method yields an equivalence between *v* and *x* (s(*v*) = s(*x*) = 3), though the individual rankings between *v* and *x* have not changed anywhere. According to the independence condition, the social relation between *v* and *x* should be the same as before which does not hold in our example. The reason for this violation is that the Copeland scores measure the "success" of an alternative in relation to *all* other options.

Example 4.2 with its modification according to voter 2's preference change can also be used to show that Dodgson's method does not satisfy Arrovian independence. Before the modification, alternative *v* is the unique Condorcet winner. After voter 2's change to *x w y v z*, we obtain a top cycle comprising alternatives *v*, *x*, *w*, and *y*. Now, both *v* and *x* need one binary inversion to become a Condorcet winner so that both are socially equivalent according to Dodgson, though, as stated before, the individual preference relationships between *v* and *x* are exactly the same before and after voter 2's preference change.

As already mentioned in the previous section, the Borda rule in its broad version does not satisfy independence. We now show that the plurality rule does not satisfy this property either. Consider the following example with three alternatives and seven voters (again the number of voters for each ranking is listed on top of each ordering):

Example 4.3a

2	2	2	1
x	z	y	y
z	x	x	z
y	y	z	x

Given that a plurality decision requires at least 40% of the votes, alternative *y* is the winner. Now consider the following profile:

Example 4.3b

2	2	2	1
x	x	y	y
z	z	x	z
y	y	z	x

Alternative *x* becomes the plurality winner though there has been no change in the individuals' preferences in relation to *y* vs. *x* and *y* vs. *z*. Since plurality with run-off is in some sense a refined version of the one-stage plurality rule and since the latter does not satisfy independence, the former does not satisfy this property either. That the result of the run-off procedure as a two-stage rule depends on irrelevant alternatives can be seen from the following modification of 4.3 above:

Example 4.4a

2	3	2	1
x	z	y	y
z	x	x	z
y	y	z	x

None of the three alternatives achieves at least 40 % of the votes so that *y* and *z* are taken to the next round where the following situation occurs:

Example 4.4b

2	3	2	1
z	z	y	y
y	y	z	z

The position of the now irrelevant alternative *x* in the rankings of the first two voters makes *z* the unique winner.

As stated in section 3, Balinski and Laraki's majority judgement scheme is an ordinal method with a focus on the median grade of each available alternative. The authors state (2007, p. 8725) that "the majority ranking is IIA (i.e., satisfies independence of irrelevant alternatives (W.G.)) in the sense of Arrow: the order between two competitors depends only on their respective grades". The following example shows that it is not just the respective grades and their relative value which count but also the absolute value of the respective grades. Consider an example with two candidates and five voters, where the ordinal grades increase from a to f:

Example 4.5a

	1	1	1	1	1
candidate x	a	a	d	d	e
candidate y	b	b	c	e	f

According to the criterion to choose the alternative with the higher median grade, candidate *x* will be picked. Now consider a modification where for each voter the *relative* ranking between *x* and *y* from 4.5a is left completely untouched:

Example 4.5b

	1	1	1	1	1
x	a	a	b	d	e
y	b	c	a	e	f

In this variant, the median grade is higher for *y* than for *x* which means that the absolute values of the grades exercise a non-negligible influence on the final outcome. Whether this feature already reveals a certain cardinal "touch" or not may be a matter of debate that we do not want to pursue any further. At any rate, the Balinski-Laraki method takes more into account than just the relative ranking between any two alternatives, which is a characteristic that lies at the heart of Arrow's original formulation of his independence condition.

Range voting as a cardinal rule satisfies independence in the same manner as any utilitarian rule fulfils this property. If the n-tuple of scores for two alternatives *x* and *y*, let's say, are the same in two profiles (of scores) not necessarily restricted to *x* and *y*, then the social relation between *x* and *y* will be identical for both profiles. The scores of other alternatives have no influence on the relation between *x* and *y*. The reader will note that this specification, by using (cardinal) scores, marks a considerable deviation from Arrow's original formulation in terms of ordinal rankings.

5. Partition Consistency

As already mentioned in the introduction, this condition, also called *reinforcement*, says that if an electorate is split into two disjoint subgroups, represented by sub-profiles of n_1 and n_2 voters, let's say, then the voting outcome over the entire electorate of $n_1 + n_2$ voters should be identical to the intersection of outcomes for the two subgroups of voters n_1 and n_2, respectively, given that the intersection is nonempty. It is easy to see that the simple majority rule, because of its additive structure, satisfies this condition as long as no majority cycles occur, neither in the two subgroups nor over the entire set of voters. If cycles occur and the assumption is made that all elements constituting a voting cycle are socially indifferent or, expressed differently, are elements of the respective choice sets, then partition consistency is no longer guaranteed. This is shown in the following example with four alternatives and 3 + 5 voters:

Example 5.1

group 1			group 2				
1	1	1	1	1	1	1	1
x	y	z	y	z	z	v	v
y	z	v	z	v	y	x	x
z	v	x	v	y	v	y	y
v	x	y	x	x	x	z	z

For group 1, there is a preference cycle comprising all four alternatives. For group 2, there is a preference cycle over *y*, *z*, and *v* so that one obtains choice sets {*x*, *y*, *z*, *v*} and {*y*, *z*, *v*}, respectively. The nonempty intersection is {*y*, *z*, *v*}. However, for the entire set of voters there exists a Condorcet winner, namely *y*.

