A New Ontological Interpretation *of* the Wave Function

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Abstract

In this paper, we propose an ontological interpretation of the wave function in terms of random discontinuous motion of particles. According to this interpretation, the wave function of an N-body quantum system describes the state of random discontinuous motion of N particles, and in particular, the modulus squared of the wave function gives the probability density that the particles appear in every possible group of positions in space. We present three arguments supporting this new interpretation of the wave function. These arguments are mainly based on an analysis of the mass and charge properties of a quantum system. It is realized that the Schrödinger equation, which governs the evolution of a quantum system, contains more information about the system than the wave function of the system, such as the mass and charge properties of the system, which might help understand the ontological meaning of the wave function. Finally, we briefly analyze possible implications of the suggested ontological interpretation of the wave function for the solutions to the measurement problem.

The wavefunction gives not the density of stuff, but gives rather (on squaring its modulus) the density of probability. Probability of what exactly? Not of the electron being there, but of the electron being found there, if its position is 'measured'. Why this aversion to 'being' and insistence on 'finding'? The founding fathers were unable to form a clear picture of things on the remote atomic scale. (Bell 1990)

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1 Introduction

Quantum mechanics is a physical theory about the wave function and its time evolution. There are two main problems in the conceptual foundations of quantum mechanics. The first one concerns the physical meaning of the wave function. The second one is the measurement problem, which concerns the time evolution of the wave function during a measurement. Although the meaning of the wave function should be ranked as the first interpretative problem of quantum mechanics, it has been treated as a marginal problem, especially compared with the measurement problem. There are already several realistic alternatives to quantum mechanics which give basically satisfactory solutions to the measurement problem. However, these theories at their present stages have not yet succeeded in making sense of the wave function.

During recent years, more and more research has been done on the ontological status and meaning of the wave function (see, e.g. Monton 2002; Lewis 2004; Gao 2011a, 2011b; Pusey, Barrett and Rudolph 2012; Ney and Albert 2013). In particular, Pusey, Barrett and Rudolph (2012) demonstrated that under certain assumptions the wave function of a quantum system is a representation of the physical state of the system. This raises a further question: what physical state does the wave function describe? and what is the existing form of the system? Conventionally, wave function realism regards the physical entity described by the wave function of an N-body quantum system as a continuous field existing in a fundamental 3N-dimensional space (Albert 1996). However, this view has at least two serious problems: the so-called "problem of perception" and "problem of lacking invariances" (Monton 2002; Lewis 2004; Solé 2013)[1]. Facing these difficulties, Monton (2002, 2013) and Lewis (2004, 2013) suggested that the wave function describes certain property of discrete particles in our ordinary three-dimensional space. The question is then which property of particles the wave function describes. In this paper, we will propose a concrete interpretation of the wave function in terms of particle ontology in three-dimensional space, and present a few arguments supporting this new interpretation.

The plan of this paper is as follows. In Section 2, we will introduce an ontological interpretation of the wave function in terms of random discontinuous motion of particles. According to this interpretation, the wave function of an N-body quantum system describes the state of random discontinuous motion of N particles, and in particular, the modulus squared of the wave function gives the probability density that the particles appear in every possible group of positions in space. At a deeper level, the wave function may represent the dispositional property of the particles that

[1] See Maudlin (2013) for other criticisms of this view.

determines their motion. In Section 3, we will present three arguments supporting this new interpretation of the wave function. It is realized that the Schrödinger equation, which governs the evolution of a quantum system, contains more information about the system than the wave function of the system, which might help understand the ontological meaning of the wave function. An important piece of information is the mass and charge properties of a quantum system, which are responsible for the gravitational and electromagnetic interactions between systems. Our main approach is to analyze these properties. In Section 4, we will briefly discuss possible implications of the suggested ontological interpretation of the wave function for the solutions to the measurement problem. Conclusions are given in the last section.

2 Interpretation

Our suggested interpretation of the wave function in terms of random discontinuous motion of particles can be basically formulated as follows (Gao 2011a, 2011b, 2013). For an N-body quantum system with properties such as masses m_1, m_2, ... m_N, there are N particles whose masses are m_1, m_2, ... m_N, respectively. At each instant, this system of N particles can be represented by a point in a 3N-dimensional configuration space. During an infinitesimal time interval around each instant, these particles perform random discontinuous motion in 3-dimensional space, and correspondingly, this point performs random discontinuous motion in the configuration space. The probability density that this point appears in each position $(x_1, x_2, ...x_N)$ or the probability density that particle 1 appears in position x_1 and particle 2 appears in position x_2, ..., and particle N appears in position x_N is $|\psi(x_1, x_2, ...x_N, t)|^2$. Loosely speaking, such motion forms a "cloud" in the configuration space, and the state of the system is represented by the density and flux density of the cloud, $\rho(x_1, x_2, ...x_N, t)$ and $j(x_1, x_2, ...x_N, t)$, where

$$\rho = |\psi|^2, \tag{1}$$

$$j_k = \frac{\hbar}{2mi}[\psi^* \nabla_k \psi - \psi \nabla_k \psi^*]. \tag{2}$$

The density and flux density of the cloud satisfy the continuity equation $\partial\rho/\partial t + \nabla j = 0$. Correspondingly, the wave function ψ can be uniquely expressed by ρ and j (except for a constant phase factor). In this way, the wave function provides a complete description of the state of random discontinuous motion of particles.

