Relativity without Light: A Further Suggestion

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The role of the light postulate in special relativity is reexamined. The existing theory of relativity without light shows that one can deduce Lorentz-like transformations with an undetermined invariant speed based on homogeneity of space and time, isotropy of space and the principle of relativity. However, since the transformations can be Lorentzian or Galilean, depending on the finiteness of the invariant speed, a further postulate is needed to determine the speed in order to establish a real connection between the theory and special relativity. In this paper, I argue that a certain discreteness of space-time, whose existence is strongly suggested by the combination of quantum theory and general relativity, may result in the existence of a maximum and invariant speed when combing with the principle of relativity, and thus it can determine the finiteness of the speed in the theory of relativity without light. According to this analysis, the speed constant c in special relativity is not the actual speed of light, but the ratio between the minimum length and the shortest time of discrete space-time. This suggests a more complete theory of relativity, the theory of relativity in discrete space-time, which is based on the principle of relativity and the constancy of the minimum size of discrete space-time.

1. Introduction

Special relativity was originally based on two postulates: the principle of relativity and the constancy of the speed of light. But, as Einstein later admitted to some extent (Einstein 1935), it is an incoherent mixture (Stachel 1995); the first principle is universal in scope, while the second is only a particular property of light, which has obvious electrodynamical origins in Maxwell's theory. In fact, there has been a lasting attempt that tries to drop the light postulate from special relativity, which can be traced back to Ignatowski (1910) (see also Torretti 1983; Brown 2005)[1]. It has been found that based on homogeneity of space and time, isotropy of space and the principle of relativity, one can deduce Lorentz-like transformations with an undetermined invariant speed. Unlike special relativity that needs to assume the constancy of the speed of light, an invariant speed naturally appears in the theory, which has been called relativity without light. This is a surprise indeed.

Since the value of the invariant speed can be infinite or finite, the theory of relativity without

1. A more detailed reference list in chronological order is: Ignatowski (1910, 1911a, 1911b); Frank and Rothe (1911, 1912); Pars (1921); Kaluza (1924); Lalan (1937); Dixon (1940); Weinstock (1965); Mitavalsky (1966); Terletskii (1968); Berzi and Gorini (1969); Gorini and Zecca (1970); Lee and Kalatos (1975); Lévy-Leblond (1976); Srivastava (1981); Mermin (1984); Schwartz (1984, 1985); Singh (1986); Sen (1994); Field (1997); Coleman (2003); Pal (2003); Sonego and Pin (2005); Gannett (2007); Silagadze (2007); Certik (2007); Feigenbaum (2008).

light actually allows two possible transformations: Galilean and Lorentzian. An empirical element is still needed to determine the invariant speed and further eliminate the Galilean transformations. This raises serious doubts about the connection between the theory and special relativity. Some authors insisted that the light postulate in special relativity is still needed to derive the Lorentz transformations (Pauli 1921; Resnick 1967; Miller 1981). Others doubted that the theory is indeed relativistic in nature (Brown 2005). However, it can be argued that the empirical element may not refer to any properties of light in an essential way (see, e.g. Lévy-Leblond 1976; Mermin 1984). Therefore, the existing theory of relativity without light is definitely an advance, but admittedly there is still a step away between it and the Lorentz transformations in special relativity; resorting to experience to determine its invariant speed is just a makeshift. The challenge for future work is two-fold. On the one hand, we need to further determine the invariant speed, not by experience but by some deeper postulates (e.g. postulates about space-time). If successful, this will establish a more complete theory of relativity without light, which can be taken as a further development of special relativity; On the other hand, we need to re-interpret the constant c in special relativity. It should be not (only) the speed of light. What, then, is its real meaning? These two problems are intimately connected as a matter of fact. The purpose of this paper is to try to solve them.

