# Can Wavefunction Collapse Conserve Energy?

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**Abstract**

The answer may be yes. A discrete model of energy-conserved wave-function collapse is proposed. It is shown that the model is consistent with existing experiments and our macroscopic experience.

## 1 Introduction

In standard quantum mechanics, it is postulated that when the wave function of a quantum system is measured by a macroscopic device, it no longer follows the linear Schrödinger equation, but instantaneously collapses to one of the wave functions that correspond to definite measurement results. However, this collapse postulate is ad hoc, and the theory does not tell us why and how a definite measurement result appears (Bell 1990). A promising solution to this measurement problem is dynamical collapse theories, in which the collapse evolution is dynamical and integrated with the normal Schrödinger evolution into a unified dynamics (Ghirardi 2008). However, the existing dynamical collapse models are plagued by the serious problem of energy non-conservation (Pearle 2007, 2009). For instance, in the CSL (Continuous Spontaneous Localization) model, the collapse due to an external noise field narrows the wave function in position space, thereby producing an increase of energy. Although it is expected that the conservation laws may be satisfied when the contributions of the noise field to the conserved quantities are taken into account (Pearle 2004; Bassi, Ippoliti and Vacchini 2005), a complete solution has not yet been found, and it is still unknown whether such a solution indeed exists.

In this paper, we will propose a discrete model of energy-conserved wave-function collapse, and show that the model is consistent with existing experiments and our macroscopic experience. It has been demonstrated that the energy-driven collapse models that conserve energy can't explain the emergence of definite measurement results (Pearle 2004). However, this important result does not imply that all energy-conserved collapse models are inconsistent with experiments. A detailed analysis of this paper will demonstrate that some energy-conserved collapse models can be consistent with existing experiments

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and our macroscopic experience. The key is to assume that the energy uncertainty driving the collapse of the entangled state of a many-body system is not the uncertainty of the total energy of all sub-systems as the energy-driven collapse models assume, but the sum of the absolute energy uncertainty of every sub-system.

## 2 A discrete model of energy-conserved wavefunction collapse

Consider a multi-level system with a constant Hamiltonian. Its initial state is:

$$|\psi(0)\rangle = \sum_{i=1}^{m} c_i(0) |E_i\rangle, \tag{1}$$

where $|E_i\rangle$ is the energy eigenstate of the Hamiltonian of the system, $E_i$ is the corresponding energy eigenvalue, and $c_i(0)$ satisfies the normalization relation $\sum_{i=1}^{m} |c_i(0)|^2 = 1$.

It is assumed that this superposition of energy eigenstates collapses to one of the eigenstates after a discrete dynamical process, and the collapse evolution satisfies the conservation of energy at the ensemble level[1]. This collapse process is composed of a finite number of discrete tiny collapses. The properties of the tiny collapses is assumed as follows. At each discrete instant $t = nt_P$ (where $t_P$ is the discrete unit of time), the probability of the tiny collapse happening in each energy branch $|E_i\rangle$ is $P_i(t) \equiv |c_i(t)|^2$, and this collapse slightly increases the probability of the energy branch and decreases the probabilities of all other branches pro rata. Then during a finite time interval much larger than $t_P$, the probability of each energy branch will undergo a discrete and stochastic evolution. In the end, the probability of one branch will be close to one, and the probabilities of other branches will be close to zero. In other words, the initial superposition will randomly collapse to one of the energy branches in the superposition. Since it has been generally conjectured that the Planck scale is the minimum spacetime scale[2], we will assume that the size of each discrete instant, $t_P$, is the Planck time in our following analysis.

Now we will give a concrete analysis of this dynamical collapse process. Since the linear Schrödinger evolution does not change the energy probability distribution, we may only consider the influence of dynamical collapse on the energy probability distribution. Suppose at a discrete instant $t = nt_P$ the tiny

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[1]It can be proved that only when the collapse states are energy eigenstates of the total Hamiltonian for each identical system in an ensemble, can energy be conserved at the ensemble level for wavefunction collapse (see Pearle 2000 for a more detailed analysis). Note that for the linear Schrödinger evolution under an external potential, energy is conserved but momentum is not conserved even at the ensemble level, and thus it is not momentum conservation but energy conservation that is a more universal restriction for wavefunction collapse. Moreover, as we will see later, the conservation of energy may also hold true at the individual level for the collapse evolution of some special wave functions.

