On Uffink's alternative interpretation of protective measurements

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Abstract

Protective measurement is a new measuring method introduced by Aharonov, Anandan and Vaidman (1993). By a protective measurement, one can measure the expectation value of an observable on a single quantum system, even if the system is initially not in an eigenstate of the measured observable. Aharonov, Anandan and Vaidman attributed this feature of protective measurements to a physical manifestation of the wave function of the system. This interpretation was challenged by Uffink (1999, 2012). He argued that only observables that commute with the system's Hamiltonian can be protectively measured, and a protective measurement of an observable that does not commute with the system's Hamiltonian does not actually measure the observable, but measure another related observable that commutes with the system's Hamiltonian. In this paper, we argue that there are several errors in Uffink's arguments, and his alternative interpretation of protective measurements is untenable.

1 Introduction

Aharonov, Anandan and Vaidman (AAV) introduced a new measuring method called protective measurements in 1993 (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993). By a protective measurement, one can measure the expectation value of an observable on a single quantum system, even if the system is initially not in an eigenstate of the measured observable. This remarkable feature makes protective measurements quite distinct from conventioanl impulse measurements, and as AAV have argued, it may have important implications on the meaning of the wave fucntion, e.g. it implies that the wave function should be given an ontological interpretation, as the wave function of a single quantum system can be directly observed by means of protective measurements (Aharonov, Anandan and Vaidman 1993).

Although numerous objections to the validity and meaning of protective measurements were raised (see, e.g. Unruh 1994; Rovelli 1994; Ghose and Home

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1995), most of these objections have been answered (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999; Vaidman 2009). A unique exception seems to be Uffink's (1999) objection[1]. Uffink argued that only observables that commute with the systems Hamiltonian can be protectively measured, and moreover, a protective measurement of an observable does not actually measure the observable, which may not commute with the system's Hamiltonian, but measure another related observable that commutes with the system's Hamiltonian (Uffink 1999, 2012). This alternative interpretation of protective measurements has been accepted by some authors (e.g. Parwani 2005; Dickson 2007; Saunders 2010; Paraoanu 2011), and if it is true, it will "protect the interpretation of the wave function against protective measurements" as Uffink expected.

In this paper, we will argue that there are several errors in Uffink's arguments, and his alternative interpretation of protective measurements is untenable. The paper is organized as follows. In section 2, we first introduce the basic principle of protective measurements. Then, in section 3, we examine Uffink's proofs of his conclusion that only observables that commute with the system's Hamiltonian can be measured protectively. It is shown that there are errors in Uffink's original proof and improved proof. In section 4, Uffink's alternative interpretation of protective measurements is further examined. We argue that Uffink's arguments for his interpretation are problematic. In section 5, we examine Uffink's analysis of a thought experiment. It is argued that a correct analysis of the experiment does not support Uffink's alternative interpretation of protective measurements. Conclusions are given in the last section.

2 Protective measurements

Protective measurement is a method for measuring the expectation value of an observable on a single quantum system. Like conventional impulse measurement, protective measurement also uses the standard measuring procedure, but with a weak, adiabatic coupling and an appropriate protection. Its general method is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction, and then make the measurement adiabatically. This permits protective measurement to be able to measure the expectation values of observables on a single quantum system.

As a simple example of protective measurement, consider a quantum system in a discrete nondegenerate energy eigenstate $|E_n\rangle$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed. The interaction Hamiltonian for a protective measurement of an observable O in this state involves the same interaction Hamiltonian as the standard measuring procedure:

$$H_I = g(t)O \otimes P, \tag{1}$$

[1]In a recent review of my manuscript "Protective Measurement and the Meaning of the Wave Function" (Gao 2011a), the reviewer said, "the manuscript fails to deal with the most important of such objections, i.e. J. Uffink in Phys. Rev. A 60: 3474-3481 (1999), a paper that argues against AAV that the concept of protective measurements has no implication for the interpretation of the wave function." (Gao 2011b). Although Vaidman (2009) regarded Uffink's objection as a misunderstanding of what the protective measurement is, he gave no concrete rebuttal.

where P is the momentum conjugate to the pointer variable X of an appropriate measuring device. The time-dependent coupling strength $g(t)$ is also a smooth function normalized to $\int dt g(t) = 1$. But different from conventional impulse measurements, for which the interaction is very strong and almost instantaneous, protective measurements make use of the opposite limit where the interaction of the measuring device with the system is weak and adiabatic. Concretely speaking, the interaction lasts for a long time T, and $g(t)$ is very small and constant for the most part, and it goes to zero gradually before and after the interaction.