In the following, we will give a strict description of random discontinuous motion of particles based on the measure theory. For the sake of simplicity but without losing generality, we analyze one-dimensional motion of a particle that corresponds to the point set in two-dimensional space and time. The results can be readily extended to the situation of three-dimensional motion of many particles.

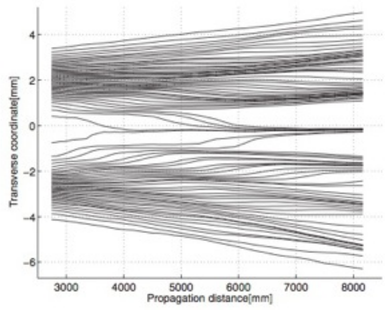

Fig.1 The description of random discontinuous motion of a single particle

Consider the state of random discontinuous motion of a particle in finite intervals Δt and Δx around a space-time point (t_i,x_j) as shown in Fig. 1. The positions of the particle form a random, discontinuous trajectory in this square region[2]. We study the projection of this trajectory in the t-axis, which is a dense instant set in the time interval Δt. Let W be the discontinuous trajectory of the particle and Q be the square region $[x_j, x_j+\Delta x]\times[t_i, t_i+\Delta t]$. The dense instant set can be denoted by $\pi_t(W \cap Q) \in \Re$, where π_t is the projection on the t-axis. According to the measure theory, we can define the Lebesgue measure:

$$M_{\Delta x,\Delta t}(x_j, t_i) = \int_{\pi_t(W\cap Q)\in\Re} dt. \tag{3}$$

Since the sum of the measures of all such dense instant sets in the time interval Δt is equal to the length of the continuous time interval Δt, we have:

$$\sum_j M_{\Delta x,\Delta t}(x_j, t_i) = \Delta t. \tag{4}$$

Then we can define the measure density as follows:

$$\rho(x, t) = \lim_{\Delta x,\Delta t\to 0} M_{\Delta x,\Delta t}(x, t)/(\Delta x \cdot \Delta t). \tag{5}$$

This quantity provides a strict description of the position distribution of the particle or the relative frequency of the particle appearing in an infinitesimal

[2]Recall that a trajectory function $x(t)$ is essentially discontinuous if it is not continuous at every instant t. A trajectory function $x(t)$ is continuous if and only if for every t and every real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that whenever a point t_0 has distance less than δ to t, the point $x(t_0)$ has distance less than ε to $x(t)$.

space interval dx around position x during an infinitesimal interval dt around instant t, and it satisfies the normalization relation $\int_{-\infty}^{+\infty} \rho(x,t)dx = 1$ by Eq. (4). According to the above interpretation of the wave function, we have $\rho(x,t) = |\psi(x,t)|^2$, where $\psi(x,t)$ is the (nomalized) wave function of the particle. We call $\rho(x,t)$ position measure density or position density in brief. Note that the existence of the above limit relies on the continuity of the evolution of $|\psi(x,t)|^2$.

We may further define position flux density $j(x,t)$ through the relation $j(x,t) = \rho(x,t)v(x,t)$, where $v(x,t)$ is the velocity of the local position density. It describes the change rate of the position density. Due to the conservation of measure, $\rho(x,t)$ and $j(x,t)$ satisfy the continuity equation:

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0. \tag{6}$$

The position density $\rho(x,t)$ and position flux density $j(x,t)$ provide a complete description of the state of random discontinuous motion of a single particle.

The description of the motion of a single particle can be extended to the motion of many particles. At each instant, a quantum system of N particles can be represented by a point in a $3N$-dimensional configuration space. Then, similar to the single particle case, the state of the system can be represented by the joint position density $\rho(x_1, x_2, ...x_N, t)$ and joint position flux density $j(x_1, x_2, ...x_N, t)$ defined in the configuration space. They also satisfy a continuity equation. The joint position density $\rho(x_1, x_2, ...x_N, t)$ represents the relative frequency of particle 1 appearing in position x_1 and particle 2 appearing in position x_2, ... and particle N appearing in position x_N. When these N particles are independent, the joint position density can be reduced to the direct product of the position density for each particle, namely $\rho(x_1, x_2, ...x_N, t) = \prod_{i=1}^{N} \rho(x_i, t)$.

From a logical point of view, for the random discontinuous motion of particles, the particles may have an instantaneous property (as a probabilistic instantaneous condition) that determines the probability density that the particles appear in every possible group of positions in space; otherwise the particles would not "know" how frequently it should appear in each group of positions in space. This property is usually called indeterministic disposition or propensity in the literature[3]. On the other hand, since the wave function in quantum mechanics is defined at instants, not during an

[3]Note that the propensity here denotes single case propensity. In addition, it is worth stressing that the propensities possessed by particles relate to their objective motion, not to the measurements on them. By contrast, according to the existing propensity interpretations of quantum mechanics, the propensities a quantum system has relate only to measurements; a quantum system possesses the propensity to exhibit a particular value of an observable if the observable is measured on the system. See also Belot (2012) and Esfeld et al (2013) for the particle propensity interpretation in Bohmian mechanics.

infinitesimal time interval around a given instant, it should be regarded not simply as a description of the state of motion of particles, but more suitably as a description of the dispositional property of the particles that determines their random discontinuous motion at a deeper level. In particular, the modulus squared of the wave function determines the probability density that the particles appear in every possible group of positions in space. By contrast, the density and flux density of the particle cloud in the configuration space, which are defined during an infinitesimal time interval around a given instant, are only a description of the state of the resulting random discontinuous motion of particles, and they are determined by the wave function. In this sense, we might say that the motion of particles is "guided" by their wave function in a probabilistic way.