[2]Note that the existing arguments, which are based on some sort of combination of quantum theory and general relativity (see, e.g. Garay 1995 for a review), do not imply but only suggest that space and time are discrete. Moreover, the meanings and realization of discrete spacetime are also different in the existing models of quantum gravity.

collapse randomly happens in an energy branch $|E_i\rangle$, and the probability of the branch $P_i(t)$ changes to

$$P_i^i(t + t_P) = P_i(t) + \Delta P_i, \tag{2}$$

where the superscript $i$ denotes this tiny collapse event, and $\Delta P_i$ is a functional of $P_i(t)$. Due to the conservation of probability, the probability of another branch $P_j(t)$ $(j \neq i)$ correspondingly turns to be[3]

$$P_j^i(t + t_P) = P_j(t) - \frac{P_j(t)\Delta P_i}{1 - P_i(t)}, \tag{3}$$

where the superscript $i$ still denotes this random event. The probability of this tiny collapse happening in the energy branch $|E_i\rangle$ at the instant is $p(E_i, t) = P_i(t)$. Then we can work out the diagonal density matrix elements of the evolution[4]:

$$
\begin{aligned}
\rho_{ii}(t + t_P) &= \sum_{j=1}^{m} p(E_j, t) P_i^j(t + t_P) \\
&= P_i(t)[P_i(t) + \Delta P_i] + \sum_{j \neq i} P_j(t)[P_i(t) - \frac{P_i(t)\Delta P_j(t)}{1 - P_j(t)}] \\
&= \rho_{ii}(t) + P_i(t)[\Delta P_i - \sum_{j \neq i} P_j(t)\frac{\Delta P_j(t)}{1 - P_j(t)}].
\end{aligned}
\tag{4}
$$

Here we shall introduce the first rule of dynamical collapse in our model, which says that the probability distribution of energy eigenvalues for an ensemble of identical systems is constant during the dynamical collapse process. It can be seen that this rule is entailed by the principle of energy conservation at the ensemble level. By this rule, we have $\rho_{ii}(t + t_P) = \rho_{ii}(t)$ for any $i$. This leads to the following set of equations:

$$
\begin{aligned}
\Delta P_1(t) - \sum_{j \neq 1} \frac{P_j(t)\Delta P_j(t)}{1 - P_j(t)} &= 0, \\
\Delta P_2(t) - \sum_{j \neq 2} \frac{P_j(t)\Delta P_j(t)}{1 - P_j(t)} &= 0, \\
&\phantom{=} \ldots \\
\Delta P_m(t) - \sum_{j \neq m} \frac{P_j(t)\Delta P_j(t)}{1 - P_j(t)} &= 0.
\end{aligned}
\tag{5}
$$

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[3]One can also obtain this result by first increasing the probability of one branch and then normalizing the probabilities of all branches. This means that $P_i(t + t_P) = \frac{P_i(t)+\Delta}{1+\Delta}$ and $P_j(t + t_P) = \frac{P_j(t)}{1+\Delta}$ for any $j \neq i$. In this way, we have $\Delta P_i = \frac{\Delta}{1+\Delta}(1 - P_i(t))$ and $\Delta P_j = \frac{\Delta}{1+\Delta} P_j(t)$ for any $j \neq i$.

[4]The density matrix describes the ensemble of states which arise from all possible random events.

By solving this equations set (e.g. by subtracting each other), we find the following relation for any $i$:

$$\frac{\Delta P_i}{1 - P_i(t)} = k, \tag{6}$$

where $k$ is an undetermined dimensionless quantity that relates to the state $|\psi(t)\rangle$.

By using Eq. (6), we can further work out the non-diagonal density matrix elements of the evolution. But it is more convenient to calculate the following variant of non-diagonal density matrix elements:

$$
\begin{aligned}
\rho_{ij}(t + t_P) &= \sum_{l=1}^{m} p(E_l, t) P_i^l(t + t_P) P_j^l(t + t_P) \\
&= \sum_{l \neq i,j} P_l(t)[P_i(t) - kP_i(t)][P_j(t) - kP_j(t)] \\
&\quad + P_i(t)[P_i(t) + k(1 - P_i(t))][P_j(t) - kP_j(t)] \\
&\quad + P_j(t)[P_j(t) + k(1 - P_j(t))][P_i(t) - kP_i(t)] \\
&= (1 - k^2)\rho_{ij}(t).
\end{aligned} \tag{7}
$$

Since the usual collapse time, $\tau_c$, is defined by the relation $\rho_{ij}(\tau_c) = \frac{1}{2}\rho_{ij}(0)$, we may use a proper approximation, where $k$ is assumed to be the same as its initial value during the time interval $[0, \tau_c]$, to simplify the calculation of the collapse time. Then we have:

$$\rho_{ij}(t) \approx (1 - k^2)^n \rho_{ij}(0). \tag{8}$$

The corresponding collapse time is in the order of:

$$\tau_c \approx \frac{1}{k^2} t_P, \tag{9}$$

In the following, we shall analyze the formula of $k$ defined by Eq. (6). To begin with, the probability restricting condition $0 \leqslant P_i(t) \leqslant 1$ for any $i$ requires that $0 \leqslant k \leqslant 1$. When $k = 0$, no collapse happens, and when $k = 1$, collapse happens instantaneously. Note that $k$ cannot be smaller than zero, as this will lead to the negative value of $P_i(t)$ in some cases. For instance, when $k$ is negative and $P_i(t) < \frac{|k|}{1+|k|}$, $P_i(t + t_P) = P_i(t) + k[1 - P_i(t)]$ will be negative and violate the probability restricting condition. That $k$ is positive indicates that at each discrete instant only the probability of one branch increases and the probabilities of all other branches decrease, which is consistent with our previous assumption.