Let the total Hamiltonian of the combined system be

$$H(t) = H_S + H_A + g(t) O \otimes P, \tag{2}$$

where H_S and H_A are the Hamiltonians of the measured system and the measuring device, respectively. Let the initial state of the pointer at $t = 0$ be $|\phi(x_0)\rangle$, which is a Gaussian wave packet of eigenstates of X with width w_0, centered around the eigenvalue x_0. Then the state of the combined system after T is

$$|t = T\rangle = e^{-i \int_0^T H(t) dt} |E_n\rangle |\phi(x_0)\rangle . \tag{3}$$

By ignoring the switching on and switching off processes[2], the full Hamiltonian (with $g(t) = 1/T$) is time-independent and no time-ordering is needed. Then we obtain

$$|t = T\rangle = e^{-iHT} |E_n\rangle |\phi(x_0)\rangle , \tag{4}$$

where $H = H_S + H_A + \frac{O \otimes P}{T}$. We further expand $|\phi(x_0)\rangle$ in the eigenstate of H_A, $|E_j^a\rangle$, and write

$$|t = T\rangle = e^{-iHT} \sum_j c_j |E_n\rangle |E_j^a\rangle . \tag{5}$$

Let the exact eigenstates of H be $|\Psi_{k,m}\rangle$ and the corresponding eigenvalues be $E(k, m)$, we have

$$|t = T\rangle = \sum_j c_j \sum_{k,m} e^{-iE(k,m)T} \langle \Psi_{k,m} | E_n, E_j^a \rangle |\Psi_{k,m}\rangle. \tag{6}$$

Since the interaction is very weak, the Hamiltonian H of Eq.(2) can be thought of as $H_0 = H_S + H_A$ perturbed by $\frac{O \otimes P}{T}$. Using the fact that $\frac{O \otimes P}{T}$ is a small perturbation and that the eigenstates of H_0 are of the form $|E_k\rangle |E_m^a\rangle$, the perturbation theory gives

$$\begin{aligned} |\Psi_{k,m}\rangle &= |E_k\rangle |E_m^a\rangle + O(1/T), \\ E(k, m) &= E_k + E_m^a + \frac{1}{T} \langle A \rangle_k \langle P \rangle_m + O(1/T^2). \end{aligned} \tag{7}$$

Note that it is a necessary condition for Eq.(7) to hold that $|E_k\rangle$ is a nondegenerate eigenstate of H_S. Substituting Eq.(7) in Eq.(6) and taking the large T limit yields

[2]The change in the total Hamiltonian during these processes is smaller than $O \otimes P/T$, and thus the adiabaticity of the interaction will not be violated and the approximate treatment given below is valid. For a more strict analysis see Dass and Qureshi (1999).

$$|t=T\rangle \approx \sum_j e^{-i(E_nT+E_j^aT+\langle O\rangle_n\langle P\rangle_j)}c_j\,|E_n\rangle\,|E_j^a\rangle\,. \tag{8}$$

When P commutes with the free Hamiltonian of the device, i.e., $[P,H_A]=0$, the eigenstates $|E_j^a\rangle$ of H_A are also the eigenstates of P, and thus the above equation can be rewritten as

$$|t=T\rangle \approx e^{-iE_nT}\,|E_n\rangle\,e^{-iH_AT-i\langle O\rangle_nP}\,|\phi(x_0)\rangle\,. \tag{9}$$

It can be seen that the third term in the exponent will shift the center of the pointer $|\phi(x_0)\rangle$ by an amount $\langle O\rangle_n$:

$$|t=T\rangle \approx e^{-iE_nT-iH_AT}\,|E_n\rangle\,|\phi(x_0+\langle O\rangle_n)\rangle. \tag{10}$$

This shows that the center of the pointer shifts by $\langle O\rangle_n$ at the end of the interaction. For the general case where $[P,H_A]\neq 0$, we can also obtain the similar result[3].

