Are there limits of objectivity in the quantum world?

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

November 23, 2018

Abstract

Healey recently argued that a version of the extended Wigner's friend Gedankenexperiment due to Masanes establishes a contradiction between the universal applicability of unitary quantum theory and the assumption of definite outcomes [Found. Phys. 48, 1568 (2018)]. In this Comment, I show that Healey's analysis is problematic and his conclusion is not true.

In a recent paper, Healey presented a penetrating analysis of the extended Wigner's friend Gedankenexperiment [1]. The Gedankenexperiment aims to show that the universal applicability of unitary quantum theory is inconsistent with the assumption that a well-conducted measurement always has a definite physical outcome. Healey analyzed three arguments related to the Gedankenexperiment. He argued that Brukner's argument [2] and Frauchiger and Renner's argument [3] fail to derive the inconsistency, while the argument due to Lluis Masanes [4] does establish a contradiction between the universal applicability of unitary quantum theory and the assumption of definite outcomes. In this Comment, I will show that Healey's analysis of Masanes' argument is problematic and his conclusion is not true.

Masanes' argument is set in the context of a Wigner's friend variant of the Bell experiment. In the Gedankenexperiment as formulated by Healey [1], there are two observers Carol and Dan and two superobservers Alice and Bob. A superobserver can undo a measurement, and the existence of such superobservers is permitted in principle by unitary quantum theory. Each of Alice and Bob are in their own separate laboratories, totally physically isolated. Carol occupies her own separate laboratory, initially totally physically isolated from Alice's, and Dan occupies his own separate laboratory, initially totally physically isolated from Bob's. Carol and Dan share a EPR pair of spin-1/2 particles, 1 and 2, in the spin singlet state:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2). \tag{1}$$

Dan first measures the spin of particle 2 at angles d, and then Carol measures the spin of particle 1 at angles c, and then Alice undoes Carol's measurement and measures the spin of particle 1 at angles a, and finally Bob undoes Dan's measurement and measures the spin of particle 2 at angles b. To make this possible, after performing Carol's measurement particle 1 is transferred out of her lab and into Alic's lab, and after performing Dan's measurement particle 2 is transferred out of his lab and into Bob's lab. Note that the restores the states of Alice and the particles to their initial states,

Suppose each measurement result is $+1$ or -1, corresponding to spin up or spin down. Then when the above situation is repeated very many times, each time with a different EPR pair, we will build up a joint probability distribution of results for the four outcomes in each trial, $\rho(a, b, c, d)$. The existence of a joint probability distribution then implies for the marginals:

$$|corr(a, b) + corr(b, c) + corr(c, d) - corr(a, d)| \leq 2, \tag{2}$$

where $corr(a, b), corr(b, c), corr(c, d)$ and $corr(a, d)$ are the statistical correlation functions for each pair of these outcomes.

Given the time order of these measurements, we can calculate the correlation functions $corr(a, b), corr(c, d)$ and $corr(a, d)$ according to quantum mechanics, which are $corr(a, b) = -cos(a - b)$, $corr(c, d) = -cos(c - d)$, and $corr(a, d) = -cos(a - d)$. For example, after Dan measures the spin of particle 2 at angles d, Carol measures the spin of particle 1 at angles c. Then, quantum mechanics predicts that the correlation function $corr(a, d)$ satisfies the relation $corr(a, d) = -cos(a - d)$. But we cannot derive the similar relation for the correlation function $corr(b, c)$, since when Bob measures the spin of particle 2 at angles b, Carol's measurement on particle 1 has been undone and his outcome no longer exists.

Here Healey used a trick permitted by special relativity under a certain condition. He calculated the correlation function $corr(b, c)$ in another inertial frame, in which Carol's measurement precedes Dan's and Bob's precedes Alice's. This is possible when Alice's and Carol's measurements and Bob's and Dan's measurements are spacelike separated, and Alice's lab and Carol's lab, which are at rest relative to each other, and Bob's lab and Dan's lab, which are at rest relative to each other, are in relative motion. In this frame, since when Bob measures the spin of particle 2 at angles b, the result of Carol's measurement of the spin of particle 1 at angles b has not been erased, quantum mechanics predicts that the correlation function $corr(b, c)$ satisfies the relation $corr(b, c) = -cos(b - c)$. Since the expectation values of the same joint measurements observed in two inertial frames should be the same, the correlation functions such as $corr(b, c)$ are invariant under changes of frame. Then, in every inertial frame, we have $corr(a, b) = -cos(a - b)$, $corr(b, c) = -cos(b - c)$, $corr(c, d) = -cos(c - d)$, and $corr(a, d) = -cos(a - d)$. Thus the above inequality (2) will become a Bell inequality:

$$|E(a,b) + E(b,c) + E(c,d) - E(a,d)| \leq 2, \tag{3}$$

where $E(a, b) = -cos(a-b)$, $E(b, c) = -cos(b-c)$, $E(c, d) = -cos(c-d)$, and $E(a, d) = -cos(a - d)$. It is well known that quantum mechanics predicts violation of a Bell inequality for certain choices of directions a, b, c, d. Thus we have derived a contradiction.

Healey argued that this contradiction results from the combination of the universal applicability of unitary quantum theory and the assumption of definite outcomes. Thus, he concluded, "the universal applicability of unitary quantum theory implies (with probability approaching 1) that there is no consistent assignment of values to the (supposedly definite, physical) outcomes of the measurements in the sequence of trials there considered." [1]. Healey further thought that this resuslt should make us reconsider the extent and nature of the objectivity of measurement outcomes.

However, there is a potential loophole in Healey's argument, which may make it invalid. The key is to notice that we *can* derive the correlation function $corr(b, c)$ in the original inertial frame based on the known correlation functions $corr(a, b), corr(c, d)$ and $corr(a, d)$ according to quantum mechanics, which turns out to be not $E(b, c)$ as Healey calculated, and thus the above inequality (2) is not violated. Let me use a simple example to illustrate how to derive the correlation function $corr(b, c)$ in the original inertial frame.

Consider the case where $c = b$ and $d = a$. In the original inertial frame, Dan first measures the spin of particle 2 at angles d, and then Carol measures the spin of particle 1 at angles c, and then Alice undoes Carol's measurement and measures the spin of particle 1 at angles a, and finally Bob undoes Dan's measurement and measures the spin of particle 2 at angles b. Then, according to quantum mechanics, when the result of Carol is +1, the conditional probability of Dan's result being +1 is $sin^2(\frac{c-d}{2})$, the conditional probability of Alice's result being −1 when Dan's result is +1 is 1, and the conditional probability of Bob's result being +1 when Alice's result is −1 is $cos^2(\frac{a-b}{2})$. When the result of Carol is +1, the conditional probability of Dan's result being −1 is $cos^2(\frac{c-d}{2})$, the conditional probability of Alice's result being +1 when Dan's result is −1 is 1, and the conditional probability of Bob's result being +1 when Alice's result is +1 is $sin^2(\frac{a-b}{2})$. Then, when the result of Carol is +1, the total conditional probability of Bob's result being +1 is $2sin^2(\frac{a-b}{2})cos^2(\frac{a-b}{2})$. Note that $c-d = b-a$ when $c = b$ and $d = a$.

Similarly, when the result of Carol is $+1$, the total conditional probability of Bob's result being -1 is $1 - 2sin^2(\frac{a-b}{2})cos^2(\frac{a-b}{2})$, which is not equal to 1 in general. This already shows that the relation $corr(b,c) = -cos(b-c)$, which means that the results of Bob and Carol are always anti-correlated, is wrong in the original inertial frame.

On the other hand, when the result of Carol is -1, the total conditional probability of Bob's result being -1 is $2sin^2(\frac{a-b}{2})cos^2(\frac{a-b}{2})$, and the total conditional probability of Bob's result being $+1$ is $1 - 2sin^2(\frac{a-b}{2})cos^2(\frac{a-b}{2})$. Then we can calculate the correlation function $corr(b,c)$, which turns out to be $corr(b,c) = 4sin^2(\frac{a-b}{2})cos^2(\frac{a-b}{2}) - 1$.