Next, $k$ is proportional to the duration of a discrete instant. The change of the probability of each branch is an accumulating process. When the duration of a discrete instant is zero, no collapse happens. When the duration of a discrete instant, $t_P$, is longer, the probability will change more. Thus we have $k \propto t_P$.

Thirdly, $k$ is also proportional to the energy uncertainty of the superposition of energy eigenstates. First, from a dimensional analysis $k$ should be proportional to an energy term in order to cancel out the dimension of time. Next, the energy term should be the energy uncertainty of the superposition defined in

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an appropriate way. When the energy uncertainty is zero, i.e., when the state is an energy eigenstate, no collapse happens. When the energy uncertainty is not zero, collapse happens. Moreover, the larger the energy uncertainty is, the larger the change of the probability of each branch at an instant is, namely the larger $k$ is. Therefore, $k$ is proportional to the energy uncertainty of the super-position. How to define the energy uncertainty then? Since $k$ is invariant under the swap of any two branches $(P_i, E_i)$ and $(P_j, E_j)$ according to Eq. (6), the most natural definition of the energy uncertainty of a superposition of energy eigenstates is[5]:

$$\Delta E = \frac{1}{2} \sum_{i,j=1}^{m} P_i P_j |E_i - E_j|. \tag{10}$$

For the simplest two-level system, we have

$$\Delta E = P_1 P_2 |E_1 - E_2|. \tag{11}$$

Then after omitting a coefficient in the order of unity, we can get the formula of $k$ in the first order:

$$k \approx \Delta E t_P / \hbar. \tag{12}$$

This is the second rule of dynamical collapse in our model. By inputting Eq. (12) into Eq. (9), we can further get the collapse time formula:

$$\tau_c \approx \frac{\hbar E_P}{(\Delta E)^2}, \tag{13}$$

where $E_P = h/t_P$ is the Planck energy, and $\Delta E$ is the energy uncertainty of the initial state.

Here it is worth pointing out that $k$ must contain the first order term of $\Delta E$. For the second order or higher order term of $\Delta E$ will lead to much longer collapse time for some common measurement situations, which contradicts experiments (Gao 2006a, 2006b). Besides, a similar analysis of the consistency with experiments may also provide a further support for the energy-conserved collapse model in which the collapse states are energy eigenstates. First of all, if the collapse states are not energy eigenstates but momentum eigenstates, then the energy uncertainty will be replaced by momentum uncertainty in the collapse time formula Eq. (13), namely $\tau_c \approx \frac{\hbar E_P}{(\Delta pc)^2}$. As a result, the collapse time will be too short to be consistent with experiments for some situations. For example, for the ground state of hydrogen atom the collapse time will be about several days. Note that the second order or higher order term of $\Delta p$ will also lead to much longer collapse time for some common measurement situations, which contradicts experiments.

Next, if the collapse states are position eigenstates[6], then the collapse time formula Eq. (13) will be replaced by something like $\tau_c \approx \frac{l^2 t_P}{(\Delta x)^2}$, where $l$ is certain length scale relating to the collapsing state. No matter what length scale $l$

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[5]Note that the common RMS (mean square root) uncertainty also satisfies the swap symmetry. Thus it still needs to be studied what the exact form of $k$ is.

[6]In continuous space and time, a position eigenstate has infinite average energy and cannot be physically real. But in discrete space and time, position eigenstates will be the states whose spatial dimension is about the Planck length, and they may exist.

is, the collapse time of a momentum eigenstate will be zero as its position uncertainty is infinite. This means that the momentum eigenstates of any quantum system will collapse instantaneously to one of its position eigenstates and thus cannot exist. Moreover, the superposition states with very small momentum uncertainty will also collapse very quickly even for microscopic particles. These results are apparently inconsistent with quantum mechanics. Although it may be possible to adjust the length scale $l$ to make the model consistent with existing experience, the collapse time formula will be much more complex than that in the above energy-conserved collapse model. Let's give a little more detailed analysis here. There are two universal length scales for a quantum system: its Compton wavelength $\lambda_c$ and the Planck length $l_P$. It is obvious that both of them cannot be directly used as the length scale in the collapse time formula $\tau_c \approx \frac{l^2 t_P}{(\Delta x)^2}$. Then the formula can only be written in a more complex form: $\tau_c \approx (\frac{\lambda_c}{l_P})^\alpha \cdot \frac{\lambda_c^2 t_P}{(\Delta x)^2}$. Moreover, experiments such as the SQUID experiments and our everyday macroscopic experience require $\alpha \approx 8$. It seems very difficult to explain this unusually large exponent in theory. To sum up, the collapse states can hardly be position eigenstates when considering the consistency with experiments and the simplicity of theory.