This paper is organized as follows. Section 2 gives a clear introduction of the theory of relativity without light. I raise the problem about how to determine the finiteness of the invariant speed in the theory by theoretical considerations. In Section 3, I propose a possible solution. As the existing theory of relativity without light implies, the existence of an invariant speed may result from the properties of space-time (e.g. homogeneity of space and time). Inspired by this result, I argue that a certain discreteness of space-time, whose existence is suggested by the combination of quantum theory and general relativity, may further account for the finiteness of the invariant speed. In such discrete space-time, there exists a finite speed that is maximum and invariant in all inertial frames. This may also provide a reasonable interpretation of the constant c in special relativity; it is not the actual speed of light, but the ratio between the minimum length and the shortest time of discrete space-time. Section 4 further suggests a more complete theory of relativity, the theory of relativity in discrete space-time. The connection between the new suggestion and some existing theories, such as doubly special relativity, is also briefly discussed.

2. Relativity without light

There are many different deductions of the Lorentz-like transformations without resorting to the light postulate. Yet the assumptions they are based on are basically the same, namely homogeneity of space and time, isotropy of space and the principle of relativity. Here I will introduce a very clear and simple deduction (see also Pal 2003).

Consider two inertial frames S and S', where S' moves with a speed v relative to S and when $t=0$ the origins of the two frames coincide. The space-time transformation equations in two-dimensional space-time can be written as follows:

$$x' = X(x,t,v) \qquad (1)$$

$$t' = T(x,t,v) \qquad (2)$$

where x', t' denote the space and time coordinates in the frame S', and x, t denote the space and time coordinates in the frame S. Now I will invoke the above assumptions to derive the space-time transformations.

(1) Homogeneity of space and time

The homogeneity of space requires that the length of a rod does not depend on its position in an inertial frame. Suppose there is a rod in the frame S, which ends are at positions x_1 and x_2 ($x_2 > x_1$). Due to the homogeneity of space, the length of the rod is the same when its ends are at positions $x_1 + \Delta x$ and $x_2 + \Delta x$. Correspondingly, the length of the rod in the frame S' is also the same for these two situations. Then we have:

$$X(x_2 + \Delta x, t, v) - X(x_1 + \Delta x, t, v) = X(x_2, t, v) - X(x_1, t, v) \quad (3)$$

or

$$X(x_2 + \Delta x, t, v) - X(x_2, t, v) = X(x_1 + \Delta x, t, v) - X(x_1, t, v) \quad (4)$$

Dividing both sides by Δx and taking the limit $\Delta x \to 0$, we get:

$$\left.\frac{\partial X(x,t,v)}{\partial x}\right|_{x_2} = \left.\frac{\partial X(x,t,v)}{\partial x}\right|_{x1} \quad (5)$$

Since the positions x_1 and x_2 are arbitrary, the partial derivative must be constant. Therefore, the function $X(x,t,v)$ will be a linear function of x. In a similar way, $X(x,t,v)$ is also a linear function of t due to the homogeneity of time, and the same for $T(x,t,v)$. In conclusion, the homogeneity of space and time requires that the space-time transformations are linear with respect to both space and time.

Considering that the origins of the two frames S and S' coincide when $t = 0$, we can write down the linear space-time transformations in a matrix notation:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} A_v & B_v \\ C_v & D_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \quad (6)$$

where A_v, B_v, C_v, D_v are only functions of the relative velocity v. Furthermore, since the origin of S' moves at a speed v relative to the origin of S, i.e., $x' = 0$ when $x = vt$, we also have the following relation:

$$B_v = -vA_v \quad (7)$$

(2) Isotropy of space

The isotropy of space demands that the space-time transformations do not change when the x-axis is reversed, i.e., both x and v change sign, and so does x'. Applying this limitation to

Equation (6) we have:

$$\begin{cases} A_{-v} = A_v \\ B_{-v} = -B_v \\ C_{-v} = -C_v \\ D_{-v} = D_v \end{cases} \quad (8)$$