The Pareto-extension rule does not satisfy partition consistency either. This can be easily seen from the following profile of two alternatives and six voters:

Example 5.2

group 1			group 2		
1	1	1	1	1	1
x	y	x	x	x	x
y	x	y	y	y	y

The choice set for group 1 is {*x*, *y*}, for group 2 it is {*x*}, so that the nonempty intersection just contains alternative *x*. However, the choice set over the entire electorate is {*x*, *y*}.

Jerry Kelly (1988) already showed that the Copeland method does not satisfy consistency either. Consider the following example with five voters, split into two groups, and three alternatives:

Example 5.3

group 1		group 2		
1	1	1	1	1
x	y	x	y	z
y	z	y	z	x
z	x	z	x	y

The choice sets for group 1 and group 2 are {*x*, *y*} and {*x*, *y*, *z*}, respectively. The nonempty intersection reads {*x*, *y*}, the choice set for the entire electorate, however, is {*x*, *y*, *z*}.

To show that the Dodgson method does not satisfy partition consistency, we can use example 5.1 from above again. For subgroup 1, *y* and *z* need one inversion each to become Condorcet winners, for subgroup 2, *y*, *z*, and *v* need one inversion so that the intersection of choice sets is {*y*, *z*}. However, as we already know, *y* is the sole Condorcet winner.

We mentioned earlier on that partition consistency is one of the constitutive axioms in Young's (1974) characterization of the Borda rule. Because of its simple additive structure, the plurality rule satisfies consistency as well. If *x*, let's say, is top more often than any other alternative in group 1 and the same holds for *x* in relation to group 2, then *x* is top more often than any other alternative in the union of the two groups. A violation of consistency can only happen if the quota for being declared a winner is different for each of the subgroups than it is for the entire set of voters, but such a requirement does not appear very intuitive.

Plurality with run-off, however, does not satisfy partition consistency. Consider the following example with 23 voters in group 1 and 13 voters in group 2; there are three alternatives:

Example 5.4a
group 1

5	2	6	2	7	1
x	x	y	y	z	z
y	z	z	x	x	y
z	y	x	z	y	x

Assume that at least 50 % of the top ranks are necessary for a candidate to win already in the first round. This is not the case. Therefore, *x* is deleted in round 1. In the second round, candidate *y* beats *z*.

Example 5.4b
group 2

4	7	2
x	y	z
z	z	x
y	x	y

Candidate *y* receives at least 50 % of the top-ranked votes in the first round so that this candidate wins. The intersection of the choice sets for both groups just contains *y*.

The preference profile for the complete electorate is given as follows:

Example 5.4c complete electorate	5	6	13	2	9	1
	x	x	y	y	z	z
	y	z	z	x	x	y
	z	y	x	z	y	x

No candidate achieves 50 % of the top ranks in the first round. Candidate *z* is therefore deleted and in the second round, *x* beats *y* by a simple majority. This shows that partition consistency is violated.

Majority judgement does not satisfy consistency either. Consider the following situation with two alternatives and seven voters, where the grades increase from a to f:

Example 5.5	group 1			group 2			
	1	1	1	1	1	1	1
x	a	b	b	d	e	e	e
y	b	c	c	c	f	f	f

For groups 1 and 2 the median grade for *y* is higher than for *x* so that the intersection of choices from both groups just contains *y*. For the entire group of seven voters, the median grade is higher for *x* than for *y* so that alternative *x* is to be chosen.

It is easily seen that range voting satisfies partition consistency. If the score difference between some *x* and *y*, let's say, summed over all voters in subgroup 1 is strictly positive and if the same holds true for the voters in group 2, then, clearly, *x* will have a higher aggregate score than *y* over the entire electorate. We would like to refer to Pivato's (2014) recent axiomatic analysis of this rule where partition consistency or reinforcement is explicitly used in a characterization of this voting scheme.

6. Condorcet Consistency and Condorcet Loser

A voting procedure satisfies *Condorcet consistency* whenever, given that a Condorcet winner exists, this candidate is elected or is in the rule's choice set. Clearly, by definition, the simple majority rule has this property. The same applies to the Copeland and Dodgson rule which are based on majority counting. In the Copeland scheme, we look for the maximal number of instances that an alternative *x*, let's say, beats or ties other available alternatives majority-wise. Clearly, a *Condorcet loser* who is beaten by all other candidates, can never come up as a Copeland winner. It appears at first sight that the Dodgson procedure where we look for the minimal number of inversions such that an alternative will become a Condorcet winner, has the same property. However, Hannu Nurmi (1987, p. 52) showed that Dodgson's method fails in this respect. Consider his example of 15 voters and four alternatives:

Example 6.1

5	3	2	5
x	y	v	z
y	z	y	v
v	x	z	x
z	v	x	y

All four alternatives need three inversions in order to become a Condorcet winner. Therefore, the choice set is $\{x, y, z, v\}$. However, v is a Condorcet loser so that Dodgson's method does not satisfy the requirement never to pick a Condorcet loser.

Sen's Pareto-extension rule trivially satisfies Condorcet consistency but unfortunately, its choice set may also contain a Condorcet loser. This is immediately visible from the following simple situation of three alternatives and three voters:

Example 6.2

1	1	1
x	x	z
y	z	y
z	y	x

Alternative y loses against every other alternative majority-wise but is element of the choice set under the Pareto-extension rule.

That the Borda rule does not necessarily choose a Condorcet winner is one of the central arguments that adherents of pairwise rules bring forward against Borda's rank counting. Consider the following example for five voters and four alternatives:

Example 6.3

1	1	1	1	1
y	y	x	y	x
x	z	z	x	v
z	x	v	v	z
v	v	y	z	y

Alternative y is majority winner, but alternative x is the candidate with the highest aggregate Borda score. Donald Saari (1995) showed that the Borda count never picks a Condorcet loser.

