Distinct Quantum States Cannot Be Compatible with a Single State of Reality

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Abstract

It is demonstrated that, based on an analysis of protective measurements, distinct quantum states cannot be compatible with a single state of reality.

Recently Lewis et al (2012) demonstrated that additional assumptions such as preparation independence are always necessary to rule out a ψ-epistemic model, in which the quantum state is not uniquely determined by the underlying physical state. Their conclusion is based on an analysis of conventional projective measurements. Here we will demonstrate that protective measurements (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993), which are distinct from projective measurements, already shows that distinct quantum states cannot be compatible with a single state of reality.

Projective measurements are one kind of measurements, for which the coupling between the measuring device and the measured system is very strong and almost instantaneous, and the measurement results are the eigenvalues of the measured observable. Due to the resulting collapse of the wave function, such impulsive measurements cannot measure the actual physical state of the measured system (when the system is not in one of the eigenstates of the measured observable). This seems to leave space for ψ-epistemic models (Lewis et al 2012). However, it has been known that the coupling strength and the measuring time can be adjusted for a standard measurement procedure, and there also exist other kinds of measurements such as weak measurements and protective measurements (Aharonov, Albert and Vaidman 1988; Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993)[1]. Protective measurement uses a weak and long duration coupling interaction and an appropriate procedure to protect the measured system from being disturbed. A general scheme is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction (in some situations the protection is provided by the measured system itself), and then make the measurement adiabatically so that the state of the system neither collapses nor becomes entangled with the measuring device appreciably. In this way, such

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[1]Note that weak measurements have been implemented in experiments (Lundeen et al 2011), and it can be reasonably expected that protective measurements can also be implemented in the near future with the rapid development of quantum technologies.

protective measurements can measure the expectation values of observables on a single quantum system (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009).

An immediate implication of protective measurements is that the result of a protective measurement, namely the expectation value of the measured observable in the measured state, reflects the actual physical state of the measured system[2], as the system is not disturbed after this result has been obtained[3]. This is in accordance with the fundamental assumption that the result of a measurement that does not disturb the measured system reflects the actual property or state of the system. Moreover, since the wave function can be reconstructed from the expectation values of a sufficient number of observables, the wave function of a quantum system is a representation of the physical state of the system[4].

This result can be illustrated with a specific example (Aharonov and Vaidman 1993). Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed. We take the measured observable A_n to be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{1}$$

An adiabatic measurement of A_n then yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv = |\psi_n|^2, \tag{2}$$

where $|\psi_n|^2$ is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can adiabatically measure another observable $B_n = \frac{1}{2i}(A_n\nabla + \nabla A_n)$. The measurement yields

$$\langle B_n \rangle = \frac{1}{v_n} \int_{V_n} \frac{1}{2i}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)dv = \frac{1}{v_n} \int_{V_n} |j(x)|^2 dv. \tag{3}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ and $j(x)$ everywhere in space.

[2]Several authors, including the inventors of protective measurements, have obtained the similar conclusion as given here, though they are based on somewhat different arguments (Aharonov and Vaidman 1993; Anandan 1993; Dickson 1995).

[3]For a realistic protective measurement whose measuring interval T is finite, there is always a tiny probability proportional to $1/T^2$ to obtain a different result, and after obtaining the result the measured state also collapses to the state corresponding to the result. However, the key point here is that when the measurement obtains the expectation value of the measured observable, the state of the measured system is not disturbed. Moreover, the above probability can be made arbitrarily small in principle when T approaches infinity, as well as negligibly small in practice by making T sufficiently large.

[4]This implication is independent of whether the wave function of the system is known beforehand for protective measurements. The reason is that even though we know the wave function, which is an abstract mathematical object, we still don't know its physical meaning. A further analysis of what physical state the wave function represents has been given by Gao (2011).

Since the measured system is not disturbed after the above measurement results, namely the density $\rho(x)$ and flux density $j(x)$, have been obtained, these results reflect the actual physical state of the measured system. Moreover, since the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for an overall phase factor), it is also uniquely determined by the underlying physical state. Note that there might also exist other components of the underlying physical state, which are not measureable by protective measurements and not described by the wave function (e.g. the positions of the Bohmian particles in the de Broglie-Bohm theory). In this case, however, the wave function is still uniquely determined by the underlying physical state, though the wave function is not a complete representation of the physical state.

