A contradiction in Bohm's theory

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Abstract

It is argued that the result assumption of Bohm's theory, which says that particle configurations represent measurement results, contradicts the predictions of the Schrödinger equation.

Bohm's theory is a typical example of the hidden-variable interpretation of quantum mechanics (Bohm, 1952). Why add hidden variables such as positions of Bohmian particles to quantum mechanics? Among other reasons, it is usually thought that adding these variables which have definite values at all times is enough to ensure the definiteness of measurement results and further solve the measurement problem of quantum mechanics. For example, the existing no-go theorems for hidden-variable theories, such as the Kochen-Specker theorem (Kochen and Specker, 1967), consider only whether observables can be assigned sharp values or whether there exist such hidden variables. However, if these hidden variables are not eligible to represent measurement results, then even though they have definite values at all times, their existence does not help solve the measurement problem.

This may be indeed the case. In this paper, I will argue that in Bohm's theory, and in a hidden-variable theory in general, the result assumption, which says that the hidden variables such as particle configurations represent measurement results, contradicts the predictions of the Schrödinger equation.

In Bohm's theory, a measurement interaction is described by the Schrödinger equation (with a potential term whose concrete form is determined by the system-device interaction) that governs the time evolution of the wave function. As a result, different measurement results are correlated in a lawful way with different wave functions of the measuring device, which may be called result wave functions, after a measurement. For example, in the Stern-Gerlach experiment which measures the spin of a system, the measurement is realized by the spin-magnetic field interaction, which is described by the Schrödinger equation for the wave function. Thus the measurement result being spin-up or spin-down is correlated with a result wave function of the measuring device.

Bohm's theory assumes that the particle configuration of the measuring device actually represents the measurement result. In Goldstein's words, "results of measurement are registered configurationally" (Goldstein, 2017). But it is the correlation between measurement results and result wave functions that permits that the predictions of Bohm's theory can be consistent with the Born rule, which is also formulated with the wave function. Consider a measurement of the x-spin of a spin one-half system. According to Bohm's theory, the probability of finding the spin of the system being x-spin up or x-spin down is equal to the probability of the particle configuration of the measuring device being in the support of the result wave function $\langle x \,|up\rangle_M$ or $\langle x \,|down\rangle_M$, which ensures the consistency of the theory with the Born rule (see, e.g. Lazarovici et al, 2018). In other words, when the particle configuration of the measuring device is in the support of the result wave function $\langle x \,|up\rangle_M$, the measurement result is x-spin up, and when the particle configuration of the measuring device is in the support of the result wave function $\langle x \,|down\rangle_M$, the measurement result is x-spin down. This just means that each measurement result is correlated with each result wave function of the measuring device, or in other words, each result wave function of the measuring device corresponds to each measurement result.[1]

Note that the wave function is real in Bohm's theory, and thus it exists for a single (quasi-isolated) system such as a measuring device and can correspond to a single measurement result. Besides, it is the whole result wave function, not any truncated result wave function such as a wave function with compact support, that corresponds to the result, since the latter is in fact a superposition of different result wave functions.

[1]In fact, due to the limits of accuracy permitted by Bohm's theory (Dürr, Goldstein and Zanghì, 1992), the pointer of a measuring device that denotes the measurement result should be also described by the (effective) wave function of the pointer, not by the more precise particle configuration of the pointer; the latter cannot be further observed by any device or observer (and may also be far away from the center of the pointer wave packet), while a measurement result should be observable in a certain way after all. Note that what a position measurement reveals in Bohm's theory is neither the initial particle configuration of the measured system nor the final particle configuration of the measured system, but the final (effective) wave function in which the Bohmian particles of the system reside. In addition, an analysis of psychophysical supervenience also supports the above conclusion. Since the behaviour of the Bohmian particles is identical to that of the wave function in which they reside within the limits of accuracy permitted by Bohm's theory, if the mental state of an observer with a certain record supervenes on the particle configuration of her brain and its evolution over time, it will also supervene on the corresponding result wave function of her brain and its evolution over time (see also Brown and Wallace, 2005). In other words, different measurement results will correspond to different result wave functions.

Now I will derive a contradiction in Bohm's theory. In Bohm's theory, the preferred bases or the result wave functions of a measuring device, such as $\langle x \,|up\rangle_M$ and $\langle x \,|down\rangle_M$, are not localized states with compact support, but localized states with infinitely long tails, in configuration space. Then, since the Bohmian particles can be anywhere in which the amplitude of the wave function is not zero, the particle configuration of a measuring device can be in any point in the configuration space no matter which result wave function the wave function of the device is. This means that there is no one-to-one correspondence from the particle configurations of a measuring device to the result wave functions of the device or the measurement results.[2] Moreover, since the overlap between two result wave functions is finite, there is always a non-zero probability that two result wave functions of a measuring device correspond to the same particle configuration of the device.