The next example shows that the plurality rule does not necessarily pick a Condorcet winner either. There are three alternatives and nine voters in this example:

Example 6.4

4	3	2
x	y	z
y	z	y
z	x	x

Given that 40% of first-place votes are sufficient to become a plurality winner, alternative x is the winner, while candidate y is Condorcet winner. Unfortunately, it is easy to construct an example where the plurality winner is a Condorcet loser. The next example for four alternatives and five voters shows exactly this:

Example 6.5

1	1	1	1	1
x	x	y	z	v
y	v	z	v	y
z	z	v	y	z
v	y	x	x	x

Alternative *x* is the plurality winner but loses majority-wise against every other candidate.

Plurality with run-off does not satisfy Condorcet consistency but fortunately does not face the Condorcet-loser dilemma. Consider the following profile for a case of 24 voters:

Example 6.6

9	4	2	7	2
y	x	x	z	v
x	y	v	x	x
z	z	z	y	y
v	v	y	v	z

None of the alternatives reaches at least 40 % of the first-place votes. In the second round, after deleting *x* and *v*, *y* wins over *z*. However, x is the Condorcet winner. It should be clear from this profile that the run-off variant of plurality will never pick a Condorcet loser.

The following example with seven voters or judges and three alternatives, with ordinal grades increasing from a to h, shows that majority judgement may not choose a clear majority winner but may pick a Condorcet loser instead:

Example 6.7

	1	1	1	1	1	1	1
x	a	a	a	e	f	f	f
y	b	b	b	d	g	g	g
z	c	c	c	b	h	h	h

Alternative *z* is the majority winner by 6:1 votes, *x* is the Condorcet loser by the same margin but wins because of the highest median grade. This shows that concentrating exclusively on the median grade may be treacherous. Is the example above far-fetched and therefore does injustice to majority judgement? We do not think so. Apparently, given the scale from a to h, some of the judges are only willing to assign low grades – they may be rather critical, whereas an equal number of judges is generous or euphoric. This can happen.

Range voting as a cardinal procedure may face the same problem that we just depicted in relation to majority judgement. Let there be three alternatives, five voters and a cardinal integer scale increasing from 1 to 8. Consider the following preferences (or assignment of grades):

Example 6.8	1	1	1	1	1
x	3	4	2	3	2
y	2	2	3	1	8
z	1	3	4	4	1

Given these rankings, *x* would be the Condorcet winner, *y* would be the Condorcet loser. However, alternative *y* wins this contest because of the very high grade that the fifth voter attached to this alternative. This fact is unpleasant and can be made smaller (in terms of probability of occurrence) when the range of admissible scores is considerably restricted, but it cannot be fully avoided.

7. Monotonicity, Positive Responsiveness, and the No-Show Paradox

The property of monotonicity exists in various forms. In its weak version, also called *nonnegative responsiveness*, it says that if the social outcome in relation to some particular preference profile is *x*, let's say, and if there is some voter who lifts the winning alternative up in his or her ordering, leaving the relative ranking of any pair of alternatives outside *x* untouched, the social outcome will continue to be *x*. *Strong monotonicity* or positive responsiveness that we already discussed in section 2 requires that if *x* is one of the socially chosen objects under a particular profile and if *x* is moved up, then *x* will be uniquely chosen under the modified profile. In the case when a Condorcet winner exists, the simple majority rule satisfies monotonicity. From May's axiomatization of this rule (for the case of two alternatives) we know that positive responsiveness is one of the characterizing properties of this procedure. Sen's Pareto-extension rule trivially satisfies monotonicity. In section 2, we showed that this scheme does not fulfil positive responsiveness if there are at least three voters.

The Copeland method satisfies monotonicity. If a Copeland winner is moved up somewhere in a profile, its score in relation to the other alternatives will either stay the same or even go up, given that the relationships among the other options remain the same. However, the Copeland rule does not fulfil the stronger version of positive responsiveness. Consider the following example of four alternatives and five voters:

Example 7.1	1	1	1	1	1
	x	v	y	z	v
	z	x	v	y	x
	y	y	x	v	z
	v	z	z	x	y

We obtain s(*x*) = s(*v*) = 2 so that *x* and *v* are socially indifferent (note that the monotonicity property does not require a unique social outcome). If in the ranking of voter 1, candidate *v* moves up by exchanging positions with *y*, *v* is able to increase its score. However, positive responsiveness is not satisfied. Likewise, consider the case that *x* is moved up to the level of *v* in the ranking of voter 2; this does not turn *x* into a unique winner.

The Dodgson method does not satisfy monotonicity. This was shown by Peter Fishburn (1977) for the case of five alternatives and 19 voters:

Example 7.2

5	8	4	2
x	v	y	y
y	x	w	x
z	z	v	v
v	w	x	w
w	y	z	z

According to Dodgson's proposal, alternative *x* would need three inversions to become a Condorcet winner, alternatives *v* and *y* would need four reversals, and the other candidates would need even more. Therefore, *x* is the Dodgson winner. Suppose now that the last two voters move *x* up so that their strict preference is *x y v w z*. Now, candidate *v* would only need two reversals to become the Condorcet winner; *x* still needs three inversions. Candidate *x* has been moved up but now loses to *v*, a violation of monotonicity. Fishburn (1982) showed that Dodgson's method satisfies monotonicity in the case of less than five alternatives.

