Protective measurements and the meaning of the wave function in the de Broglie-Bohm theory

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Abstract

There are three possible interpretations of the wave function in the de Broglie-Bohm theory: taking the wave function as corresponding to a physical entity or a property of the Bohmian particles or a law. In this paper, we argue that the first interpretation is favored by an analysis of protective measurements.

The de Broglie-Bohm theory is an alternative to standard quantum mechanics initially proposed by de Broglie (1928) and later developed by Bohm (1952). According to the theory, a complete description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The wave function follows the linear Schrödinger equation and never collapses. The motion of the particles, which are usually called Bohmian particles, follows the so-called guiding equation. Although the de Broglie-Bohm theory is mathematically equivalent to quantum mechanics, there is no clear consensus with regard to its physical interpretation. In particular, the interpretation of the wave function in the theory has been debated by its proponents. According to a recent review (Belot 2012), there are mainly three interpretations of the wave function in the de Broglie-Bohm theory: taking the wave function as corresponding to a physical entity different from the Bohmian particles, taking the wave function as corresponding to a property of the Bohmian particles, and taking the wave function as corresponding to a law. In this paper, we will argue that the first interpretation of the wave function is favored by an analysis of protective measurements.

The meaning of the wave function is often analyzed in the context of conventional (impulsive) measurements, for which the coupling between the measuring device and the measured system is very strong and almost instantaneous, and the measurement results are the eigenvalues of the measured observable. Due to the resulting collapse of the wave function, such impulsive measurements cannot measure the actual physical state of the measured system and determine what its wave function represents (when the system is not in one of the eigenstates of the measured observable). Fortunately, it has been known that the coupling

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strength and the measuring time can be adjusted for a standard measurement procedure, and there also exist other kinds of measurements such as weak measurements and protective measurements (Aharonov, Albert and Vaidman 1988; Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993)[1]. Protective measurement uses a weak and long duration coupling interaction and an appropriate procedure to protect the measured system from being disturbed. A general scheme is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction (in some situations the protection is provided by the measured system itself), and then make the measurement adiabatically so that the state of the system neither collapses nor becomes entangled with the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system. Since the principle of protective measurements is independent of the controversial collapse postulate, their results as predicted by quantum mechanics can be used to examine the no-collapse alternatives to quantum mechanics such as the de Broglie-Bohm theory.

An immediate implication of protective measurements is that the result of a protective measurement, namely the expectation value of the measured observable in the measured state, reflects the actual physical state of the measured system[2], as the system is not disturbed after this result has been obtained[3]. This is in accordance with the fundamental assumption that the result of a measurement that does not disturb the measured system reflects the actual property or state of the system. Moreover, since the wave function can be reconstructed from the expectation values of a sufficient number of observables, it is a representation of the physical state[4].

This result can be illustrated with a specific example (Aharonov and Vaidman 1993). Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed. We take the measured observable A_n to be (normalized) projection operators on small spatial regions V_n having volume v_n:

[1]Note that weak measurements have been implemented in experiments (Lundeen et al 2011), and it can be reasonably expected that protective measurements can also be implemented in the near future with the rapid development of quantum technologies.

[2]Several authors, including the inventors of protective measurements, have obtained the similar conclusion as given here (Aharonov and Vaidman 1993; Anandan 1993; Dickson 1995). However, their arguments seem to rely on the presupposition that protective measurements are completely reliable. As pointed out notably by Dass and Qureshi (1999), this presupposition is wrong, as a realistic protective measurement can never be performed on a single quantum system with absolute certainty. Our argument here avoids this problem.

[3]For a realistic protective measurement whose measuring interval T is finite, there is always a tiny probability proportional to $1/T^2$ to obtain a different result, and after obtaining the result the measured state also collapses to the state corresponding to the result. However, the key point here is that when the measurement obtains the expectation value of the measured observable, the state of the measured system is not disturbed. Moreover, the above probability can be made arbitrarily small in principle when T approaches infinity, as well as negligibly small in practice by making T sufficiently large.

