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Abstract. Jean Nicod (1893–1924) is a French philosopher and logician who worked with Russell during the First World War. His PhD, with a preface from Russell, was published under the title *La géométrie dans le monde sensible* in 1924, the year of his untimely death. The book did not have the impact he deserved. In this paper, I discuss the methodological aspect of Nicod's approach. My aim is twofold. I would first like to show that Nicod's definition of various notions of equivalence between theories anticipates, in many respects, the (syntactic and semantic) model-theoretic notion of interpretation of a theory into another. I would secondly like to present the philosophical agenda that led Nicod to elaborate his logical framework: the defense of rationalism against Bergson's attacks.

§1. Introduction. The philosopher and logician Jean Nicod is a forgotten figure of the French intellectual life of the beginning of the Twentieth Century. His early death (Nicod died on February 16, 1924, at the age of 30, from tuberculosis, in Geneva) partly explains the neglect of his work. Nicod was awarded the *agrégation de philosophie* in 1914, and went to Cambridge to work under the supervision of Russell during the First World War. It is at this time he published his most famous paper, a presentation of an axiomatization of the propositional calculus which contains only one connector and one axiom [14].[1] After the war, Nicod came back to France, wrote several articles and completed his PhD, which, at the time in France, had to contain two parts, the *thèse principale*, which Nicod devoted to the geometry and the sensible world, and the *thèse complémentaire*, which dealt with the problem of induction. These two works were first published in French (1924), then in English (1930).

In this paper, I will focus on the *thèse principale* [17]. Nicod's work is divided in three parts: the first (32 pages), entitled "Geometrical order," is about logic and methodology; the second (36 pages), entitled "Sensible terms and relations," aims at describing the sensible world, namely, at identifying the most basic sensible terms and relations, and at setting out the principles that govern these entities; the third part, the most developed one (84 pages), entitled "Some sensible geometries," shows how, starting from the axiomatic of the "sensible world" given in Part II, it is possible to extract various "formal geometries," in the sense in which it was set up in the methodological Part I. In what follows, I will show that this organization is grounded on an original, precise and subtle view of what theories are, and of how one can define

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[1] Russell will use the system in the second edition of *Principia Mathematica* [9, pp. 434–435].

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a notion (in fact, a family of notions) of equivalence between theories. Nicod explains his view in Part I; it is then this Part that will be the primary focus of my attention here.

To give a first overview of what Nicod is trying to do, let's start with the idea, taken for granted at the beginning of the XXth century, that an axiomatic theory can be applied to various fields—in Nicod's language, that a formal geometry can be satisfied by various "systems of meaning." In this then standard approach, the fixed starting point is the axiomatic system, and what vary are the models that instantiate it. But is it possible to reverse the process—to start with a fixed model (in this case, the sensible world), and to consider all the axiomatic systems that could describe it? This question does not immediately make sense: how does one define the collection of all the theories that describe a given model? And, even if one could make sense of it, what is the point of reversing the process? There are many known advantages in applying a given set of axioms to different fields, ranging from theoretical unification to economy of thought. It is difficult to see what would be gained by multiplying the forms of description of one and the same domain (even if the domain was the sensible world).

In what follows, I will first explain how Nicod manages to give a precise meaning to this inversion of the standard process by elaborating a relation of equivalence between theories, which prefigures the model-theoretic notion of interpretation (and mutual interpretation) between theories. Secondly, I will show that Nicod's motivation comes from his wish to oppose Bergson's anti-intellectualism, especially his claim that the introduction of spatial and geometrical language over-intellectualizes, and then betrays, the nature of sensible experience. Nicod wants to show that, even if, as Bergson claims, the basic elements of the sensible experience are not geometrical, some features of this experience can only be detected through a geometrization. Far from betraying it, spatial concepts make visible certain aspects of sensible experience which, otherwise, would go unnoticed.

Sections 2–5 follow the plan of [17, Part I]. In Section 2, I explain the distinction between theory, system of meaning and domain. In Section 3, I present the notion of equivalence between theories, and in Section 4, I explain how theoretical equivalence gives rise to a relation between systems of meaning called conjugation. In Section 5, I show that the theories that are equivalent to each other in Nicod's sense, and therefore also their models, can differ greatly from each other. Sections 6 and 7 give the philosophical motivation of Nicod's construction. Section 6 argues that there is wide agreement between Nicod and Bergson regarding the description of the sensible world. Section 7 explains how Nicod's development on theoretical equivalence is instrumental in his rejection of Bergson's way of articulating geometry and sensible experience.

§2. Theories, systems of meaning and domains. In chapter 49 of [20], Russell explained that the axiomatic systems of the various geometrical spaces available in the literature at the time can be regarded as purely logical definitions, once the different non-logical constants (undefinables) are replaced in the postulates by variables of the appropriate types. In his *thèse*, Nicod endorsed Russell's point of view.

In [17, Part I, chap. 1], Nicod begins by emphasizing the independence of a geometry from any kind of intuition. Basing his development on the axiomatic system T_e of the Euclidean space with three non-logical constants, the set of "points" (noted hereafter p_e), the three-term relation "collinearity" ($Coll_e$), and the binary relations between pairs of points "congruent" (C_e), he explains that all the theorems of Euclidean geometry can be proved without resorting to any figure, and even without assigning any meaning to the three non-logical constants. From this diagnosis, common to many mathematicians at the time (Pasch and Hilbert are the most famous examples), Nicod goes further to recover the full strength of Russell's analysis. Speaking of the T_e-primitive terms, he explains [17, p. 6]:

> Instead of granting these terms definite, but unknown, meanings, we may regard them as variables, simple instruments enabling to express the following universal truth. Let a class π, a relation R having as terms three members of π, and a relation S having as terms two pairs of members of π, satisfy the axioms and postulates [...]. Then under theses conditions, π, R, S, also satisfy the theorems.

Thus, instead of saying that a certain geometrical proposition Φ_e (containing non-logical constants) is a theorem of T_e, Nicod (following Russell) makes the following moves:

1. The relations $Coll_e$ and C_e are replaced by the variables X_1 and X_2 (of the appropriate types) in the T_e-postulates and in Φ_e at each of their occurrences.
2. The set π, used in T_e to fix the first-order quantificational domain, is replaced by a class variable at each of its occurrences in the T_e-postulates and in Φ_e. Let's label the resulting propositional functions obtained from (1) and (2) T_e^L and Φ_e^L.
3. Instead of considering Φ_e as an isolated proposition, and asking whether or not Φ_e is a consequence of T_e, Nicod considers the conditional propositional function $T_e^L \to \Phi_e^L$, and asks whether or not its universal closure is a logical truth.

This kind of systematic rewriting of the axiomatic system T_e can be generalized. To refer to an axiom system T with domain p and relational constants relational constants $R_1, \ldots, R_n$, Nicod usually writes $T(p, R_1, \ldots, R_n)$.[2] Nicod proposes here to construct T^L, called in the literature the "ramsified" version of T, in which $p, R_1, \ldots, R_n$ have been replaced in all the axioms of T by variables of the appropriate types $x, X_1, \ldots, X_n$. In this framework, instead of saying that a certain geometrical (non-logical) proposition Φ is a logical consequence of the T-axioms, one says that the purely higher-order logical proposition (Φ^L being the "ramsified" version of Φ)

$$\forall x, \forall X_1, \ldots, \forall X_n (T^L \to \Phi^L)$$

is a logical truth, that is, is derivable from the logical principles contained in the *Principia Mathematica*. The idea that a formal axiomatic theory is nothing else than the purely logical propositional function one obtains once the initial device has been "ramsified" was then widely shared. As we saw, it was endorsed by Russell. But it was also defended by Frege, and it has also been vindicated by Carnap, Gödel and Tarski in certain of their works dating from the twenties and early thirties. All these authors

2 See for instance [17, p. 74]: "Consider any geometry, for example that in which the primitive terms are *points* and the sole primitive relation is *congruence* .... Let $G(p, C)$ be the system's set of axioms, expressed in terms of a class p and a relation C."

treated the axioms as n-ary propositional functions, and represented the mathematical primitives of a theory by free variables, in a higher-order logical setting.[3]

In this sort of framework, the formal theory T^L is purely logical, and it does not contain any non-logical constant. But semantic notions like satisfaction or truth in a model can yet be introduced, in a substitutional way so to speak. That is, to take again Nicod's example, the interpreted axiomatic system $T_e(p_e, Coll_e, C_e)$, or more precisely the series of (non-logical) constants $\langle p_e, Coll_e, C_e \rangle$, provides us with an ersatz of a T_e^L-model: $\langle p_e, Coll_e, C_e \rangle$ can be said to satisfy the purely logical theory T_e^L. Of course, there are other triplets which instantiate T_e^L, as for instance the "arithmetical triplet" that Nicod presents in [17, p. 7].[4] To the purely logical theory T^L, Nicod gives the name of formal theory, and he calls the sequence of constants[5] $\langle p, R_1, \dots, R_n \rangle$ that instantiate it a *système de sens*, a system of meaning. At this stage, Nicod draws an interesting analogy between logic and algebra. The relation between a formal theory and their systems of meaning is compared to the relation between an algebraic equation and their solutions. The formal theory T^L, like an equation, sets formal conditions on the variables (the unknowns); and the various instantiating systems of sense correspond to distinct solutions to the equation [17, p. 32]:[6]

> The set of axioms which begins our book thus becomes the datum for an entirely new problem, in which the geometric expressions [the variables] form the unknowns: *are there meanings which, if ascribed to the geometric expressions appearing in these axioms, transform them into truths*? Each system of meanings which resolves this problem is an illustration, an example, an interpretation, or, better still, a *solution* of the set of axioms concerned.

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3 On this, see [24]. For instance, speaking of Carnap (p. 455): "The relevant convention, in all the works considered, is that of treating the primitive mathematical vocabulary of a theory in terms of variable expressions. Axiomatic theories are formalized as sentential functions, i.e., open formulas in the modern sense. The primitive terms of a theory are thus represented as (higher-order) variables and not, as is standard practice today, in terms of non-logical constants. Consequently, semantic notions like satisfaction or truth in a model are defined for pure logical languages, without non-logical terminology. In cases where genuinely mathematical or "applied languages" are considered,... the semantic evaluation of sentences is done indirectly, via the method of "variabilization," i.e., the translation into a pure language where free variables (of the right type) are substituted for the non-logical constants."