It is very easy to see that the Borda rule satisfies positive responsiveness. Consider two voters and three alternatives:

Example 7.3

1	1
x	y
y	x
z	z

Both *x* and *y* are Borda winners. If *x* is moved up to the level of *y* in the second voter's ranking, *x*'s Borda score increases while *y*'s score remains the same or decreases to 1.5 after the change, if equally high positions receive the arithmetic mean of adjacent ranks. Therefore, *x* becomes the unique Borda winner.

The plurality rule which only considers the top rank in each voter's ordering trivially satisfies monotonicity. Consider the following example for six voters and four alternatives:

Example 7.4

1	1	1	1	1	1
x	x	y	y	y	z
y	z	z	x	z	y
z	v	x	z	v	x
v	y	v	v	x	v

If at least 50 % of the top ranks are needed such that an alternative be socially chosen, *y* is in the choice set. If the sixth voter switches the positions of *z* and *y* so that *y* moves to the top, *y* is again in the choice set. In order to check for the fulfilment of strong monotonicity or positive responsiveness, one would have to allow first that the plurality rule can generate socially equivalent objects (which is, of course, admissible) and second that individuals are allowed to manifest indifference between objects. The latter, however, is against the convention that plurality rankings are linear orderings. For the case that weak orderings were allowed, imagine that voter 6 first had the ordering *x y v z* so that *x* and *y* become socially equivalent and that this voter then decided to move *y* to the top, on a par with *x*. Then *y* becomes the sole winner. Notice that under the plurality rule the preferential improvement of some alternative as required by the condition of positive responsiveness would have to occur at or just below the top rank. The plurality rule is completely unresponsive to preference changes which occur "further down" in the profile.

When plurality with run-off is considered, even the monotonicity property is violated. Dan Felsenthal and Nicolaus Tideman (2013) give the following example with three alternatives and 43 voters:

Example 7.5	7	9	14	13
	x	x	y	z
	y	z	z	x
	z	y	x	y

Plurality with run-off will delete *z* in the first round and *x* will beat *y* in the second round. Felsenthal and Tideman now suppose that 5 out of the 14 voters with the ranking *y z x* change their ranking to *x y z* , thus increasing the support for *x*. Now candidate *y* will be eliminated in the first round and *z* will beat *x* in the second round. The increase of support for *x* by five voters did not strengthen *x*'s position but made *x* lose against *z*.

As observed by Balinski and Laraki, majority judgement is monotonic, but since this rule focuses on the median grade alone, it does not satisfy the strong monotonicity property. Consider the following example of two alternatives and five judges, with grades increasing from a to g:

Example 7.6	1	1	1	1	1
x	a	b	c	d	g
y	b	a	c	e	f

Alternatives *x* and *y* are indistinguishable in terms of their median grade so that they are both equivalent. If the fourth judge increases his or her grade for *x* from d to e, *x* and *y* are still equivalent. Only if the third judge upgraded alternative *x* from c to d, let's say, *x* would be uniquely chosen. So in order to satisfy strong monotonicity, this requirement would have to be redefined in terms of changes with respect to the median grade only, a clear weakening of this condition.

Range voting satisfies strong monotonicity or positive responsiveness quite naturally. If for a given common scale, *x* and *y*, let's say, receive the same sum of scores so that they are deemed equivalent and if one voter lifts *y* up by at least one rank, this alternative will win over *x*.

Let us now consider situations with a variable electorate. The phenomenon that we wish to discuss has been termed the *no-show paradox*. It describes situations where a voter or a group of voters may reach a better outcome by not showing up at the polls than by taking part in the election and expressing their sincere preference rankings. The no-show paradox clashes with the idea of participation which says that a voter should never lose by joining the electorate and reporting his or her true preferences. Do all voting procedures suffer from this rather unpleasant feature which, according to Nurmi (1999, p. 50) "undermines the very rationale of voting"? The answer is "no", but Hervé Moulin (1988) has given a general proof that if there are at least four candidates, all Condorcet consistent voting rules are subject to the no-show paradox.

Clearly, in the case that a Condorcet winner exists, a voter who has this candidate "high up" in his or her ranking would, by abstaining from the election, endanger and not strengthen the status of this candidate. And a voter whose ranking does not show much sympathy for the Condorcet winner would, by not showing up, possibly make the position of this candidate even stronger.

Felsenthal (2012) has recently provided an example which demonstrates that the Copeland method is vulnerable to the no-show paradox. Consider four alternatives and 33 voters:

Example 7.7	11	2	12	4	2	2
	x	y	y	z	v	v
	y	z	z	x	x	y
	z	x	v	v	y	x
	v	v	x	y	z	z

In this situation there is a top cycle comprising *x*, *y*, and *z* which are all preferred to *v*. So Copeland's procedure gives a tie with s(x) = s(y) = s(z) = 2. If the two voters with the ranking *y z x v* decided not to participate in the election, Copeland's method will generate a tie between *y* and *z* which is clearly better for the voters who decided to abstain.

Felsenthal and Tideman (2013) illustrated Moulin's general result by providing an example for the Dodgson method. Consider a situation with 110 voters and five alternatives:

Example 7.8	42	26	21	11	10
	y	x	w	w	x
	x	w	v	x	y
	z	z	y	y	w
	v	y	x	v	z
	w	v	z	z	v

The number of inversions needed for *y* to become a Condorcet winner is 8 which is smallest among all given alternatives. If the last 10 voters who rank *x* on top decide to abstain from voting, *x* will indeed be elected by the Dodgson rule, with 14 inversions between *x* and *y* against 18 inversions between *w* and *y*.