Based on the above analysis, the state of the multi-level system at instant $t = n t_P$ will be:

$$|\psi(t)\rangle = \sum_{i=1}^{m} c_i(t) e^{-iE_i t/\hbar} |E_i\rangle, \tag{14}$$

Besides the linear Schrödinger evolution, the collapse dynamics adds a discrete stochastic evolution for $P_i(t) \equiv |c_i(t)|^2$:

$$P_i(t + t_P) = P_i(t) + \frac{\Delta E}{E_P}[\delta_{E_s E_i} - P_i(t)], \tag{15}$$

where $\Delta E$ is the energy uncertainty of the state at instant $t$ defined by Eq. (10), $E_s$ is a random variable representing the branch where the discrete tiny collapse happens, and its probability of assuming $E_i$ at instant $t$ is $P_i(t)$. When $E_s = E_i$, $\delta_{E_s E_i} = 1$, and when $E_s \neq E_i$, $\delta_{E_s E_i} = 0$.

This equation of dynamical collapse can be extended to the entangled states of a many-body system. The difference only lies in the definition of the energy uncertainty $\Delta E$. It is assumed that for a non-interacting or weakly-interacting many-body system in an entangled state, for which the energy uncertainty of each sub-system can be properly defined, $\Delta E$ is the sum of the absolute energy uncertainty of all sub-systems, namely

$$\Delta E = \frac{1}{2} \sum_{l=1}^{n} \sum_{i,j=1}^{m} P_i P_j |E_{li} - E_{lj}|, \tag{16}$$

where $n$ is the total number of the entangled sub-systems, $m$ is the total number of energy branches in the entangled state, and $E_{li}$ is the energy of sub-system $l$ in the $i$-th energy branch of the state. Correspondingly, the collapse states are the product states of the energy eigenstates of the Hamiltonian of each sub-system. It should be stressed here that $\Delta E$ is not defined as the uncertainty of the total energy of all sub-systems as in the energy-driven collapse models (see, e.g. Percival 1995, 1998; Hughston 1996). For each sub-system has its

own energy uncertainty that drives its collapse, and the total driving "force" for the whole entangled state should be the sum of the driving "forces" of all sub-systems, at least in the first order approximation. Although these two kinds of energy uncertainty are equal in numerical values in some cases (e.g. for a strongly-interacting many-body system), there are also some cases where they are not equal. For example, for a superposition of degenerate energy eigenstates of a non-interacting many-body system, which may arise during a common measurement process, the uncertainty of the total energy of all sub-systems is exactly zero, but the absolute energy uncertainty of each sub-system and their sum may be not zero. As a result, the superpositions of degenerate energy eigenstates of a many-particle system may also collapse. As we will see later, this is an important feature of our model, which can avoid Pearle's (2004) serious objections to the energy-driven collapse models.

It can be seen that the equation of dynamical collapse, Eq.(15), has an interesting property, scale invariance. After one discrete instant $t_P$, the probability increase of the branch $|E_i\rangle$ is $\Delta P_i = \frac{\Delta E}{E_P}(1 - P_i)$, and the probability decrease of the neighboring branch $|E_{i+1}\rangle$ is $\Delta P_{i+1} = \frac{\Delta E}{E_P}P_{i+1}$. Then the probability increase of these two branches is

$$\Delta(P_i + P_{i+1}) = \frac{\Delta E}{E_P}[1 - (P_i + P_{i+1})]. \tag{17}$$

Similarly, the equation $\Delta P = \frac{\Delta E}{E_P}(1 - P)$ holds true for the total probability of arbitrarily many branches (one of which is the branch where the tiny collapse happens). This property of scale invariance may simplify the analysis in many cases. For instance, for a superposition of two wavepackets with energy difference, $\Delta E_{12}$, much larger than the energy uncertainty of each wavepacket, $\Delta E_1 = \Delta E_2$ , we can calculate the collapse dynamics in two steps. First, we use Eq.(15) and Eq.(11) with $|E_1 - E_2| = \Delta E_{12}$ to calculate the time of the superposition collapsing into one of the two wavepackets[7]. Here we need not to consider the almost infinitely many energy eigenstates constituting each wavepacket and their probability distribution. Next, we use Eq.(15) with $\Delta E = \Delta E_1$ to calculate the time of the wavepacket collapsing into one of its energy eigenstates. In general, this collapse process is so slow that its effect can be ignored.