(3) Principle of relativity

The principle of relativity requires that the inverse space-time transformations assume the same form as the original transformations. This means that the transformations from S' to S assume the same functional forms as the transformations from S to S'. Moreover, the combination of the principle of relativity with isotropy of space further implies reciprocity (Berzi and Gorini 1969; Budden 1997; Torretti 1983), namely that the speed of S' relative to S is the negative of the speed of S relative to S'. Thus we have:

$$\begin{cases} A_{-v} = \dfrac{D_v}{A_v D_v - B_v C_v} \\ B_{-v} = \dfrac{-B_v}{A_v D_v - B_v C_v} \\ C_{-v} = \dfrac{-C_v}{A_v D_v - B_v C_v} \\ D_{-v} = \dfrac{A_v}{A_v D_v - B_v C_v} \end{cases} \quad (9)$$

Combining the conditions (8) and (9) we can get:

$$D_v = A_v \quad (10)$$

$$C_v = \frac{A_v^2 - 1}{B_v} \quad (11)$$

Then considering Equation (7) the space-time transformations can be formulated in terms of only one unknown function A_v, namely

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} A_v & -vA_v \\ -\dfrac{A_v^2 - 1}{vA_v} & A_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \quad (12)$$

or

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = A_v \begin{pmatrix} 1 & -v \\ -\dfrac{A_v^2 - 1}{vA_v^2} & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \quad (13)$$

Now consider a third frame S'' which moves with a speed u relative to S', and we have:

$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} = A_u A_v \begin{pmatrix} 1 & -u \\ -\frac{A_u^2 - 1}{uA_u^2} & 1 \end{pmatrix} \begin{pmatrix} 1 & -v \\ -\frac{A_v^2 - 1}{vA_v^2} & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$= A_u A_v \begin{pmatrix} 1 + u\frac{A_v^2 - 1}{vA_v^2} & -(u+v) \\ -\frac{A_u^2 - 1}{uA_u^2} - \frac{A_v^2 - 1}{vA_v^2} & 1 + v\frac{A_u^2 - 1}{uA_u^2} \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \qquad (13)$$

The principle of relativity demands that this transformation assumes the same form as the transformation from S to S', and thus the two diagonal elements of the matrix also satisfy Equation (10), namely they are equal. Thus we have:

$$1 + v\frac{A_u^2 - 1}{uA_u^2} = 1 + u\frac{A_v^2 - 1}{vA_v^2} \qquad (14)$$

or

$$\frac{A_u^2 - 1}{u^2 A_u^2} = \frac{A_v^2 - 1}{v^2 A_v^2} \qquad (15)$$

Since u and v are arbitrary, this equation means that its both sides are constants. Denoting this constant by K and considering the condition $A_v = 1$ when $v = 0$, we have:

$$A_v = \frac{1}{\sqrt{1 - Kv^2}} \qquad (16)$$

Therefore, we deduce the final space-time transformations in terms of the homogeneity of space and time, isotropy of space and the principle of relativity, namely:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \frac{1}{\sqrt{1 - Kv^2}} \begin{pmatrix} 1 & -v \\ -Kv & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \qquad (17)$$

The velocity addition law can be further deduced. Suppose the speed of the frame S'' relative to S' is w. Then using Equation (16) and Equation (13), in which the first diagonal element of the matrix is A_w by definition, we can directly deduce the velocity addition law, namely:

$$w = \frac{u + v}{1 + Kuv} \qquad (18)$$

It can be seen that $K^{-1/2}$ is an invariant speed, independent of any inertial frame. The possible values of K can be determined as follows. Equation (16) indicates $A_v > 0$ for any v. Moreover, the first diagonal element of the matrix in Equation (13) further demands $A_v \geq 1$, for if $A_v < 1$ then for some values of u and v (e.g. $u >> v$) we can get $A_w < 0$. Therefore, we have $K \geq 0$ according to Equation (16).