Sen's Pareto extension rule is immune to the no-show paradox. Expressing a strict preference for some *x* over some *y* with *x* being the top element in a voter's ranking always makes sure that *x* belongs to the choice set. Not articulating this preference increases the danger that *x* will no longer be an element of the set of socially chosen objects. To illustrate this point, assume that 99 voters prefer *y* to *x* and one voter has the opposite strict preference. If this voter abstains, *x* will no longer be socially equivalent to *y*.

The Borda method and the plurality rule are also immune to the no-show paradox. Both schemes register and aggregate points for all feasible alternatives. If voters choose not to show up, points for their preferred alternatives get lost which may turn out to be pivotal when it comes to summation.

Felsenthal and Tideman (2013) report an example which shows that plurality with run-off succumbs to the no-show paradox. Consider the following example with 21 voters and three alternatives:

Example 7.9	6	1	2	4	3	5
	x	x	y	y	z	z
	y	z	x	z	x	y
	z	y	z	x	y	x

Candidate *y* is eliminated in the first round. In the second round, *z* is elected. If two of the voters with strict preference *x y z* do not show up, *x* will be deleted in the first round and *y* will be finally selected. This outcome is definitely better than *z* for the voters who abstained.

Felsenthal and Moshé Machover (2008) showed that Balinski and Laraki's majority judgement is vulnerable to the no-show paradox even for the case of only two candidates. Let there be candidates *x* and *y*, seven voters and ordinal grades starting from a and going up to f:

Example 7.10	1	1	1	1	1	1	1
x	a	a	a	d	e	e	e
y	b	b	b	c	f	f	f

According to majority judgement, *x* wins. If voters 1 and 2 decide to abstain from voting, candidate *y* will be elected. And this is the preferred outcome for the two who refused to submit their true ranking. Should their votes be disregarded anyway since the grades which they assign to *x* and *y* are much lower than the grades that most of the other voters assign? This would be a major interference. The mechanism behind majority judgement simply does not allow us to do so.

Range voting is similar to the Borda count. A preferred alternative receives a higher score than a candidate who is less liked. An abstention would potentially weaken the status of the preferred option.

8. Manipulability

Not showing up at an election and thus abstaining from casting one's true ballot can be viewed as a type of manipulative behaviour. We have seen in example 7.7 that if the two voters with the strict preference *y z x v* do not announce their preference ranking, they will reach a better outcome. This is exactly what manipulative behaviour wants to achieve. Nevertheless, we prefer to distinguish between the no-show strategy and manipulative voting since strictly speaking, the latter stands for the deliberate announcement of a false or dishonest preference relation which is not the case when an individual decides not to show up at the polls with his or her honest preferences.

Allan Gibbard (1973) and Mark Satterthwaite (1975) showed that any "reasonable" voting mechanism is manipulable if there are at least three alternatives. In the case of only two options, the simple majority rule, for example, is non-manipulable or *strategy-proof*. Moulin

(1980) proved that if the domain of permissible individual preference orderings is restricted to profiles which are *single-peaked* (Duncan Black, 1948), a class of generalized Condorcet winners exists which is immune to manipulative behaviour. A single-peaked structure is such that there is a peak, the point of highest desirability, and on either side of this peak, given that the peak does not lie at the extreme left or the extreme right of the spectrum of objects which are arranged along the horizontal line, individual desirability declines. There is no cardinality involved, the framework is purely ordinal. However, Moulin's result is "shaky" in the sense that both the true preferences of the voters and their professed preferences have to belong to the single-peaked domain. Jean-Marie Blin and Satterthwaite (1976) were among the first to discover this problem, which was again recently discussed by Maggie Penn et al. (2011) within the context of coalition formation and strategic voting. Since there are no "laws" which would prevent any individual voter from announcing a preference ordering which is incompatible with the property of single-peakedness, there is a risk that even situations where according to Moulin's result a Condorcet winner exists are manipulable. Consider the following example of four alternatives and five voters, where the order along the horizontal line is x y z v:

Example 8.1

1	1	1	1	1
x	y	z	v	v
y	x	y	z	z
z	z	x	y	y
v	v	v	x	x

This is a case of single-peaked preferences with a unique Condorcet winner, namely *z*. If the first voter now decides to profess the strict ordering *x y v z* instead of the true one, all four alternatives form a preference cycle. If in such a situation, a random mechanism would be picked to select a unique outcome, then instead of a 100 % certainty that *z* will be declared the winner, there would be a 50 % chance for voter 1 that *x* or *y* will be the final outcome and both *x* and *y* are definitely better than *z* according to the sincere preference ranking of this voter; and with a 75 % chance, the outcome will be at least as good for voter 1 as the result where every voter had been truthful. Only if the first voter were extremely risk-averse in the sense that his or her worst option should under all circumstances be prevented from coming about, the above argument would not hold.

Similar to the simple majority rule, Sen's Pareto-extension rule is not manipulable in the case of two alternatives. In the case of more than two options, an argumentation similar to the above holds. We consider four alternatives and three voters:

Example 8.2

1	1	1
x	y	z
y	z	v
z	v	x
v	x	y

According to Sen's rule, the choice set contains *x, y,* and *z*. So from the perspective of the third individual, the probability to end up in one of the two lowest-ranked alternatives is 2/3. If voter 3 slightly modifies her strict preference to *v z x y,* the choice set will be {*x, y, z, v*}.

