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Research Article

Dynamical Behavior of a Novel Impulsive Switching Model for HLB with Seasonal Fluctuations

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This paper studies a new model for Huanglongbing with seasonal fluctuations. Switching coefficients and switching control schemes are considered in this model. The main purpose of this paper is to study the effects of switching control schemes on dynamics of the model. Firstly, we theoretically investigate the basic reproductive number and its computation formulae for general impulsive switching model with periodic environment. Secondly, the basic reproductive number and global dynamics of the impulsive switching model for Huanglongbing are analyzed. Finally, numerical results indicate that spring and autumn are the optimum seasons for killing psyllids, and winter is the optimum season for removing infected trees.

1. Introduction

Citrus Huanglongbing (HLB), also known as citrus greening, is one of the most devastating diseases of citrus worldwide [1]. The Asiatic citrus psyllid and *Diaphorina citri* Kuwayama are the only two known vectors of the debilitating citrus HLB [2]. Nearly 50 countries are affected by this disease especially in Asian, African, and American countries, such as Brazil, USA, and China. It was estimated by the University of Florida in 2012 that, in Florida, HLB had resulted in the loss of 6611 jobs from 2006 throughout 2011, 1.3 billion in revenue to growers, and 3.63 billion in economic activity [3]. In São Paulo, 64.1% of the commercial citrus blocks and 6.9% of the citrus trees were affected by HLB in 2012 [4]. Till now, in China, the damaged area of citrus is more than 80% of the total cultivated area [5]. Unfortunately, there currently is no cure for HLB nor is there any naturally occurring citrus cultivar that is resistant to HLB.

HLB is a vector-transmitted bacterial infection through psyllids [6]. Since the pioneering work of MacDonald and Barbour on schistosomiasis [7, 8], many mathematical models have been proposed in analyzing the spread and control of vector-borne diseases, such as malaria, dengue fever, schistosomiasis, West Nile disease, HLB (see [9–14] and references therein). In [8], Barbour formulated a mathematical model of schistosomiasis as follows:

$$\begin{aligned}
\frac{dI_h(t)}{dt} &= aS_hI_v - \mu_1 I_h,\\
\frac{dI_v(t)}{dt} &= bS_vI_h - \mu_2 I_v,\\
\frac{dS_h(t)}{dt} &= \mu_1 - aS_hI_v - \mu_1 S_h,\\
\frac{dS_v(t)}{dt} &= \mu_2 - bS_vI_h - \mu_2 S_v,
\end{aligned} \tag{1}$$

where S_h (I_h) is the susceptible (infected) host population, S_v (I_v) is the susceptible (infected) vector population, a and b are infection rates, μ_1 (μ_2) is the natural mortality rate of the host (vector) population.

As we know, young flush (initial infection of newly developing cluster of young leaves) become infectious within 15 days after receiving an inoculum of bacteria [15], and

symptoms of HLB do not appear on leaves for months to years after initial infection. The survey results from [16, 17] indicated that the incubation period from grafting to development of HLB symptoms is 3 to 12 months under greenhouse conditions. For large trees in a field situation, the incubation period may be much longer, up to more than 5 years. This means that HLB has a long incubation period during which the plant is asymptomatic but infectious [18]. Therefore, in this paper, we classify the citrus tree into four compartments: susceptible S_h, infected and asymptomatic but not yet infectious E_h, infectious and asymptomatic I_1, and infectious and symptomatic I_2, and the psyllids vector into two compartments: susceptible class S_v and infected class I_v. Let $N_h(t)$ and $N_v(t)$ be the total numbers of citrus trees and psyllids, respectively, at time t in a grove. That is, $N_h(t) = S_h(t) + E_h(t) + I_1(t) + I_2(t)$ and $N_v(t) = I_v(t) + S_v(t)$. Assume that removed trees are immediately replaced by susceptible trees, keeping the grove size constant [19]. Thus, $N_h(t)$ is constant and denotes N_h. Inspired by the idea of Barbour's model (1), considering HLB transmission between citrus trees and psyllids, we establish the following HLB model.

$$\begin{aligned}
\frac{dE_h(t)}{dt} &= \frac{aS_hI_v}{N_h} - \mu_1E_h - \alpha E_h - dE_h,\\
\frac{dI_1(t)}{dt} &= \alpha E_h - \mu_1I_1 - \theta I_1 - dI_1,\\
\frac{dI_2(t)}{dt} &= \theta I_1 - \mu_1I_2 - dI_2,\\
\frac{dI_v(t)}{dt} &= \frac{cS_vI_1}{N_h} + \frac{bS_vI_2}{N_h} - \mu_2I_v,\\
\frac{dS_h(t)}{dt} &= \mu_1N_h - \mu_1S_h - \frac{aS_hI_v}{N_h} + dE_h + dI_1 + dI_2,\\
\frac{dS_v(t)}{dt} &= \Lambda - \frac{cS_vI_1}{N_h} - \frac{bS_vI_2}{N_h} - \mu_2S_v,
\end{aligned} \tag{2}$$

where α and θ are the conversion rates, μ_1 (μ_2) is the natural mortality rate of the citrus tree (psyllids), a is the infection rate from infected psyllids to susceptible trees, b is the infection rate from infectious and symptomatic trees to psyllids, $c = kb$ means the infection rate from infectious and asymptomatic trees to psyllids, and k ($0 < k \le 1$) is the proportional coefficient, d is the mortality rate of citrus trees due to illness, and Λ is the constant recruitment rate of psyllids.

In general, spraying insecticides over entire groves as well as eliminating infected symptomatic trees have always been implemented in controlling the spread of HLB. In Thailand, 3–6 sprays per year was required during flush periods to rehabilitate citrus production in a HLB-infected area [20]. However, the common assumption about the continuity of control activities is contradictory from the reality that the control behavior usually occurs in regular pulses [21]. Spraying insecticides is generally applied at a fixed time, and the effect of pesticide spraying depends on the time of initial spraying and frequency. By considering impulsive control strategies, system (2) can be described by impulsive differential equations as follows:

$$\begin{aligned}
&\frac{dE_h(t)}{dt} = \frac{aS_hI_v}{N_h} - \mu_1E_h - \alpha E_h - dE_h,\\
&\frac{dI_1(t)}{dt} = \alpha E_h - \mu_1I_1 - \theta I_1 - dI_1,\\
&\frac{dI_2(t)}{dt} = \theta I_1 - \mu_1I_2 - dI_2 - \gamma I_2,\\
&\frac{dI_v(t)}{dt} = \frac{cS_vI_1}{N_h} + \frac{bS_vI_2}{N_h} - \mu_2I_v,\\
&\frac{dS_h(t)}{dt} = \mu_1N_h - \mu_1S_h - \frac{aS_hI_v}{N_h} + dE_h + dI_1 + dI_2 + \gamma I_2,\\
&\frac{dS_v(t)}{dt} = \Lambda - \frac{cS_vI_1}{N_h} - \frac{bS_vI_2}{N_h} - \mu_2S_v.\\
&\qquad t \in (t_{k-1}, t_k],\\
&I_v(t^+) = (1-p)I_v(t),\\
&S_v(t^+) = (1-p)S_v(t),\\
&\qquad t = t_k.
\end{aligned} \tag{3}$$

where γ is the removal rate of infected symptomatic trees and p is the killing rate of psyllids by insecticide spraying.