3 Arguments

In the following, we will present three arguments supporting the above interpretation of the wave function in terms of random discontinuous motion of particles. Our main approach is to analyze the mass and charge properties of a quantum system.

3.1 Argument one

A better way to investigate the relationship between the wave function and the physical entity it describes is not only analyzing the structure of the wave function itself, but also analyzing the whole Schrödinger equation, which governs the evolution of the studied quantum system. The Schrödinger equation contains more information about the system than the wave function of the system, an important piece of which is the mass and charge properties of the system that are responsible for the gravitational and electromagnetic interactions between systems[4]. Based on an analysis of these properties, we will give a heuristic argument that what the wave function of an N-body system describes is not one physical entity, either a continuous entity or a discrete particle, in a 3N-dimensional space, but N physical entities in 3-dimensional space, and these entities are not continuous entities but discrete particles.

First of all, in the Schrödinger equation for an N-body quantum system, there are N mass parameters m_1, m_2, ... m_N (as well as N charge parameters etc). These parameters are not natural constants, but properties of the system; they may be different for different systems. Moreover, it is arguably that different mass parameters represent the same mass property of different

[4]In this sense, the wave function is not a complete description of the system (even though one assumes that the 3N-dimensional space it lives on has a rich structure that can group the 3N coordinates), since it contains no information about the masses and charges of the system.

physical entities. If a system has N mass parameters, then it will contain N physical entities. Therefore, an N-body quantum system contains N physical entities, and the wave function of the system describes the state of these physical entities. Next, these N entities exist in 3-dimensional space, not in a 3N-dimensional configuration space. The reason is that in the Schrödinger equation for an N-body quantum system, each mass parameter m_i is only correlated with each group of three coordinates (x_i, y_i, z_i) of the 3N coordinates in configuration space. Thirdly, these N entities are not continuous entities, which are completely described by density and flux density. The reason is that the density and flux density of N continuous entities, which are defined in 3-dimensional space, are not enough to constitute the (entangled) wave function defined in a 3N-dimensional space.

Therefore, it is arguably that the wave function of an N-body system describes the state of N discrete particles with mass and charge in 3-dimensional space. Concretely speaking, at a given instant, the positions of these N particles in 3-dimensional space are represented by a point in a 3N-dimensional configuration space. During an infinitesimal time interval around the instant, these particles move in the real space, and correspondingly, this point moves in the configuration space, and its motion forms a cloud in the configuration space, which is described by density and flux density or the wave function composed of these two quantities.

3.2 Argument two

An analysis of the mass and charge distributions of a quantum system and their origin may provide further support for the existence of particles, and may also help find how these particles move. In the following, we will mainly analyze one-body systems.

First of all, we will argue that for a one-body quantum system with mass m and charge Q, the corresponding physical entity described by its wave function, $\psi(x,t)$, is massive and charged, and the effective mass and charge density in each position x is $|\psi(x,t)|^2 m$ and $|\psi(x,t)|^2 Q$, respectively. The existence of effective mass and charge distributions can be seen from the Schrödinger equation that governs the evolution of the system. The Schrödinger equation for the system in an external electrostatic potential $\varphi(x)$ is

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = [-\frac{\hbar^2}{2m}\nabla^2 + Q\varphi(x)]\psi(x,t). \qquad (7)$$

The electrostatic interaction term $Q\varphi(x)\psi(x,t)$ in the equation indicates that the physical entity described by $\psi(x,t)$ has electrostatic interaction with the external potential in all regions where $\psi(x,t)$ is nonzero. The existence of electrostatic interaction with an external potential in a given region means that there exists electric charge distribution in the region, which has efficiency to interact with the potential and is responsible for the interaction. Therefore, the physical entity described by $\psi(x,t)$ is charged in all regions where $\psi(x,t)$ is nonzero. In other words, for a charged one-body quantum system, the corresponding physical entity described by its wave function has effective charge distribution in space. Similarly, the existence of effective mass distribution can be seen from the Schrödinger equation for a one-body quantum system in an external gravitational potential:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=[-\frac{\hbar^2}{2m}\nabla^2+mV_G(x)]\psi(x,t). \tag{8}$$

The gravitational interaction term $mV_G(x)\psi(x,t)$ in the equation indicates that the (passive gravitational) mass of the system distributes throughout the whole region where its wave function $\psi(x,t)$ is nonzero. In other words, the physical entity described by the wave function also has effective mass distribution.

