Individuation and Identity in quantum mechanics

Debajyoti Gangopadhyay

Abstract

Developments of quantum mechanics during the first three decades of the last century led most fundamentally to different orders of difficulties in the spatio-temporal modes of description. And, as a consequence, ambiguities in the meaning of quantum mechanical *are* in sight for the first time.

So the question (from logico-philosophical grounds) we were left as back as late 20s of the last century – a question *methodologically prior* to all other questions - How to talk about quantum particles [in a Language] which are *not* instantiation of spatio-temporal *is* or *ONE* of their particular kind? Though not popular among the physicists, philosophers often use the phrase *failure of principles of Individuation* or *ontological priorities of individual* to describe this situation.

Within the intended scope of this volume, we need not give the technical details of this *failure* here. We are basically narrating here a non-technical story outline of the way quantum mechanical language messes with standard logic and set theory embedded with the concept of *unambiguous is*, and the possible consequences.

1: In what sense do quantum entities exist?

Impact on Foundation of Logic and Set Theory

" how can we talk about what happens at this level if there are no names ..? "

French and Krause ' *Identity in Physics* ,2006

Failure of spatio-temporal modes of description is equivalent to an ambiguity in the sense of spatially individuated 'one'. Failure to make sense of *ONE* is methodologically equivalent to failure of *discernability* of quantum entities as *distinct / distinguishable individual* with numerical identity (for our present purpose we need not bother to keep *distinguishability* and *individuality* conceptually distinct) in the sense of , for example , Lowe - *"* … an object that is differentiated from the others of its kind in such a fashion that it and they are apt to constitute a countable plurality with each member of such plurality counting for just one , a unit of its kind.*"* [E. J. Lowe ' 1994]

Consequently Leibnizian *Principle of Identity of Indiscernible* (PII) -

$$\forall x \; and \; \forall y \big[\forall P \big(P(x) \leftrightarrow P(y) \big) \big] \rightarrow x \equiv y$$

underlying standard logic and mathematics , seems to collapse**[1]** .

This implies, for a collection of quantum particles, possibility of saving only the notion of *cardinality* but not *ordinality*. These all have direct impact for foundation of Logic and Set theory particularly in question of unambiguous applicability of logical quantifiers in extensional sense and the binary membership function. Quantum entities do not seem to *exist* in a sense standard logic can make room for!

2: How about (re)creating discernability criteria?

Under what conditions are entities different or faithful to PII ?

Following the difficulties to make any straightforward sense of what can possibly *count as one* (discernibility)/ PII in quantum mechanical context, one of the immediate next questions is –

How it is *still* possible to make sense of *one* in a collection even if there is no discernable difference between them?

or, in other words,

How it is possible to treat *quantum entities as distinct individuals* though there is no *empirically discernable* difference between them? That is to say, how to talk about truly *indiscernible* but *distinct* quantum physical objects in a Language? This has an oxymoronic flavor.

2.1: Treating *indiscernibility* within *classical framework*

So long we can continue to talk in terms of a quality something can possess by virtue of *being where it is* , answer to these questions is not far away from our intuitive reach . In fact , 'being where it is ' can be understandably described as a kind of accidental feature that can vary with time , but surely *exclusive* in a sense that , this feature can't be shared with others(can be seen as a philosophical forerunner of Pauli exclusion principle , 1925). This can be taken as motivating grounds to talk in terms of discernible *relational attributes* **[2]** arguing that an entity can be attributed a certain measure of individuation by virtue of *where it is in a structure as a whole* - though not intrinsic but claiming a certain *positional distinction* in a large relational structure . Unfortunately, *spatial symmetry* conditions render this ineffective again to save PII, as whatever

can be said about any point – even about its relational distinction, can be said about the each of them..

One of the interesting responses to save *discernability* (and thus PII) in pre-Kantian Western philosophy (needless to say , without any reference to quantum mechanics or even Physics in broad sense) is in terms of an *individuating feature transcending empirical discernability* , that can even be made a counterfactual sense , a primitive *thisness* or *haecceity* - an unique *'trait'* of distinction that can't be shared even with identical copies (conceptual coinage of *Haecceity* is attributed to the medieval Scottish philosopher *John Duns Scotus* to address some confusion in Greek metaphysics.) . It is perhaps this kind of *non-empirical uniqueness* which can psychologically assure the authenticity and value, for example, of our wedding ring, to survive among its exactly identical 'cloned' copies if ever made!