Lastly, we want to stress another important point. In our model, the energy eigenvalues are assumed to be discrete for any quantum system. This result seems to contradict quantum mechanics, but when considering that our universe has a finite size (i.e. a finite event horizon), the momentum and energy eigenvalues of any quantum system in the universe may be indeed discrete[8]. The reason is that all quantum systems in the universe are limited by the finite horizon, and thus no free quantum systems exist in the strict sense. For example, the energy of a massless particle (e.g. photon) can only assume discrete values $E_n = n^2 \frac{hc}{4R_U}$, and the minimum energy is $E_1 = \frac{hc}{4R_U} \approx 10^{-33}eV$, where

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[7]Note that most collapse states in an ensemble of identical systems keep the shape of the wavepacket almost precisely.

[8]There might exist a subtle connection here. It seems that the energy-conserved wavefunction collapse in discrete time requires a finite event horizon to ensure the energy eigenvalues of any system are discrete. On the other hand, it seems that discrete spacetime permits the existence of quantum fluctuations of spacetime (as a possible form of dark energy) to lead to acceleration and finite event horizon (Gao 2005). In any case, the existence of a cosmological constant also leads to the existence of a finite event horizon.

$R_U \approx 10^{25}m$ is the radius of the horizon of our universe[9]. Besides, for a free particle with mass $m_0$, its energy also assumes discrete values $E_n = n^2 \frac{h^2}{32 m_0 R_U^2}$. For instance, the minimum energy is $E_1 \approx 10^{-72} eV$ for free electrons, which is much smaller than the minimum energy of photons[10].

It is interesting to see whether this tiny discreteness of energy makes the collapse dynamics more abrupt. Suppose the energy uncertainty of a quantum state is $\Delta E \approx 1 eV$, and its energy ranges between the minimum energy $E_1$ and $1eV$. Then we can get the maximum energy level $l_{max} \approx \sqrt{\frac{1eV}{10^{-33}eV}} \approx 10^{16}$. The probability of most energy eigenstates in the superposition will be about $P \approx 10^{-16}$. During each discrete instant $t_P$, the probability increase of the energy branch with tiny collapse is $\Delta P \approx \frac{\Delta E}{E_P}(1 - P) \approx 10^{-28}$. This indicates that the probability change during each discrete instant is still very tiny. Only when the energy uncertainty is larger than $10^{23} eV$ or $10^{-5} E_P$, will the probability change during each discrete instant be sharp. Therefore, the collapse evolution is still very smooth for the quantum states with energy uncertainty much smaller than the Planck energy.

# 3 On the consistency of the model and experiments

In this section, we will analyze whether the discrete model of energy-conserved wavefunction collapse is consistent with existing experiments and our macroscopic experience. Note that Adler (2002) has already presented a detailed consistency analysis in the context of energy-driven collapse models, and as we will see below, most of his analysis also applies to our model.

## 3.1 Maintenance of coherence

First of all, the model satisfies the constraint of predicting the maintenance of coherence when this is observed. Since the energy uncertainty of the state of a microscopic particle is very small in general, its collapse will be too slow to have any detectable effect in present experiments on these particles. For example, the energy uncertainty of a photon emitted from an atom is in the order of $10^{-6} eV$, and the corresponding collapse time is $10^{25}s$ according to Eq. (13) of our collapse model, which is much longer than the age of the universe, $10^{17}s$. This means that the collapse states (i.e. energy eigenstates) are never reached for a quantum system with small energy uncertainty even during a time interval as long as the age of the universe. As another example, consider the SQUID experiment of Friedman et al (2000), where the coherent superpositions of macroscopic states consisting of oppositely circulating supercurrents are observed. In the experiment, each circulating current corresponds to the collective motion of

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[9]Note that the present upper bound on the photon mass is about $m_\gamma < 10^{-18} eV/c^2$ (Nakamura et al, 2010).

[10]Whether this heuristic analysis is (approximately) valid depends on the application of the final theory of quantum gravity to our finite universe. However, it is worth noting that the existence of discrete energy levels for a free quantum system limited in our universe is also supported by the hypothetical holographic principle, which implies that the total information within a universe with a finite event horizon is finite. If the energy of a quantum system is continuous, then the information contained in the system will be infinite.

about $10^9$ Cooper pairs, and the energy uncertainty is about $8.6 \times 10^{-6} eV$. Eq. (13) predicts a collapse time of $10^{23} s$, and thus maintenance of coherence is expected despite the macroscopic structure of the state[11]. For more examples see Adler (2002).

## 3.2 Rapid localization in measurement situations

In the following, we will investigate whether the discrete model of energy-conserved wavefunction collapse can account for the emergence of definite measurement results.