4 This is the usual Cartesian model, where the domain of quantification is $\mathbb{R}^3$ and where collinearity and congruence are defined in terms of relations between coordinates.

5 These constants are non-logical in the sense that they do not belong to the list of constants occurring in *Principia* ramified type theory. But they could nevertheless be defined in the *Principia* type theory. For instance, real numbers can occur as constants in a system of meaning, even though Nicod, following Russell (see for instance [16]), would consider real numbers as logical entities.

6 See also [17, p. 7]: "Let us try to discover, or at least conceive, one or several systems of meanings satisfying our [purely logical] axioms: we shall call such a system of meanings *a solution of this group of axioms*." As we will see, Nicod takes the example of the cubic equation $x^3 + 3x^2 + 4x + 3 = 0$, whose set of solutions $\langle a_0, a_1, a_2 \rangle$ (in $\mathbb{C}$) would correspond to the system of meaning of the formal conditions set in an abstract theory.

As we will see below, Nicod used, in a quite refined way, this algebraic picture.[7] But for now, I would just like to emphasize that this comparison strengthens the idea that there is a difference in level, as there is one in Model theory today, between the formal theory (the algebraic formula) on the one hand, and its interpretations (the numerical solutions) on the other. For Nicod, an interpreted theory (T) is merely an application of a purely logical form (T^L) to a certain "matter" (a system of meaning).

There is however another notion of theory, that of a "domain" (*domaine*), which is never completely and clearly explained in [17], but which it is important to recognize as such, and to distinguish from the idea of interpreted theory that we have just commented on. From what Nicod says about the two examples of domains he mentions, namely the domain of numbers and the sensible domain, it seems that a domain is merely an interpreted theory whose "ramsified" version does not allow for any variation. Let me explain.

In his book, Nicod doesn't say much about the numerical domain, except that this domain provides us with the usual arithmetical interpretations of geometries. Nicod is more explicit on the other domain, the sensible domain, to which Part II is dedicated. Like the arithmetical domain, the sensible domain provides solutions to formal geometrical theories—in physics, says Nicod, but also in daily life, the words "point," "line," and "surface" are used to talk about the parts of the sensible space that surrounds us and which organizes our perception.[8] Of course, these two domains are very different. Sense-data are given to us through the senses, which is not the case of numbers. Besides, the laws governing the relations between sensations are empirical,[9] while the arithmetical rules which govern the numerical world are a priori. The only thing the two domains have in common seems, thus, that they provide meanings to the formal theories Nicod considers in *La géométrie*.

But this fact raises a question, since domains are also theories. This is true of the arithmetical domain. Nicod's numerical interpretations of geometries rely on the properties of the real numbers, and therefore assumes the availability of a theory of the real numbers field. The assumption of the theoretical character of domains is even more explicit in the case of the sensible domain, since Nicod sketches the construction of a theory of the sensible world in Part II. He uses the symbol " $E(s, R_1, \ldots, R_n)$" (s being the set of sense-data, $R_1, \ldots, R_n$ being relations between sense-data) to speak of the sensible domain, that is, exactly the same notation he uses to designate the standard interpreted geometrical theories in Part I (*Ibid.*, p. 74):

> Let s the class of my sense terms; let $R_1, \ldots, R_n$ list the various relations I observe among them, and let $E(s, R_1, \ldots, R_n)$ be the set of laws which I shall be led to regard as inductively probable.

The two examples of domains Nicod gives are thus examples of interpreted theories.[10] But then, if it is the case, what is the difference between a domain and an interpreted theory? Couldn't we analyze the domain $E(s, R_1, \ldots, R_n)$ as the application

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7 One finds a similar comparison between axiomatization and algebra in the early works of Norbert Wiener (see [31]), who was then also a disciple of Russell.

8 See [17, pp. xvii–xx, 35].

9 See [17, p. 74].

10 With "logical" constants, for the arithmetical domain, with "non-logical" constants, for the sensible domain; see footnote 5 above.

of the formal theory $E^L = E(x, X_1, ..., X_n)$ to the system of meaning $\langle s, R_1, ..., R_n \rangle$, exactly as we have done for T_e before? Absolutely not: abstracting the form of a domain in order to instantiate it by another system of meaning is never considered by Nicod. It is of course possible to "ramsify" $E(s, R_1, ..., R_n)$ in order to create the formal E^L, as one can always do with an interpreted theory. And we will see below that this is in fact something that Nicod does in a few very specific circumstances. But what Nicod never does is to consider E^L as a fixed formal framework that can have various solutions. Everything happens as if, when a theory is regarded as a domain, the theory always kept its intended interpretation. Everything happens as if separating E^L from $\langle s, R_1, ..., R_n \rangle$ for applying it to another "matter" doesn't really make sense.

So, let me summarize what we have so far. There are two different notions of a theory in [17]:

1. The standard interpreted theory, like T_e, which is composed of two elements:
   (a) the form, which is expressed by the "ramsified" logical version T_e^L (in Model theory, this would correspond to the theory),
   (b) the matter, which is conveyed by the constants $\langle p_e, Coll_e, C_e \rangle$ of T_e (in Model theory, this would correspond to the model).
2. The domain, like the sensible domain E, in which one does not separate an abstract form E^L from a matter $\langle s, R_1, ..., R_n \rangle$—or more precisely, in which one does not apply E^L to systems of meaning that are different from the intended one.

To better understand Nicod's concept of domain, and the relationship between domain and theory, we must continue reading Part I, and go into the central problem that Nicod raises in chapter 2.

§3. Equivalence between theories: transformability, containment and inseparability. Let's take two distinct interpreted theories that axiomatize the very same structure; is it possible to define a general relation of equivalence between the "ramsified" version of the two theories? For instance, the Euclidean space can be axiomatized in a system with only one relation of congruence between two couples of points $T_c(p_e, C_e)$, or in a system (first developed by the Italian mathematician Mario Pieri) with a five-term relation of sphericity $T_s(p_e, S_e)$. According to Nicod, there is a sense in which one can say that $T_c(p_e, C_e)$ and $T_s(p_e, S_e)$ axiomatize the same space. But how do we define, in the most general terms, the equivalence between T_c^L and T_s^L? This is the central problem of [17, Part I], and chapters 2–4 are entirely dedicated to this issue.[11] Nicod looks at it in a step-by-step fashion, each time combining general developments and discussions of examples. In chapter 2, he presents his notion of equivalence between theories. In chapter 3, he explains how this notion can be used to construct new interpretations of theories. In chapter 4, he extends the scope of his previous construction by referring to a specific example from Whitehead.

The first notion Nicod introduces is that of transformability of a theory into another theory [17, p. 13]:

[11] Chapter 2 begins as follows (p. 9):"We could end this introduction here without taking note of the fact that geometry can be put into more than one form, for the reason that all these forms are equivalent. However, we shall adopt a more general viewpoint by taking their plurality into account and by seeking the precise nature of their equivalence."

> Suppose... we are given two propositional functions $F_1(x_1, \ldots, \alpha_1, \ldots, R_1, \ldots)$, $F_2(x_2, \ldots, \alpha_2, \ldots, R_2, \ldots)$ in which the indeterminates can be individuals x_1, x_2, ..., classes α_1, α_2,..., and relations R_1, R_2,... The number of terms appearing in the two functions and their logical types need not necessarily be the same...
> We establish a correspondence between each of the indeterminates of F_2 and a certain *logical* function of the indeterminates F_1, that is, an expression containing only logical terms apart from these indeterminates, by setting
>
> $$x_2' = f(x_1, \ldots, \alpha_1, \ldots, R_1, \ldots), \text{ etc.}$$
> $$\alpha_2' = g(x_1, \ldots, \alpha_1, \ldots, R_1, \ldots), \text{ etc.}$$
> $$R_2' = h(x_1, \ldots, \alpha_1, \ldots, R_1, \ldots), \text{ etc.}$$
>
> Now replace in F_2 all the expressions *f*, *g*, *h* by the simple terms x_2',..., α_2',..., R_2',... If we find that we have
>
> $$F_1(x_1, \ldots, \alpha_1, \ldots, R_1, \ldots) \leftrightarrow F_2(x_2', \ldots, \alpha_2', \ldots, R_2', \ldots)$$
>
> we shall say that F_1 is *transformable* into F_2.

Note first that F_1 and F_2 are propositional functions (Nicod explains that any axiomatic system can be replaced by a unique function, which is the conjunction of all its postulates), that is, they correspond to formal theories, in which all the constants are replaced by variables. Thus, to come back to our example, Nicod's reasoning applies to $T_c^L(\pi, X_c)$ and $T_s^L(\pi, X_s)$, and not to T_c and T_s. Now, Nicod's key idea is to specify some logical functions that translate each F_2-indeterminate into F_1-indeterminates, so that

$$F_1(x_1, \ldots, \alpha_1, \ldots, R_1, \ldots) \leftrightarrow F_2(x_2', \ldots, \alpha_2', \ldots, R_2', \ldots),$$

is a logical truth. Here, $x_2', \ldots, \alpha_2', \ldots, R_2', \ldots$ are the values of the translation functions, that "transform" F_1- variables into F_2-variables. Note that these translation functions are logical, in the sense that they do not contain any non-logical constants, and do not contain any free variables other than those of F_1. If one can find such translation functions, one says that F_1 is transformable into F_2: this means that "it is *possible* to 'translate' the simple expressions of a system into a compound expressions of another in such a way that the axioms of the first system (and consequently all its propositions) become translated into propositions of the second system" (*Ibid.*, p. 13).