Haecceity or primitive '*thisness'* is also identified with the notion of *self-identity* – indeed very peculiar as a property, because something, if not identical with anything else, must not escape being identical with itself!

Another way explored at length in recent literature is to talk in terms of *weak discernability* , following Quine , on the basis of *irreflexive* relations (see S. Saunders , Physics and Leibniz Principle '2003) .

However, these all standpoints presuppose, in one way or another, the validity of 'being *somewhere'* in space time.In fact, *relation* can't seem to provide grounds for individuation since they presuppose *relata* which requires *prior* individuation in space time.

But , though none of these answers are perfectly immune to save PII - in one version or another , there is no logical injunction to think that (presupposing of course the validity of *being where it is*), given all intrinsic properties are same , there seems to be space for other (may be hidden or non-empirical or even counterfactual) properties and relations not considered in original (universe of)discourse , in a sense , for example , described by *Quine*

" … Ontology is doubly relative. Specifying the universe of a theory makes sense only relative to some background theory and only relative to some choice of manual of translation of the one theory into the other. Identity is thus a piece with ontology. Accordingly, it is involved in some relativity. Imagine a fragment of economic theory. Suppose its universe comprise persons, but its predicates are incapable of distinguishing between persons whose income is equal. The interpersonal relation of equality enjoys, within the theory, the subsitutivity

property of the identity relation itself; the two relations are indistinguishable. It is only relative to a background theory , in which more can be said of personal identity than equality of income , that we are able even to the above account of the fragment of economic theory , hinging as the account does on a contrast between persons and incomes . " [Quine , 1969]

This is precisely what happens when we treat indistinguishability / indiscernibility within a classical framework such as first order *ZF(Zermelo Fraenkel set theory* with axiom of choice)which encompasses classical logic. Technically speaking certain mathematical structures (built in ZF) can be considered as non-rigid so that once we work within these structures , we can regard some objects as indiscernible relative to all the *predicates* and *relations* defined in the structure (Quine's *group of people* indiscernible with respect to the income predicate). It can be proved that, in ZF every structure can be extended to a rigid structure. This result is of considerable importance as it assures that, it is *always possible* to distinguish between two distinct objects whatsoever - if not in the original domain of discourse or structure where they are being described, then in some of its *rigid extension* in Quine's sense.

So it is not difficult to appreciate that, within ZF (as such within a mathematical structure based on ZF) , there can't be any genuine indiscernibility (indiscernible objects which differ *solo numero*) , but only indiscernible within framework of a given structure.
So it is subsequently a question of possibility to extending or modifying this structure (rigid extension) to accommodate new properties and relations not considered in original discourse. Discernibility is possible to be regained, in principle, so far we talk about classical situation.

In fact Leibniz's Principle (PII) is often proposed to be replaced by *Hilbert-Bernays (- Quine) axiom*, where every primitive predicate is mentioned, quantifying in each argument place -

$$\forall x \forall y \{ \; x = y \; \equiv \; [\ldots \wedge (F_i x \equiv F_i y) \wedge \ldots$$

$$\ldots \; \wedge \forall z \; ((G_J xz \equiv G_J yz) \wedge (G_J zx \equiv G_J zy) \;) \wedge \ldots.$$

$$\ldots \wedge \forall z \; \forall w \; (\; (H_k \; xzw \equiv H_k \; yzw) \wedge (H_k \; zxw \equiv H_k \; zyw)$$

$$\wedge (H_k \; zwx \equiv H_k \; zwy) \;) \wedge \ldots.] \}$$

However the situation is substantially different in quantum mechanics because of the intrinsic restriction imposed by uncertainty relation.

3. Different responses of the quantum Physicists to this situation...

" We have taken over from previous theory [classical mechanics] the idea of a particle and all technical language concerning it . It constantly drives our mind to ask for information which has obviously no significance ..."

Schrodinger '1957 in *What is an elementary particle*

3. 1: Early responses of the founding fathers of quantum mechanics

Though the process of *troubled individuation was* recognized by the founding fathers as soon as the mathematical machinery was at hand by 1926 - 27... , the pre-war mainstream debates were guided mainly within the framework of what Schrodinger himself described later as "*idea of particle and all technical language"* taken over from previous theory!