Let's first see a simple position measurement experiment. Consider an initial state describing a particle in a superposition of two locations (e.g. a superposition of two Gaussian wavepackets separated by a certain distance). After the measurement interaction, the position measuring device evolves to a superposition of two macroscopically distinguishable states:

$$(c_1\psi_1 + c_2\psi_2)\varphi_0 \rightarrow c_1\psi_1\varphi_1 + c_2\psi_2\varphi_2, \tag{18}$$

where $\psi_1$, $\psi_2$ are the states of the particle in different locations, $\varphi_0$ is the initial state of the position measuring device, and $\varphi_1$, $\varphi_2$ are the different outcome states of the device. For an ideal measurement, the two particle/device states $\psi_1\varphi_1$ and $\psi_2\varphi_2$ have precisely the same energy spectrum. Then it appears that this superposition will not collapse according to the energy-conserved collapse model.

However, this is not the case. The key is to see that the two states of the particle in the superposition are detected in different parts of the measuring device, and they interact with different atoms or molecules in these parts. Thus we should rewrite the device states explicitly as $\varphi_0 = \chi_A(0)\chi_B(0)$, $\varphi_1 = \chi_A(1)\chi_B(0)$, and $\varphi_2 = \chi_A(0)\chi_B(1)$, where $\chi_A(0)$ and $\chi_B(0)$ denote the initial states of the device in the parts A and B, respectively, and $\chi_A(1)$ and $\chi_B(1)$ denote the outcome states of the device in the parts A and B, respectively. Then we have

$$(c_1\psi_1 + c_2\psi_2)\chi_A(0)\chi_B(0) \rightarrow c_1\psi_1\chi_A(1)\chi_B(0) + c_2\psi_2\chi_A(0)\chi_B(1) \tag{19}$$

This reformulation clearly shows that there exists energy difference between the sub-systems in the different outcome states of the device. Since there is always some kind of measurement amplification from the microscopic state to the macroscopic outcome in the measurement process, there is a large energy difference between the states $\chi_A(0)$, $\chi_B(0)$ and $\chi_A(1)$, $\chi_B(1)$. As a result, the total energy difference $\Delta E = |\Delta E_A| + |\Delta E_B|$ is also very large, and it will

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[11]A potentially more promising case is provided by certain long-lived nuclear isomers, which have large energy gaps from their ground states (see Adler 2002 and references therein). For example, the metastable isomer of $^{180}$Ta, the only nuclear isomer to exist naturally on earth, has a half-life of more than $10^{15}$ years and an energy gap of $75 keV$ from the ground state. According to Eq. (13), a coherent superposition of the ground state and metastable isomer of $^{180}$Ta will spontaneously collapse to either the isomeric state or the ground state, with a collapse time of order 20 minutes. It will be a promising way to test our collapse model by examining the maintenance of coherence of such a superposition.

result in the rapid collapse of the above superposition into one of its branches according to the energy-conserved collapse model[12].

Let's see a more realistic example, a photon being detected via photoelectric effect (e.g. by a single-photon avalanche diode). In the beginning of the detection, the spreading spatial wave function of the photon is entangled with the states of a large number of surface atoms of the detector. In each local branch of the entangled state, the total energy of the photon is wholly absorbed by the electron in the local atom interacting with the photon. This is clearly indicated by the term $\delta(E_f - E_i - \hbar\omega)$ in the transition rate of photoelectric effect. The state of the ejecting electron is a (spherical) wavepacket moving outward from the local atom, whose average direction and momentum distribution are determined by the momentum and polarization of the photon. The small energy uncertainty of the photon will also be transferred to the ejecting electron[13].

This microscopic effect of ejecting electron is then amplified (e.g. by an avalanche process of atoms) to form a macroscopic signal such as the shift of the pointer of a measuring device. During the amplification process, the energy difference is constantly increasing between the branch in which the photon is absorbed and the branch in which the photon is not absorbed near each atom interacting with the photon. This large energy difference will soon lead to the collapse of the whole superposition into one of the local branches, and thus the photon is only detected locally. Take the single photon detector - avalanche photodiode as a typical example. Its energy consumption is sharply peaked in a very short measuring interval. One type of avalanche photodiode operates at $10^5$ cps and has a mean power dissipation of 4mW (Gao 2006a). This corresponds to an energy consumption of about $2.5 \times 10^{11} eV$ per measuring interval $10^{-5}s$. By using the collapse time formula Eq. (13), where the energy uncertainty is $\Delta E \approx 2.5 \times 10^{11} eV$, we find the collapse time is $\tau_c \approx 1.25 \times 10^{-10}s$. This collapse time is much smaller than the measuring interval.