To help us understanding what is going on, Nicod gives first an algebraic analogy. Let's take the propositional function $x^3 + 3x^2 + 4x + 3 = 0$ as F_1, the propositional function $y^3 + y + 1 = 0$ as F_2, and set the translation function as $y' = x + 1$;[12] then, one can easily show that $x^3 + 3x^2 + 4x + 3 = 0 \leftrightarrow y'^3 + y' + 1 = 0$, and therefore that F_1 is transformable into F_2. As a second example, Nicod explains how one can transform T_c^L into T_s^L by defining sphericity as the "relation holding among five points such that there is a sixth point forming five congruent pairs with them" (*Ibid.*, p. 13). In this

[12] For the algebraic analogy to work, it is necessary to fix the interpretation of addition and product once and for all, and to integrate the symbols of algebraic operations in the logical apparatus. Thus, for the sake of the analogy, F_1, F_2 and the translation function can be regarded as purely logical expressions.

phrase, the two non-logical constants "points" and "congruence" should be replaced by the T_c^L-indeterminates π and X_c in order to form the purely logical function

$$X'_s = h(\pi, X_c).$$

Thanks to this translation, one can show that the postulates of both theories are preserved, that is:

$$T_c^L(\pi, X_c) \leftrightarrow T_s^L(\pi, h(\pi, X_c)).$$

According to Nicod, this result gives a definite meaning to our vague intuition that T_c and T_s (more precisely, T_c^L and T_s^L) axiomatize the "same" object, that they are "equivalent."

In fact, Nicod is even more precise, since he introduces two other notions of inter-theoretical equivalence: the relation of containment, one the one hand, and the relation of inseparability, on the other. Let me first focus on containment. For F_1 to be transformable into F_2, $F_1 \to F_2$ is not sufficient; one must also have $F_2 \to F_1$. This means that, translations must not only preserve F_1-theoremhood, but also F_1 -non-theoremhood. Containment is a weaker relation than transformability: saying that F_1 contains F_2 amounts at saying that condition $F_1 \to F_2$ is satisfied, without requiring $F_2 \to F_1$.[13] Nicod's last notion of equivalence, inseparability, is defined in terms of containment. Two theories F_1 and F_2 are inseparable when each contains the other [17, p. 13]:

> Finally, when the functions F_1 and F_2 contain each other, we shall say that they are *inseparables*. This is the definition of the general formal relationship between two systems of equivalent principles, whether or not expressed in terms of the same primitive expressions: this relationship *constitutes* their equivalence.

In order for two theories F_1 and F_2 to be inseparable, it is thus required that there be a translation of F_2-indeterminates into F_1-indeterminates which preserves F_1-theoremhood, but also that there be a translation of F_1-indeterminates into F_2-indeterminates which preserves F_2-theoremhood.

To summarize, Nicod defines three related notions of equivalence between two formal theories, all based on the existence of logical translation functions between the indeterminates of the two theories that preserve a certain notion of theoremhood. This construction anticipates in many aspects the syntactic characterization of interpretation between theories one finds in Model theory today. Recall that a (first-order) $\mathcal{L}$-theory T is syntactically interpretable in a (first-order) $\mathcal{L}^*$-theory T^* if and only if the primitives of the interpreted theory T can be translated into formulas of the interpreting theory T^* so that the translation of every theorem of T is a

[13] The way Nicod defines containment is a bit different [17, p. 14]: "If we now denote by F_1', F_1'',... the *logical consequences* of F_1, that is the theorems which result from the axioms F_1, and which naturally are functions of the same indeterminates, it may happen that it is no longer the function F_1 itself which is transformable into F_2, but one of the function F_1', F_1'',... which derive from it (a single letter being used to represent the logical product of several theorems, as before that of several axioms). To indicate this more general formal relationship of which the first is a special case, we shall say that the function F_1 *contains* the function F_2."

theorem of T^*.[14] Now, there are differences between this definition and Nicod's view. Firstly, the modern characterization applies to theories in which non-logical constants occur, whereas Nicod's characterization applies to the "ramsified" versions of those theories. Secondly, the modern notion is restricted to first-order theory which is not the case in Nicod. And finally, in Nicod, the translation functions are not regarded as metalinguistic relations between the terms of two languages (that can be subjected to different sorts of constraints[15]). For Nicod, translation functions are logical propositional functions. But even if the differences between Nicod's and the Model-theoretic views should not be undermined, one can only be struck by the similarity between the two approaches. In both case, the same insight according to which two axiomatic systems T and T^* are notational variants from each other is captured by devising translation functions (going from the primitive terms of the one to the primitive terms of the other) that preserve theoremhood. The example of the relation between T_c and T_s Nicod uses to illustrate his reasoning is perfectly suited to illustrate the modern Model-theoretic notion. One can even match the various notions of equivalence that Nicod elaborates to the various species of interpretability that Model-theorists distinguish today: the distinction between containment and transformability corresponds to the one between interpretability and faithful interpretability, while the distinction between containment and inseparability corresponds to the one between interpretability and mutual interpretability.[16]

§4. Material consequence of equivalence: conjugation and logical construction. In chapter 3, Nicod shows that the various notions of inter-theoretical equivalence he defines in chapter 2 can be used to derive, from an interpretation of a given abstract theory T^{*L}, an interpretation of another abstract theory T^L equivalent to T^{*L}. To understand the reasoning, it is helpful to come back to the analogy between axiomatic theories and equations. Nicod claims that just as $x^3+3x^2+4x+3=0$ can be transformed into $y^3+y+1=0$ by setting $y=x+1$, the theory T_c^L can be transformed into the theory T_s^L by setting $X_2' = h(\pi, X_1)$ (see above). Now, a change of variable is useful in algebra to find solutions to complicated equations: by transforming $x^3+3x^2+4x+3=0$ into $y^3+y+1=0$, one can resolve it. Even if the two equations have no common solution, a solution to the one provides a solution to the other. Nicod claims that the same holds for formal theories: from one "solution" (system of meaning) of a theory T^{*L}, one can always find one "solution" (system of meaning) of a theory T^L, equivalent to T^{*L}.[17]

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14 On the syntactic definition of interpretation between theories, see [5, 11].

15 For instance, it seems reasonable to require that the translation function be recursive, but there are possible alternatives in how to limit the use of metalinguistic resources in the definition of the translation functions, and these alternatives give rise to different notions of interpretation. For more detail, see [5, pp. 115–116].

16 See [5, pp. 115–117].

17 [17, p. 16]: "The two equations $x^3+3x^2+4x+3=0$ and $y^3+y+1=0$, which can be transformed onto one another by setting $y=x+1$, have no common solution, but nevertheless each solution of the one provides a solution of the other. Likewise, no interpretation can possibly satisfy both the axioms of congruence [...] and those of sphericity [...] at the same time, but each interpretation X_1, satisfying the axioms of congruence, supplies an interpretation Y_1' satisfying the axioms of sphericity."

Let me explain. Let $F_1(X_1)$ and $F_2(X_2)$ be two formal theories containing only one indeterminate (if the indeterminates were numerous, the reasoning would still be the same), such that F_2 is contained in F_1. Nicod first remarks that "in general, the expressions X_1 and X_2 do not admit a common meaning," and that this incompatibility is "manifest in the case in which X_1 and X_2 are of different logical types, where X_1 for example is congruence, a relation between two pairs of terms, and X_2 is sphericity, a relation among five terms" [17, p. 15]. Nicod then explains (*Ibid.*):[18]

> Because $F_1(X_1)$ contains $F_2(X_2)$, there is a logical function
>
> $$X_2' = f(X_1)$$
>
> such that $G(X_1) \leftrightarrow F_2(X_2')$ is a consequence of $F_1(X_1)$. Now suppose that a certain interpretation A_1 satisfies $F_1(X_1)$, in other words, we have $F_1(A_1)$. Construct the interpretation
>
> $$A_2 = f(A_1).$$
>
> This second interpretation, formulated logically from the first satisfies F_2 in view of the fact that f is a logical function. For $F_2(A_2)$ arises from $F_1(A_1)$, and $F_1(A_1)$ is true.

A_1 is an F_1-interpretation, *i.e.*, a system of meaning that satisfies F_1. Nicod explains that, when another theory F_2 is contained in F_1, one can use the function f which translates the F_2-indeterminate in terms of the F_1-indeterminate to construct a system of meaning satisfying F_2: $A_2 = f(A_1)$ is an interpretation of F_2. Indeed, F_2 is contained in F_1 means that a certain consequence G of F_1 [19] is such that $\forall X_1(G(X_1) \leftrightarrow F_2(f(X_1)))$ is true; now, $F_1(A_1)$ is true, and thus $G(A_1)$ is true; from this, it follows that $F_2(f(A_1))$ is true—in other words, that $A_2 = f(A_1)$ is an interpretation of the formal theory F_2.

To give some flesh to the reasoning, let's take some examples. The equation $x^3+3x^2+4x+3=0$ is contained in the equation $y^3+y+1=0$, by setting $y=x+1$. Now, if $\langle a_0,a_1,a_2\rangle$ is the set of solutions of $x^3+3x^2+4x+3=0$, then the set $\langle a_0+1,a_1+1,a_2+1\rangle$ is the solution of the equation $y^3+y+1=0$. This sounds obvious, but note that, in such a reasoning, one goes from a relation between equations (formal theories) to a relation between solutions (systems of meaning). And the same transfer from one level to another applies in the case of relations between theories. T_s^L is contained in T_c^L, which means that X_s (coding the relation of sphericity) can be translated by a purely logical function $h(\pi, X_c)$ in T_c^L. Now from any interpretation of congruence A_c (from the arithmetical relation of congruence, for instance), one can generate an interpretation $A_s = h(\mathbb{R}^3, A_c)$ of the T_s^L-indeterminate X_s, by 'mechanically' applying the translation rule to the (arithmetical) relation A_c.[20]

18 I have slightly altered the text to remain uniform in the notation.

19 This is how I understand the symbol $G(X_1)$ that Nicod does not define. The passage is a bit difficult, since Nicod's clause doesn't seem to cohere with the explanation given in footnote 13 of what it means for F_1 to contain F_2. It seems that Nicod should have said that $G(X_1)$ is a consequence of $F_1(X_1)$, not that $G(X_1) \leftrightarrow F_2(X_2')$ is a consequence of $F_1(X_1)$.

20 If A_1 is the relations between two couples (a,b) and (c,d), where a,b,c and d are triplets of real numbers (I write $a=(x_a,y_a,z_a)$) such that $D_{a-b} = D_{c-d}$, where $D_{a-b} = (x_b-x_a)^2+(y_b-y_a)^2+(z_b-z_a)^2$, then one can interpret the relation variable X_s in T_s as the relation that

Here again, what is important is that Nicod derives from a relation between formal theories a "semantical" relation between certain interpretations of these theories. As he says, the equivalence relation between two theories T and T^* gives us the means to construct, in a uniform way, from a "solution" of T, a "conjugate" solution of T^* (*Ibid.*, p. 16):

> From a 'solution' of a system of axioms, we can thus logically construct the 'solution' of all systems inseparable from it. And *logically*, that is to say without introducing any subject-matter. All these conjugate values have the same elements of meaning.