3 Must the observable measured protectively commute with the system's Hamiltonian?

The above example demonstrates that one can measure the expectation value of an observable on a single quantum system by a protective measurement. The observable does not necessarily commute with the system's Hamiltonian, and the system is not necessarily in an eigenstate of the observable either. AAV attributed this feature of protective measurements to a physical manifestation of the wave function of the system (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993, 1996). This interpretation of protective measurements was challenged by Uffink (1999, 2012). He tried to prove that only observables that commute with the systems Hamiltonian can be protectively measured. His proof can be basically formulated as follows.

Uffink first defined an operator $U_{\rm app}$ that brings about the approximate evolution Eq. (9) exactly for all vectors of the form $|\phi_n\rangle\,|\chi\rangle$, i.e.:

$$U_{\rm app}\ :\ |\phi_n\rangle\,|\chi\rangle \longrightarrow e^{-i\tau E_n}\,|\phi_n\rangle\,e^{-i(H_A\tau+\langle O\rangle_nP)}\,|\chi\rangle\,. \tag{11}$$

Moreover, he gave an explicit expression for $U_{\rm app}$ with another observable $\tilde{O}=\sum_n P_nOP_n$, where $P_n=|\phi_n\rangle\langle\phi_n|$ is a projector on the eigenstates of the Hamiltonian H_S:

$$U_{\rm app}=e^{-i(H_S+H_A)\tau-i\tilde{O}\otimes P} \tag{12}$$

Since $[\tilde{O},H_S]=0$, it immediately follows that $[U_{\rm app},H_S]=0$, or in other words:

$$U^\dagger_{\rm app}H_SU_{\rm app}=H_S. \tag{13}$$

This means that H_S is conserved under the evolution $U_{\rm app}$.

Uffink then tried to prove that $U_{\rm app}$ is a good approximation to U only if the observable O commutes with the system's Hamiltonian H_S. To say that the approximation involved in Eq. (9) is good means that

[3]For details see Dass and Qureshi (1999) and Gao (2011c).

$$\|(U - U_{\text{app}})\,|\phi_n\rangle\,|\chi\rangle\,\| \to 0 \;\text{ if }\; \tau \to \infty. \tag{14}$$

By a series of derivations, Uffink (1999) proved that this happens only if for almost all values of p:

$$\langle\phi_m|e^{i(\tau H_S + pO)} H_S e^{-i(\tau H_S + pO)}\,|\phi_n\rangle \to E_n\delta_{mn} \tag{15}$$

Uffink (1999) thought that this is equivalent to

$$e^{i\tau(E_m - E_n)}\langle\phi_m|e^{ipO} H_S e^{-ipO}\,|\phi_n\rangle \to E_n\delta_{mn}. \tag{16}$$

which means that for almost all $p \in I\!R$,

$$e^{ipO} H_S e^{-ipO} = H_S, \tag{17}$$

which further implies:

$$[O, H_S] = 0. \tag{18}$$

Then Uffink (1999) concluded that the observable whose expectation value is obtained by protective measurement must commute with the system's Hamiltonian.

However, as admitted also by Uffink (2012), there is an error in the most crucial step of the proof, namely the derivation from Eq. (15) to Eq. (16). In the derivation, it is implicitly assumed that the two operators O (the observable) and H_S (the system Hamiltonian) are commutative. The exponential function satisfies the equality $e^{X+Y} = e^X e^Y$ only if the two operators X and Y commute. But the aim of the proof is to prove the commutativity of these two operators. Thus Uffink's proof is circular because it presupposes what it sets out to prove.

Uffink (2012) provided an improved proof. He used the Baker-Campbell-Hausdorff theorem to expand $e^{ipA} H_S e^{-ipA}$, where A is defined as $A = \frac{\tau}{p} H_S + O$:

$$e^{ipA} H_S e^{-ipA} = \sum_{k=0}^{\infty} \frac{(ip)^k}{k!} H_k \tag{19}$$

where $H_0 = H_S$, $H_1 = [A, H_S]$, $H_2 = [A, [A, H_S]]$, $H_k = [A, H_{k-1}]$. Correspondingly, Eq. (15) becomes

$$\langle\phi_m|e^{ipA} H_S e^{-ipA}\,|\phi_n\rangle \to E_n\delta_{mn}. \tag{20}$$

Since H_k only contains terms proportional at most to $p^{-(k-1)}$, when assuming $|p|$ to be very small, we may only investigate the first two terms of the series expansion Eq. (19) for an approximate calculation of the total sum.