[4]This implication is independent of whether the wave function of the system is known beforehand for protective measurements. The reason is that even though we know the wave function, which is an abstract mathematical object, we still don't know its physical meaning. An initial analysis of what physical state the wave function represents has been given by Gao (2013).

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \quad (1)$$

An adiabatic measurement of A_n then yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv, \quad (2)$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can measure another observable $B_n = \frac{\hbar}{2mi}(A_n \nabla + \nabla A_n)$. The measurement yields

$$\langle B_n \rangle = \frac{1}{v_n} \int_{V_n} \frac{\hbar}{2mi} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) dv = \frac{1}{v_n} \int_{V_n} j(x) dv. \quad (3)$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can obtain $\rho(x)$ and $j(x)$ everywhere in space[5]. Since the measured system is not disturbed after the above measurement results, namely the density $\rho(x)$ and flux density $j(x)$, have been obtained, these results reflect the actual physical state of the measured system. Moreover, since the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for an overall phase factor), it also represents the underlying physical state.

Now we can examine the three interpretations of the wave function in the de Broglie-Bohm theory in terms of the above analysis of protective measurements. It can be seen that the analysis favors the first interpretation and disfavors the other two interpretations. If the wave function corresponds to a law, then the results of the above protective measurements made throughout the whole space can hardly be explained. A law, unlike a physical entity, has no direct manifestation in the physical space. By contrast, if taking the wave function as a representation of the state of a physical entity, then these results can readily be explained as the manifestation of the entity. It is obvious that the physical entity is not localized in one position but distributed throughout space, and it is not the Bohmian particle of the measured system[6]. Therefore, at least for one-body systems, the wave function represents the state of a physical entity which is distinct from the Bohmian particle in the de Broglie-Bohm theory.

It is worth noting that the above argument is not influenced by the concept of effective wave function in the de Broglie-Bohm theory (cf. Esfeld et al 2012). First, it is logically possible that the universal wave function can be decoupled into a direct product of the wave functions of some subsystems and the wave function of all others. Then these subsystems can be used as measured systems and measuring devices for protective measurements in the above argument. Next, experiments show that we can always prepare a system whose wave function is a pure state, which means that the universal wave function can be

[5]When the interaction Hamiltonian of these protective measurements is physically realized by the electromagnetic or gravitational interaction between the measured system and the measuring device, it can be argued that what the protective measurements measure is the charge or mass density and flux density of a physical entity. For details see Gao (2013).

[6]It has been shown that during the protective measurements the Bohmian particle of the measured system remains still and does not generate the measurement results (Aharonov, Englert and Scully 1999; Drezet 2006).

decoupled into a direct product of the wave function of this system and the wave function of all other systems. Moreover, we can also prepare a measured system and a measuring device whose wave functions are independent of each other and not entangled with the wave function of all others, which is required by a protective measurement (as well as by a conventional measurement). Thirdly, even though the (effective) wave function of a system may be taken as encoding the information of the Bohmian particles of all other systems as argued by Esfeld et al (2012), its direct manifestation in the physical space (as revealed by protective measurements) can hardly be accounted for according to the nomological interpretation of the wave function.

Lastly, we will briefly discuss the meaning of the wave function for many-body systems, without referring to which our analysis will be incomplete. It is well known that the wave function of a many-body system lives not in real space but in the configuration space. This seems to pose some difficulties when interpreting the wave function as representing the state of a physical entity (see, e.g. Belot 2012; Esfeld et al 2012). However, these difficulties arise largely because of directly taking the physical entity as a field. A further analysis of protective measurements suggests that the wave function may describe the state of certain ergodic motion of particles in real space[7], and at a deeper level, it may represent the property of these particles that determines their motion (Gao 2013)[8]. In this view, the physical entity described by the wave function of a one-body system, which is distributed throughout space and measurable by protective measurements, is formed by the ergodic motion of a particle, for which the probability density that this particle appears in every position is equal to the modulus squared of its wave function there.

Appendix: Mathematical formulation of protective measurement

Protective measurement, in the language of standard quantum mechanics, is a method to measure the expectation value of an observable on a single quantum system (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009). As a typical example, we consider a quantum system in a discrete nondegenerate energy eigenstate $|E_n\rangle$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed[9].