In conclusion, we have demonstrated that, without resorting to nontrivial assumptions such as preparation independence, the wave function or quantum state is uniquely determined by the underlying physical state, and thus distinct quantum states cannot be compatible with a single state of reality. This improves the interesting result obtained by Pusey, Barrett and Rudolph (2012). Certainly, the quantum state also plays an epistemic role by giving the probability distribution of the results of projective measurements according to the Born rule. However, this role is secondary and determined by the complete quantum dynamics that describes the measuring process, e.g. the collapse dynamics in dynamical collapse theories.

Appendix: Mathematical formulation of protective measurement

Protective measurement, in the language of standard quantum mechanics, is a method to measure the expectation value of an arbitrary observable on a single quantum system (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009). For a conventional impulsive measurement, the coupling interaction between the measured system and the measuring device is of short duration and strong. By contrast, protective measurement uses a weak and long duration coupling interaction and an appropriate procedure to protect the measured system from being disturbed. A general scheme is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction (in some situations the protection is provided by the measured system itself), and then make the measurement adiabatically so that the state of the system neither changes nor becomes entangled with the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system, and in particular, the physical state of the system, which is described by its wave function, can also be measured as expectation values of certain observables.

As a typical example, we consider a quantum system in a discrete nondegenerate energy eigenstate $|E_n\rangle$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed[5].

[5]As will be shown below, before the protective measurement we only need to know the measured state is a discrete nondegenerate energy eigenstate of the Hamiltonian of the system, and we need not to know the measured state or the Hamiltonian of the system or the measured

According to the standard von Neumann procedure, measuring an observable A in this state involves an interaction Hamiltonian

$$H_I = g(t)PA \tag{4}$$

coupling the measured system to an appropriate measuring device, where P is the momentum conjugate to the pointer variable X of an appropriate measuring device. The time-dependent coupling strength $g(t)$ is a smooth function normalized to $\int dtg(t) = 1$ during the interaction interval T, and $g(0) = g(T) = 0$. The initial state of the pointer at $t = 0$ is supposed to be $|\phi(x_0)\rangle$, which is a Gaussian wave packet of eigenstates of X with width w_0, centered around the eigenvalue x_0.

For a conventional impulsive measurement, the interaction H_I is of very short duration and so strong that it dominates the rest of the Hamiltonian (i.e. the effect of the free Hamiltonians of the measuring device and the measured system can be neglected). Then the state of the combined system at the end of the interaction can be written as

$$|t = T\rangle = e^{-\frac{i}{\hbar}PA} |E_n\rangle |\phi(x_0)\rangle . \tag{5}$$

By expanding $|E_n\rangle$ in the eigenstates of A, $|a_i\rangle$, we obtain

$$|t = T\rangle = \sum_i e^{-\frac{i}{\hbar}Pa_i} c_i |a_i\rangle |\phi(x_0)\rangle , \tag{6}$$

where c_i are the expansion coefficients. The exponential term shifts the center of the pointer by a_i:

$$|t = T\rangle = \sum_i c_i |a_i\rangle |\phi(x_0 + a_i)\rangle . \tag{7}$$

This is an entangled state, where the eigenstates of A with eigenvalues a_i get correlated to measuring device states in which the pointer is shifted by these values a_i. Then by the collapse postulate of standard quantum mechanics, the state will instantaneously and randomly collapse into one of its branches $|a_i\rangle |\phi(x_0 + a_i)\rangle$ with probability $|c_i|^2$. This means that the measurement result can only be one of the eigenvalues of measured observable A, say a_i, with a certain probability, say $|c_i|^2$. The expectation value of A is then obtained as the statistical average of eigenvalues for an ensemble of identically prepared systems, namely $\langle A\rangle = \sum_i |c_i|^2 a_i$.

Different from the conventional impulsive measurements, for which the interaction is very strong and almost instantaneous, protective measurements make use of the opposite limit where the interaction of the measuring device with the system is weak and adiabatic, and thus the free Hamiltonians cannot be neglected[6]. Let the Hamiltonian of the combined system be

$$H(t) = H_S + H_D + g(t)PA, \tag{8}$$

state is one of a known collection of energy eigenstates. In this case, by a conventional impulsive measurement we can only measure the energy of the system, and we cannot measure the expectation value of any other observable of the system (as well as the wave function of the system).

[6]For a more detailed derivation of protective measurement see Dass and Qureshi (1999).

where H_S and H_D are the free Hamiltonians of the measured system and the measuring device, respectively. The interaction lasts for a long time T, and $g(t)$ is very small and constant for the most part, and it goes to zero gradually before and after the interaction.