The effective mass and charge distributions manifest more directly during a protective measurement, which can measure the expectation values of observables on a single quantum system (Aharonov and Vaidman 1993, Aharonov, Anandan and Vaidman 1993). Consider an ideal protective measurement of the charge of a quantum system with charge Q in an infinitesimal spatial region dv around x_n. This is equivalent to measuring the following observable:

$$A=\begin{cases}Q, & \text{if } x_n\in dv,\\ 0, & \text{if } x_n\notin dv.\end{cases} \tag{9}$$

During the measurement, the wave function of the measuring system, $\phi(x,t)$, will obey the following Schrödinger equation:

$$i\hbar\frac{\partial\phi(x,t)}{\partial t}=-\frac{\hbar^2}{2M}\nabla^2\phi(x,t)+k\frac{e\cdot|\psi(x_n,t)|^2dvQ}{|x-x_n|}\phi(x,t), \tag{10}$$

where M and e are the mass and charge of the measuring system, respectively, and k is the Coulomb constant. From this equation, it can be seen that the property of the measured system in the measured position x_n that has efficiency to influence the measuring system is $|\psi(x_n,t)|^2dvQ$, the effective charge there[5]. This is also the result of the protective measurement,

[5]Note that even in standard quantum mechanics, it is also assumed that the above interaction term indicates that there is a charge $|\psi(x_n,t)|^2dvQ$ in the region dv. If there exists no effective charge in the measured position which is responsible for the shift of the pointer of the measuring device there, then a new entity existing elsewhere (which is different from the entity described by the wave function) and a new dynamics for the entity (which is different from the Schrödinger equation) will be needed for a realist explanation of the shift of the pointer. For example, suppose the measured wave function is localized in two widely-separated regions and the measurement is made in one region. If there is nothing in the measured region, then the result of the protective measurement made there

$\langle A\rangle = |\psi(x_n,t)|^2 dvQ$. When divided by the volume element, it gives the effective charge density $|\psi(x,t)|^2 Q$[6].

Now we will analyze the physical origin of the effective charge distribution[7]. What kind of entity or process generates the effective charge distribution in space or the physical efficiency of the quantity $|\psi(x,t)|^2 dvQ$? It can be expected that the answer will help understand the ontological meaning of $|\psi(x,t)|^2$ and the wave function itself. There are two possibilities: the effective charge distribution of a one-body system can be generated by either (1) a continuous charge distribution with density $|\psi(x,t)|^2 Q$ or (2) the motion of a discrete point charge Q with spending time $|\psi(x,t)|^2 dvdt$ in the infinitesimal spatial volume dv around x in the infinitesimal time interval $[t, t+dt]$[8]. Correspondingly, the underlying physical entity is either a continuous entity or a discrete particle. For the first possibility, the charge distribution exists throughout space at the same time, while for the second possibility, at every instant there is only a localized, point-like particle with the total charge of the system, and its motion during an infinitesimal time interval forms the effective charge distribution. Concretely speaking, at a particular instant the charge density of the particle in each position is either zero (if the particle is not there) or singular (if the particle is there), while the time average of the density during an infinitesimal time interval around

or the shift of the pointer of the measuring device there can only be explained by the existence of certain entity in other regions via action at a distance. This will require a wholly new theory different from quantum theories, which will not be considered here.

[6]Similarly, we can protectively measure another observable $B = \frac{\hbar}{2mi}(A\nabla + \nabla A)$. The measurements will give the electric flux density $j_Q(x,t) = \frac{\hbar Q}{2mi}(\psi^* \nabla\psi - \psi\nabla\psi^*)$ everywhere in space (Aharonov and Vaidman 1993).

[7]Historically, the charge density interpretation for electrons was originally suggested by Schrödinger in his fourth paper on wave mechanics (Schrödinger 1926). Schrödinger clearly realized that the charge density cannot be classical because his equation does not include the usual classical interaction between the densities. Presumably since people thought that the charge density could not be measured and also lacked a consistent physical picture, this interpretation was soon rejected and replaced by Born's probability interpretation. Now protective measurements help re-endow the effective charge distribution of an electron with reality. The question is then how to find a consistent physical explanation for it. Our following analysis may be regarded as a further development of Schrödinger's original idea to some extent. For more discussions on Schrödinger's charge density interpretation see Bacciagaluppi and Valentini (2009) and Gao (2013).

[8]Note that the expectation value of an observable at a given instant such as $\langle A\rangle = |\psi(x_n,t)|^2 dvQ$ is either the physical property of a quantum system at the precise instant (like the position of a classical particle) or the limit of the time-averaged property of the system at the instant (like the standard velocity of a classical particle). These two interpretations correspond to the above two possibilities. For the later, the observable assumes an eigenvalue at each instant, and its value spreads all eigenvalues during an infinitesimal time interval. Moreover, the spending time in each eigenvalue is proportional to the modulus squared of the wave function of the system there. In this way, such ergodic motion generates the expectation value of the observable in an infinitesimal time interval (cf. Aharonov and Cohen 2014). We will discuss later whether this picture of ergodic motion applies to properties other than position.

the instant gives the effective charge density. Moreover, the motion of the particle is ergodic in the sense that the integral of the formed charge density in any region is equal to the expectation value of the total charge in the region.