Consequently, the probability to end up in one of the two lowest-ranked options has decreased to ½ for the third voter. Therefore, a case of successful manipulation is given, except that the earlier argument holds, namely that voter 3 is extremely risk-averse and only focuses on the worst outcome(s).
Example 4.2 in section 4 can be used to show that both the Copeland rule and Dodgson's method are manipulable. The original preference profile yields a unique winner *v*, the outcome according to Copeland and Dodgson after the second voter's preference change is {*v*, *x*}, which is definitely better for this voter, given her sincere preference ranking. Another example to show that Dodgson's rule is manipulable is the following situation with three voters and four alternatives:

Example 8.3

1	1	1
x	y	z
y	z	v
z	v	x
v	x	y

According to Dodgson, the choice set is {*y*, *z*}. If the first voter increases the support for *y* by announcing *y x z v*, alternative *y* becomes the unique Condorcet winner which is definitely better for this voter than {*y*, *z*}.

The Borda rule has "a reputation" for being easily manipulable. Jean-Charles de Borda was very aware of this when he exclaimed that this rule was meant for honest men only. Consider the following situation of four voters and five alternatives:

Example 8.4a

1	1	1	1
x	y	y	z
z	x	x	y
y	v	w	x
v	w	z	v
w	z	v	w

According to the Borda count, *y* wins and *x* is second.

Example 8.4b: voters 2, 3, and 4 have the same ranking as before but this time, voter 1 dishonestly declares that *x* is best, then *z*, followed by *v*, then *w* and, finally, *y*. Now *x* comes out as the winner which is the top element in the ranking of voter 1.

Consider a situation where according to the plurality's focus on tops only, *x* receives more first-rank votes than any other alternative *y*. In such a situation, no single voter has the possibility to turn his or her top candidate into a winner and topple *x* by announcing an insincere ranking. However, there still is a possibility of manipulation. A voter may try to avoid an outcome that according to his or her truthful preference ranking is considered as bad by registering a "false" top candidate. To demonstrate this kind of strategic behaviour, Keith Dowding and Martin van Hees (2008) refer to the presidential election in the US in 2000 where a group of people who were supporters of Ralph Nader voted for Al Gore rather than for Nader (they clearly preferred Gore to Bush) because they believed that Nader had no real chance of being elected president. So by voting for Gore, namely moving a less preferred option to the top of their ranking, this group hoped that their second-place candidate would win the election.

The class of rules that embodies the Nader-Gore type of manipulation is denoted *top-monotonic* by Dowding and van Hees. The plurality rule is, perhaps, the most prominent member of this class. The authors take this example from the real world to argue that, from a democratic perspective, manipulation should not be too worrying after all. The Nader-Gore case of manipulation is, of course, quite different from the kind of manipulative behaviour depicted in example 8.4 where voter 1 penalizes candidate *y* for the simple reason that by doing so, his or her most preferred alternative will win.

In order to show that plurality with run-off is manipulable, consider a modification of the earlier example 7.5 with three alternatives and eleven voters:

Example 8.5	2	2	4	3
	x	x	y	z
	y	z	z	x
	z	y	x	y

Let the rule be such that if in the first round, no alternative achieves at least 50 % of the votes, the alternative with the lowest number of top ranks is deleted, which is candidate *z* in the situation above. In the second round, alternative *x* is picked which is bad for the four voters with the ranking *y z x*. If one of those four voters announces the ranking *z x y*, then in the first round *y* is deleted and *z* wins against *x* in the second round, an outcome which the manipulating voter prefers to the result under truthfulness.

Balinski and Laraki (2007) argue that majority judgement is "minimally manipulative". It is true that this rule does not suffer that much from judges that assign an extremely high or extremely low grade to some candidate, a problem that range voting and other cardinal rules such as utilitarianism face. But majority judgement can still be successfully manipulated by a single judge as can be seen from the following example with two alternatives, five judges and grades rising from a to j:

Example 8.6	1	1	1	1	1
x	b	d	f	g	h
y	d	b	c	i	j

The median grade for *x* is f, for *y* this grade is d so that alternative *x* is chosen. If the first judge who prefers *y* to *x* changes his or her grade for *y* from d to g, then *y* will be the winner.

Exactly this phenomenon can also happen under range voting. Extreme or relatively high grades, whether truthful or not, cast by a single voter can always change the voting outcome. In his characterization of range voting, Pivato (2014) speaks of "minority overrides". Therefore, adherents of this voting rule often argue in favour of relatively narrow upper and lower bounds of the scores that voters are allowed to assign. This can constrain the extent of manipulation but will not be able to prevent it altogether.

9. The Wickedness Issue

We are now in a position to come back to the wickedness problem, the central theme of this paper. In our introduction we had referred to Rittel and Webber who asserted that in the realm of public policy “there are no true or false answers” (1973, p. 163). This is due to different value sets that groups of individuals have when decisions on various public policies have to be taken. The authors argue that there can be “better-or-worse” solutions or “good enough” proposals but these solutions can by no means be viewed as definitive or objective. Table 9.1 which summarizes our analysis of the previous sections will help to look somewhat more deeply into the wickedness issue of social choice.

Our problem is somewhat akin to that of an engineer who, for example, has to select the best tool(s) in order to perform a particular job or has to decide which of the newly devised engines should enter serial production. The engineering literature has indeed studied social choice approaches in order to do the selection process in a consistent and effective way (see, for example, Scott and Antonsson (1999) and Franssen (2005)).

How should one go about solving our problem? One could, for example, formulate priorities and then see which of the nine rules that we investigated satisfy these. We shall actually do this, keeping in mind that priorities can hardly be set objectively. One could look for “local” vector dominance and check whether some rule(s) is (are) at least as good as some other rule(s) in relation to the various criteria we have formulated above and better at at least one instance. We shall also follow this line of thought in what follows. To hope for “global” dominance in the sense that one rule performs best in relation to all criteria appears to be futile.