So quite understandably, founding fathers hardly talked in terms of *failure* of standard logic and Set theory. Consequently, the question of quantum indiscernibility though addressed and acknowledged, right from the beginning, but received inadequate formal treatments on behalf of the founding fathers at least from logico-philosophical grounds.

3.1.1: Hilbert space formulation (1932) by von Neumann as an instantiation of construction on standard set theory

It is most curious to note that, the two eminent German mathematicians , David Hilbert and R . Courant, not physicists, had written a book entitled *Methods of Mathematical Physics* (published 1924)), which contained every mathematical method, devices and special details required for the later development of wave mechanical version of Schrodinger during 1926.

Understandably , all the concepts of Hilbert space formulation (von Neumann 1932) have standard set theoretical analogues , and in that way , quantum mechanics (1924-27) formally dressed up in the language of Hilbert space lend itself automatically within the framework - presupposing all the standard mathematics (read standard set theory and classical logic with *identity*), though the underlying basis (set theory …) of the language was already turned out to be *inadequate* .

As a result, the major interpretational variants by the physicists within the framework of Hilbert space formulation (HSF) continued to be guided by strategies within the framework of standard mathematics encompassing standard logic and set theory. [And] in that way, the major variants happened to be different modes of typical instantiations of what usually happen when indiscernibility is treated within a

classical framework, such as *ZF set theory* which encompasses classical logic. This amounts to say that, standard quantum mechanics talks about indiscernibility within a framework which is intrinsically meant to talk about *discernable* individual! Kind of oxymoronic flavor can't be overlooked here.

Let us define a structure as $\aleph = < \mathcal{F} , \mathcal{E} , \mathfrak{H}_i , \xi >$

Where $\mathcal{F}$ is a model of standard functional analysis , $\mathcal{E}$ is the set of permissible events , $\mathfrak{H}_i$ are the observable part of the theory to formalize and , ξ is a mapping which attribute to each ***event*** $\epsilon \mathcal{E}$, a Hilbert Space in F as well as the scope of semantics (defined by Born's Rule) delineated by Hermitian operators on Hilbert Space .

As is well known that structure of this kind are axiomatizable within ZF by Set theoretical predicates in the sense of Suppes (Suppes 2002) . However as $\aleph$ is a structure in **ZF**, the notion of *individual* and *identity* continue to play underneath as bedrock. As a result, as we already discussed, we can't talk about legitimate indistinguishable objects within the scope of this structure or any of its rigid extension.

So quantum mechanics doesn't seem to have a language of its own, but only a fragment of functional analysis extended *illegitimately*, which though seems to have served our purpose, but at the (often overlooked) cost of a mess about *what is objective* and *what is subjective* in this language. E T Jaynes stated this clearly though in a different context -

" ... But our present (quantum mechanical) formalism is not purely epistemological ; it is a peculiar mixture describing in part realities of Nature , in part incomplete human information about Nature - all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble . Yet we think that unscrambling is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple. " [E T Jaynes ' 1991]

3.2: Is it at all safe to treat quantum indiscernibility within such classical mathematical framework?

"you can't mark an electron, you can't paint it red. Indeed you must not even think of it as marked. "

" .. we have been compelled to dismiss the idea that… a particle is an individual entity which retains its sameness forever. Quite the contrary, we are obliged to assert that the ultimate constituents of matter have no sameness at all."

Schrodinger, 1953

Schrodinger's clear injunction in question of " marking " or "painting " an electron can be translated in Quinean terms as - *Quantum states, though maximally informative, is at the same time logically incomplete*. The information is *maximal* as we can't, in principle, extend them (by *marking* or *painting*) to a *richer* knowledge within the framework of quantum mechanics.

Stated in other words, it is not possible, in principle, to *extend* the information about a quantum state to any richer *background theory* in which, according to Quine "more *can be said* "than it is possible in *base theory*.