In the following, we will argue that the existence of a continuous charge distribution may lead to inconsistency. If the charge distribution is continuous and exists throughout space at the same time, then any two parts of the distribution, like two electrons, will arguably have electrostatic interaction described by the interaction potential term in the Schrödinger equation. However, the existence of such electrostatic self-interaction for a quantum system contradicts the superposition principle of quantum mechanics (at least for microscopic systems such as electrons). Moreover, the existence of the electrostatic self-interaction for the effective charge distribution of an electron is incompatible with experimental observations either. For example, for the electron in the hydrogen atom, since the potential of the electrostatic self-interaction is of the same order as the Coulomb potential produced by the nucleus, the energy levels of hydrogen atoms would be remarkably different from those predicted by quantum mechanics and confirmed by experiments if there existed such electrostatic self-interaction. By contrast, if there is only a localized particle at every instant, it is understandable that there exists no such electrostatic self-interaction for the effective charge distribution formed by the motion of the particle. This is consistent with the superposition principle of quantum mechanics and experimental observations.

Here is a further clarification of this argument. It can be seen that the non-existence of self-interaction of the charge distribution poses a puzzle. According to quantum mechanics, two charge distributions such as two electrons, which exist in space at the same time, have electrostatic interaction described by the interaction potential term in the Schrödinger equation, but for the effective charge distribution of an electron, any two parts of the distribution have no such electrostatic interaction. Facing this puzzle one may have two choices. The first one is simply admitting that the non-existence of self-interaction of the effective charge distribution is a distinct feature of the laws of quantum mechanics, but insisting that the laws are what they are and no further explanation is needed. However, this choice seems to beg the question and unsatisfactory in the final analysis. A more reasonable choice is to try to explain this puzzling feature, e.g. by analyzing its relationship with the existent form of the effective charge distribution. The effective charge distribution has two possible existing forms after all. On the one hand, the non-existence of self-interaction of the distribution may help determine which possible form is the actual one. For example, one possible form is inconsistent with this distinct feature, while the other possible form is consistent with it. On the other hand, the actual existent form of the effective charge distribution may also help explain the non-existence of self-interaction of the distribution. This is just what the above argument has done. The analysis establishes a connection between the non-existence of self-interaction of the effective charge distribution and the actual existent form of the distribution. The reason why two wavepackets of an electron, each of which has part of the electron's charge in efficiency, have no electrostatic interaction is that these two wavepackets do not exist at the same time, and their effective charges are formed by the motion of a localized particle with the total charge of the electron. Since there is only a localized particle at every instant, it is understandable that there exists no electrostatic self-interaction of the effective charge distribution formed by the motion of the particle. By contrast, if the two wavepackets with charges, like two electrons, existed at the same time, then they would also have the same form of electrostatic interaction as that between two electrons[9].

This analysis of electrostatic self-interaction also applies to many-body systems. We can also protectively measure the charge density (and electric flux density) of a many-body system in our three-dimensional space. A protective measurement of the observable $\sum_{i=1}^{N} A_i$ on an N-body system whose wave function is $\psi(x_1, x_2, ...x_N, t)$ yields

$$\sum_{i=1}^{N} \langle A_i \rangle = \sum_{i=1}^{N} \int ... \int Q_i |\psi(x_1, ...x_{i-1}, x, x_{i+1}, ...x_N, t)|^2 dv_1 ... dv_{i-1} dv_{i+1} ... dv_N dv, \tag{11}$$

where Q_i is the charge of the i-th subsystem. When divided by the volume element dv, it yields the charge density in space. Like a one-body system, the effective charge distribution of an N-body system are arguably generated by the ergodic motion of N charged particles, where the spending time of particle 1 with charge Q_1 in an infinitesimal spatial volume dv_1 around x_1 and particle 2 with charge Q_2 in an infinitesimal spatial volume dv_2 around x_2 ... and particle N with charge Q_N in an infinitesimal spatial volume dv_n around x_N is $|\psi(x_1, x_2, ...x_N, t)|^2 dv_1 ... dv_N dt$ in the infinitesimal time interval $[t, t + dt]$, or equivalently, the spending time of the N particles in an infinitesimal volume dV around each position $(x_1, x_2, ...x_N)$ in the 3N-dimensional configuration space in the infinitesimal time interval $[t, t + dt]$ is $|\psi(x_1, x_2, ...x_N, t)|^2 dV dt$.

Which sort of ergodic motion? This is a further question that needs to be answered. If the ergodic motion of particles is continuous, then it can only form the effective mass and charge distributions during a finite time interval around a given instant[10]. But according to quantum mechanics, the effective

[9]Note that this argument does not assume that charges which exist at the same time are classical charges and they have classical interaction. By contrast, the Schrödinger-Newton equation, which was proposed by Diósi (1984) and Penrose (1998), treats the mass distribution of a quantum system as classical.

[10]For other objections to classical ergodic models see Aharonov and Vaidman (1993)

mass and charge distributions at a given instant are required to be formed by the ergodic motion of particles during an infinitesimal time interval around the instant. Thus it seems that the ergodic motion of particles cannot be continuous but must be discontinuous. This is at least what the existing theory says. This conclusion can also be reached by analyzing a specific example. Consider an electron in a superposition of two energy eigenstates in two separate boxes. In this example, even if one assumes that the electron can move with infinite velocity, it cannot *continuously* move from one box to another due to the restriction of box walls. Therefore, any sort of continuous motion cannot generate the effective charge distribution that exists in both boxes[11].