The semantic nature of Nicod's approach obviously brings us to compare it to the semantic definition of interpretation one finds in contemporary Model theory. Remember that a (first-order) theory T is semantically interpretable in a (first-order) theory T^* iff, whatever the model $\mathcal{M}$ of T considered, one can define, in a uniform way, from every model $\mathcal{M}^*$, a quotient structure isomorphic to it.[21] One key resemblance between the two reasonings is that the rules of translation are used to create, in a uniform way, from any model of the interpreting theory, a model of the other. That is, the idea in both cases is no longer to relate T-sentences to T^*-sentences so as to preserve theoremhood, but to relate T-structures to T^*-structures so as to preserve satisfaction. There is however an important difference (not taking into account the restriction of the case of first-order theories in the modern approach[22]) between the two views: Nicod does not seek to provide a new semantical definition, alternative to the ones presented above, of the notion of equivalence between theories. His goal is just to emphasize that equivalence between formal theories T and T^* gives us a uniform method to construct a model of the interpreted theory T as soon as a model of T^* is given. There is no doubt however that Nicod distinguishes sharply the (formal or syntaxic) level of the equivalence between formal theories from the (applied or semantic) level of conjugation between systems of meaning.

Nicod's account is surely inspired by Russell's method of logical constructions. Russell gave several examples of his method, but he never really characterized it in a general way. The following passage is the closest we can find to such a general definition [23, p. 326]:

> When some set of supposed entities has neat logical properties, it turns out, in a great many instances, that the supposed entities can be replaced by purely logical constructions composed of entities which have not such neat properties. In that case, in interpreting a body of propositions hitherto believed to be about the supposed entities, we can substitute the logical structures without altering any detail of the body of propositions in question.

---

holds between five triplets of real numbers a, b, c, d, e such that there is a $p = (x, y, z)$ in $\mathbb{R}^3$, such that $D_{p-a} = D_{p-b} = D_{p-c} = D_{p-d} = D_{p-e}$.

21 See, for more on this, [5, 11]. On Nicod's view about quotient structure see the next section.

22 In the first-order case, the syntactic and semantic notions of interpretation are extensionally equivalent. See [5, pp. 114–119].

Russell starts with some supposed classes of entities $e_1, \ldots$ with some given "neat"[23] properties $P_1, \ldots$ and relations $R_1, \ldots$, that he proposes to replace by some "purely logical constructions" composed of other entities $e_1^*, \ldots$, with different properties $P_1^*, \ldots$ and relations $R_1^*, \ldots$. The "body of propositions" about the supposed entities is $T(e_1, \ldots, P_1, \ldots, R_1, \ldots)$, while the "body of propositions" specifying the principles governing the basic entities is $T^*(e_1^*, \ldots, P_1^*, \ldots, R_1^*, \ldots)$. For instance, T can be a theory which deals with physical bodies, and T^* a theory which deals with sense-data; Russell would then maintain that it is possible to replace the reference to the physical bodies and their properties in T by reference to logical constructions composed of sense-data and their properties. In Nicod's terms, Russell is merely describing the possibility of constructing from a given system of meaning of a certain theory T^*, another conjugate system of meaning for a theory T. This requires finding some logical translation functions to show that T^L is contained in T^{*L}. It demands, secondly, to apply these translation functions to a model of T^{*L} in order to generate, "mechanically" so to speak, a model of T^L which replaces the initial "neat" one. Russell's reasoning, however, is purely semantical: when he describes the method of construction, he considers only the relation between interpretations (the second part of the process). Nicod's originality is to give a general and abstract characterization of Russell's method, one that clearly separates the syntactic (or formal) dimension from the semantic (or applied) dimension that Russell confused in his presentation. It is likely that the analogy between algebraic equations and theories played a decisive role in this achievement. Indeed, one finds in [15] an explicit comparison between change of variables in algebra and Russell's logical construction (p. 83):

> The predictions of physics, in their current form, are similar to equations which would offer sensory facts as a function of non-sensory entities, such as space or matter. The philosophical physicist seeks to eliminate the latter from the statement of scientific predictions. He changes unknowns: from equations giving sensory facts as a function of non-sensory facts, he derives the expression of the latter as a function of the former.

Let me summarize Nicod's reasoning. Having distinguished purely logical formal theories from interpreted theories and systems of meaning, Nicod seeks to clarify the sense in which two interpreted theories can be said to have the same content. For instance, the theory of congruence T_c and the theory of sphericity T_s are two axiomatizations of the Euclidean space. But how can we define in a general way this "equivalence" between T_c and T_s? Nicod's definitions of transformability, containment and inseparability are answers to this question: two formal theories are equivalent if and only if translation rules from the one into the other that preserve theoremhood exist. From this first result, Nicod derives another one: when two formal theories are equivalent, then, from a model of the one, one can construct, in a purely logical way, a model of the other.

One may wonder, however, whether Nicod's answer is not too strong with regard to the question raised. Theories that are commonly regarded as mere notational variants

[23] Russell does not say what he means by "neat logical properties," but this is not important to us. On this, see [13] and section 7 below.

from each other, as T_s^L and T_c^L are, are equivalent in Nicod's sense; but is the reverse true? Should we consider that if two theories are equivalent, they axiomatize the same structure? The connection between Nicod's development and Russell's method of logical construction shows that the issue is of importance. According to Russell, the theory of the sensible world is contained in the theory of the physical bodies, and vice versa. But Russell would never have said that the two theories are about the same object. On the contrary, the whole point of Russell's method is to replace the "supposed" entities by the logical constructions, without assuming that there is an identity between the two things.[24] It seems then that the theories that are equivalent in Nicod's sense are much more different from each other, and much more varied, than the example of T_c and T_s suggest.

In Model theory, the fact that theories T and T^* are mutually interpretable is also sometimes taken as an indication that T and T^* relate to the "same" object or express the same "content" (in a relevant sense of the "same"). But, as Sean Walsh emphasized,[25] two arguments can be leveled against this understanding. First, it turns out that mutually interpretable theories can be very different from each other. For instance, complete first-order theory of the real numbers are interpretable in the Peano axioms,[26] and projective geometry and Euclidean geometry are mutually interpretable.[27] It is difficult to find any reasonable notion of "content" or "object" that can support the idea that the content of theory of reals is a part of the content of the theory of integers, and that projective space is basically the same thing as Euclidean space. Second, incompatible theories can be mutually interpretable. For instance, (first-order) Hyperbolic geometry (which contains a negation of the parallel axiom) and (first-order) Euclidean geometry are mutually interpretable (See [26] and below). In this case, how could one claim that the two theories have the same content? Following Walsh, we will call the first problem *Plethora* and the second *Coherence*. It seems that *Plethora* and *Coherence* apply to Nicod's notion of inseparability. Was Nicod aware that the class of theories equivalent to any given theory T is very large, contains theories very different from, and sometimes incompatible with, T? And if this is the case, which meaning did Nicod give to an equivalence result? In the following section, I will tackle these two issues.

§5. Domain and interpretation. Before embarking on this path, let us go back to the last chapter of [17, Part I], devoted to a discussion of Whitehead's geometry of volumes.[28] The connection with what precedes is that, in chapter 4, Nicod considers

[24] Quine perfectly understood this point when he said [19, pp. 258–259]: "This construction [of the ordered pair] is paradigmatic of what we are most typically up to when in a philosophical spirit we offer an "analysis" or "explication" of some hitherto inadequately formulated "idea" or expression. We do not claim synonymy. We do not claim to make clear and explicit what the users of the unclear expression had unconsciously in mind all along. We do not expose hidden meanings, as the words "analysis" and "explication" would suggest; we supply lacks. We fix on the particular functions of the unclear expression that make it worth troubling about, and then devise a substitute, clear and couched in terms to our liking that fills those functions."

[25] See [5, 29].

[26] See [29, p. 92].

[27] See [26].

[28] On the geometry of volumes, see [22, chap. 4] and [30].

equivalent theories that do not have the same quantificational domain, and therefore are very different one from another. Let me quote Nicod's introduction of the topic [17, p. 21]:

> The time has come for us to point out that all the geometric systems of which we have spoken up to now contain an expression in common. Although the primitive relation may vary, the primitive terms remain in all cases the same: these are the *points*. Could the point be an indispensable element?... It is untrue that [geometry] regards 'point' as a simple term. We can conceive systems which introduce point as a composite term, composed of terms more easily interpretable in nature.

Whitehead's theory T_v, as presented by Nicod, contains two indefinables, the primitive terms volume (V) and the primitive relation of congruence between two pairs of volumes (C_v).[29] Nicod first defines volume and volume-congruence in terms of point (P) and point-congruence (C_p). Following the lead of Whitehead, Nicod seeks to reverse the process, and defines the indefinables of T_c^L in terms of T_v^L [17, p. 24]:

> Let us [...] whenever possible insert the expressions just defined [V and C_v] into the theorems of the geometry of points: certain statements no longer explicitly contain points or relations between points, but only volumes and relations between volumes. Let us make a separate list of these statements in their new form and then take them on their own merits. In other words, let us stop describing volumes in terms of points, and instead let us take *volume* and the relations [...] of volumes as primitive expressions with entirely undefined meanings. Let us call this set of propositions, which can be divided into axioms and theorems, the *properties of volumes*. [...] Do the properties of volumes comprise the whole of geometry? Do these properties, which are formally included in the properties of points, formally include the latter?

Whitehead's solution is to define the points as sequences of nested volumes growing indefinitely smaller so as to converge toward a limit (Whitehead's "abstractive sets"). If the limit existed, then the limit would correspond to the point. But Whitehead does not assume the existence of the limit, and suggests instead to replace the point with abstractive classes. This proposal does not work, however. As many abstractive sets converge toward the same limit, one cannot identify any one of them to the point-limit. Whitehead then has to introduce an equivalence relation between those abstractive classes, and to define a point as an equivalence class between abstractive classes. Next, he introduces various relations between volumes and succeeds to define the relation of congruence between points in terms of relations between volumes. There are several technical niceties in Whitehead's approach (in particular, one genuine difficulty with the definition of abstractive class that is pointed out and corrected by Nicod).[30] I will

29 I won't deal here in detail with Whitehead's system. For a rich presentation of the issues at stake, see [28].

30 On this see [28, sec. 4].

set that aside here. From our perspective, the important thing is that we end up with the following result: T_c^L is contained in T_v^L, and as the reverse is also true, the two theories are inseparable. Thus [17, p. 28]:

> The geometries of point and volume are... *inseparable*. Each satisfactory interpretation of point and of the relations between points furnishes a satisfactory and more complex interpretation of volume and of the relations between volumes, and each interpretation of volume and of relations between volumes furnishes a satisfactory and more complex interpretation of point and relations between points. Hence, geometry does not in any way demand of nature volumes made of points rather than points made of volumes.