This analysis relies on the precondition that the result wave functions have infinitely long tails in configuration space. If each result wave function is a localized state with compact support in configuration space, then there will be a one-to-one correspondence from the particle configurations to the result wave functions. However, such localized states with compact support cannot exist in an arbitrary short time interval, since the Schrödinger evolution will instantaneously turn them into localized states with infinitely long tails.

Then, what does the non-existence of a one-to-one correspondence from the particle configurations to the result wave functions mean? It seems to mean that the result assumption contradicts the predictions of the Schrödinger equation in Bohm's theory. According to the Schrödinger equation, when a measuring device measures the same observable of two systems being in different (non-degenerate) eigenstates of the observable, the post-measurement states of the device will be two different result wave functions which correspond to two different results. But according to the result assumption, the measuring device may obtain the same result with a non-zero probability, since it is possible that the particle configurations of the measuring device are the same after these two measurements.

This contradiction is more obvious when considering observers. Since the result assumption of Bohm's theory implies that the mental state of an observer supervenes on the particle configuration of the brain of the observer (Stone, 1994; Maudlin, 1995; Brown and Wallace, 2005; Lewis, 2007), different mental states of an observer must correspond to different particle configurations of her brain. But according to the above analysis, two mental states of an observer with different records may correspond to

[2] A similar conclusion applies to Bohm's result assumption, which says that the branch of the wave function "occupied" by the Bohmian particles represents the measurement result. Due to the lack of a one-to-one correspondence from the particle configurations to the result branches, there is no unambiguious way to define the occupation.

the same particle configuration of her brain with a non-zero probability.[3]

The above contradiction also exists in other hidden-variable theories in which the interactions between quantum systems including measurements are described by the Schrödinger equation for the wave function, while the measurement result is represented by the hidden variables and two result wave functions may correspond to the same values of the hidden variables. By contrast, in theories in which the interactions between quantum systems are described by the Schrödinger equation for the wave function, and the measurement result is also represented by the wave function, such as collapse theories, the contradiction does not exist.

Admittedly, since the overlap between two result wave functions is very small, the possibility that two result wave functions correspond to the same particle configuration is also very small. In fact, it may never happen in practical situations, just like the violation of the second law of thermodynamics. However, the above contradiction exists no matter how small the possibility is, if only it is not zero. In other words, the nature of the contradiction is that certain assumptions of a hidden-variable theory such as Bohm's theory are incompatible, and the incompatibility is independent of experience.

There are two possible ways to avoid the above contradiction in Bohm's theory. The first way is more conservative in the sense that it keeps the Bohmian laws of motion unchanged and only revises the result assumption of the theory. For example, it is not the configuration of the Bohmian particles at an instant, but the trajectory of the Bohmian particles in configuration space during a time interval that represents the measurement result. It is understandable that there is a one-to-one correspondence from the total history of the Bohmian particles to the result wave function in general. Different result wave functions will lead to different trajectories of the Bohmian particles by the guiding equation, although these particles may reach the same position in configuration space after these measurements. However, an opponent may argue that it is not the trajectory of the pointer of a measuring device during a measurement, but the final position of the pointer after a measurement that indicates the measurement result; a measurement result is always encoded in a configurations of things after all. Thus, it seems that the one-to-one correspondence from the total history of the Bohmian particles to the result wave function, although it exists, might

[3]Note that although different result wave functions of the brain of an observer have a small overlap in configuration space, they already correspond to the mental states of the observer with different records. In other words, the measurement has finished in the brain of the observer, and no further measurements are needed. Certainly, if we make a further position measurement on these result wave functions, then since they have an overlap in configuration space, we may obtain the same result with a nonzero probability. But this does not mean that two orthogonal states cannot be distinguished with certainty. This analysis also applies to a practical spin measurement, for which there is always a position measurement on the (spatial) result wave function.

not help avoid the above contradiction.

The second way to avoid the contradiction is more revolutionary in the sense that it keeps the result assumption unchanged but revises the Bohmian laws of motion. In such a revised theory, the interactions between quantum systems including measurements are described not by the Schrödinger equation for the wave function, but by a new equation of motion for the Bohmian particles. Then it can be expected that the particle configuration of the measuring device may have a one-to-one correspondence with the value of the measured quantity. In other words, it will be indeed the configuration of Bohmian particles, not the wave function, that represents the measurement result. In this case, the above contradiction does not appear. No doubt, the revised theory will be significantly different from Bohm's theory. One may worry about the issue of whether such a theory can be constructed so that its predictions are consistent with existing experiments and our experience. In view of the empirical success of the Schrödinger equation and the Born rule, this worry is quite reasonable.

To sum up, I have argued that the result assumption of Bohm's theory, which says that particle configurations represent measurement results, contradicts the predictions of the Schrödinger equation. There are two possible ways to avoid the contradiction by revising either the result assumption or the Bohmian laws of motion, but it remains to be seen if any of them is a feasible approach.

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