Since quantum mechanics does not provide further information about the positions of the particles at each instant, the discontinuous motion of particles described by the theory is also essentially random. Moreover, the spending time of the N particles of an N-body system around N positions in 3-dimensional space being proportional to the modulus squared of the wave function of the system there means that the (objective) probability density for the particles to appear in the positions is also proportional to the modulus squared of the wave function there (and for normalized wave functions they are equal). This ensures that the motion of these particles forms the right mass and charge distributions. In addition, the N particles as a whole may also have an indeterministic disposition or propensity (as a probabilistic instantaneous condition) which determines the probability density for them to appear in the N positions in space.

To sum up, based on an analysis of the mass and charge distributions of a quantum system and their origin, we have argued that the physical entities described by the wave function are discrete, localized particles, and the ergodic motion of the particles, which forms the effective mass and charge distributions measurable by protective measurements, is discontinuous and random, and the probability density for the particles to appear in every group of positions is equal to the modulus squared of the wave function of the system there.

3.3 Argument three

In the following, we will give another argument supporting the existence of particles and their discontinuous motion based on an analysis of entangled states.

Consider a two-body system whose wave function is defined in a six-

and Aharonov, Anandan and Vaidman (1993).

[11]One may object that this is merely an artifact of the idealization of infinite potential. However, even in this ideal situation, the ergodic model should also be able to generate the effective charge distribution by means of some sort of ergodic motion of the electron; otherwise it will be inconsistent with quantum mechanics.

dimensional space. We first suppose the wave function of the system is localized in one position $(x_1, y_1, z_1, x_2, y_2, z_2)$ in the space at a given instant. This wave function can be decomposed into a product of two wave functions which are localized in positions (x_1, y_1, z_1) and (x_2, y_2, z_2) in our ordinary three-dimensional space, respectively. It is uncontroversial that this wave function describes two independent physical entities, which are localized in positions (x_1, y_1, z_1) and (x_2, y_2, z_2) in our three-dimensional space, respectively. Moreover, as argued above, the Schrödinger equation that governs the evolution of the system may further indicate that these two physical entities have masses m_1 and m_2 (as well as charges Q_1 and Q_2 etc), respectively.

Next, suppose the wave function of the two-body system is localized in two positions $(x_1, y_1, z_1, x_2, y_2, z_2)$ and $(x_1', y_1', z_1', x_2', y_2', z_2')$ in the six-dimensional space at a given instant. This is an entangled state, which can be generated from a non-entangled state by the Schrödinger evolution of the system. The existence of similar entangled states has also been confirmed by experiments. In this case, there are still two physical entities with the same masses and charges, since these properties of the system do not change during the evolution, and it is arguably that the Schrödinger evolution does not create or annihilate physical entities either. According to the above analysis, the wave function of the two-body system being localized in position $(x_1, y_1, z_1, x_2, y_2, z_2)$ means that physical entity 1 with mass m_1 and charge Q_1 exists in position (x_1, y_1, z_1) in three-dimensional space, and physical entity 2 with mass m_2 and charge Q_2 exists in position (x_2, y_2, z_2) in three-dimensional space. Similarly, the wave function of the two-body system being localized in position $(x_1', y_1', z_1', x_2', y_2', z_2')$ means that physical entity 1 exists in position (x_1', y_1', z_1') in three-dimensional space, and physical entity 2 exists in position (x_2', y_2', z_2') in three-dimensional space. Moreover, since the physical entities described by the wave function exist in the region of space where their wave function is not zero, the wave function of the two-body system being localized in both positions $(x_1, y_1, z_1, x_2, y_2, z_2)$ and $(x_1', y_1', z_1', x_2', y_2', z_2')$ means that the above two physical situations both exist in reality. The question is: In what form?

An obvious existent form is that physical entity 1 exists in both positions (x_1, y_1, z_1) and (x_1', y_1', z_1'), and physical entity 2 exists in both positions (x_2, y_2, z_2) and (x_2', y_2', z_2'). However, since the physical entities described by the wave function do not exist in the region of space where the wave function is zero, when physical entity 1 exists in (x_1, y_1, z_1), physical entity 2 cannot exist in (x_2', y_2', z_2'), and when physical entity 1 exists in (x_1', y_1', z_1'), physical entity 2 cannot exist in (x_2, y_2, z_2), or vice versa. In other words, the wave function that describes this existent form should be localized in four positions $(x_1, y_1, z_1, x_2, y_2, z_2)$, $(x_1', y_1', z_1', x_2', y_2', z_2')$, $(x_1, y_1, z_1, x_2', y_2', z_2')$, and $(x_1', y_1', z_1', x_2, y_2, z_2)$ in the six-dimensional space.