For $k = 0$, we get

$$\langle\phi_m|H_0\,|\phi_n\rangle = \langle\phi_m|H_S\,|\phi_n\rangle = E_n\delta_{mn}. \tag{21}$$

This means that Eq. (20) can only hold if the contributions from the terms with $k \geqslant 1$ in Eq. (19) vanish in the limit $\tau \to \infty$.

For $k = 1$, there is a contribution to the series expansion of the left-hand side of Eq. (20):

$$ip\langle\phi_m|H_1\,|\phi_n\rangle = ip\langle\phi_m|[O, H_S]\,|\phi_n\rangle\,. \tag{22}$$

Note that this term does not depend on τ, and thus it will not be affected by the limit $\tau \to \infty$.

Then Uffink (2012) concluded that the condition Eq. (20) can only hold for the chosen value of p if the term with $k = 1$ is exactly zero, namely:

$$\langle \phi_m | [O, H_S] | \phi_n \rangle = 0, \tag{23}$$

which further implies that $[O, H_S] = 0$, i.e. the observable O commutes with the system's Hamiltonian H_S.

There is an obvious problem here. Since the terms for $k > 1$ in the series expansion Eq. (19) also contain other terms independent of τ, even if the sum of all terms independent of τ is zero, we cannot obtain the result that the term with $k = 1$ is zero withour an additional justification. This loophole may be closed by noticing that Eq. (20) holds true for almost all values of p[4]. However, Uffink's new proof has a more serious problem. It is that the series expansion of the left-hand side of Eq. (20) is valid only when τ is very small. When τ is very large and even goes to infinity, the series does not converge. For instance, $\langle \phi_m | H_1 | \phi_n \rangle$, $\langle \phi_m | H_2 | \phi_n \rangle$ ... will all go to infinity if τ goes to infinity. Thus we cannot conclude that the term with $k = 1$, as well as the sum of all terms independent of τ, is zero[5]. In fact, we can give a counterargument by reduction to absurdity. When $[O, H_S] = 0$, which is the result that Uffink tried to prove, is true, then the left-hand side of Eq. (15) will be equal to the right-hand side of Eq. (15) for any value of τ. But this contradicts Eq. (15), according to which these two sides are equal only when τ goes to infinity.

There is also a general arguement against Uffink's proofs. The validity of first order perturbation theory and the adiabatic theorem, which have been widely used and confirmed in quantum mechanics, already implies that Uffink's attempt cannot succeed. For according to these theories, Eq. (15) can be satisfied when the two operators O and H_S are noncommutative (see Section 2). In other words, if Eq. (15) can be satisfied only when the two operators O and H_S are commutative as Uffink tried to prove, then either first order perturbation theory or the adiabatic theorem will be wrong.

4 Uffink's alternative interpretation of protective measurements

In order to explain away the remarkable features of protective measurements, Uffink (1999, 2012) proposed an alternative explanation for what happens in a protective measurement.

As we know, for a protective measurement, the interaction between the measured system and the measuring device is produced by a very small interaction term, i.e. $g(t)O \otimes P$, that works for a very long time. The smallness is responsible for the fact that $|\phi_n\rangle$ remains unchanged, and the long time permits that a non-vanishing effect of the interaction builds up in the state of the device. According to Uffink's explanation, the effect that builds up in the course of

[4] I am grateful to an anonymous reviewer of this journal for pointing out this.

[5] A simple example is that $e^{-x} = 1 - x + ...$ is valid only when x is small and the series converges. If x goes to infinity, then it is obvious that the left-hand side, which is zero, is not equal to the term independent of x on the right-hand side, which is 1.

time is due only to the part of O that commutes with H_S (namely $\tilde{O}$). It is only the operator $\tilde{O}$ whose expectation value is revealed, and the procedure is insensitive to the remainder $O - \tilde{O}$, i.e. the part of O that does not commute with H_S. In short, Uffink's alternative explanation of a protective measurement is that the procedure does not actually measure the observable O, which may not commute with the system's Hamiltonian H_S, but the related observable $\tilde{O}$, which commutes with the system's Hamiltonian H_S. We write the explicit form of $\tilde{O}$ again:

$$\tilde{O} = \sum_n P_n O P_n, \tag{24}$$

where $P_n = |\phi_n\rangle\langle\phi_n|$ is a projector on the eigenstates of the Hamiltonian H_S.