(A) Priorities

Many researchers will probably argue that aggregation rules which may pick a Condorcet loser are hardly acceptable for social choice. If this is a knock-out criterion, then several of the rules that we considered have to go, namely Sen’s Pareto-extension rule, Dodgson’s method, the plurality rule, majority judgement, and range voting. If Condorcet consistency is required on top, plurality with run-off and the Borda rule have to exit as well so that we are left with the Copeland method and the simple majority rule. The latter, however, can lead to a cyclical social preference or an empty choice set under unrestricted domain. If criterion (1) is added to criteria (4) and (5), the Copeland method becomes the only acceptable method for collective choice. This method, however, does not satisfy partition consistency so that if this criterion is taken as an additional requirement, we are left empty-handed.

If among the criteria we have just examined Condorcet consistency is put aside (a move that is hardly acceptable for adherents of Condorcet), Borda’s rule stands out as an attractive voting procedure. The Borda count is, in addition, positively responsive and is not affected by the no-show paradox. Apart from Arrow’s independence condition, which the Borda scheme does not satisfy, the question for or against this rule really boils down to the issue of Condorcet consistency. Let us discuss the following situation with seven voters and four alternatives:

Example 9.1	1	1	1	1	1	1	1
	x	x	x	y	v	y	z
	z	v	y	x	y	x	y
	v	z	v	z	x	v	x
	y	y	z	v	z	z	v

Here *y* is the Condorcet winner while *x* is the Borda winner (with 15 points against 12 for *y*).

choice mechanisms ordinal (o) / cardinal (c)	(1) no cycles of social preference – non-empty choice set	(2) independ-ence of irrevelant alternatives	(3) partition consistency – reinforce-ment	(4) Condorcet consistency	(5) never chooses Condorcet loser	(6) monotoni-city	(7) positive responsive-ness	(8) no-show paradox does not occur	(9) being strategy-proof
simple majority rule (o)	-	+	-	+	+	+	+	+ *)	+ **)
Sen's Pareto extension rule (o)	+	+	-	+	-	+	-	+	-
Copeland method (o)	+	-	-	+	+	+	-	-	-
Dodgson's method (o)	+	-	-	+	-	-	-	-	-
(broad) Borda rule (o)	+	-	+	-	+	+	+	+	-
Plurality rule (o)	+	-	+	-	-	+	-	+	-
Plurality rule with run-off (o)	+	-	-	-	+	-	-	-	-
majority judgment (o)	+	+	-	-	-	+	-	-	-
range voting (c)	+	+	+	-	-	+	+	+	-

*) under appropriate domain restriction

**) only under domain restriction for both true and announced preferences

Table 9.1

Alternative *y* wins against all other options with 4:3 votes, *x* loses against *y* but wins against *z* and *v* with 6:1 each. So when looking at *all* alternatives and *all* pairwise comparisons, *x* appears to be a strong contestant. Wouldn't then *x*, in spite of violating Condorcet consistency, be the better choice in this situation? We think that this example is by no means far-fetched or pathological. Clearly, we do not deny that there are voting profiles where the choice of the Condorcet instead of the Borda winner gains much more intuitive support. Nevertheless, the question arises whether the fulfilment of Condorcet consistency should always have absolute priority. If this query is conceded, even the better-or-worse comparison to which we referred cannot be resolved in a quick and simple way.

Much simpler, perhaps, is a decision against rules that do not satisfy the monotonicity property. If this criterion is postulated as a minimum requirement, plurality with run-off and Dodgson's method fail, the latter in the case of more than four alternatives.

Various researchers argue that the no-show paradox is so much against the idea of democratic participation and voting that rules that succumb to this phenomenon should not be serious candidates for collective choice. If this view is accepted, Copeland's procedure, Dodgson's method, the plurality rule with run-off, and majority judgement have to be deleted from further consideration. The Borda count and simple majority voting, the latter on a restricted domain, do not fall a prey to the no-show paradox. The same is true for the Pareto-extension rule, plurality and range voting. The latter will be eliminated by those who find any form of cardinality unacceptable. The plurality rule is found intolerable by many because of its total neglect of detailed information on ranks other than the top rank. Sen's Pareto-extension rule produces too many instances of equivalence even in cases of only mild disagreement among the voters. So again, the Borda count stands out quite favourably.

Strategy-proofness is not a good priority criterion since all rules violate it to a higher or lower degree. An exception is simple majority voting under the condition that both the true and the professed preferences are restricted to single-peakedness so that a non-empty set of Condorcet winners exists. If Arrow's independence axiom is proposed as a priority criterion, the simple majority rule, Pareto-extension, majority judgement, and range voting are the only admissible candidates. If the monotonicity condition is added, all four rules continue to pass. Once monotonicity is strengthened to positive responsiveness, Pareto-extension and majority judgement drop out.

Range voting does not run into the cyclicity problem that simple majority voting has. It is the only cardinal rule that we discuss and if this is a knock-out criterion or if it is argued that it is impossible for a person to translate her underlying preferences over a given number of alternatives into ranks on a common scale that was proposed, then this rule has to go. We do not want to claim that range voting should replace the concept and procedure of a social welfare function which is meant to represent the general will of society, but for committee decisions where an evaluation of competing candidates or alternative projects takes place, this scheme appears to make a lot of sense. As pointed out before, it is not as inflexible as the linear rank-order structure that the Borda count uses. The following example is meant to show the degree of flexibility that range voting offers. Consider a situation of six voters and three alternatives:

Example 9.2	1	1	1	1	1	1
	x	x	y	y	z	z
	z	z	x	x	y	y
	y	y	z	z	x	x

According to the Borda count, all three alternatives are equivalent. There exists a Condorcet cycle with *y* preferred to *x*, *x* preferred to *z*, and *z* preferred to *y*. If the plurality rule with run-off is applied, all three alternatives can be the final outcome, depending on which alternative is deleted first. If *z* is eliminated first, the outcome is *y*; if *x (y)* is eliminated first, the outcome is *z* (*x*). The ordinal arrangement of the rankings in our example depicts a situation of perfect symmetry. Imagine now that the three candidates have to be assigned to ranks on a scale from 8 to 1. The assignment of each of the six orderings of our example can be done in 56 different ways, from ranks 8, 7, 6 for *x, z, y* up to ranks 3, 2, 1 for the first person's linear ranking, and likewise for the other five voters. In other words, range voting with a commonly binding scale offers a multitude of constellations which all represent the profile in example 9.2. The perfect symmetry which the ordinal representation depicts will only occur with a very low probability, given that the voters decide independently with no prior knowledge of how the other voters will determine their assignment of scores or ranks. It is in this sense that range voting allows for a high degree of flexibility in expressing the preferences of the individuals involved.

(B) "Local" vector dominance

Table 9.1 illustrates that the endeavour to obtain a "globally" best aggregation procedure is condemned to fail. If we lived in a world where the preferences of all individuals always obeyed Sen's (1970) *value restriction*, let's say, the simple majority rule would be a good candidate for global vector dominance. Once we content ourselves with local improvements, there are two clear instances of dominance among the procedures we investigated. One is the case of Copeland versus Dodgson. With respect to two criteria to which many researchers attach great importance, namely the monotonicity property and the requirement never to pick a Condorcet loser, Copeland is superior to Dodgson. In relation to the other criteria, both methods fare equally well – or equally badly. The other case is Borda versus the plurality rule and plurality with run-off. Local vector dominance may thus provide a good argument to declare Dodgson's method as well as plurality (without or with run-off) as inferior or even inadmissible. An argument to question the superior role of the Borda method would have to refer to the degree of manipulability which these schemes concede. We quoted Dowding and van Hees (2008) who asserted that the kind of manipulability that exists under the plurality rule can hardly be viewed as objectionable while Borda's method permits many instances of malevolent down-grading. Finally, there is an instance of vector dominance between range voting and majority judgement. Range voting fares better with respect to partition consistency, responsiveness, and the no-show paradox but one should keep in mind that range voting, in contrast to majority judgement, represents a cardinal measure which some may find unnecessarily demanding or even unacceptable.

10. Concluding Remarks

We have seen in this paper that among the aggregation procedures which we considered there is no uniquely best method, given nine criteria that are often discussed in the social choice literature. Of course, there are other collective choice methods and also additional criteria that we did not discuss but the situation in an "enlarged scenery" would be similar to the one depicted here. Table 9.1 may, perhaps, incite the reader to count, for each method, the number of cases of fulfilment of the criteria given and deduct from this the number of cases where a violation occurs. However, such a step would only make sense if one had good reasons to attach the same degree of importance to all criteria. This assumption would most probably meet the opposition of many scholars. On the other hand, if an equal weighting is denied, the question arises which criteria deserve a higher weight and which should receive less weight. This is a wicked or if you prefer another term, an intricate problem with no quick resolution. Is, for example, Condorcet consistency of ultimate concern or is it rather the avoidance of the no-show paradox on which our primary attention should focus? Concerning the latter property, Balinski and Laraki (2010, chapt. 16) argue that the no-show paradox is "of little real importance" (p. 286) nor are they worried that their method, for example, fails to satisfy partition consistency. They are rather articulate when they write (p. 291): "It is a relatively simple matter to invent extreme examples that seem, at first glance, to make *any* method look bad. …To be useful, examples must show how one or another important property is violated, for in that case the logical consequences of the property may be investigated". This is exactly what we were trying to do in this paper. Was our analysis biased? If this were to hold, it would in the same way apply to many aggregation rules discussed in this paper. We stated above that if the Condorcet-loser property, for example, were a knock-out criterion, several quite different rules would have to exit immediately. We leave it to our readers to resolve this issue for themselves.

One might have taken a completely different view than we have chosen in order to distinguish and discriminate among different criteria and, consequently, different decision rules. One might have argued that properties and decision schemes should be related to context, namely to the nature of the decision problem. Would we then have achieved clear-cut results? We have our doubts. Should the plurality rule, for example, be used in large-scale elections as is the case in France for the presidential elections and should the Borda rule perhaps be applied in smaller bodies or committees? Baujard et al. (2013) showed in a field experiment during the 2012 election in France that plurality with run-off favours candidates who elicit "strong feelings" whereas evaluative rules such as approval voting and range voting favour candidates who attract the support of a wide span of the electorate. Leininger (1993) had shown that in the parliamentary vote on the site of the German capital after the unification, a type of majority rule with run-off made Berlin the site of both Parliament and Government whereas plurality and Borda would have left both Parliament and Government in Bonn. Can we infer from these two findings that plurality is the wrong decision scheme for presidential elections in France and Borda's scheme is the wrong form of decision mechanism for parliamentary voting?

Table 9.1 has enabled us to detect cases of vector dominance. This is a feature which is helpful and nice since it allows us to make some "local" better-or-worse statements. But it does not help us to reach a globally definitive verdict which Rittel and Webber generally denied for public policy issues. As long as there is no meta-theory which would permit us to do comparison and evaluation from an as it were elevated standpoint, the problem remains unresolved. One can, however, view the social choice situation from a thoroughly positive angle as well: there apparently is an "embarras de richesses" when it comes to methods and procedures of preference aggregation in order to derive a societal verdict.

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