Although the above two situations cannot exist at the same time at a single instant, they may exist "at the same time" during an infinitesimal time interval around the instant[12]. Concretely speaking, the situation in which physical entity 1 is in (x_1, y_1, z_1) and physical entity 2 is in (x_2, y_2, z_2) exists in one part of continuous time, and the situation in which physical entity 1 is in $(x_1^{'}, y_1^{'}, z_1^{'})$ and physical entity 2 is in $(x_2^{'}, y_2^{'}, z_2^{'})$ exists in the other part. The restriction is that the temporal part in which each situation exists cannot be a continuous time interval during an arbitrarily short time interval; otherwise the wave function describing the state in the time interval will be not the original superposition of two branches, but one of the branches. This means that the set of the instants when each situation exists is not a continuous set but a discontinuous, dense set. At some discontinuous instants, physical entity 1 with mass m_1 and charge Q_1 exists in position (x_1, y_1, z_1), and physical entity 2 with mass m_2 and charge Q_2 exists in position (x_2, y_2, z_2), and at other discontinuous instants, physical entity 1 exists in position $(x_1^{'}, y_1^{'}, z_1^{'})$, and physical entity 2 exists in position $(x_2^{'}, y_2^{'}, z_2^{'})$. By this way of time division, the above two situations exist "at the same time" during an arbitrarily short time interval or during an infinitesimal time interval around the given instant.

This way of time division implies a picture of discontinuous motion for the involved physical entities. It is as follows. Physical entity 1 with mass m_1 and charge Q_1 jumps discontinuously between positions (x_1, y_1, z_1) and $(x_1^{'}, y_1^{'}, z_1^{'})$, and physical entity 2 with mass m_2 and charge Q_2 jumps discontinuously between positions (x_2, y_2, z_2) and $(x_2^{'}, y_2^{'}, z_2^{'})$. Moreover, they jump in a precisely simultaneous way. When physical entity 1 jumps from position (x_1, y_1, z_1) to position $(x_1^{'}, y_1^{'}, z_1^{'})$, physical entity 2 always jumps from position (x_2, y_2, z_2) to position $(x_2^{'}, y_2^{'}, z_2^{'})$, or vice versa. In the limit situation where position (x_2, y_2, z_2) is the same as position $(x_2^{'}, y_2^{'}, z_2^{'})$, physical entities 1 and 2 are no longer entangled, while physical entity 1 with mass m_1 and charge Q_1 still jumps discontinuously between positions (x_1, y_1, z_1) and $(x_1^{'}, y_1^{'}, z_1^{'})$. This means that the picture of discontinuous motion also exists for one-body systems. As argued before, since quantum mechanics does not provide further information about the positions of physical entities at each instant, the discontinuous motion described by the theory is also essentially random.

The above analysis can be extended to an arbitrary entangled wave function of an N-body system. Since each physical entity is only in one position in space at each instant, it may well be called particle. Here the concept of particle is used in its usual sense. A particle is a small localized object with mass and charge etc, and it is only in one position in space at an instant. Therefore, the physical entities described by the wave function such as physical entities 1 and 2 are localized particles. Moreover, the motion of

[12]This means that the state of the physical entity described by the wave function is defined during an infinitesimal time interval around a given instant, like the standard velocity in classical mechanics. We have discussed this point when analyzing the origin of mass and charge distributions of a quantum system in the last section.

these particles is not continuous but discontinuous and random in nature, and especially, the motion of entangled particles is precisely simultaneous.

4 Discussions

We have been analyzing random discontinuous motion of particles in position space. Does the picture of random discontinuous motion exist for other observables such as momentum and energy? Since there are also momentum wave functions etc in quantum mechanics, it seems tempting to assume that the above interpretation of the wave function in position space also applies to the wave functions in momentum space etc. This means that when a particle is in a superposition of the eigenstates of an observable, it also undergoes random discontinuous motion among the eigenvalues of this observable. For example, a particle in a superposition of momentum eigenstates also undergoes random discontinuous motion among all momentum eigenvalues. At each instant the momentum of the particle is definite, randomly assuming one of the momentum eigenvalues with probability given by the modulus squared of the wave function at this momentum eigenvalue, and during an infinitesimal time interval around the instant the momentum of the particle spreads throughout all momentum eigenvalues.

However, there is also another possibility, namely that the picture of random discontinuous motion exists only for position, while momentum and energy etc are not instantaneous properties of a particle and they do not undergo random discontinuous change either. There are several reasons supporting this possibility. The first is that our previous arguments for random discontinuous motion of particles apply only to position, not to other observables such as momentum and energy etc. For example, since the interaction Hamiltonian for a many-particle system relates to the positions of these particles, not to their momenta and energies, the previous analysis of electrostatic self-interaction applies only to position. Next, the Kochen-Specker theorem requires that under certain reasonable assumptions only a certain number of observables can be assigned definite values at all times (Kochen and Specker 1967). This strongly suggests that the picture of random discontinuous motion exist only for a certain number of observables. Moreover, since there are infinitely many other observables and these observables arguably have the same status, this may further imply that the picture of random discontinuous motion does not exist for any observable other than position. Lastly, the meaning of observables as Hermitian operators acting on the wave function lies in the corresponding ways to decompose (and also to measure) the same wave function. For example, position and momentum reflect two ways to decompose the same spatial wave function. In this sense, the existence of random discontinuous motion for momentum will be redundant.

Therefore, it seems more reasonable to assume that the picture of random discontinuous motion exists only for position. On this view, the position of a particle is the only instantaneous property of the particle defined at instants (besides its wave function), while momentum and energy are properties relating to the state of motion of the particle (e.g. momentum and energy eigenstates), which is formed by the motion of the particle during an infinitesimal time interval around a given instant[13]. Certainly, when a particle is in a momentum or energy eigenstate, we may still say that the particle has definite momentum or energy, whose value is the corresponding eigenvalue. Moreover, when a particle is in a momentum or energy superposition state and the momentum or energy branches are well separated in space, we may also say that the particle has definite momentum or energy in each separated region.