Furthermore, in endemic areas, removing of citrus trees is always based predominantly on the presence of visible symptoms [22]. All of the trees showing HLB symptoms should be removed 3 times in each year [20]. These imply that the infection rates and the removal rate vary with season fluctuations. Thus, it is necessary to consider that some coefficients of model (3) are time-varying and switching in time. Suppose that some parameters are modeled as switching parameters and governed by a switching rule $\sigma(t)$: $(t_{k-1}, t_k] \to \{1, 2, \dots, m\} = \mathscr{P}$, $k = 1, 2, \dots$, where m is the number of the subsystems and $\sigma(t)$ is a piecewise continuous switching rule such that $\sigma(t) = i_k \in \mathscr{P}$ for all $t \in (t_{k-1}, t_k]$. The switching times $\{t_k\}$ satisfy $t_{k+1} > t_k > 0$ and $\lim_{k\to\infty} t_k = \infty$. Define the set of all switching rules by $\mathscr{S}$. Motivated by above fact, we yield the switching HLB model with impulsive control:

$$\begin{aligned}
&\frac{dE_h(t)}{dt} = \frac{a_\sigma S_hI_v}{N_h} - \mu_1E_h - \alpha E_h - dE_h,\\
&\frac{dI_1(t)}{dt} = \alpha E_h - \mu_1I_1 - \theta I_1 - dI_1,\\
&\frac{dI_2(t)}{dt} = \theta I_1 - \mu_1I_2 - dI_2 - \gamma_\sigma I_2,\\
&\frac{dI_v(t)}{dt} = \frac{c_\sigma S_vI_1}{N_h} + \frac{b_\sigma S_vI_2}{N_h} - \mu_2I_v,\\
&\frac{dS_h(t)}{dt} = \mu_1N_h - \mu_1S_h - \frac{a_\sigma S_hI_v}{N_h} + dE_h + dI_1 + dI_2 + \gamma_\sigma I_2,\\
&\frac{dS_v(t)}{dt} = \Lambda - \frac{c_\sigma S_vI_1}{N_h} - \frac{b_\sigma S_vI_2}{N_h} - \mu_2S_v,
\end{aligned}$$

$$
\begin{aligned}
&t \in (t_{k-1}, t_k],\\
&I_v(t^+) = (1-p_\sigma)I_v(t),\\
&S_v(t^+) = (1-p_\sigma)S_v(t),\\
&t = t_k,
\end{aligned}
\tag{4}
$$

where p_σ $(0 \le p_\sigma \le 1)$ are the killing rates of psyllids by insecticide spraying at time t_k $(k = 1, 2, \dots)$. The initial conditions for system (4) satisfy $E_h(t_0^+) = E_h(t_0) \ge 0$, $I_1(t_0^+) = I_1(t_0) \ge 0$, $I_2(t_0^+) = I_2(t_0) \ge 0$, $I_v(t_0^+) = I_v(t_0) \ge 0$, $S_h(t_0^+) = S_h(t_0) > 0$, and $S_v(t_0^+) = S_v(t_0) > 0$. The model flow diagram is depicted in Figure 1.

The spread of infectious diseases is influenced by many factors, such as the behavior of the human population and the environment in which it spread [23]. Consequently, it is more realistic to consider the periodic switching rule. Following the idea of [23], we assume that the switching rule σ satisfies $t_k - t_{k-1} = \omega_k$ with $\omega_{k+m} = \omega_k$, and then $\omega = \sum_{k=1}^{m} \omega_k$ is the period of switch σ. Assume that $a_{i_k} = a_k$, $b_{i_k} = b_k$, $c_{i_k} = c_k$, $m_{i_k} = m_k$, and $p_{i_k} = p_k$ for $t \in (t_{k-1}, t_k]$, and $a_{k+m} = a_k$, $b_{k+m} = b_k$, $c_{k+m} = c_k$, $m_{k+m} = m_k$, and $p_{k+m} = p_k$. Define $\mathcal{I}_p$ as the set of periodic switching rule.

Note that models (2) and (3) can be considered as special cases of model (4). If the parameters of model (4) are constant and not switching in time, that is, there is only one independent subsystem $(m = 1)$, then model (4) yields to model (3). Further, if control strategies are not in use, in the case where the killing rate of psyllids (p) and the removal rate of infected symptomatic trees (γ) are zero, then model (3) reduces to model (2). Our main purpose is to explore the effects of switching control schemes on the dynamics properties of model (4).

The rest of this paper is organized as follows: In Section 2, some basic notations and useful results are given. In Section 3, the threshold condition and global asymptotic stability of the disease-free periodic solution of system (4) are studied. Furthermore, sufficient condition for persistence of the disease is derived. Numerical simulations are given in Section 4. Brief discussion and conclusion are presented in Section 5.

2. Some Useful Results

2.1. Some Useful Results for Linear Impulsive Switching System. Before investigating system (4), we will present some notations and state some results for linear impulsive switching system with periodic environment.

Define $\mathbb{R}_+ = \{x \in \mathbb{R} \mid x \ge 0\}$, $\mathbb{R}_+^n = \{(x_1, \dots, x_n) \in \mathbb{R}^n \mid x_i \ge 0, i = 1, 2, \dots, n\}$. Let $r(B)$ be the spectral radius of matrix B.

Consider a linear impulsive switching differential system:

$$
\begin{aligned}
&\dot{x}(t) = A_\sigma(t)x(t), \quad t \in (t_{k-1}, t_k],\\
&x(t_k^+) = P_\sigma x(t_k), \quad t = t_k, k \in \mathbb{N},\\
&x(t_0^+) = x_0, \quad t_0 \ge 0,
\end{aligned}
\tag{5}
$$

where $x = (x_1, x_2, \dots, x_n)^T$, $A_\sigma, P_\sigma \in \mathbb{R}^{n\times n}$, $\sigma \in \mathcal{I}$.

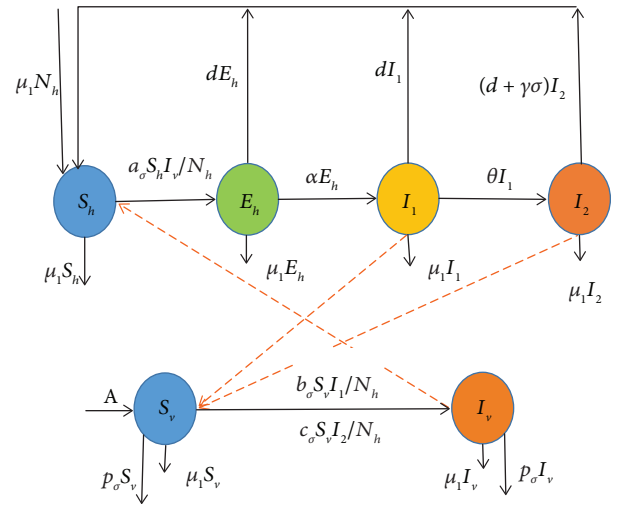

Figure 1: A schematic of model (4) showing transitions to different categories for trees and psyllids. Black arrows show the transitions between compartments. Orange dashed arrows show the necessary interactions between trees and psyllids to obtain transmission.

