Comment on "Distinct Quantum States Can Be Compatible with a Single State of Reality"

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November 23, 2012

In their Letter [1], Lewis *et al.* demonstrated that additional assumptions such as preparation independence are always necessary to rule out a ψ-epistemic model, in which the quantum state is not uniquely determined by the underlying physical state. Here we point out that these authors ignored the important work of Aharonov, Anandan and Vaidman on protective measurements [2-5], and their conclusion, which is based only on an analysis of conventional projective measurements, is not true.

Projective measurements are one kind of measurements, for which the coupling between the measuring device and the measured system is very strong and almost instantaneous, and the measurement results are the eigenvalues of the measured observable. Due to the resulting collapse of the wave function, such impulsive measurements cannot measure the actual physical state of the measured system. This seems to leave space for ψ-epistemic models [1]. Thanks to the work of Aharonov et al, however, it has been known that the coupling strength and the measuring time can be adjusted for a standard measurement procedure, and there also exist other kinds of measurements such as weak measurements [6] and protective measurements [2-5] (Note that weak measurements have been implemented in experiments [7], and it can be reasonably expected that protective measurements can also be implemented in the near future with the rapid development of quantum technologies). In particular, the actual physical state of the measured system can be measured by a series of protective measurements, and the wave function turns out to be a one-to-one representation of the physical state [2-5]. Therefore, the ψ-epistemic models, in which the wave function or quantum state is not uniquely determined by the underlying physical state, can be ruled out without resorting to nontrivial assumptions beyond those required for a well-formed ontological model.

A general method of protective measurements is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction, and then make the measurement adiabatically so that the state of the system neither collapses nor becomes entangled with the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system [2-5]. As a simple example, consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed. We take

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the measured observable A_n to be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{1}$$

An adiabatic measurement of A_n then yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv = |\psi_n|^2, \tag{2}$$

where $|\psi_n|^2$ is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can adiabatically measure another observable $B_n = \frac{1}{2i}(A_n\nabla + \nabla A_n)$. The measurement yields

$$\langle B_n \rangle = \frac{1}{v_n} \int_{V_n} \frac{1}{2i}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) dv = \frac{1}{v_n} \int_{V_n} |j(x)|^2 dv. \tag{3}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ and $j(x)$ everywhere in space. Since the measured system is not disturbed during the above adiabatic measurement, the measurement results, namely the density $\rho(x)$ and flux density $j(x)$, reflect the actual physical state of the measured system. Moreover, since the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for a constant phase factor), it is uniquely determined by the underlying physical state, which are more directly represented by $\rho(x,t)$ and $j(x,t)$.

Therefore, the epistemic interpretation of the wave function is strongly disfavored by protective measurements. Certaintly, the wave function also plays an epistemic role in defining the distribution of possible results of a projective measurement according to the Born rule. However, this role is secondary and determined by the complete quantum dynamics that describes the measuring process, e.g. the collapse dynamics in dynamical collapse theories.

References

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