Finally, we note that spin is a more distinct property. Since the spin of a free particle is always definite along one direction, the spin of the particle does not undergo random discontinuous motion, though a spin eigenstate along one direction can always be decomposed into two different spin eigenstates along another direction. But if the spin state of a particle is entangled with its spatial state due to interaction and the branches of the entangled state are well separated in space, the particle in different branches will have different spin, and it will also undergo random discontinuous motion between these different spin states. This is the situation that usually happens during a spin measurement.

5 Implications

In this section, we will briefly discuss possible implications of the new interpretation of the wave function for the solutions to the measurement problem.

It can be seen that random discontinuous motion of particles, unlike the continuous motion of particles in the de Broglie-Bohm theory or Bohmian mechanics, does not provide a solution to the measurement problem. This is not against expectation, since it only provides an ontological interpretation of the wave function, and what the precise laws of motion are still needs to be determined. However, as we will argued below, this ontological interpretation of the wave function might also have implications for the existing solutions to the measurement problem.

[13]Note that the particle position here is different from the position property represented by the position observable in quantum mechanics, and the latter is also a property relating only to the state of motion of the particle such as position eigenstates. In addition, for random discontinuous motion the position of a particle in a position superposed state is indeterminate in the sense of usual hidden variables, though it does have a definite value at each instant. Another way to see this is to realize that random discontinuous motion of particles alone does not provide a way to solve the measurement problem. For further discussions see Gao (2013).

An important aspect of the measurement problem is to explain the origin of the Born probabilities or the probabilities of measurement results. According to our interpretation of the wave function in terms of random discontinuous motion of particles, the ontological meaning of the modulus squared of the wave function of an electron in a given position is that it represents the probability density that the electron as a particle appears in this position, while according to the Born rule, the modulus squared of the wave function of the electron in the position also gives the probability density that the electron is found there. It is hardly conceivable that these two probabilities have no connection. On the contrary, it seems natural to assume that the origin of the Born probabilities is the random discontinuous motion of particles. If this assumption turns out to be true, then it will have significant implications for the solution to the measurement problem, because the existing solutions have not accommodated this assumption. In Bohmian mechanics (Goldstein 2013), the Born probabilities are epistemic. In the latest formulation of the many-worlds theory (Wallace 2012), the Born probabilities are subjective. In dynamical collapse theories, although the Born probabilities are objective, it is usually assumed that the randomness originates from a classical noise field independent of the wave function of the studied system (Ghirardi 2011). In short, none of these main solutions to the measurement problem assumes that the Born probabilities originate from the wave function itself.

Therefore, if the Born probabilities originate from the objective probabilities inherent in the random discontinuous motion of particles described by the wave function, then all these realistic alternatives to standard quantum mechanics need to be reformulated. The reformulation may be easier for some alternatives, but more difficult for others. For example, it is relatively easy to find a dynamical collapse model where the chooser or the noise source that collapses the wave function is the underlying random discontinuous motion of particles (Gao 2013). Moreover, it seems possible or even more promising to reformulate the many-worlds theory or the many-minds theory in terms of the objective probabilities inherent in the random discontinuous motion of particles. However, it seems difficult to find a new formulation of Bohmian mechanics in which the probabilities of measurement results come from the wave function.

Certainly, if one rejects the interpretation of the wave function in terms of random discontinuous motion of particles, then the above implications will be totally irrelevant. However, these analyses at least indicate that understanding the origin of the Born probabilities may be a key to solving the measurement problem. Moreover, if one rejects this interpretation of the wave function, then one must reject either the arguments supporting the interpretation or the basic realistic assumption used in these arguments. This realistic assumption is that the wave function of a quantum system at each instant describes the state of a physical entity or many physical entities either at the instant or during an infinitesimal interval around the instant. A formulation of Bohmian mechanics does reject this assumption, and it assumes a nomological interpretation of the wave function (Goldstein 2013). On the other hand, if one accepts this realistic assumption, then one needs to find loopholes in our arguments in order to avoid the above implications for the solutions to the measurement problem.

6 Conclusions

The physical meaning of the wave function is an important interpretative problem of quantum mechanics. Notwithstanding more than eighty years' developments of the theory, it remains hot topic of debate. In this paper, we propose a new approach for solving this problem, which is to analyze the mass and charge properties of a quantum system. It is realized that the Schrödinger equation, which governs the evolution of a quantum system, contains more information about the system than the wave function of the system, such as the mass and charge properties of the system, which, as we have argued, may help understand the ontological meaning of the wave function. These analyses lead to a new ontological interpretation of the wave function in terms of random discontinuous motion of particles. According to this interpretation, quantum mechanics, like Newtonian mechanics, also deals with the motion of particles in space and time. Microscopic particles such as electrons are still particles, but they move in a discontinuous and random way. The wave function describes the state of random discontinuous motion of particles, and in particular, the modulus squared of the wave function gives the probability density that the particles appear in every possible group of positions in space. Quantum mechanics, in this way, is essentially a physical theory of the laws of random discontinuous motion of particles. It is a further and also harder question what the precise laws are, e.g. whether the wave function undergoes a stochastic and nonlinear collapse evolution.

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