Particularly, if $\sigma \in \mathcal{I}_p$, system (5) can be rewritten as follows:

$$
\begin{aligned}
&\dot{x}(t) = A_k(t)x(t), \quad t \in (t_{k-1}, t_k],\\
&x(t_k^+) = P_k x(t_k), \quad t = t_k, k \in \mathbb{N},\\
&x(t_0^+) = x_0, \quad t_0 \ge 0,
\end{aligned}
\tag{6}
$$

where $A_{k+m}(t) = A_k(t)$, $P_{k+m} = P_k$, and $t_k - t_{k-1} = \omega_k$ with $\omega_{k+m} = \omega_k$, and then $\omega = \sum_{k=1}^{m} \omega_k$ is the period of switch σ.

Let $\Psi_{A_k}(t, s)(t \ge s)$ be the evolution operator of the linear ω-periodic system

$$
\dot{x}(t) = A_k(t)x(t), \quad x \in \mathbb{R}^n. \tag{7}
$$

Denote

$$
\Phi_{A_k P_k}(\omega) := \prod_{k=1}^{m} \left(P_{m-k+1}\Psi_{A_k}(t_{m-k+1}, t_{m-k})\right). \tag{8}
$$

Lemma 1. *If $\eta = (1/\omega) \ln r(\Phi_{A_k P_k}(\omega))$, then there exists a positive ω-periodic vector function $v(t)$ such that $\exp(\eta t)v(t)$ is a solution of system (6).*

Since the proof is similar to that of Lemma 1 in [24], so one omits it.

Lemma 2. *If $r(\Phi_{A_k P_k}(\omega)) < 1$, then the trivial solution of system (6) is asymptotically stable.*

Using the similar method in [25], this result can be easily proved (not shown in this paper).

2.2. R_0 for General Impulsive Periodic System with Switching Parameters. Consider a general impulsive switching system with periodic environment:

$$\begin{aligned} \dot{x}(t) &= f^k(x), \quad t \in (t_{k-1}, t_k], \\ x(t^+) &= \psi^k(x(t)), \quad t = t_k, k \in \mathbb{N}, \\ x(t_0^+) &= x_0, \quad t_0 \geq 0, \end{aligned} \tag{9}$$

where $f^k : \mathbb{R}_+^n \to \mathbb{R}^n$, $\psi^k : \mathbb{R}_+^n \to \mathbb{R}_+^n$, $f^{k+m} = f^k$, and $\psi^{k+m} = \psi^k$.

Following [26], we split the compartments by two types with the first q compartments $\{x_1, x_2, \dots, x_q\}$ the infected individuals and $\{x_{q+1}, x_{q+2}, \dots, x_n\}$ the uninfected individuals. And denote $X = (x_1, x_2, \dots, x_q)$, $Y = (x_{q+1}, x_{q+2}, \dots, x_n)$, $\psi^k = (h^k, g^k)^T$, $h^k = (\psi_1^k, \psi_2^k, \dots, \psi_q^k)$, and $g^k = (\psi_{q+1}^k, \dots, \psi_n^k)$. Define

$$X_s = \{x \in \mathbb{R}_+^n \mid x_i = 0, i = 1, \dots, q\}. \tag{10}$$

We can rewrite system (9) as:

$$\begin{aligned} \dot{x}(t) &= \mathscr{F}^k(x(t)) - \mathscr{V}^k(x(t)), \quad t \in (t_{k-1}, t_k], \\ X(t^+) &= h^k(x(t)), \\ Y(t^+) &= g^k(x(t)), \\ & t = t_k, k \in \mathbb{N}, \\ x(t^+) &= x_0, \quad t_0 \geq 0, \end{aligned} \tag{11}$$

where $\mathscr{F}^k(x)$ are the newly infected rates, $\mathscr{V}^{k+}(x)$ are the input rates of individuals by other means, and $\mathscr{V}^{k-}(x)$ are the rates of transfer of individuals out of compartments; then, $\mathscr{V}^k(x) = \mathscr{V}^{k-}(x) - \mathscr{V}^{k+}(x)$ represent the set transfer rates out of compartments. Thus, $f^k(x) = \mathscr{F}^k(x) - \mathscr{V}^k(x)$. We assume that system (11) satisfies $\mathscr{F}^{k+m} = \mathscr{F}^k$, $\mathscr{V}^{k+m} = \mathscr{V}^k$, $h^{k+m} = h^k$, and $g^{k+m} = g^k$, and system (11) has a disease-free periodic solution $x^*(t)$.

We make the following assumptions, which share the same biological meanings as those by Wang and Zhao [27] and Yang and Xiao [28].

(H1) If $x_i \geq 0$, then the function $\mathscr{F}_i^k(x)$, $\mathscr{V}_i^{k+}(x)$, and $\mathscr{V}_i^{k-}(x)$ are nonnegative and continuous on $\mathbb{R}_+^n$ and continuously differential with respect to x for $i = 1, \dots, n$.

(H2) If $x_i = 0$, then $\mathscr{V}_i^{k-}(x) = 0$. Particularly, if $x \in X_s$, then $\mathscr{V}_i^{k-}(x) = 0$ for $i = 1, \dots, q$.

(H3) $\mathscr{F}_i^k(x) = 0$ for $q + 1, \dots, n$.

(H4) If $x \in X_s$, then $\mathscr{F}_i^k(x) = \mathscr{V}_i^{k+}(x) = 0$ for $i = 1, \dots, q$.

(H5) The pulse on the infected compartments must be uncoupled with the uninfected compartments; that is, $h^k(x(t_k))$ is essentially $h^k(X(t_k))$, and $h^k(\mathbf{0}) = \mathbf{0}$.