One important point needs to be stressed here. Although a measured particle is detected locally in a detector (e.g. the spatial size of its collapse state is in the order of the size of an atom), its wave function does not necessarily undergo the position collapse as assumed by standard quantum mechanics in an ideal position measurement, and especially, energy can be conserved during the localization process according to our model. The reason can be summarized as follows. The wave function of the measured particle is usually a spherical wave (e.g. a spherically symmetric wave function) in three-dimensional space. Its momentum is along the radial direction, but the local and random measurement result distributes along the sphere, perpendicular to the radial direction. During the detection, the measured particle interacts with a single atom of the detector by an ionizing process in each local branch of the entangled state of the whole system including the particle and the atoms in the detector. The particle is usually absorbed by the atom or bound in the atom, and its energy is wholly transferred to the newly-formed atom and the ejecting electrons during the

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[12]Since the uncertainty of the total energy of the whole entangled system is still zero, the energy-driven collapse models will predict that no wavefunction collapse happens and no definite measurement result appears for the above measurement process, which contradicts experimental observations (Pearle 2004).

[13]In more general measurement situations, the measured particle (e.g. electron) is not annihilated by the detector. However, in each local branch of the entangled state of the whole system, the particle also interacts with a single atom of the detector by an ionizing process, and its total energy is also wholly transferred to the atom and the ejecting electrons.

ionizing process in each branch. Then the amplification process such as an avalanche process of atoms introduces very large energy difference between the detected branch and the empty branch, and as a result, the whole superposition will soon collapse into one of its local branches in a random way according to the energy-conserved collapse model[14]. After the collapse, the state of the measured particle is localized in the spatial region of one atom. Moreover, since all local branches of the entangled state of the particle and the detector have the same energy spectrum, the collapse process also conserves energy at the individual level.

## 3.3 Emergence of the classical world

In this subsection, we will analyze whether the discrete model of energy-conserved wavefunction collapse is consistent with our macroscopic experience. It seems that there is an apparent inconsistency here. According to the model, when there is a superposition of a macroscopic object in an identical physical state (an approximate energy eigenstate) at two different, widely separated locations, the superposition does not collapse, as there is no energy difference between the two branches of the superposition. But the existence of such superpositions is obviously inconsistent with our macroscopic experience; the macroscopic objects are localized. This common objection has been basically answered by Adler (2002). The crux of the matter lies in the influences of environment. The collisions and especially the accretions of environmental particles will quickly increase the energy uncertainty of the entangled state of the whole system including the object and environmental particles, and thus the initial superposition will soon collapse to one of the localized branches according to our model. Accordingly, the macroscopic objects can always be localized due to environmental influences. Note that the energy uncertainty here denotes the sum of the absolute energy uncertainty of each sub-system in the entangled state as defined in our model[15].

As a typical example, we consider a dust particle of radius $a \approx 10^{-5} cm$ and mass $m \approx 10^{-7} g$. It is well known that localized states of macroscopic objects spread very slowly under the free Schrödinger evolution. For instance, for a Gaussian wave packet with initial (mean square) width $\Delta$, the wave packet will spread so that the width doubles in a time $t = 2m\Delta^2/\hbar$. This means that the double time is almost infinite for a macroscopic object. If the dust particle had no interactions with environment and its initial state is a Gaussian wave packet with width $\Delta \approx 10^{-5} cm$, the doubling time would be about the age of the universe. However, if the dust particle is in interaction with environment, the situation turns out to be very different. Although the different components that couple to the environment will be individually incredibly localised, collectively they can have a spread that is many orders of magnitude larger. In other words, the state of the dust particle and the environment could be a superposition of zillions of very well localised terms, each with slightly different positions, and which are collectively spread over a macroscopic distance (Bacciagaluppi 2008). According to Joos and Zeh (1985), the spread in an environment full

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[14]In a similar way, a spherically symmetric wave function will be detected as one linear track in a cloud chamber (cf. Mott 1929).

[15]The uncertainty of the total energy of the whole system is still very small even if the influences of environment are counted. Thus no observable collapse happens for the above situation according to the energy-driven collapse models (Pearle 2004).

of thermal radiation only is proportional to mass times the cube of time for large times, namely $(\Delta x)^2 \approx \Lambda m\tau^3$, where $\Lambda$ is the localization rate depending on the environment, defined by the evolution equation of density matrix $\rho_t(x, x') = \rho_0(x, x')e^{-\Lambda t(x-x')^2}$. For example, if the above dust particle interacts with thermal radiation at $T = 300K$, the localization rate is $\Lambda = 10^{12}$, and the overall spread of its state is of the order of $10m$ after a second (Joos and Zeh 1985). If the dust particle interacts with air molecules, e.g. floating in the air, the spread of its state will be much faster.