According to the standard von Neumann procedure, measuring an observable A in this state involves an interaction Hamiltonian

[7] Note that the motion of the Bohmian particles is not ergodic in the de Broglie-Bohm theory (Aharonov, Erez and Scully 2004).

[8] This view is also supported by the analyses of Monton (2002, 2006) and Lewis (2004, 2011) from other angles. For example, according to Monton (2006), "the wave function can correspond to a property possessed by the system of all the particles in the universe".

[9] As will be shown below, before the protective measurement we only need to know the measured state is a discrete nondegenerate energy eigenstate of the Hamiltonian of the system, and we need not to know the measured state or the Hamiltonian of the system or the measured state is one of a known collection of energy eigenstates. In this case, by a conventional impulsive measurement we can only measure the energy of the system, and we cannot measure the expectation value of any other observable of the system (as well as the wave function of the system). Note also that for a protective measurement the measured observable does not necessarily commute with the system's Hamiltonian either.

$$H_I = g(t)PA \tag{4}$$

coupling the measured system to an appropriate measuring device, where P is the momentum conjugate to the pointer variable X of an appropriate measuring device. The time-dependent coupling strength $g(t)$ is a smooth function normalized to $\int dtg(t) = 1$ during the interaction interval T, and $g(0) = g(T) = 0$. The initial state of the pointer at $t = 0$ is supposed to be $|\phi(x_0)\rangle$, which is a Gaussian wave packet of eigenstates of X with width w_0, centered around the eigenvalue x_0.

For a conventional impulsive measurement, the interaction H_I is of very short duration and so strong that it dominates the rest of the Hamiltonian (i.e. the effect of the free Hamiltonians of the measuring device and the measured system can be neglected). Then the state of the combined system at the end of the interaction can be written as

$$|t = T\rangle = e^{-\frac{i}{\hbar}PA} |E_n\rangle |\phi(x_0)\rangle . \tag{5}$$

By expanding $|E_n\rangle$ in the eigenstates of A, $|a_i\rangle$, we obtain

$$|t = T\rangle = \sum_i e^{-\frac{i}{\hbar}Pa_i} c_i |a_i\rangle |\phi(x_0)\rangle , \tag{6}$$

where c_i are the expansion coefficients. The exponential term shifts the center of the pointer by a_i:

$$|t = T\rangle = \sum_i c_i |a_i\rangle |\phi(x_0 + a_i)\rangle . \tag{7}$$

This is an entangled state, where the eigenstates of A with eigenvalues a_i get correlated to measuring device states in which the pointer is shifted by these values a_i. Then by the collapse postulate of standard quantum mechanics, the state will instantaneously and randomly collapse into one of its branches $|a_i\rangle |\phi(x_0 + a_i)\rangle$ with probability $|c_i|^2$. This means that the measurement result can only be one of the eigenvalues of measured observable A, say a_i, with a certain probability, say $|c_i|^2$. The expectation value of A is then obtained as the statistical average of eigenvalues for an ensemble of identically prepared systems, namely $\langle A\rangle = \sum_i |c_i|^2 a_i$.

Different from the conventional impulsive measurements, for which the interaction is very strong and almost instantaneous, protective measurements make use of the opposite limit where the interaction of the measuring device with the system is weak and adiabatic, and thus the free Hamiltonians cannot be neglected. Let the Hamiltonian of the combined system be

$$H(t) = H_S + H_D + g(t)PA, \tag{8}$$

where H_S and H_D are the free Hamiltonians of the measured system and the measuring device, respectively. The interaction lasts for a long time T, and $g(t)$ is very small and constant for the most part, and it goes to zero gradually before and after the interaction.