(H6) $r(\Phi_{M_k Q_k}(\omega)) < 1$, where $\Phi_{M_k Q_k}(\omega) = \prod_{k=1}^m (Q_{m-k+1} \Psi_{M_k}(t_{m-k+1}, t_{m-k}))$, and $\Phi_{M_k Q_k}(t)$ is the fundamental solution matrix of the following system:

$$\begin{aligned} \dot{Z}(t) &= M_k(t) Z(t), \quad t \in (t_{k-1}, t_k], \\ Z(t_k^+) &= Q_k Z(t_k), \quad t = t_k, k \in \mathbb{N}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} M_k(t) &= \left(\frac{\partial f_i^k(x^*(t))}{\partial x_j} \right)_{q+1 \leq i,j \leq n}, \\ Q_k &= \left(\frac{\partial \psi_i^k(x^*(t_k))}{\partial x_j} \right)_{q+1 \leq i,j \leq n}. \end{aligned} \tag{13}$$

From (H2)–(H4), the derivatives of $\mathscr{F}^k(x^*(t))$ and $\mathscr{V}^k(x^*(t))$ can be parted as follows:

$$\begin{aligned} D\mathscr{F}^k(x^*(t)) &= \begin{pmatrix} F_k(t) & 0 \\ 0 & 0 \end{pmatrix}, \\ D\mathscr{V}^k(x^*(t)) &= \begin{pmatrix} V_k(t) & 0 \\ J_k & -M_k(t) \end{pmatrix}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} F_k(t) &= \left(\frac{\partial \mathscr{F}_i^k(x^*(t))}{\partial x_j} \right)_{1 \leq i,j \leq q}, \\ V_k(t) &= \left(\frac{\partial \mathscr{V}_i^k(x^*(t))}{\partial x_j} \right)_{1 \leq i,j \leq q}. \end{aligned} \tag{15}$$

Furthermore, it follows from (H5) that h^k are the functions of $X(t_k)$. So the derivatives of $\psi^k(x^*(t_k))$ can be separated as follows:

$$D\psi^k(x^*(t_k)) = \begin{pmatrix} P_k & 0 \\ \Gamma_k & Q_k \end{pmatrix}, \tag{16}$$

where $P_k \in \mathbb{R}^{q \times q}$ and $\Gamma_k \in \mathbb{R}^{(n-q) \times q}$ defined by

$$\begin{aligned} P_k &= \left(\frac{\partial \psi_i^k(x^*(t_k))}{\partial x_j} \right)_{1 \leq i,j \leq q}, \\ \Gamma_k &= \left(\frac{\partial \psi_i^k(x^*(t_k))}{\partial x_j} \right)_{q+1 \leq i \leq n, 1 \leq j \leq q}. \end{aligned} \tag{17}$$

(H7) $r(\Phi_{-V_k(t)P_k}(\omega)) < 1$.

In addition, from Assumption (H7) and Lemma 2, we can see that the trivial solution of the following linear switching system with impulses

$$\begin{aligned} \dot{y}(t) &= -V_k y(t), \quad t \in (t_{k-1}, t_k], \\ y(t^+) &= P_k y(t), \quad t = t_k, k \in \mathbb{N}, \end{aligned} \tag{18}$$

is asymptotically stable. According to Remark 3.5 in Sect. III. 7 of [29], we have that there exist constants $K > 0$ and $\rho > 0$, such that

$$\|Y(t,s)\| \leq K \exp\{-\rho(t-s)\}, \quad \forall t \geq s, s \in \mathbb{R}, \tag{19}$$

where $Y(t,s)$ is the evolution operator of system (18).

Similar to the notation and definition of [24], we define the so-called next infection operator L,

$$\begin{aligned} L\phi(t) &= \int_{-\infty}^{t} Y(t,s)F(s)\phi(s)\mathrm{d}s \\ &= \int_{0}^{\infty} Y(t,t-a)F(t-a)\phi(t-a)da, \quad \forall t \in \mathbb{R}_+, \end{aligned} \tag{20}$$

where $\phi(s)$ is a ω-periodic function from $\mathbb{R}$ to $\mathbb{R}_+^q$ and denotes the initial distribution of infections individuals, and $F(t) = F_k(t)$ when $t \in (t_{k-1}, t_k]$.

Now, we define the basic reproductive number $\mathscr{R}_0$ for system (11) as follows:

$$\mathscr{R}_0 = r(L). \tag{21}$$

In order to calculate the implicit expression $\mathscr{R}_0$ by numerical simulation, we consider the auxiliary ω-periodic switching system with impulses:

$$\begin{aligned} \dot{U}(t) &= \left[-V_k(t) + \frac{F_k(t)}{\lambda}\right] U(t), \quad t \in (t_{k-1}, t_k], \\ U(t^+) &= P_k U(t), \quad t = t_k, k \in \mathbb{N}, \end{aligned} \tag{22}$$

where $\lambda \in (0, \infty)$. Set $U(t,s,\lambda)(t \geq s, s \in \mathbb{R})$ to be the evolution operator of system (22), then $U(\omega, 0, \lambda) = \Phi_{((F_k/\lambda)-V_k)P_k}(\omega)$. Following the idea in [28], we have following results.

Lemma 3. *Assuming that (H1)–(H7) hold, then the following statements are valid:*

(i) If $r(\Phi_{((F_k/\lambda)-V_k)P_k}(\omega)) = 1$ has a positive solution λ_0, then λ_0 is an eigenvalue of L, and so $\mathscr{R}_0 > 0$.

(ii) If $\mathscr{R}_0 > 0$, then $\lambda = \mathscr{R}_0$ is the unique solution of $r(\Phi_{((F_k/\lambda)-V_k)P_k}(\omega)) = 1$.

(iii) $\mathscr{R}_0 = 0$ if and only if $r(\Phi_{((F_k/\lambda)-V_k)P_k}(\omega)) < 1$ for all $\lambda > 0$.

By applying Lemma 3, one knows that $\mathscr{R}_0$ for impulsive periodic switching system (11) is the solution of algebraic equation $r(\Phi_{((F_k/\lambda)-V_k)P_k}(\omega)) = 1$.

Lemma 4. *Assuming that (H1)–(H7) hold, then the following statements are valid for system (11):*

(i) $\mathscr{R}_0 = 1$ if and only if $r(\Phi_{(F_k-V_k)P_k}(\omega)) = 1$.

(ii) $\mathscr{R}_0 > 1$ if and only if $r(\Phi_{(F_k-V_k)P_k}(\omega)) > 1$.

(iii) $\mathscr{R}_0 < 1$ if and only if $r(\Phi_{(F_k-V_k)P_k}(\omega)) < 1$.

It follows from Lemma 4 that the disease-free periodic solution $x^*(t)$ of system (11) is asymptotically stable if $\mathscr{R}_0 < 1$ and unstable if $\mathscr{R}_0 > 1$.

3. Main Results

In this section, we are going to explore the threshold condition which leads to the extinction and persistence of the disease for impulsive switching model (4) for HLB with seasonal fluctuations.

Lemma 5. *All solutions of system (4) with nonnegative initial conditions are nonnegative for all $t > t_0$ and ultimately bounded.*

The proof of Lemma 5 is simple; we omit it.