Besides his failed proofs, Uffink's main argument for his alternative explanation is that the measurement of the related observable $\tilde{O}$ on a system in an eigenstate $|\phi_n\rangle$ of H_S also yields the expectation value $\langle O\rangle_n$. However, it is obvious that this argument alone cannot determine which observable a protective measurement actually measures; it can be either $\tilde{O}$ or O. In other words, Uffink did not provide a sufficient reason to favor his explanation and reject the normal explanation. On the other hand, as we think, there are some good reasons to favor the normal explanation, namely that what a protective measurement measures is not $\tilde{O}$ but O.

First of all, as Uffink (2012) also admitted, when the measuring time τ is finite, what a protective measurement measures is O, not $\tilde{O}$. The measurement of $\tilde{O}$, which commutes with the system's Hamiltonian, results in neither entanglement between the measured system and the measuring device nor collapse of the measured state. By contrast, for a protective measurement of O, the entanglement and collapse can never be completely avoided for any finite τ. Then in the limit $\tau \to \infty$, what the protective measurement measures should be still O, not $\tilde{O}$, by continuity. Moreover, the effect that builds up in the course of a protective measurement for any finite τ is due not only to the part of O that commutes with H_S (namely $\tilde{O}$), but also to the part of O that does not commute with H_S (namely the remainder $O - \tilde{O}$), though when $\tau \to \infty$, the effect due to $O - \tilde{O}$ is close to zero.

Next, it can be argued that a protective measurement of O is still proper when the measuring time τ is finite but very long so that the adiabatic condition can be satisfied. In this case, even though entanglement and collapse cannot be completely avoided, their effects can be made arbitrarily small when τ is arbitrarily large. Thus only a very small ensemble is needed for measuring the expectation value of O by protective measurements (Dass and Qureshi 1999; Gao 2011c). This still presents a striking contrast to conventional impulse measurements, and the contrast cannot be explained away by Uffink's proposal.

Lastly, it is worth noting that in realistic situations we normally know which observable we will measure before a measurement, though in general we don't know exactly the state of the measured system and its Hamiltonian. For example, when we measure the spin of a particle, we certainly know the observable we will measure is spin before the measurement, and without this information we cannot prepare the measurement setting, e.g. a setting with a Stern-Gerlach magnet. It is the observable O, not the observable $\tilde{O}$, that we may know before a measurement, as knowing $\tilde{O}$ requires a full *a priori* knowledge of the system's

Hamiltonian, which is generally unavailable before a measurement.

5 A thought experiment

Uffink (1999, 2012) illustrated his conclusions by means of a thought experiment which had been discussed by Aharonov, Anandan and Vaidman (1993). However, his analysis of the experiment is also problematic.

In the experiment, a charged particle is in a superposition of two states localized in distant boxes L and R:

$$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|\phi_L\rangle + |\phi_R\rangle), \tag{25}$$

where $|\phi_L\rangle$ and $|\phi_R\rangle$ are the ground states of the box potentials. The question is whether a protective measurement can demonstrate that the particle is in a delocalized state. Since this superposition state degenerates with

$$|\phi_-\rangle = \frac{1}{\sqrt{2}}(|\phi_L\rangle - |\phi_R\rangle), \tag{26}$$

a protective procedure is needed to lift the degeneracy. For example, by arranging that in the region between the two boxes the potential has a large but finite constant value V as Uffink suggested, one can achieve that these two states are no longer degenerate.

Then a protective measurement of the observable:

$$O = -|\phi_L\rangle\langle\phi_L| + |\phi_R\rangle\langle\phi_R| \tag{27}$$

on this state will yield its expectation value $\langle O\rangle_+ = 0$. This measurement can be done by sending a charged test particle straight through the middle between the boxes, perpendicular to the line joining the two boxes, and the trajectory of the test particle will not deviate.