The state of the combined system after T is given by

$$|t = T\rangle = e^{-\frac{i}{\hbar}\int_0^T H(t)dt} |E_n\rangle |\phi(x_0)\rangle . \tag{9}$$

By ignoring the switching on and switching off processes[7], the full Hamiltonian (with $g(t) = 1/T$) is time-independent and no time-ordering is needed. Then we obtain

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} |E_n\rangle |\phi(x_0)\rangle , \tag{10}$$

where $H = H_S + H_D + \frac{PA}{T}$. We further expand $|\phi(x_0)\rangle$ in the eigenstate of H_D, $\left|E_j^d\right\rangle$, and write

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} \sum_j c_j |E_n\rangle \left|E_j^d\right\rangle , \tag{11}$$

Let the exact eigenstates of H be $|\Psi_{k,m}\rangle$ and the corresponding eigenvalues be $E(k, m)$, we have

$$|t = T\rangle = \sum_j c_j \sum_{k,m} e^{-\frac{i}{\hbar}E(k,m)T} \langle\Psi_{k,m}|E_n, E_j^d\rangle|\Psi_{k,m}\rangle. \tag{12}$$

Since the interaction is very weak, the Hamiltonian H of Eq.(8) can be regarded as $H_0 = H_S + H_D$ perturbed by $\frac{PA}{T}$. Using the fact that $\frac{PA}{T}$ is a small perturbation and that the eigenstates of H_0 are of the form $|E_k\rangle \left|E_m^d\right\rangle$, the perturbation theory gives

$$\begin{aligned} |\Psi_{k,m}\rangle &= |E_k\rangle \left|E_m^d\right\rangle + O(1/T), \\ E(k, m) &= E_k + E_m^d + \frac{1}{T}\langle A\rangle_k \langle P\rangle_m + O(1/T^2). \end{aligned} \tag{13}$$

Substituting Eq.(13) in Eq.(12) and taking the limit $T \to \infty$ yields

$$|t = T\rangle_{T\to\infty} = \sum_j e^{-\frac{i}{\hbar}(E_n T + E_j^d T + \langle A\rangle_n \langle P\rangle_j)} c_j |E_n\rangle \left|E_j^d\right\rangle . \tag{14}$$

For the case where P commutes with the free Hamiltonian of the device[8], i.e., $[P, H_D] = 0$, the eigenstates $\left|E_j^d\right\rangle$ of H_D are also the eigenstates of P, and thus the above equation can be rewritten as

$$|t = T\rangle_{T\to\infty} = e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T - \frac{i}{\hbar}\langle A\rangle_n P} |E_n\rangle |\phi(x_0)\rangle . \tag{15}$$

It can be seen that the third term in the exponent will shift the center of the pointer $|\phi(x_0)\rangle$ by an amount $\langle A\rangle_n$:

$$|t = T\rangle_{T\to\infty} = e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T} |E_n\rangle |\phi(x_0 + \langle A\rangle_n)\rangle. \tag{16}$$

[7]The change in the total Hamiltonian during these processes is smaller than PA/T, and thus the adiabaticity of the interaction will not be violated and the approximate treatment given below is valid.

[8]For the derivation for the case $[P, H_D] \neq 0$ see Dass and Qureshi (1999).

This indicates that the result of the protective measurement is the expectation value of the measured observable in the measured state, and moreover, the measured state is not changed by the protective measurement[9].

This strict mathematical result can also be understood in terms of the adiabatic theorem and the first order perturbation theory in quantum mechanics. By the adiabatic theorem, the adiabatic interaction during the protective measurement ensures that the measured system cannot make a transition from one discrete energy eigenstate to another. Moreover, according to the first order perturbation theory, for any given value of P, the energy of the measured energy eigenstate shifts by an infinitesimal amount: $\delta E = \langle H_I \rangle = P\langle A\rangle_n/T$, and the corresponding time evolution $e^{-iP\langle A\rangle_n/\hbar}$ then shifts the pointer by the expectation value $\langle A\rangle_n$.

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[9]It might be worth noting that there appeared numerous objections to the validity of protective measurements (see, e.g. Unruh 1994; Rovelli 1994; Ghose and Home 1995; Uffink 1999), though these objections have been answered (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999; Vaidman 2009; Gao 2012).

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