The state of the combined system after T is given by

$$|t = T\rangle = e^{-\frac{i}{\hbar}\int_0^T H(t)dt} |E_n\rangle |\phi(x_0)\rangle . \tag{9}$$

By ignoring the switching on and switching off processes[10], the full Hamiltonian (with $g(t) = 1/T$) is time-independent and no time-ordering is needed. Then we obtain

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} |E_n\rangle |\phi(x_0)\rangle , \tag{10}$$

where $H = H_S + H_D + \frac{PA}{T}$. We further expand $|\phi(x_0)\rangle$ in the eigenstate of H_D, $|E_j^d\rangle$, and write

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} \sum_j c_j |E_n\rangle |E_j^d\rangle , \tag{11}$$

Let the exact eigenstates of H be $|\Psi_{k,m}\rangle$ and the corresponding eigenvalues be $E(k,m)$, we have

$$|t = T\rangle = \sum_j c_j \sum_{k,m} e^{-\frac{i}{\hbar}E(k,m)T} \langle\Psi_{k,m}|E_n, E_j^d\rangle|\Psi_{k,m}\rangle. \tag{12}$$

Since the interaction is very weak, the Hamiltonian H of Eq.(8) can be regarded as $H_0 = H_S + H_D$ perturbed by $\frac{PA}{T}$. Using the fact that $\frac{PA}{T}$ is a small perturbation and that the eigenstates of H_0 are of the form $|E_k\rangle |E_m^d\rangle$, the perturbation theory gives

$$\begin{aligned} |\Psi_{k,m}\rangle &= |E_k\rangle |E_m^d\rangle + O(1/T), \\ E(k,m) &= E_k + E_m^d + \frac{1}{T}\langle A\rangle_k \langle P\rangle_m + O(1/T^2). \end{aligned} \tag{13}$$

Substituting Eq.(13) in Eq.(12) and taking the limit $T \to \infty$ yields

$$|t = T\rangle_{T\to\infty} = \sum_j e^{-\frac{i}{\hbar}(E_nT + E_j^dT + \langle A\rangle_n\langle P\rangle_j)} c_j |E_n\rangle |E_j^d\rangle . \tag{14}$$

For the case where P commutes with the free Hamiltonian of the device[11], i.e., $[P, H_D] = 0$, the eigenstates $|E_j^d\rangle$ of H_D are also the eigenstates of P, and thus the above equation can be rewritten as

$$|t = T\rangle_{T\to\infty} = e^{-\frac{i}{\hbar}E_nT - \frac{i}{\hbar}H_DT - \frac{i}{\hbar}\langle A\rangle_n P} |E_n\rangle |\phi(x_0)\rangle . \tag{15}$$

It can be seen that the third term in the exponent will shift the center of the pointer $|\phi(x_0)\rangle$ by an amount $\langle A\rangle_n$:

$$|t = T\rangle_{T\to\infty} = e^{-\frac{i}{\hbar}E_nT - \frac{i}{\hbar}H_DT} |E_n\rangle |\phi(x_0 + \langle A\rangle_n)\rangle. \tag{16}$$

This indicates that the result of the protective measurement is the expectation value of the measured observable in the measured state, and moreover, the measured state is not changed by the protective measurement[12].

[10] The change in the total Hamiltonian during these processes is smaller than PA/T, and thus the adiabaticity of the interaction will not be violated and the approximate treatment given below is valid.

[11] For the derivation for the case $[P, H_D] \neq 0$ see Dass and Qureshi (1999).

[12] It might be worth noting that there appeared numerous objections to the validity of protective measurements (see, e.g. Unruh 1994; Rovelli 1994; Ghose and Home 1995; Uffink 1999), and these objections have been answered (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999; Vaidman 2009; Gao 2012). For a more detailed introduction to protective measurements see Gao (2013).

This strict mathematical result can also be understood in terms of the adiabatic theorem and the first order perturbation theory in quantum mechanics. By the adiabatic theorem, the adiabatic interaction during the protective measurement ensures that the measured system cannot make a transition from one discrete energy eigenstate to another. Moreover, according to the first order perturbation theory, for any given value of P, the energy of the measured energy eigenstate shifts by an infinitesimal amount: $\delta E = \langle H_I \rangle = P\langle A \rangle_n / T$, and the corresponding time evolution $e^{-iP\langle A \rangle_n/\hbar}$ then shifts the pointer by the expectation value $\langle A \rangle_n$.

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