This provides interesting points of entry into the question of Bohmean trajectory as well as some metalogical issues. But within the scope of this brief introductory article we will not get into that. **[3]**

Though *fermions* statistics, being directly endorsed by Pauli's Exclusion principle, seems to be apparently compatible (though Pauli did not endorse this compatibility) with PII, Henry Margenau was among the early few to note (1944) a violation of PII by fermions themselves. The violation was fundamentally about incompatibility of the informal statement of the principle with the later mathematical formulations **[4]**

So the fragment of Functional analysis (HSF) that is employed to talk about quantum mechanics can be described as *illegitimate* extension of classical situation as this fail to capture the true essence of indiscernibility ! Quantum mechanical structure can't be extended in non-trivial sense.

In fact Von Neumann himself was not quite happy with HSF, though his misgivings were motivated from different perspective

" … I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert Space any more .After all, Hilbert–space (as far as quantum mechanical things are concerned) was obtained by generalizing Euclidean space , footing on the principle of " conserving the validity of all formal rules ". This is

very clear, if you consider the axiomatic-geometric definition of Hilbert space, where one simply takes Weyl's axioms for a unitary-Euclidean –space, drops the condition on the existence of a finite linear basis, and replace it by a minimum of topological assumptions (completeness + separability). Thus Hilbert–space is a straightforward generalization of Euclidean space, if one considers the vectors as the essential notions."

[M. Rédei, *John von Neumann: Selected letters* , History of Mathematics **27** , American Mathematical Society , Providence , RI, (2005)]

So in view of the above discussion, one thing is clear that, *identity* and even the notion of *individual* - employed in describing quantum mechanical entities, are extrapolation of classical situation and can't be qualified as 'real ' in standard sense. They can be better described as having *mock identity* - something in *As If* sense **[5]** which can be seen as *ontological surplus* or even *toll* of unacknowledged metaphysics. **[6]**

Of course, there is *symmetrization rule* to ensure a tricky bypass, but still the semantic inadequacy is there.

These all imply, first of all, that, it is not straightforward to ***transcribe*** any classically intuitive *discernibility criteria* in quantum mechanical context as these attempts are inevitably fraught with risks of characteristic orders. Though Physics works fine, but often leads to interpretational disaster. As it was stated more generally by French and Krause (2006) –

" … although there are different 'theories' of the quantum domain , all the forms of 'quantum mathematics' considered until recently , in so far as they have been constructed within the resources of standard set theories , *remain compromised*(stress mine) with a theory of identity which leads to *philosophical problems* (stress mine) when it comes to the form of indistinguishability …"

So, one of the crucial messages that can be figured out in view of all these foundational debates is fundamentally about different notes of caution or *philosophical compromise* regarding concepts having no 'rational' justification , (and consequently) which need not be bothered any more to *take formally into account*. These are the concepts Schrodinger famously warned having *no real* significance, but still enjoying a *formal hostage* within our present theories. In fact , this very formal hostage accepted (as is standard practice) as starting point , is likely to lead to these so called 'tolls' or *overlooked (ontological) excess baggage* at some further stages of formulations.

So there are clearly two options left as described by Krause

" to continue using standard classical language at the expenses of using symmetry conditions , as usual , or to go deep into the philosophical problem of finding an adequate language to express the fact that we really should begin with indiscernible entities in the first place . "

[Krause , 2011]

3.3: Attempt to develop an adequate language of non-individuals

During the last 50 years or so there is albeit slow but steady developments in the line inspired by Schrodinger by different logicians as well as mathematicians, for example, like Heintz Post, Yuri Manin[7] and others acknowledging the semantic inadequacy of standard Set theory . What is being tried is basically to develop different versions of metaphysics of *non-individual* which is not subscriptive to particle ontology or discernibility right from the beginning; particle concept itself along with the possibility of creating *tag* or *label* for it was considered to be ontologically surplus.

Schrodinger's original spirit of dispensing altogether with the notions of *identity* and *sameness* is captured by hosting non-individuals in the ontology of the theory at the cost of assuming **background theory**(***meta***language in which we can speak about our object language and describe the Semantic concepts) as a *quasi-Set Theory instead of ordinary set theory underlying Hilbert space formulation* .

In a quasi-Set theory א, the property " being identical with a" , for a certain term **a** , which we can write as $\mathbf{P_a(x)} =_D \mathbf{x=a}$, *can't be considered* among the properties of the object **a** . For the elements of א , non-individuality is taken into account by making room for entities for which it doesn't make sense to assert that they are identical to themselves or different from each other in a class . So classical theory of Identity(CTI) is not allowed to be applicable for them .CTI Characterizes the objects as individual in a sense that they can always be distinguished from each other either for having a certain property or by existence of a set to which it belongs to , but not in others . In other words, their membership function is clearly bi-valent .