Once again, the parallelism between Nicod's development and Model theory is striking. Indeed, today, it is common to expand a restricted semantic notion of interpretability based on the concept of a definable structure (a structure $\mathcal{M}$ is said to interpretable in a structure $\mathcal{M}^*$ if it is isomorphic to a structure that is definable in $\mathcal{M}^*$) to a broader notion based on the concept of definable quotient structure ($\mathcal{M}$ is interpretable in a structure $\mathcal{M}^*$ in this enlarged sense if it is isomorphic to a quotient structure that is definable in $\mathcal{M}^*$).[31] Nicod follows the same path in his book. He first focuses on the easy example of the equivalence between T_s and T_c, before extending his reasoning to more difficult cases like T_v and T_c, where the first-order quantificational domain of one theory is a quotient of the domain of the other, and where the interpretation of the identity relation must be appropriately adjusted.

Nicod's notion of equivalence is then broad, as broad as the Model-theoretical notion of interpretation. *Plethora* and *Coherence* therefore apply within the framework developed in [17]. Was Nicod aware of this?

It seems so. The transition from the first easy example about T_s and T_c we presented in the preceding section to the difficult one we just present shows that Nicod knew how large the range of variation between equivalent theories could be. Above all, Nicod explicitly acknowledged that incompatible theories can be equivalent. This case is in fact very important in [17], since it has been put forward by Poincaré in *La Science et l'Hypothèse*. Using Nicod's terminology, Poincaré explains, in the famous passage about the Hyperbolic-Euclidean dictionary,[32] that Hyperbolic and Euclidean formal theories are inseparable and that one can then construct an interpretation from Hyperbolic geometry from any interpretation of Euclidean geometry—and vice versa. This shows that Euclidean and Hyperbolic geometries, two incompatible theories, are nevertheless equivalent. Nicod was then aware of *Plethora* and *Coherence*, and did not identify the content of equivalent theories.

In chapter 3, we find a discussion in which Nicod wonders to what extent two theories interpreted by conjugated models (systems of meaning)[33] are about the same thing [17, p. 16]:

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[31] See for instance [29, pp. 86–87].

[32] [18, p. 50]: "Let us now take Lobatschewsky's theorems and translate them by the aid of this dictionary, as we would translate a German text with the aid of a German-French dictionary. We shall then obtain the theorems of ordinary geometry."

[33] From now on, I will use interchangeably "model" and "system of meaning." The first expression, in addition to being shorter and more familiar, makes it easier to keep in mind

> We can... say that two inseparable systems of axioms are true of the same realities. But let us remember that this expression is still undefined, that what we call the same reality provides the raw material for a multitude of logically different entities, whose different laws nevertheless betoken the same order. Were this not the case, we would be confronted with a clash between the identity of the realities in which inseparable systems of axioms hold and the mutual incompatibility of their solutions.

Nicod speaks here about two inseparable theories whose (conjugate) models are constructed from the same basic structure. Thus, contrary to what we could think at first, it is not just two, but three theories that are considered in this passage: the two inseparable theories T_1^L and T_2^L, and another one T_D^L, which contains the first two, and whose system of meaning provides the basis for the construction of the two others. T_D^L comes then with a fixed interpretation, that we will call M_D. As T_1 is contained in T_D, one can, by applying the translation rules $F_{1\to D}$ to the basic model M_D, construct in a purely logical way a model $M_1 = F_{1\to D}(M_D)$ of T_1; and of course, we can do the same to construct a model $M_2 = F_{2\to D}(M_D)$ of T_2.[34] With this equipment, one can understand what Nicod means in this passage. M_1 and M_2 are two interpretations, and thus two "realities," described by two theories (perhaps very different from each other, and even incompatible). The consequences of *Plethora* and *Coherence* are then plainly acknowledged: the interpreted theories T_1 and T_2 are not about the same thing. But if T_1 and T_2 are not true of the same reality, they are true "of the same realities," in the sense that M_1 and M_2 are made of the same "raw material" coming from the basis M_D. Of course, M_D neither satisfies T_1^L, nor T_2^L; but the fact that M_D provides the elements of meaning, the "realities," M_1 and M_2 are made of, is taken into account in Nicod's view. Schema 1 summarizes the relations among the different items. The thin arrows from the theories T_D, T_1 and T_2 represent the relation of interpretation (or satisfaction). The thick arrows from M_D to M_1 and M_2 imply that the two systems of meaning M_1 and M_2 are constructed from M_D by using the fact that T_D^L contains both T_1 and T_2.

It is precisely at this stage that the notion of domain, we already encountered in Section 2, resurfaces. Nicod uses the concept to characterize the relation between the theories T_1^L and T_2^L on the one hand, and M_D, on the other: to say that a given domain D is a domain of a theory T^L is to say that D "provides the raw material" for constructing models of T. In other words, in Schema 1, M_D is a domain for T_1 and T_2. And, although M_D is a model of T_D, it is not a model of T_1 and T_2. The relation connecting theories to domain is complicated: when we say that T_1 and T_2 have the same domain, we are saying that their models have a certain relation with a third one, that we call their domain. This last relation is very indeterminate: the way the systems of meaning of T_1 and T_2 are constructed from M_D is not specified. The crucial point is then to sharply distinguish the property "being a model" from the property "being a domain." In Schema 1, M_D is a model of T_D^L, and a domain of T_1^L and T_2^L.

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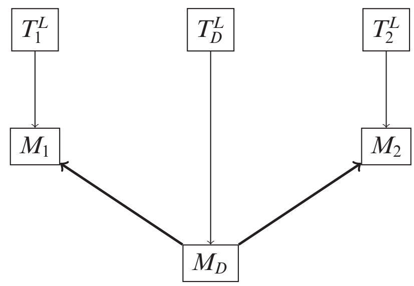
the distinction from the term "formal theory" that the word "system" (present in the term "axiomatic system") could obscure.

34 One can alternatively, once we have M_1, use the translation rule $F_{2\to 1}$ to construct another model of T_2; or, once we have M_2, use the translation function $F_{1\to 2}$ to construct a model of T_1. All these variations do not change the point Nicod wants to make.

T_1^L and T_2^L contained in T_D^L



Schema 1. Domain and system of meaning.

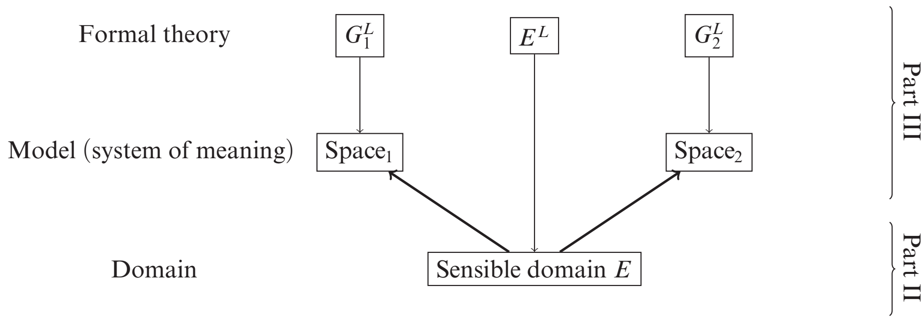
Later in chapter 3, Nicod criticizes Poincaré for having blurred this distinction. From the inseparability of the incompatible Hyperbolic and Euclidean geometries, Poincaré concludes, says Nicod, that "the question of the validity of the one or the other type is devoid of meaning" (p. 17). But (*Ibid.*):

> Doubtless, if we consider a fixed domain, such as the number system, or the physical world, all these geometrical system [the Hyperbolic and the Euclidean ones] hold at the same time, *because their interpretations remain indeterminate*. But tell us what arithmetic, or physical, relations you call congruence and rectilinearity: the problem of determining whether these relations behave in a Euclidean or a non-Euclidean manner could not be clearer.

Let T_1^L and T_2^L be the "ramsified" versions of Hyperbolic and Euclidean geometries; let $T_D(M_D)$ be the standard (interpreted) theory of the field of real numbers. One can, from the field of real numbers M_D, logically construct a model of Hyperbolic M_1 and Euclidean M_2 spaces. According to Nicod, from the fact that the two incompatible theories T_1 and T_2 have the same domain M_D, one cannot, as Poincaré suggests, conclude that T_1 and T_2 are neither true or false, namely, that conventionalism is right. It is by fixing the meaning of T_1^L- and T_2^L-indeterminates, *i.e.*, by constructing M_1 and M_2, the "arithmetic" T_1^L- and T_2^L-models, that one can determine the truth values of the geometrical sentences. When Poincaré considers the common reality to which Euclidean and Hyperbolic geometries apply, he is only considering the domain M_D from which the models M_1 and M_2 are constructed, and not these models themselves. In other words, for Nicod, Poincaré's conventionalism results from a confusion between model and domain.

My aim is not here to assess the strength of Nicod's criticism of Poincaré.[35] I refer to Nicod's objection only to show how the concept of domain is used in [17]. Let me recall that we characterized in Section 2 a domain as a theory with a fixed interpretation. Then, from Sections 3 to 5, we defined certain relations of equivalence between formal theories and their models, and completely forgot what we had said about the domain. We are now in position to put the pieces together. Nicod's definitions of various

[35] Thus, it is likely that a conventionalist would not accept the parallelism that Nicod is making between what holds in the numerical domain and what holds in the sensible domain.



Schema 2. The sensible domain.

relations of equivalence between theories are not intended to spell out a robust notion of theoretical content. As we have seen Nicod acknowledged *Plethora* and *Coherence*, and he never claimed that two equivalent theories are notational variants of each other. Nicod's real aim is to distinguish between two notions of interpretation: a strict one, that corresponds to the relation between a formal theory and a system of meaning; and a large, much more indeterminate one, according to which two theories having the same domain, (*i.e.*, two systems of meaning constructed from a common third one) can be said to have the same interpretation.