Referring to [21], we can get that system (4) has a unique disease-free periodic solution $x^*(t) = (0, 0, 0, 0, S_h^*(t), S_v^*(t))$, where $S_h^*(t)$ and $S_v^*(t)$ are the unique periodic solution of the following systems, respectively:

$$\frac{dS_h(t)}{dt} = \mu_1 N_h - \mu_1 S_h(t), \tag{23}$$

and

$$\begin{aligned} \frac{dS_v(t)}{dt} &= \Lambda - \mu_2 S_v(t), \quad t \neq t_k, \\ S_v(t^+) &= (1-p_k)S_v(t), \quad t = t_k. \end{aligned} \tag{24}$$

We can easily obtain that Assumptions (H1)–(H5) hold for system (4). Next, we will show that Assumptions (H6) and (H7) hold. By (13), (15), and (17), we can calculate M_k, Q_k, F_k, V_k, and P_k of system (4), which are represented as the following form:

$$M_k(t) = \begin{pmatrix} -\mu_1 & 0 \\ 0 & -\mu_2 \end{pmatrix},$$

$$Q_k = \begin{pmatrix} 1 & 0 \\ 0 & 1-p_k \end{pmatrix},$$

$$F_k(t) = \begin{pmatrix} 0 & 0 & 0 & a_k \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \dfrac{c_k S_v^*(t)}{N_h} & \dfrac{b_k S_v^*(t)}{N_h} & 0 \end{pmatrix},$$

$$V_k(t) = \begin{pmatrix} \mu_1 + \alpha + d & 0 & 0 & 0 \\ -\alpha & \mu_1 + \theta + d & 0 & 0 \\ 0 & -\theta & \mu_1 + d + \gamma_k & 0 \\ 0 & 0 & 0 & \mu_2 \end{pmatrix},$$

$$P_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-p_k \end{pmatrix}. \tag{25}$$

By calculating, we get

$$\Phi_{M_k Q_k}(\omega) = \begin{pmatrix} \exp(-\mu_1\omega) & 0 \\ 0 & \prod_{k=1}^{m}(1-p_k)\exp(-\mu_2\omega) \end{pmatrix}, \tag{26}$$

and

$$\Phi_{-V_k P_k}(\omega) = \begin{pmatrix} J_1 & 0 & 0 & 0 \\ * & J_2 & 0 & 0 \\ * & * & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{pmatrix}, \tag{27}$$

where $J_1 = \exp\{-(\mu_1 + d + \alpha)\omega\}$, $J_2 = \exp\{-(\mu_1 + d + \theta)\omega\}$, $J_3 = \exp\{\sum_{k=1}^{m} - (\mu_1 + d + \gamma_k)\omega_k\}$, and $J_4 = \prod_{k=1}^{m}(1-p_k)\exp(-\mu_2\omega)$. There is no need to calculate the exact forms of $*$, as they are not required in the analysis that follows. Obviously, $r(\Phi_{M_k Q_k}(\omega)) < 1$ and $r(\Phi_{-V_k P_k}(\omega)) < 1$. Thus, Assumptions (H6) and (H7) hold.

Theorem 1. *If $\mathscr{R}_0 < 1$, then the disease-free periodic solution $x^*(t)$ of system (4) is globally asymptotically stable, whereas it is unstable if $\mathscr{R}_0 > 1$.*

Proof 1. From Lemma 4, one has that the unique disease-free periodic solution $x^*(t)$ is unstable if $\mathscr{R}_0 > 1$, and $x^*(t)$ is locally stable if $\mathscr{R}_0 < 1$. Therefore, one only needs to show the global attractivity of $x^*(t)$ for $\mathscr{R}_0 < 1$.

From Lemma 4, we get $r(\Phi_{(F_k - V_k)P_k}(\omega)) < 1$ since $\mathscr{R}_0 < 1$. So we can choose a sufficiently small $\varepsilon_1 > 0$ such that

$$r\left(\Phi_{(F_k - V_k + M_{\varepsilon_1 k})P_k}(\omega)\right) < 1,$$

$$\text{where } M_{\varepsilon_1 k} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{c_k\varepsilon_1}{N_h} & \frac{b_k\varepsilon_1}{N_h} & 0 \end{pmatrix}. \tag{28}$$

From system (4), we have that

$$\begin{aligned} &\frac{dS_v(t)}{dt} \le \Lambda - \mu_2 S_v(t), \quad t \ne t_k, \\ &S_v(t^+) = (1-p_k)S_v(t), \quad t = t_k. \end{aligned} \tag{29}$$

By comparison theorem in impulsive differential equations, for the abovementioned ε_1, we have that there exists a $\bar{T}_1 > 0$ such that

$$S_v(t) \le S_v^*(t) + \varepsilon_1, \quad \text{for } t > \bar{T}_1. \tag{30}$$

According to system (4) and inequality (30), we can get that for $t > \bar{T}_1$,

$$\begin{aligned} &\frac{dE_h(t)}{dt} \le a_k I_v - \mu_1 E_h - \alpha E_h - dE_h, \\ &\frac{dI_1(t)}{dt} = \alpha E_h - \mu_1 I_1 - \theta I_1 - dI_1, \\ &\frac{dI_2(t)}{dt} = \theta I_1 - \mu_1 I_2 - dI_2 - \gamma_k I_2, \\ &\frac{dI_v(t)}{dt} \le \frac{c_k(S_v^*(t)+\varepsilon_1)I_1}{N_h} + \frac{b_k(S_v^*(t)+\varepsilon_1)I_2}{N_h} - \mu_2 I_v, \\ &\qquad t \in (t_{k-1}, t_k], \\ &E_h(t^+) = E_h(t), \\ &I_1(t^+) = I_1(t), \\ &I_2(t^+) = I_2(t), \\ &I_v(t^+) = (1-p_k)I_v(t), \\ &\qquad t = t_k. \end{aligned} \tag{31}$$

Consider the following comparison system:

$$\begin{aligned} &\frac{d\bar{J}(t)}{dt} = \left(F_k(t) - V_k(t) + M_{\varepsilon_1 k}\right)\bar{J}(t), \quad t \ne t_k, \\ &\bar{J}(t^+) = P_k\bar{J}(t), \quad t = t_k, \end{aligned} \tag{32}$$

where $\bar{J}(t) = (\bar{E}_h(t), \bar{I}_1(t), \bar{I}_2(t), \bar{I}_v(t))^T$.

In view of Lemma 1, there exists a positive ω-periodic vector function $v_1(t)$ such that $\bar{J}(t) = v_1(t)\exp(\varsigma t)$ is a solution of system (32), where $\varsigma = \ln r(\Phi_{(F_k - V_k + M_{\varepsilon_1 k})P_k}(\omega)) < 0$. So $\bar{J}(t) \to 0$, as $t \to \infty$. It follows (28) that $\lim_{t\to\infty}\bar{E}_h(t) = 0$, $\lim_{t\to\infty}\bar{I}_1(t) = 0$, $\lim_{t\to\infty}\bar{I}_2(t) = 0$, and $\lim_{t\to\infty}\bar{I}_v(t) = 0$. By the comparison theorem in impulsive differential equations, we have $\lim_{t\to\infty}E_h(t) = 0$, $\lim_{t\to\infty}I_1(t) = 0$, $\lim_{t\to\infty}I_2(t) = 0$, and $\lim_{t\to\infty}I_v(t) = 0$. By the theory of asymptotic semiflows, we can get

$$\begin{aligned} &\lim_{t\to\infty} S_h(t) = S_h^*(t), \\ &\lim_{t\to\infty} S_v(t) = S_v^*(t). \end{aligned} \tag{33}$$

Hence, the disease-free periodic solution $x^*(t)$ is globally asymptotically stable.