As Krause put it –"non-individuals, taken as indistinguishable in the object theory cannot be distinguished even in the background theory for they lack the concept of identity …"(Krause 2009)

4. Conclusion

We can conclude that a question can be figured out to have still survived receiving diverse treatments from both the physicists and philosophers during the last 80 years or so – " how is it possible to discern quantum mechanical entities ? "

We have seen this question to have taken shape gradually during the entire history of developments of quantum mechanical historiography , leaving an impression that this is a convoluted issue which belongs to the fuzzy border of physics ,philosophy as well as formal logic , and calls for a clear standpoint regarding the network of belief in *individuation* and *identity* developed by us in a slowly varying universe , and this 'belief' hierarchy (based on these notions) infiltrates or spills over non-trivially in our formal artifacts aimed towards understanding reality . But the punch line of the whole issue is that, being human we can't stop committing to this *hierarchy of belief* in course of the way we become *Self* by acquisition of language; that is to say, we can't stop believing in spatio-temporal modes of description involving unambiguous notion of individuation and identity, while quantum mechanics taught us a formidable trouble in maintaining that.

So we can legitimately ask at this stage, that to what extent can we believe in our world of everyday experience – the conventional reality, or, the mathematics based on it? Stated more precisely, how to reconcile the *failure* of spatio-temporal modes of description in quantum mechanics with *our* familiar spatio-temporal modes of *individuation* is the next great problem physics has to make sense.

Notes and References -

[1] Leibniz famously pointed out that if certain objects are not identical, there must be some quality (a given property) that distinguishes them . In section 9 of his *Discourse on Metaphysics*, he notes that, "it is not true that two substances can resemble each other completely and differ only in number [*solo numero*]".

Stated otherwise, no two objects can share all the intrinsic qualitative properties.

But standard quantum mechanics seems to advocate an ontology *admitting truly indiscernible, but distinct entities..!*

In fact, 'truly indiscernible, but distinct' sounds like a perfect oxymoron and standard logic can't certainly make room of this situation!

[2] *John Stachel , Structural Realism and contextual Individuality '2005*]

[3] Debajyoti Gangopadhyay , *Logical aspects of creating discernibility in Bohmean mechanics* , forthcoming

[4] Margenau, H. *The exclusion principle and its philosophical importance, Philosophy* of Science. 11 (1944)…; a recent discussion of the issue can be found in *Massimi*, M. *Exclusion principle and the Identity of Indiscernibles : a response to Margenau's argument ,* BJPS. 52, 2001*]*

[5] Philosophy of *AS IF* is one of the most interesting versions of *fictionalism* which can be traced to the German philosopher Hans Vaihinger(1852 -1933) during the early decades of the last century . His influential book *Die Philosophie des Als ob,*based on his 1877 dissertation, was published in 1911 ..

[6] Redhead and Teller, *Particles, particle labels and quanta: the toll of unacknowledged metaphysics* , Foundation of Physics , Vol 21 ,1991 , pp. 43-62).

[7] Manin was very explicit about the need of a *new* language

" I would like to point out that it is rather an extrapolation of common sense physics , whether we can distinguish things , count them , put them in order, etc.. New quantum physics has shown us models of entities with quite different behavior. Even sets of photons in a looking-glass box, or of electrons in a nickel piece are much less Cantorean than the sets of grains of sands. The twenty century return to middle age scholastics taught us a lot about formalisms. Probably it is time to look outside again. Meaning is what really matters."