This second sense is crucial because it supports Nicod's whole project. In the *thèse*, the sensible world $E(s, R_1, \ldots, R_n)$ is regarded as a domain, in the sense of Schema 1. Indeed, Nicod presents various formal geometrical theories G_1^L, G_2^L,... that are contained in E^L,[36] and explains how, for any such abstract geometry, it is possible to extract, from the sensible domain $\langle s, R_1, \ldots, R_n \rangle$, some systems of meaning that satisfy G_1^L, G_2^L. Part II of [17] is completely devoted to the description of the sensible domain $E(s, R_1, \ldots, R_n)$. In Part III, Nicod studies geometries that have $E(s, R_1, \ldots, R_n)$ as their common domain. The distinction between model and domain is thus absolutely central in [17]: it would be a terrible misinterpretation to believe that, when Nicod speaks of *La géométrie dans le monde sensible*, he is arguing that sensible experience is a model of a certain geometry. Nicod's claim is much weaker: it is only that the sensible world is a domain of certain geometries. Schema 2, which is just an instantiation of Schema 1 (M_D is replaced by E, T_1 and T_2 by the geometries G_1 and G_2), gives us the organization of [17], and shows that the articulation between Part II and Part III is completely grounded on the distinction model/domain.

§6. Nicod against Bergson: setting. In the two following sections, I would like to clarify Nicod's reasons for distinguishing between model and domain. More precisely, I will show that this distinction is the basis of an argument against Bergson. It is difficult today to imagine the role of Bergson in France in the first decades of the

[36] I explain in Section 2 that it makes no sense to "ramsify" E to give to the abstract form E^L another interpretation than the one that is intended. But here, one "ramsifies" E only to apply the schema presented by Nicod. And, in this application, E is never regarded as a form that can be diversely interpreted; on the contrary, it is to provide interpretations to other theories that we need to "ramsify" E^L.

XXth Century. His anti-intellectualism was not endorsed by all, but even his fiercest opponents recognized the importance of Bergson's philosophy.[37] Bergson was one of the most well-known French philosophers in England and the US,[38] and even Russell gave in to fashion, by devoting an entire paper,[39] several long passages of [22],[40] and two reviews to Bergson.[41] Nicod's *thèse* must be put in this context: *La géométrie dans le monde sensible* is a sophisticated response to Bergson's anti-intellectualism.

Let me first give a rough idea of Bergson's view of the sensible experience and its relation to time and space. Here is how Russell characterizes it in [21 p. 331]:

> Mathematics conceives change, even continuous change, as constituted by a series of state; Bergson, on the contrary, contends that no series of states can represent what is continuous, and that in change a thing is never in any state at all. This view that change is constituted by a series of changing states he calls cinematographic; this view, he says, is natural to the intellect, but is radically vicious. True change can only be explained by true duration; it involves an interpenetration of past and present, not a mathematical succession of static states. This is what is called a "dynamic" instead of "static" view of the world.

According to Bergson, there is a gap between the sensible experience ("true change" and "true duration") and the picture that science and common sense make of it. It is the introduction, by the intellect and imagination, of the notion of geometric space that is the real cause of the "vicious" distortion one finds in the "static" view.[42] Two assumptions are central in Bergson's approach: first, sensible experience does not have any mathematical, and in particular geometrical structure; second, geometrical space can be defined as a product of reason and imagination that is projected into the sensible world. Let's deal with these two points one by one and show how Nicod reacts to them.

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37 See [1].

38 Bergson had been invited several times in the UK and the US, and all his major works were translated in English early on.

39 See [21].

40 In [22], there are at least four places where Russell discussed Bergson's philosophy: in Lecture I (pp. 18–41), Lecture V (pp. 138–158), Lecture VI (pp. 179–188), and Lecture VIII (pp. 232–240).

41 The overall tone is critical, but it would be wrong to believe that Russell didn't get anything out of it. On the relation between Russell and Bergson, see the headnotes to the papers on Bergson in [25, pp. 309–346].

42 As a sample of Bergson's developments, let me quote [3, pp. 100–101]: "There are [...] two possible conceptions of time, the one free from all alloy, the other surreptitiously bringing in the idea of space.... We can conceive of succession without distinction, and think of it as a mutual penetration, an interconnection and organization of elements, each one of which represents the whole, and cannot be distinguished or isolated from it except by abstract thought. Such is the account of duration which would be given by a being who... had no idea of space. But, familiar with the latter idea and indeed beset by it, we introduce it unwittingly into our feeling of pure succession; we set our states of consciousness side by side in such a way as to perceive them simultaneously, no longer in one another, but alongside one another; in a word, we project time into space, we express duration in terms of extensity, and succession thus takes the form of a continuous line or a chain, the parts of which touch without penetrating one another."

6.1. Geometry and sensible experience. In his 1914 discussion, Russell concedes to Bergson that "when we observe the motion of the seconds hand [of a watch], we do not merely see first a position and then another—we see something as directly sensible as color" (p. 144). For Russell too, then, the experience of a change does not (always) reduce to a "static" succession of sensations of states. But Russell's concession is limited. It does not preclude the possibility of representing continuous change as a series of states. And it does not imply that the part/whole relation cannot be applied to sense-data, nor that the notions of sensible space and sensible time are devoid of content.[43] What Bergson's objection shows, according to Russell, is that the ultimate sensible atoms have vague boundaries, and that one can be acquainted to a sense-datum A and a sense-datum B without being able to say if A is identical to or different from B.[44] For Russell, then, contrary to what Bergson claims, the part/whole relation and the notions of space and time apply to the sensible world.[45]

In Part II of [17], in which he sketches a description of the sensible experience, Nicod is getting closer to Bergson. The picture he draws of the sensible world is indeed based on a relation of interiority between sense-data that does not behave like a relation of spatio-temporal composition. Here is the way Nicod introduces the notion [17, p. 37]:

> I follow with my eye the flight of an eagle crossing my field of vision in a slow and continuous glide, the whole of which I perceive as a single visual term. In the middle of its flight, the eagle flaps its wings once. Between the one event, namely the flap of the bird's wings, and the other, larger event, namely its flight, I perceive a very clear and doubtless very simple relation which I express by saying that the first of these two sense terms is *interior* to the second.

Nicod makes clear that "the relation of interiority does not imply the logical relation of component to composite, and [that] the most extended and prolonged sense-data of the richest internal variety may appear as simple term" [17, p. 43]. Thus, Nicod's sense-data do not resemble at all Russell's atomic components of the sensible experience: the extended and prolonged flight of the eagle should be regarded as a simple item, which does not break down into different independent elements. Nicod also insists on the fact that the relation of interiority is "a simple relation which is irresolvable into relations of extension (space) and duration (time)" (p. 46).[46] The relation of interiority is not the product of a more fundamental spatial and/or temporal inclusion. In [17], there is thus no intrinsic sensible spatio-temporal pattern, as it was the case in Russell.

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43 [22, p. 158]: "There cannot be change... unless there is something different at one time from what there is at some other time [and] change, therefore, must involve relations and complexity, and must demand analysis."

44 On this, see [32].

45 Russell considers that Bergson's rejection of analysis comes from his implicit endorsement of the idealistic thesis of internal relations [22, p. 150]: "The view urged explicitly by Bergson, and implied in the doctrines of many philosophers, is, that a motion is something indivisible, not validly analyzable into a series of states. This is part of a much more general doctrine, which holds that analysis always falsifies, because the parts of a complex whole are different, as combined in that whole, from what they would otherwise be."

46 See also *Ibid.*, p. 40: "[interiority is] a more concrete, more undifferentiated nexus, which is antecedent to the division of relations with respect to extension and duration."

Nicod believes that basing his theory of the sensible world on the relation of interiority allows him to accommodate Bergson's approach. At the end of [17, Part II, chap. 1], he explains (p. 45):

> There is a contrast between the technician's analytic attention, directed towards sensible details that are difficult to grasp owing to their minuteness, and the artist's synthetic attention directed, on the contrary, to broad and rich terms whose apprehension is made difficult by the richness and breadth of their extension and duration. At one extreme we have the discernment of point-instants, and at the other the apprehension of all experience as one single term.... Once involved in this dilemma, we may decide in favor of analysis as did Leibniz, in which case the reality of sense terms will crumble into dust; or, as Bergson does, in favor of synthesis, so that reality will only belong to the totality of immediate experience.... But we only mislead ourselves by making reason intervene in situations to which it is indifferent: this mystery of the sensible whole not being the sum of its parts vanishes as soon as we realize that these are not true parts, and that interiority in extension and duration does not constitute a rational relation.

Nicod then describes his own position as an intermediate between Russell's atomistic approach and Bergson's integrative perspective. It would be compatible with Russell's view (when one replaces the relation of interiority with a relation of spatio-temporal composition); but it is in reality more in line with Bergson's holistic approach, insofar as the interpreted theory $E(s, R_1, \ldots, R_n)$ that Nicod sketches does not contain any relation of composition, any spatial or temporal relation—in brief, any geometrical notion.[47] By using the technical apparatus explained above, one can summarize Nicod's agreement with Bergson in these terms: the sensible world is a model (system of meaning) of a theory E which does not contain any geometrical notion among its primitive terms. Of course, it could be argued that Nicod's "neutrality" is an illusion. Orthodox Russellians could say that, since E contains relations, Nicod does not really deviate from Russell's position. And Bergsonians could argue that Nicod's description of the sensible world is still far too "static." But for us, the key point lies elsewhere: Nicod's sensible world does not contain any spatial structure, and sensible experience can be completely described without resorting to geometry.

6.2. The positional model. As I explained above, Bergson considers that the "static" view is based on the fact that reason and imagination illegitimately introduce spatial pattern into the description of the sensible experience. I say "reason and imagination" because Bergson's concept of space has a hybrid status: it is connected to the intellect defined as the faculty of analysis and decomposition, but also to the imagination defined as an interface between the intellect and the sensible experience. Indeed, it is crucial for Bergson that the space introduced by reason applies spontaneously, and "contaminates," so to speak, sensibility, and for this to happen, space must not be a

47 Note that no attempt is made in [17] to characterize in general terms geometry as a specific family of theories. Nicod seems here, like Bergson, to rely on our intuitive pre-understanding of the word. His reasoning seems to be: "Whatever Bergson calls a geometrical concept, it is true that the sensible experience does not contain any."