Theorem 2. *If $\mathscr{R}_0 > 1$, then the disease is uniformly persistent for system (4); that is, there is a positive constant $\epsilon > 0$, such that $\liminf_{t\to\infty}E_h(t) > \epsilon$, $\liminf_{t\to\infty}I_1(t) > \epsilon$, $\liminf_{t\to\infty}I_2(t) > \epsilon$, and $\liminf_{t\to\infty}I_v(t) > \epsilon$.*

Proof 2. Denote $\tilde{K} = \{(E_h, I_1, I_2, I_v, S_h, S_v) \in \mathbb{R}_+^6\}$, $K_0 = \{(E_h, I_1, I_2, I_v, S_h, S_v) \in \tilde{K} : E_h > 0, I_1 > 0, I_2 > 0, I_v > 0, S_h \geq 0, S_v \geq 0\}$, and $\partial K_0 = \tilde{K} \setminus K_0$. Let $u(t, t_0, x_0)$ be the unique solution of system (4) with the initial value $x_0 = (E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0)$ at time t_0.

Define Poincaré map $P : \tilde{K} \to \tilde{K}$ associated with system (4) as follows:

$$P(x_0) = u(t_0 + \omega^+, x_0), \quad \forall x_0 \in \tilde{K}, \forall t_0 \in \mathbb{R}_+. \tag{34}$$

Set

$$\begin{aligned} M_\partial = \{ & (E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0) \\ & \in \partial K_0 \mid P^m(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0) \\ & \in \partial K_0, \forall m \in \mathbb{Z}_+ \}. \end{aligned} \tag{35}$$

One claims that

$$M_\partial = \{(0, 0, 0, 0, S_h, S_v) \mid S_h \geq 0, S_v \geq 0\}. \tag{36}$$

Obviously, $\{(0, 0, 0, 0, S_h, S_v) \mid S_h \geq 0, S_v \geq 0\} \subseteq M_\partial$. Next, one wants to show

$$M_\partial \setminus \{(0, 0, 0, 0, S_h, S_v) \mid S_h \geq 0, S_v \geq 0\} = \varnothing. \tag{37}$$

If (37) does not hold, then there exists a point $(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0) \in M_\partial \setminus \{(0, 0, 0, 0, S_h, S_v) \mid S_h \geq 0, S_v \geq 0\}$. Next, for four initial values E_h^0, I_1^0, I_2^0, and I_v^0, three cases should be discussed.

Case (i). One initial value equals zero, and the others are larger than zero. Without loss of generality, one chooses $E_h^0 = 0$, $I_1^0 > 0$, $I_2^0 > 0$, and $I_v^0 > 0$. It is obvious that $S_h(t) > 0$ and $I_v(t) > 0$ for any $t \geq t_0$. Then, from the first equation of system (4), one gets $(dE_h(t)/dt) \mid_{t=t_0} = a_1 S_h(t_0) I_v(t_0)/N_h > 0$. Thus, $(E_h, I_1, I_2, I_v, S_h, S_v) \notin \partial K_0$ for $0 < t - t_0 \ll 1$. This is a contradiction. Other cases are similarly proved.

Case (ii). Two initial values equal zero, and the others are larger than zero. One lets $E_h^0 = I_1^0 = 0$, $I_2^0 > 0$, and $I_v^0 > 0$. It is obvious that $S_h(t) > 0$ and $I_v(t) > 0$ for any $t \geq t_0$. Using the same method as aforementioned, one can prove $(E_h, I_1, I_2, I_v, S_h, S_v) \notin \partial K_0$ for $0 < t - t_0 \ll 1$. This is a contradiction. Other cases can be proved similarly.

Case (iii). Three initial values equal zero, and the other is larger than zero. Set $E_h^0 = I_2^0 = I_v^0 = 0$ and $I_1^0 > 0$. It is obvious that $S_v(t) > 0$ and $I_1(t) > 0$ for any $t \geq t_0$. Then, from the fourth equation of system (4), one gets $(dI_v(t)/dt) \mid_{t=t_0} = b_1 S_v(t_0) I_1(t_0)/N_h > 0$. Thus, $(E_h, I_1, I_2, I_v, S_h, S_v) \notin \partial K_0$ for $0 < t - t_0 \ll 1$. This is a contradiction. Similarly, one can prove the other cases.

Thus,

$$M_\partial = \{(0, 0, 0, 0, S_h, S_v) \mid S_h \geq 0, S_v \geq 0\}. \tag{38}$$

In the following, one proceeds by contradiction to prove that there exists $\widehat{\xi} > 0$ such that

$$\limsup_{t\to\infty} d\left(P^m\left(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0\right), P_0\right) \geq \widehat{\xi}, \quad \forall x_0 \in \tilde{K}, m \in \mathbb{Z}_+. \tag{39}$$

where $P_0 = (0, 0, 0, 0, S_h^*(t_0), S_v^*(t_0))$.

By Lemma 4, one has $r(\Phi_{(F_k - V_k)P_k}(\omega)) > 1$ if $\mathscr{R}_0 > 1$. So one can choose $\varepsilon_2 > 0$ sufficiently small such that

$$r\left(\Phi_{(F_k - V_k - M_{\varepsilon_2 k})P_k}(\omega)\right) > 1,$$
$$\text{where } M_{\varepsilon_2 k} = \begin{pmatrix} 0 & 0 & 0 & \dfrac{a_k \varepsilon_2}{N_h} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \dfrac{c_k \varepsilon_2}{N_h} & \dfrac{b_k \varepsilon_2}{N_h} & 0 \end{pmatrix}. \tag{40}$$

If (39) does not hold, then for any $\widehat{\xi} > 0$, one obtains

$$\begin{aligned} & \limsup_{t\to\infty} d\left(P^m\left(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0\right), P_0\right) < \widehat{\xi}, \\ & \text{for some } \left(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0\right) \in \tilde{K}. \end{aligned} \tag{41}$$

Without loss of generality, one supposes that

$$d\left(P^m\left(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0\right), P_0\right) < \widehat{\xi}, \quad \forall \widehat{\xi} > 0, m \in \mathbb{Z}_+. \tag{42}$$

By the continuity of the solution with respect to initial values, one has that there exists sufficiently small $\widehat{\xi}$ such that

$$\begin{aligned} & \left\| u\left(t, P^m\left(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0\right)\right) - u(t, P_0) \right\| \leq \varepsilon_2, \\ & \forall t \in [t_0, t_0 + \omega], \forall m \in \mathbb{Z}_+. \end{aligned} \tag{43}$$