Manin , Yu. I., 1976, " *Problems of present day mathematics:* I (Foundations)" in Browder, F. E.(ed.) Proceedings of Symposia in pure Mathematics 28 American mathematical society

Background Readings:

About the metaphysical notion of Individuation and Identity and how do they figure in the language of Physics as a whole, a *historical survey* -

1. *A Peek Behind the Veil of Maya* : Einstein , Schopenhauer and the historical background of the conception of Space as a ground of individuation of

Physical systems , Don Howard in Cosmos of Science , ed. John Earman and John D. Norton , University of Pittsburg Press

For more technical treatments

1. *Identity in Physics , A Historical , Philosophical and formal analysis ,* Steven French and Decio Krause (Clarendon Press , Oxford , 2006)

2. *Post H. , Individuality and Physics , The Listner ,* Vol. 70, 1963

3. Weyl H., Philosophy of Mathematics and Natural Science , Princeton University Press , 1949

4. Interpreting Bodies : Classical and Quantum objects in Modern Physics , Elina Castellani ,(ed.) , Princeton University Press , 1998

5. M. L. Dalla Chiara, R. Giuntini, M. Redei, ``The history of quantum logic'', in D. Gabbay, J. Woods (eds.), *Handbook of the History of Logic*, vol. VIII, Kluwer, Dordrecht, 2007

6. M.L. Dalla Chiara, R. Giuntini, R. Greechie, *Reasoning in Quantum Theory*, Kluwer, 2004. An "unofficial" copy of the book may be found at: http://www2.latech.edu/~greechie/2004%20Reasoning%20in%20Quantum%20Theory%20Sharp%20and%20Unsharp%20Quantum%20Logics.pdf

7. S. French: **Identity** and **individuality** in **quantum** theory (Stanford Encycl. philosophy), ii) Ian Duck and E C G Sudarshan, Toward an understanding of the Spin-Statistics Theorem, American Jl. Physics, v66, p284 (1998).

8. [Review Article] *Interpretation and Identity in Quantum Theory* , Jeremy Butterfield , 2009

9. Schrodinger E. , Science and Humanism , Cambridge University Press , 1952 (this book contains Schrodinger's final version of interpretation of quantum mechanics where he was ready to dispense with the notion of *identity* of the quantum mechanical entities , thus ignoring the thrat to violate Leibniz's *Principle of Identity of Indiscernibles* . This standpoint provides the basis of non-standard logic developed by the Brazilian group of logicians ever since the 90s of the last century.)

For different technical levels of discussion about the departure of photon as well as any quantum mechanical entities in general from the accepted particle ontology (on the logico philosophical aspects of the problem)

1. *Identity and Individuality in classical and quantum physics*, Steven French, Australasian journal of philosophy, vol 67 (1989) , pp. 432-46

2. *Quantum Physics and the Identity of indiscenibles* , Steven French and M . Readhead , British journal for the philosophy of Science ,vol 39(1988),pp. 233-46 ,[Mathematical reviews (MathSciNet): MR89k:81009]

3. *Can mathematics explain Physical phenomena,* Steven French and Otavio Bueno , British journal for the philosophy of Science ,vol 63(2012),pp. 85-113

4. *Scientific Theories, Models, and Semantic Approaches* , Otavio Bueno and Decio Krause , Principia11, 2007 ,

5. S. Saunders: Indistinguishability (Oxford handbook of philosophy of physics, available in internet), Wikipedia: Identical particles, A. Einstein: Physics and Reality (Jl. Franklin Inst. 1936).

6. Quantum Logic , John Stachel in Jessica Pfeifer and Sahotra Sarkar, eds, *The Philosophy of Science: An Encyclopedia*, vol. 2, (Routledge Press, 2006), pp. 633---644. (see attachment)

7. Towards a Bell-Kochen –Specker theorem of Identity, R Srikanth and D Gangopadhyay , 2012 arXiv:1201.5080 [quant-ph]

8. State-independent contextuality with identical particles ,Ad´an Cabello and Marcelo Terra Cunha [arXiv:1212.5501v3 [quant-ph] 27 Feb 2013]

9. *Discerning "indistinguishable"quantum systems,* Adam Caulton

10. Redhead, M., and P. Teller, Particle labels and the theory of indistinguishable particles in quantum mechanics, British Journal of Philosophy of Science, Vol. 43b (1992), pp. 201-18

11. Dalla Chiara, M. L., and Toraldo di Francia, Individuals, kinds and names in Physics, pp 261-83 in *Bridging the Gap: Philosophy, Mathematics and Physics,* edited by G. Corsi *et al* ., Kluwer , Dordrecht , 1993

12. Mittlestaed, P., Constituting , naming and identity in quantum logic , pp. 215-34 , in Recent Developments in Quantum Logic , edited by P. Mittlestaed AND e. w. Stachow , Bibliographisches Institute: Mannheim , 1985