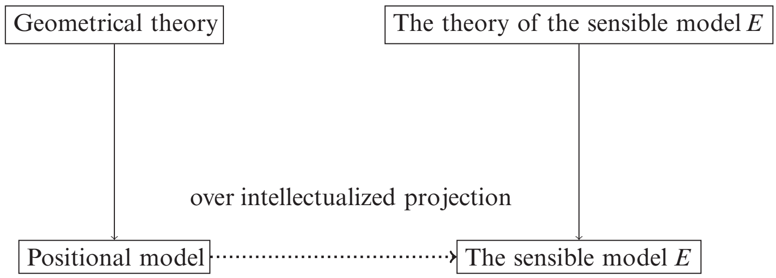
mere theoretical concept, it must have a link with imagination. In the introduction of Part III, commenting on a curious passage in which Bergson asks under what conditions a moving material point, becoming conscious of itself, would say that its perception of a succession takes the form of a straight line, Nicod insists on the fact that, for Bergson, space is a very specific thing [17, pp. 77–78]:

> [M. Bergson] does not say what he means by a *line*, but his answer to the question he poses [what is the 'form of a line'] gives sufficient indication of the meaning he assigns the term. The moving point's experience would, he says, assume the form of a line 'on condition that it could in some manner rise above the line it traverses, and apprehend simultaneously several juxtaposed points'. Whence this assertion? It arises from the fact that M. Bergson... imagines a set of simultaneous elements offering a certain primitive order which he calls juxtaposition and [from the fact that this order] is indeed a line which satisfies the geometer.

For Nicod, Bergson does not endorse a formal approach of geometry, according to which a space (a line) would be any model satisfying a certain axiomatic theory. Bergson has a particular model in mind: a space (a line) is for him "a set of simultaneous elements offering a certain primitive order" that our imagination, when governed by our intellect, creates. Nicod calls this model "positional space," because space is seen as a simultaneous network of interdependent positions. Bergson's approach is thus "semantic," in the sense that it is the specificities of the positional model that explain its philosophical harmfulness. Each position of the model can be instantiated by a piece of the sensible material, which, because of this operation, loses its interconnection with the rest. When the sensible experience is reorganized in accordance with the positional grid, the sensible world is then atomized into a myriad of elementary positions, and it is against what appears to him an over intellectualized view of experience that Bergson struggles.

Nicod does not reject Bergson's claim (this is the second point of agreement between them) that imagination can provide us with a particular model of geometry. Nicod is however cautious on the topic. Throughout Part III, the idea that men do have the faculty to form in imagination a positional model is neither endorsed nor completely dismissed.[48] Nicod recognizes with Bergson that the positional model is simple and particularly appealing. He does also agree with him that this model is "a dream of the imagination," *i.e.*, that the sensible world E does not contain positions as primitive

48 Let me quote the summary of the discussion one finds at the end of the book [17, p. 149]: "The relations of position [the positional models] provide the only simple interpretation geometry admits in nature: in the other spatial networks, all geometrical relations are to be interpreted as combinations of relations. Relations of position also provide the only interpretation of geometry which is apprehended in an instant; for no other sensible space is present in its entirety at the same time. This is why the intuitive relations of positions occupy the first place in the image that we form of a sensible structure illustrating a geometry. However, we have observed that it is conceivable that they do not exist at all and are but a dream of the imagination... In any case, even if an intuitive space of this type does exist, no matter what sway it may have over the imagination, we have convinced ourselves that inductive sensible spaces comprising temporal relations do not derive their order from it, and that if it can help us to think about them, it cannot in any way manufacture them, even for our intellect."



Schema 3. Nicod's rendition of Bergson.

terms. What Nicod rejects however is the idea that the unique way to construct a sensible interpretation of a geometrical theory is to "contaminate" the sensibility by the intellect, *i.e.*, to apply the positional model to the sensible world. We will now turn to this criticism.

Let me summarize, in a diagram, Nicod's rendition of Bergson's thought. The sensible experience is the model of a theory that has nothing to do with geometry (left column of Schema 3). The imagination, guided by the intellect, has the power to create a positional model of geometry (right column). This imagined space can "contaminate" and "fragment" the rich and abundant content of sensible experience, and this process (the thick dotted arrow between the right and the left columns) is at the source of the "static" picture of the sensible world that the intellect gives us in the science and in the common sense.

§7. Nicod against Bergson: arguments. Despite Nicod's points of agreement with Bergson, *La géométrie* can be seen as a sustained criticism of Bergson's view of the relationship between geometrical space and the sensible experience. Two arguments can be discerned in Nicod. The first one touches only superficially on the Bergsonian position. The second, more powerful, is also more interesting, in that it allows us to deepen our understanding of Nicod's concept of a domain.

Let's begin with the easy one. At the beginning of Part III, Nicod explains [17, p. 75]:

> Consider any geometry, for example that in which the primitive terms are *points* and the sole primitive is *congruence* of two pairs of points. Let $G(p,C)$ be the system's set of axioms, expressed in terms of a class p and a relation C between two pairs of members of p. Moreover, let s be the class of my sense terms; let $R_1, R_2, \ldots, R_n$ list the various relations I observe among them, and let $E(s, R_1, R_2, \ldots, R_n)$ be the set of laws which I shall be led to regard as inductively probable. Discovering an illustration, a 'solution' of the formal system $G(p,C)$ in sensible nature means constructing, in a logical manner using the relations R and the class s, a relation C_0 and a class p_0 such that $G(p_0, C_0)$ is entailed in $E(s, R_1, R_2, \ldots, R_n)$.

Unfortunately, the relation of "entailment" is not defined in *La géométrie*, but what Nicod has in mind is clear. If the "ramsified" version G^L of the theory G is contained (in the sense of Section 3) in the "ramsified" version E^L of $E(s, R_1, \ldots, R_n)$, then, from any solution (model) of E^L, one can construct, in a uniform way, a solution (model) of G^L. Thus, that $\langle p_0, C_0 \rangle$ is entailed in $\langle s, R_1, \ldots, R_n \rangle$ means that the first one is constructed, in a way that has been explained before, from the second ($\langle p_0, C_0 \rangle$ and $\langle s, R_1, \ldots, R_n \rangle$ are conjugate). Since it satisfies G^L, $\langle p_0, C_0 \rangle$ is a space; since the point p_0 and the relation of congruence C_0 are defined in terms of sensible entities, $G(p_0, C_0)$ is "about" the sensible world, in the sense that E is the domain of $G(p_0, C_0)$ (see Schema 2). Contrary to what Bergson claims, it seems thus possible to apply geometry to sensible experience.

Note that Nicod's claim is compatible with his idea that *E* is completely deprivate of geometrical structure, and that geometries have positional models created by the imagination. What Nicod refuses is the idea that the positional model is the only channel through which geometry can be applied to the sensible world. The distinction between model and domain allows him to clarify Bergson's vague notion of "contamination," or "projection," of the spatial grid into the sensible experience: *E* does not provide the geometrical theory G^L with a model; *E* supplies it with a domain. This means that, from *E*, one can define, in a logically controlled way, not one, but a host of geometrical models (see "Space $_1$" and "Space $_2$" in Schema 2). Among them, some are, perhaps, positional, but others are not; and this is the decisive point for Nicod: contrary to what Bergson claims, one does not have to pass through positional model and imagination to construct sensible spaces. In other words, Nicod blames Bergson for his laziness: having found one model of geometry (the positional one), Bergson attempts to extend it everywhere and to everything else, without realizing that this model is not the only possible one and that others can be developed.[49]

Fair as it is, this first argument is insufficient. Indeed, as *Plethora* and *Coherence* have made clear, the fact that a theory T is interpreted in (or contained in) T^* does not mean that the two theories speak about the same thing. The sensible world *E* is not a model of *G*, it is merely the domain from which a *G*-model (a space) can be defined. The sense in which we can say that $G(p_0, C_0)$ and *E* relate to the same reality is thus very loose. How would the fact that *G* is interpretable in *E* tell us anything about *E* then? And if the containment of *G* in *E* does not tell anything about *E*, then wouldn't Bergson finally be right to say that the projection of the idea of space *G* into the sensible reality *E* distorts *E* by unduly intellectualizing it? True, Bergson can be criticized for making the positional model of imagination play an excessive role, and for not understanding the distinction between model and domain. But this in no way detracts from the relevance of his fundamental insight: to geometrize the sensible experience is to betray it.

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49 Here is the continuation of the passage quoted section 6.2 [17, pp. 77–78]: "M. Bergson conjures up a peremptory image of what a line ought to look like. He imagines a set of simultaneous elements offering a certain primitive order which he calls juxtaposition and, seeing that this is indeed a line which satisfies the geometer, he does not dream that there may be some, which are altogether different, fashioned from other elements and relations. The form of a line means to him this particular appearance. Having asked himself if the succession his conscious moving point experiences would assume the form of a line from its viewpoint, he then answers: undoubtedly yes, provided that it has this aspect [*aspect*], or assumes its through some illusion. But this is to abandon thought for imagination, to slip into the arbitrary. For us, the form of a line will be solely the laws governing lines: and if, as we shall see, these laws can found in the intuitive order of an instantaneous apprehension, they can also be found in other aspects [*aspects*] of experience."

To meet this final challenge, Nicod introduces a new idea. The key passage is [17, pp. 74–75]:

> Let us illustrate this with an example. We have all seen as children those drawings which depict something we cannot distinguish at first sight, where the idea is to discern a giraffe or a lion in the contours of a landscape which at first sight seems deserted. When we have 'discovered' them, we have seen something new. The outline of this hillock was the lion's rump, and the knot in this tree trunk its eye. Into this network of lines we has reach a certain structure, namely the landscape, and now we had just read a second structure into it, namely the lion. As for the lines themselves and the elementary relations—angles, distances, intersections—which in the final analysis determine the whole drawing, we have in these the substance of the remainder, the arabesque itself, into which we can first of all read a landscape by noticing that its elements manifest a certain order when grouped in a certain way, and then a lion by observing that a different grouping brings to light a second structure. The drawing I have before me is sensible nature. The elementary links which I know how to spell out, so to speak, are the primitive relations among my data. The form I attempt to read into it is, for example, the geometry $G(p,C)$. What groups, taken as elements, make this structure G appear in the relations which stem from their composition? Could there even be several methods of grouping which satisfy this requirement—might there even be more than one way of finding a lion in the landscape?