For any $t \geq t_0$, there exists an integer $l \in \mathbb{Z}_+$ such that $t = l\omega + \widehat{t}$, where $\widehat{t} \in [t_0, t_0 + \omega]$. Then one has

$$\begin{aligned} & \left\| u\left(t, P^m\left(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0\right)\right) - u(t, P_0) \right\| \\ & = \left\| u\left(\widehat{t}, P^m\left(E_h^0, I_1^0, I_2^0, I_v^0, S_h^0, S_v^0\right)\right) - u\left(\widehat{t}, P_0\right) \right\| \leq \varepsilon_2. \end{aligned} \tag{44}$$

Therefore, one has

$$\begin{aligned} & S_h(t) \geq S_h^*(t) - \varepsilon_2, \\ & S_v(t) \geq S_v^*(t) - \varepsilon_2, \\ & \text{for all } t \geq t_0. \end{aligned} \tag{45}$$

From system (4) and inequality (45), one gets

$$\begin{aligned}
\frac{dE_h(t)}{dt} &\geq \frac{a_k(S_h^*(t)-\varepsilon_2)I_v}{N_h} - \mu_1 E_h - \alpha E_h - dE_h,\\
\frac{dI_1(t)}{dt} &= \alpha E_h - \mu_1 I_1 - \theta I_1 - dI_1,\\
\frac{dI_2(t)}{dt} &= \theta I_1 - \mu_1 I_2 - dI_2 - \gamma_k I_2,\\
\frac{dI_v(t)}{dt} &\geq \frac{c_k(S_v^*(t)-\varepsilon_2)I_1}{N_h} + \frac{b_k(S_v^*(t)-\varepsilon_2)I_2}{N_h} - \mu_2 I_v,\\
&\quad t \in (t_{k-1}, t_k],\\
E_h(t^+) &= E_h(t),\\
I_1(t^+) &= I_1(t),\\
I_2(t^+) &= I_2(t),\\
I_v(t^+) &= (1-p_k)I_v(t),\\
&\quad t = t_k, k \in \mathbb{N}.
\end{aligned} \tag{46}$$

Consider the comparison system for system (46):

$$\begin{aligned}
\frac{dZ(t)}{dt} &= \left(F_k(t) - V_k(t) - M_{\varepsilon_2 k}\right)Z(t), \quad t \neq t_k\\
Z(t^+) &= P_k Z(t), \quad t = t_k
\end{aligned} \tag{47}$$

where $Z(t) = (E_h(t), I_1(t), I_2(t), I_v(t))^T$.

By Lemma 1, one knows that there exists a positive ω-periodic vector function $v_2(t)$ such that $Z(t) = v_2(t)\exp(\zeta t)$ is a solution of system (47), where $\zeta = \ln r(\Phi_{(F_k - V_k - M_{\varepsilon_2 k})P_k}(\omega))$. From (40), one can get that $Z(t) \to \infty$ as $t \to \infty$, and $E_h(t) \to \infty$, $I_1(t) \to \infty$, $I_2(t) \to \infty$, and $I_v(t) \to \infty$ as $t \to \infty$. By the comparison theorem in impulsive differential equations, one has $E_h(t) \to \infty$, $I_1(t) \to \infty$, $I_2(t) \to \infty$, and $I_v(t) \to \infty$ as $t \to \infty$. This contradicts with the boundedness of the solutions. Thus, one has proved that (39) holds and P is weakly uniformly persistent with respect to $(K_0, \partial K_0)$.

Obviously, the Poincaré map P has a global attractor P_0. P_0 is an isolated invariant set in $\tilde{K}$ and $W^s(P_0) \cap K_0 = \varnothing$ and it is acyclic in M_∂. Every solution in M_∂ converges to P_0. According to Zhao [30], one derives that P is uniformly persistent with respect to $(K_0, \partial K_0)$. This implies that the solution of system (4) is uniformly persistent with respect to $(K_0, \partial K_0)$. This completes the proof.

4. Numerical Simulations

In this section, we first provide results from numerical simulations of model (4) that demonstrate and support our theoretical results. For these simulations, part of parameters values for model (4) are outlined in Table 1.

In [20], Zhao revealed that 1-2 sprays should be done in the period after picking and before spring sprout, in spring, summer growth, and in autumn growth. So we assume that the system is composed of four subsystems, and the switching law is periodic and satisfies

Table 1: Parameter values for system (4).

Parameter	Value	Unit	Reference
Λ	6,028,433	year^{-1}	[31]
N_h	2000	—	[31]
μ_1	0.04	year^{-1}	[32]
μ_2	3.2	year^{-1}	[33]
d	0.025	year^{-1}	[21]
α	24.33	year^{-1}	[15]
θ	1.8	year^{-1}	[16, 17]
ω	1	year	—
k	0.6	—	Estimation

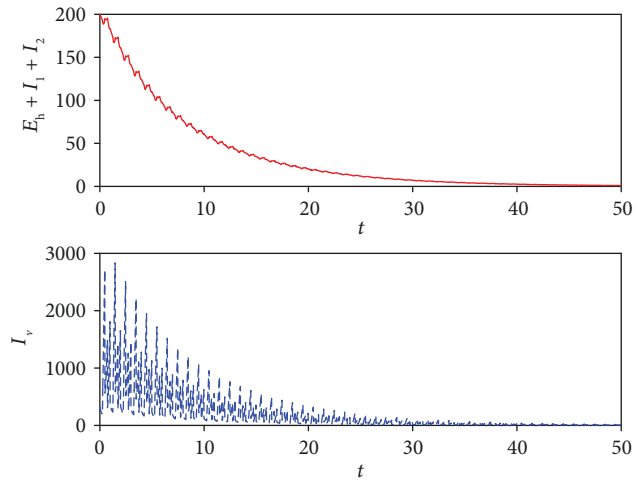

Figure 2: Time series plots of the total of infected trees and infected psyllids. The disease dies out with $\mathscr{R}_0 = 0.903702 < 1$.

$$\sigma(t) = \begin{cases} 1, & \text{if } t \in (k, k+0.25], \text{ winter},\\ 2, & \text{if } t \in (k+0.25, k+0.5], \text{ spring},\\ 3, & \text{if } t \in (k+0.5, k+0.75], \text{ summer},\\ 4, & \text{if } t \in (k+0.75, k+1], \text{ autumn},\\ & k = 0, 1, 2, \ldots. \end{cases} \tag{48}$$

Consider dynamical behavior of system (4) with initial conditions $E_h^0 = 100$, $I_1^0 = 40$, $I_2^0 = 60$, $I_v^0 = 330$, $S_h^0 = 1800$, and $S_v^0 = 10028$. The switched parameter values are used as follows: $a_1 = 0.002$, $a_2 = 0.078$, $a_3 = 0.05$, $a_4 = 0.078$, $b_1 = 0.006$, $b_2 = 0.156$, $b_3 = 0.106$, and $b_4 = 0.156$. For the control switched parameter values, we set $\gamma_1 = 0.2$, $\gamma_2 = 0.8$, $\gamma_3 = 0.2$, $\gamma_4 = 0.8$, $p_1 = 0$, $p_2 = 0.8$, $p_3 = 0.8$, and $p_4 = 0.8$. According to Lemma 3, we can get $\mathscr{R}_0 = 0.903702 < 1$ by numerical calculation, which shows that the disease dies out (see Figure 2). Set $\gamma_1 = 0.2$, $\gamma_2 = 0.4$, $\gamma_3 = 0.2$, $\gamma_4 = 0.35$, $p_1 = 0$, $p_2 = 0.2$, $p_3 = 0.15$, and $p_4 = 0.5$. We get $\mathscr{R}_0 = 1.801682 > 1$; the disease is uniformly persistent by Theorem 2, which is showed from Figure 3.