Uffink (1999) argued that the protective measurement does not demonstrate that the measured particle is in a delocalized state. His argument is as follows. Consider the case where the measurement is carried out on a charged particle prepared in a localized state $|\phi_L\rangle$. Since this state is not protected, one obtains the evolution:

$$|\phi_L\rangle|\chi\rangle = \frac{1}{\sqrt{2}}(|\phi_+\rangle + |\phi_-\rangle)|\chi\rangle \to \frac{1}{\sqrt{2}}(|\phi_+\rangle|\chi_+\rangle + |\phi_-\rangle|\chi_-\rangle), \tag{28}$$

where $|\chi\rangle$ is the initial state of the test particle, $|\chi_+\rangle$ and $|\chi_-\rangle$ are its final states in the cases when the measured particle was initially in the states $|\phi_+\rangle$ and $|\phi_-\rangle$. Since $\langle O\rangle_+ = \langle O\rangle_- = 0$, the test particle travels a straight trajectory in the state $|\chi_+\rangle$ as well as in $|\chi_-\rangle$. Thus the test particle will travel on a straight path, regardless of whether the measured particle is delocalized or not. Based on this result, Uffink concluded that the above protective experiment provides no evidence for the spatial delocalization of the measured particle.

At first sight Uffink's argument seems invulnerable. However, it is not difficult to find its problems by a careful analysis. The key is to realize that in order to measure the state of a single system, e.g. whether the system is in a delocalized state or not, the measured state must be protected beforehand in order that the state does not collapse during the measurement. If a measurement results in the collapse of the measured state, then the measurement result will not reflect the actual measured state[6]. It is obvious that in the above thought experiment the measured state $|\phi_L\rangle$ is not protected and will collapse to $|\chi_+\rangle$ or $|\chi_-\rangle$ after the measurement, which is also admitted by Uffink. Accordingly, the collapse state and the result of the measurement cannot tell us that the initial state $|\phi_L\rangle$ is localized, and thus the experiment cannot be used to support Uffink's conclusion. In other words, only when the result of the protective measurement of $|\phi_L\rangle$ is the same as the result of the protective measurement of $|\phi_+\rangle$ (for both measurements no collapse happens), can Uffink's argument hold true. But certainly these two results are different; for the former, the trajectory of the test particle deviates, while for the latter the trajectory is a straight path.

Another problem of Uffink's argument is that the result of the non-protective measurement of $|\phi_L\rangle$ is not exactly the same as the result of the protective measurement of $|\phi_+\rangle$ or $|\phi_-\rangle$. The reason is not only that the results of the protective measurements of $|\phi_+\rangle$ and $|\phi_-\rangle$ are not exactly the same, which has been noticed by Uffink, but also that a non-protective adiabatic measurement will result in wavefunction collapse as we have noted above. Since the wavefunction collapse is very tiny for the non-protective measurement of $|\phi_L\rangle$ in the above experiment, this problem may evade Uffink's scrutiny and lead him to the wrong conclusion. In order to see more clearly the problem, let's consider the protective measurement of a general state:

$$|\phi_+\rangle = a\,|\phi_L\rangle + b\,|\phi_R\rangle\,, \tag{29}$$

where $a \neq b$, and $|a|^2 + |b|^2 = 1$. Since this state degenerates with

$$|\phi_-\rangle = b^*\,|\phi_L\rangle - a^*\,|\phi_R\rangle\,, \tag{30}$$

a similar protective procedure is also needed to lift the degeneracy. For this general case, the results of the protective measurements of $|\phi_+\rangle$ and $|\phi_-\rangle$ will be obviously different. Therefore, the non-protective measurement of $|\phi_L\rangle = a^*\,|\phi_+\rangle + b\,|\phi_-\rangle$ will lead to obvious wavefunction collapse; its result will be either the result of the protective measurement of $|\phi_+\rangle$ with probability $|a|^2$ or the result of the protective measurement of $|\phi_-\rangle$ with probability $|b|^2$, and correspondingly the measured state $|\phi_L\rangle$ will collapse to one of these two states with the same probabilities. To sum up, the result of a non-protective measurement cannot reflect the actual measured state and indicate whether the measured particle is in a localized state or not due to the resulting wavefunction collapse.

6 Conclusions

Uffink's (1999, 2012) purpose is to prove that only observables that commute with the system's Hamiltonian can be measured protectively. If it is indeed the case, then this restriction will protect the interpretation of the wave function

[6]This is why a protective measurement needs a protective procedure in general; the protection permits it to be able to measure the actual state of the measured system.

against protective measurements and save the coherence of alternative interpretations. As we have argued above, however, Uffink's attempt failed[7]. Moreover, the validity of first order perturbation theory and the adiabatic theorem tell us that an *arbitrary* observable of a single quantum system can be protectively measured. As a result, protective measurements may have important implications on the meaning of the wave function[8].

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