It can be seen that the above inequality (2) is not a Bell inequality and it is not violated when using this right correlation function $corr(b,c)$. The l.h.s of the inequality (2) is:

$$\begin{aligned} & |corr(a,b) + corr(b,c) + corr(c,d) - corr(a,d)| \\ = & |cos(a-b) + 4sin^2(\frac{a-b}{2})cos^2(\frac{a-b}{2}) - 1 - cos(c-d) + cos(a-d)| \\ = & |4sin^2(\frac{a-b}{2})cos^2(\frac{a-b}{2}) - 2cos(a-b)| \\ = & 2|2sin^2(\frac{a-b}{2})cos^2(\frac{a-b}{2}) - cos^2(\frac{a-b}{2}) + sin^2(\frac{a-b}{2})| \end{aligned} \tag{4}$$

Let $\alpha = cos^2(\frac{a-b}{2})$, then we have:

$$\begin{aligned} |corr(a,b) + corr(b,c) + corr(c,d) - corr(a,d)| &= 2|2\alpha(1-\alpha) - \alpha + 1 - \alpha| \\ &= 2|1 - 2\alpha^2| \end{aligned} \tag{5}$$

Since $0 \leq \alpha \leq 1$, we have $2|1 - 2\alpha^2| \leq 2$, and thus the inequality (2) is satisfied.

This means that when calculating the four correlation functions in the same original inertial frame, then there will be no violation of the inequality (2). This result also holds true in every other inertial frame. Since the predictions of quantum mechanics is complete in each inertial frame, this result already indicates that the combination of the universal applicability of unitary quantum theory and the assumption of definite outcomes does not lead to a contadiction, and in particular, the universal applicability of unitary quantum theory does not imply that there is no consistent assignment of values to the outcomes of the measurements in the sequence of trials considered above.

There is a deeper reason why Healey's calculation of $corr(b,c)$ is problematic. It is that Healey's derivation of the relation $corr(b,c) = -cos(b-c)$ cannot go through in the non-relativistic domain. In the non-relativistic domain where quantum mechanics holds, the time order of spacelike separated events is invariant under the Galileo transformations of inertial frame, and thus we cannot derive the relation $corr(b, c) = -cos(b - c)$ according to quantum mechanics in any other inertial frame for the same reason as in the original inertial frame (i.e. because when Bob measures the spin of particle 2 at angles b, Carol's measurement on particle 1 has been undone and his outcome no longer exists). However, one should be able to derive the correlation function $corr(b, c)$ in the non-relativistic domain. The correlation functions in (non-relativistic) quantum mechanics should not depend on special relativity after all. This is just what I did above, in which the correlation function $corr(b, c)$ is derived in the non-relativistic domain in the original inertial frame, without resorting to anything beyond quantum mechanics.

Certainly, there is still a question left, namely why the correlation function $corr(b, c)$ is not invariant under changes of frame in the relativistic domain. In my view, this is indeed a deep puzzle. In order to solve the puzzle, it seems that one must admit that unitary quantum theory and special relativity are incompatible [5-7]. This is not completely beyond expectations. As Pusey noted [4], the Frauchiger and Renner's argument based on Hardy's paradox is essentially the same as the argument given by Masanes based on the Bell inequality. But the former, which does not resort to special relativity, does not lead to a puzzle, as Healey showed, while the latter, if resorting to special relativity, will lead to the above puzzle. Thus it seems that the origin of the puzzle must be special relativity. Since a further discussion of how to solve the puzzle is beyond the scope of this Comment, we stop here.

Acknowledgments

I wish to thank Richard Healey for helpful discussion at the online Workshop on Wigner's Friend 2018, which was hosted by International Journal of Quantum Foundations. This work is supported by the National Social Science Foundation of China (Grant No. 16BZX021).

References

[1] Healey, R. Quantum Theory and the Limits of Objectivity. Found. Phys. 48, 1568 (2018).

[2] Brukner, C. On the quantum measurement problem (2015). https://ar-xiv.org/abs/1507.05255. In: Bertlmann, R., Zeilinger, A. (eds.) Quantum [Un]speakables II, pp. 95-117. Springer International, Cham (2017).

[3] Frauchiger, D. and Renner, R. Quantum theory cannot consistently describe the use of itself. Nat. Commun. 9, 3711 (2018).

[4] Pusey, M. Is QBism 80% complete, or 20%, talk given at the Information-Theoretic Interpretations of Quantum Mechanics workshop, Western University, Canada. (2016).

[5] Lazarovici, D. and Hubert, M. How Quantum Mechanics can consistently describe the use of itself. arXiv:1809.08070 (2018).

[6] Gao, S. Unitary quantum theories are incompatible with special relativity. http://philsci-archive.pitt.edu/15357/ (2018).

[7] Leegwater, G. When Greenberger, Horne and Zeilinger meet Wigner's Friend. arXiv:1811.02442 (2018).