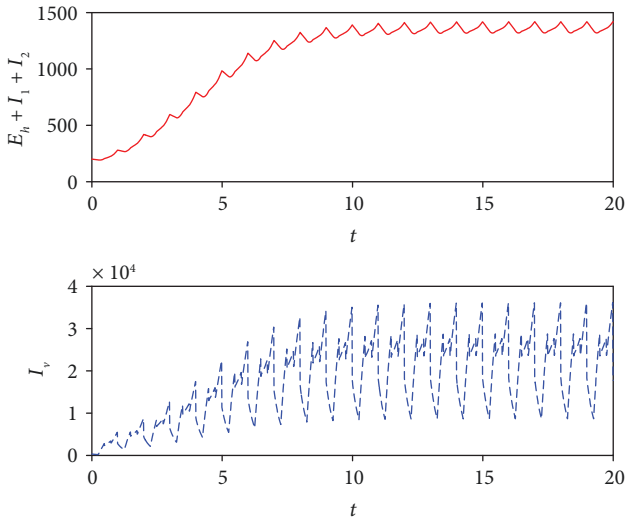

FIGURE 3: Time series plots of the total of infected trees and infected psyllids. The disease is persistent with $\mathscr{R}_0 = 1.801682 > 1$.

TABLE 2: The effect of parameters p_σ and γ_σ on the disease control.

	p_1	p_2	p_3	p_4	γ_1	γ_2	γ_3	γ_4	$\mathscr{R}_0$
Baseline scenario	0.3	0.8	0.3	0.8	0.2	0.8	0.2	0.8	0.878143
Strategy I	0.3	0.8	0.3	0.8	0.8	0.2	0.8	0.2	0.868525
Strategy II	0.8	0.3	0.8	0.3	0.2	0.8	0.2	0.8	0.994301

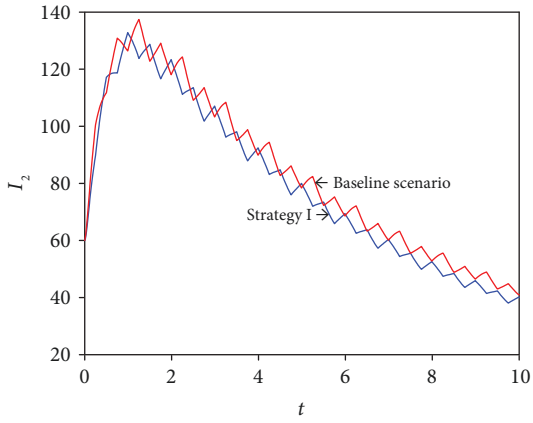

FIGURE 4: Time series plot of the total of infected trees for the baseline scenario and Strategy I.

Switching parameters have an effect on the peak size of infected individuals for switching epidemic models. Next, we consider the effect of varying switching removal rates γ_σ and insecticide spraying rates p_σ to evaluate the effectiveness of various control measures, while holding the other switched parameters constant. In Table 2, we give two different control projects to compare with the baseline scenario, which is denoted by Strategy I and Strategy II.

Figures 4 and 5 show the numerical simulations of the baseline scenario, Strategy I, and Strategy II. If we compare the baseline scenario and Strategy I (see rows 1 and 2), the evaluation implies that the baseline scenario is worse than

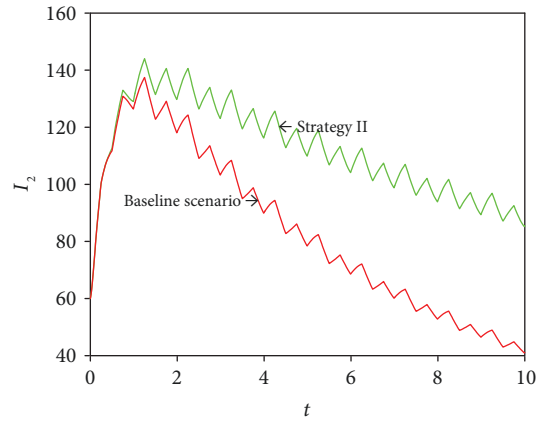

FIGURE 5: Time series plot of the total infected trees for the baseline scenario and Strategy II.

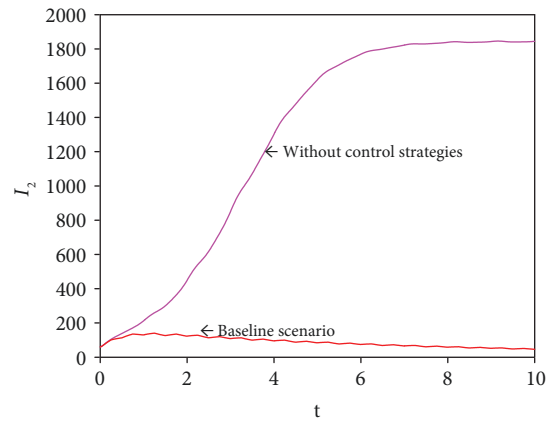

FIGURE 6: Time series plot of the total infected trees for the baseline scenario and without control strategies.

Strategy I (larger final and peak sizes and $\mathscr{R}_0$). If we compare the baseline scenario and Strategy II (see rows 1 and 3), the evaluation suggests that the baseline scenario is better than Strategy II (lower final and peak sizes and $\mathscr{R}_0$). This illustrates that Strategy I is the best control project, and the most effective control strategy is spraying in spring and autumn and removing in winter.

By calculating, $\mathscr{R}_0 = 5.023193$ in the absence of control strategies. We can observe from Figure 6 that the disease breaks out rapidly. This illustrates that removing infected trees and spraying pesticides play an important role in controlling the spread of HLB.

5. Conclusions

By introducing switching parameters into a general impulsive HLB model, a novel impulsive switching model for HLB with seasonal fluctuations has been constructed and a threshold value $\mathscr{R}_0$ with switching effect has been established to measure whether the disease is uniformly persistent. The modeling and analytic methods presented in this paper improve the classical results for the systems with impulsive

interventions. Numerical examples have been given to demonstrate the effectiveness of the results obtained.

Our numerical investigations demonstrate that the most effective season of spraying insecticide is in spring and autumn and the most effective season of removing infected trees is winter. The result strongly suggests and supports the previous observations [19, 34]. This can serve as an integrating measure to design an appropriate strategy to control HLB spread.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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