In the illustration, the "arabesque" of lines corresponds to the sensible world, and the lion hidden in it corresponds to the geometrical space. The first thing to note is that Nicod recognizes that, in a certain sense, the lion is not in the landscape: a complete description of the drawing can be made without mentioning the hidden lion and without using the concept of lion. Nicod remains faithful to the idea that the sensible world E is a model of a theory that does not contain any geometrical notion in the list of its primitives. But at the same time, it would be false to say that when I use the concept of lion to depict the picture, I describe a different drawing than the original one. What I discover when I see the lion is a new aspect of the picture. And the same holds for the concept of space: when I apply geometrical notions to the sensible world $\langle s, R_1, \ldots, R_n \rangle$, I do not misrepresent it, as Bergson claims, but bring forward an aspect of it that had not been seen before. The abstract geometry G^L contained in E^L gives us the means to isolate and fix aspects of the sensible world that escape us when we describe it as a model of E^L, that is, in purely sensible terms. We do learn something on $\langle s, R_1, \ldots, R_n \rangle$, then, when we learn that it is a domain of a certain geometry.

Nicod does not reject the idea that the sensible world is delivered to us in a pure intuition, free of any intellectual and geometric stain. What he asserts is that even if this last statement is true, the sensible realm contains within it dormant possibilities, aspects, which require a special work or a specific gift in order to be awakened. Bergson's rather simplistic opposition between the true intuition and the vicious intellect[50] leads

[50] In fact, Bergson is sometimes much more balanced, and says that intuition requires a long period of prior conceptual analysis. For a reading of Bergson along these lines, see [12].

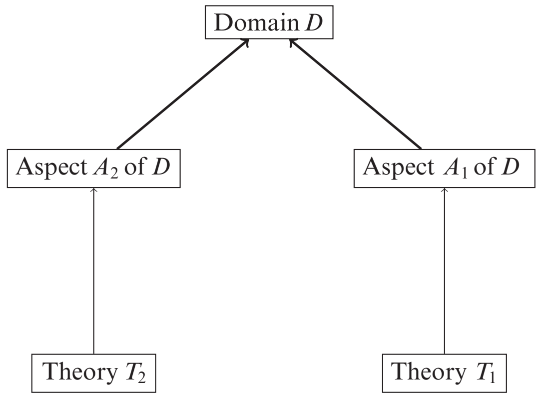
him to overlook the dimension of *Gestalt* switch and implicitness that is an essential part of our sensible experience. The move is clever, because the notion of aspect is central in Bergson's thought. Nicod's passage on the riddle figure echoes thus many developments on *Gestalt* switch in Bergson. Let me quote one of them taken from a lecture given in Oxford in 1911 [4, p. 158]:

> The great painters are men who possess a certain vision of things which has or will become the vision of all men. A Corot, a Turner,... have seen in nature many an aspect that we did not notice. Shall it be said that they have not seen but created, that they have given us products of their imagination...? It is true to a certain extent; but, if it were only that, why should we say of certain works... that they are true?... If we reflect deeply upon what we feel as we look at a Turner or a Corot, we shall find that, if we accept them and admire them, it is because we had already perceived something of what they show us. But we had perceived without seeing. It was, for us, a brilliant and vanishing vision, lost in the crowd of those visions, equally brilliant and equally vanishing, which become overcast in our ordinary experience... The painter has isolated it; he has fixed it so well on the canvas that henceforth we shall not be able to help seeing in reality what he himself saw.

Bergson says here that, thanks to the strength of their intuition, the great painters have the power to isolate and fix on the canvas some aspects of our experience that we did not see before. In the passage on the riddle figure, Nicod turns Bergson against himself: his treatment of the relation between painting and the sensible experience can be extended to the relation between geometry and the sensible world. Geometers, as great painters, succeed in isolating and fixing aspects, that, until them, escaped us. In the case of the geometers, this achievement does not come from the strength of intuition; it comes from the elaboration of a formal theory G^L, and from the realization that G^L is contained in the description of the sensible world E. But the key point is that just as the great painters do not falsify the sensible experience but reveal it, so does the geometrical intellect not falsify the sensible intuition but strengthens it. The geometrical theory is a means to uncover the intuition's latent possibilities.[51]

In more general terms, the confrontation with Bergson forced Nicod to adopt the point of view that one has when one places oneself in the interpretative theory, (*i.e.*, the sensible domain), and to abandon the point of view, which was that of Russell and Whitehead, when one places oneself in the interpreted theories (*i.e.*, geometry). Recall that, in the standard method of logical construction, one starts from entities with neat properties (a "high level" mathematized theory) that one seeks to replace by some logical constructions composed of entities which have no such neat properties. Once this replacement is done, the task is completed: the theory one started with has found a credible, down-to-Earth, interpretation. Nicod's wish to counter Bergson's

[51] At roughly the same time, the Polish logician and painter Leon Chwistek applied Russell's theory of logical construction to classify the styles of painting according to the way they depict realities. According to Chwistek, the various constructions should not be seen as different aspects of the same underlying sensible world, but as different realities. See for instance his article *The Plurality of Realities in Art* from 1918 translated in English in [6].



Schema 4. The domain-centered view.

anti-intellectualism leads him to focus on the sensible domain, and reverse the schema: assuming we have a certain domain (a theory with a fixed interpretation), and assuming that some "neat" theories that do not apply directly to it have models that are constructed within this domain, how does this fact tell us anything about the basic domain? And if it tells us something, what does it tell us? The starting point is here no longer the syntaxic-formal level (the theory), but the semantic-applied (the domain) one. The question Nicod raises is not: how can we interpret a given theory? But: what additional knowledge about the interpreting domain is provided by the fact that various theories can be interpreted from within it?[52] In Nicod, it is not the interpretations that vary, and the theory which remains fixed; what is considered constant is the domain, and what vary are the theories. This crucial shift explains why Nicod escapes the issue raised by *Plethora* and *Coherence*: Nicod's construction is not guided by the wish to identify the common content of different theories, but by the wish to explain how various theories can isolate and fix different aspects of the same domain. In Schema 4, which is an inversion of Schema 2, I have represented this shift in perspective: the sensible domain E comes first, and the various geometrical models entailed in it are regarded as aspects of E, rather than solutions of the theories G^L.

One could perhaps consider that the notion of aspect, that Nicod puts forth here, remains vague and imprecise. I would like to suggest, however, that it is not the case. Logicians and Model theorists explain the notion of interpretation and interpretative structure today in similar terms. At the beginning of his chapter on interpretation, Hodges writes [11, pp. 93–94]:

> There is a... model-theoretic slogan: structure is whatever is definable.... For example, if we have a field K, we can define the projective plane over K.... The plane comes with the field; in some

[52] In his introduction to [17], Russell insists on the inversion of perspective performed by Nicod (xiv–xv): "Dr Whitehead has examined, from the point of view of mathematical logic, how we can define in terms of empirical data the entities that traditional geometry consider as primitive.... This method starts from the knowledge of the completed mathematical system which is the object to be attained, and goes back to entities more analogous to those of sense perception. The method adopted by Nicod follows the inverse order: starting from data of perception, it tries to attain the various geometries that can be built on them."

> abstract sense it is the field, but looked at from an unusual point of view. A merit of the... slogan is that it gives us a means of controlling the host of 'implicit' features of a structure A. If these features are definable in terms of A, they must be definable by some kind of sentences. So we can consider those features definable by sentences of a certain form, or in a certain language.

Hodges insists here on the fact that there is more in a structure than what is delivered by the notion of a model of a theory. A field K is a model of a theory of field, but it is also, in a certain sense, a projective plane over K ("the plane comes with the field"), because the second structure is definable in the first (it is "entailed" in it, in Nicod's sense). The modern notion of interpretation also carries with it the shift in perspective from theories to domain, insofar as it obliges us to fix our attention on the interpreting structure, and on various other structures definable from it. When this reversal of perspective is present, then the terminologies of the *Gestalt* switch and of implicitness come quite naturally.

§8. Conclusion. In this article, we first presented Nicod's works on equivalence between theories and about model conjugation, and we have shown that, in many respects, Nicod's conception can be seen as an early occurrence of the syntactic and semantic model-theoretic notion of interpretation of a theory into another one. We have also explained how Nicod used his construction to counter Bergson anti-intellectualist attack against the geometrization of sensible experience. This journey through *La géométrie* allowed us to measure the technical mastery and philosophical depth of the promising young man that Nicod was at that time. But, in addition to the tribute paid to him, there are at least two reasons to come back to Nicod today. First, reading *La géométrie* shows that Hintikka's[53] and van Heijenoort's[54] idea according to which Model theory would come from a line which is completely distinct from the universalist tradition does not hold. Nicod clearly followed in the footsteps of Russell, the champion of the universalist tradition, when he wrote his *thèse* and elaborated on notions that would be developed much later in Model theory. Thus, Nicod's work vividly illustrates that the universalist and the Model-theoretic tradition are not like parallel lines separated in a watertight manner. The present analysis is then consistent with and reinforces the conclusions of a set of works that seeks to rewrite the pre-history of Model theory by leaving aside Hintikka–van Heijenoort's dichotomy.[55] But until now, the main focus, in these researches, has been on the issues related to the foundations of mathematics, as they have been elaborated in the Hilbert's school and the Vienna Circle (Carnap, Gödel and Tarski).[56] The second reason to read *La géométrie* is that Nicod's innovative work was driven by a philosophical agenda, and not by questions related to the foundations of mathematics. His elaboration of what can be seen, in hindsight, as an anticipation of the concept of interpretation between theories, comes from his want to defend Russell's method of logical construction against objections from Bergson. The ways of creation, whether in logic, philosophy

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53 See [10].
54 See [27].
55 See among others [2, 8, 24].
56 [7] is an important exception.

or elsewhere, are always mysterious. But in the case at hand, one could suggest that it is precisely the relative isolation and provincial character of the French intellectual scene (the excessive place given to Bergson) that enabled Nicod to see differently the connection between theories and realities.

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