Let's see whether the energy-conserved collapse in our model can prevent the above spreading of the wave packet. Suppose the dust particle is in a superposition of two identical localized states that are separated by $10^{-5}cm$ in space. The particle floats in the air, and its average velocity is about zero. At standard temperature and pressure, one nitrogen molecule accretes in the dust particle, which area is $10^{-10}cm^2$, during a time interval of $10^{-14}s$ in average (Adler 2002). Since the mass of the dust particle is much larger than the mass of a nitrogen molecule, the velocity change of the particle is negligible when compared with the velocity change of the nitrogen molecules during the process of accretion. Then the kinetic energy difference between an accreted molecule and a freely moving molecule is about $\Delta E = \frac{3}{2}kT \approx 10^{-2}eV$. When one nitrogen molecule accretes in one localized branch of the dust particle (the molecule is freely moving in the other localized branch), it will increase the energy uncertainty of the total entangled state by $\Delta E \approx 10^{-2}eV$. Then after a time interval of $10^{-4}s$, the number of accreted nitrogen molecules is about $10^{10}$, and the total energy uncertainty is about $10^8eV$. According to Eq. (13) in our collapse model, the corresponding collapse time is about $10^{-4}s$. Since the two localized states in the superposition have the same energy spectra, the collapse also conserves energy.

In the energy-conserved collapse model, the collapse states are energy eigenstates, and in particular, they are nonlocal momentum eigenstates for free quantum systems. Thus it is indeed counterintuitive that the energy-conserved collapse can make the states of macroscopic objects local. As shown above, this is due to the constant influences of environmental particles. When the spreading of the state of a macroscopic object becomes larger, its interaction with environmental particles will introduce larger energy difference between its different local branches, and this will then collapse the spreading state again into a more localized state[16]. As a result, the states of macroscopic objects in an environment will never reach the collapse states, namely momentum eigenstates, though they do continuously undergo the energy-conserved collapse. To sum up, there are two opposite processes for a macroscopic object constantly inter-

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[16]It is interesting to note that the state of a macroscopic object can also be localized by the linear Schrödinger evolution via interactions with environment, e.g. by absorbing an environmental particle with certain energy uncertainty. For example, if a macroscopic object absorbs a photon (emitted from an atom) with momentum uncertainty of $\Delta p \approx 10^{-6}eV/c$, the center-of-mass state of the object, even if being a momentum eigenstate initially, will have the same momentum uncertainty by the linear Schrödinger evolution, and thus it will become a localized wavepacket with width about $0.1m$. Note that there is no vicious circle here. The energy spreading state of a microscopic particle can be generated by an external potential (e.g. an electromagnetic potential in general) via the linear Schrödinger evolution, and especially they don't necessarily depend on the localization of macroscopic objects such as measuring devices. Thus we can use the existence of these states to explain the localization of macroscopic objects.

acting with environmental particles. One is the spreading process due to the linear Schrödinger evolution, and the other is the localization process due to the energy-conserved collapse evolution. The interactions with environmental particles not only make the spreading more rapidly but also make the localization more frequently. In the end these two processes will reach an approximate equilibrium. The state of a macroscopic object will be a wave packet narrow in both position and momentum, and this narrow wave packet will approximately follow Newtonian trajectories by Ehrenfest's theorem (if the external potential is uniform enough along the width of the packet)[17]. In some sense, the emergence of the classical world around us is "conspired" by environmental particles according to the energy-conserved collapse model.

Ultimately, the energy-conserved collapse model should be able to account for our definite conscious experience. According to recent neuroscience literature, the appearance of a (definite) conscious perception in human brains involves a large number of neurons changing their states from resting state (resting potential) to firing state (action potential). In each neuron, the main difference of these two states lies in the motion of $10^6$ $Na^+$s passing through the neuron membrane. Since the membrane potential is in the order of $10^{-2}V$, the energy difference between firing state and resting state is $\Delta E \approx 10^4 eV$. According to the energy-conserved collapse model, the collapse time of a quantum superposition of these two states of a neuron is

$$\tau_c \approx \frac{\hbar E_P}{(\Delta E)^2} \approx (\frac{2.8 MeV}{0.01 MeV})^2 \approx 10^5 s, \tag{20}$$

where the Planck energy $E_P \approx 10^{19} GeV$. When considering the number of neurons that can form a definite conscious perception is usually in the order of $10^7$, the collapse time of the quantum superposition of two different conscious perceptions will be

$$\tau_c \approx (\frac{2.8 MeV}{100 GeV})^2 \approx 10^{-9} s. \tag{21}$$

Since the normal conscious time of a human being is in the order of several hundred milliseconds, the collapse time is much shorter than the normal conscious time. Therefore, our conscious perceptions are always definite according to the energy-conserved collapse model.

# 4    Conclusions

It has been doubted whether the collapse of the wave function can conserve energy. In this paper, we propose a discrete model of energy-conserved wavefunction collapse, and show that the model is consistent with existing experiments and our macroscopic experience. The analysis demonstrates that the collapse of the wave function may conserve energy at the ensemble level.

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[17]When assuming the energy uncertainty of an object is in the same order of its thermal energy fluctuation, we can estimate the rough size of its wavepacket. For instance, for a dust particle of mass $m = 10^{-7}g$, its root mean square energy fluctuation is about $10^3 eV$ at room temperature $T = 300K$ (Adler 2002), and thus the width of its wavepacket is about $10^{-10}m$.

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