Two points need to be discussed about the above deduction. First of all, the idea that the homogeneity of space and time requires space-time transformations are linear can be traced back to Einstein, and was later developed by more authors (see, e.g. Terletskii 1968; Lévy-Leblond 1976; Berzi and Gorini 1969). However, it can be argued that the principle of relativity, together with the law of inertia, can also lead to the linearity of space-time transformations (Fock 1969; Torretti 1983; Brown 2005). Thus the homogeneity of space and time may be dropped from the assumptions needed for deduce a theory of relativity without light. Secondly, isotropy of space plays a pivotal role in the deduction. Since isotropy of space and its resulting condition of reciprocity hold only for the standard convention of simultaneity, we only deduce a theory of relativity without light consistent with the standard convention. If simultaneity is really a convention (for a different view see Malament 1977), then it seems that in order to have a theory of relativity without light we should deduce the general Edwards-Winnie transformations for any convention (Edwards 1963; Winnie 1970), not only the Lorentz-like transformations. But this seems to be an impossible task, as symmetries such as isotropy of space and reciprocity play an indispensable role in the deduction.

Now I will analyze the possible implications of the above theory of relativity without light. When $K = 0$ we obtain the Galileo transformations, while when $K > 0$ we obtain the Lorentz transformations. Thus the theory is the most general one consistent with the principle of relativity, which can accommodate both Galilean and Einsteinian relativity. But in this meaning it is not yet relativistic in nature, as the value of K or an invariant speed needs to be further determined in order to establish its connection with Einstein's relativity. Note that this does not mean we need to determine the concrete value of K such as $K = 1/c^2$. What we need to determine is only $K \neq 0$, as K and c are quantities with dimension and their values can assume the unit of number 1 in principle. Certainly we can resort to experience, also without light, to eliminate the possibility of $K = 0$, and we have more today indeed. This, however, is unsatisfactory in several aspects. First of all, we have not deduced a theory of relativity without light consistent with Einstein's relativity in this way. There is still one step left, which may be more important. This obviously departs from the initial aim of dropping the light postulate from special relativity. We hope that, by dropping the light postulate, we can still deduce a theory consistent with special relativity. Next, although we can determine the value of K by experience, there is still one deep mystery unexplained. It is why there exists an invariant and maximum speed, independent of any inertial frame. For Galilean relativity there is no such mystery, but for Einstein's relativity there is one. Lastly, the determination of K by theoretical considerations may lead us to a deeper understanding of space-time and relativity, and will probably bring a further development of special relativity. The existing theory of relativity without light is only a first step towards this direction.

To sum up, we have not had a theory of relativity without light consistent with Einstein's relativity yet. Only after answering why there is an invariant and maximum speed and thus determining the finiteness of K by a deeper postulate can we claim we have. I will provide a possible answer in the next section.

3. Discreteness of space-time and the invariance of a finite maximum speed

In special relativity, the speed of light in vacuum, denoted by c, is invariant in all inertial frames. Moreover, it is the maximum speed with which all objects can move[2]. This postulate is not a result of logical analysis but a direct representation of experience. The theory itself cannot answer why the speed of light is invariant and maximum. Now the appearance of the theory of relativity without light further urges us to understand the meaning of *c* in special relativity. The theory suggests that *c* is not (merely) the speed of light, but a universal constant of nature, an invariant speed. Furthermore, it also shows that the existence of an invariant speed partly results from the properties of space and time, e.g. homogeneity of space and time and isotropy of space. This makes us be closer to the real meaning of *c*. However, the theory cannot yet tell us the origin of *c*. In fact, it is still incomplete and cannot even establish a real connection between its invariant speed with *c*. Anyway, we need to explain exactly why there is a maximum and invariant speed.

Since speed is essentially the ratio of space interval and time interval, it is a natural conjecture that the existence of a maximum and invariant speed may result from some undiscovered property of space and time, as the existing theory of relativity without light has implied. In the following, I will argue that the property is probably the ontological discreteness of space-time.

Consider the continuous motion of an object in discrete space-time, in which there is a minimum length, denoted by L_U, and a minimum time interval, denoted by T_U. If the object moves with a speed larger than L_U / T_U, then it will move more than a minimum length L_U during a minimum time interval T_U, and thus moving L_U will correspond to a time interval shorter than T_U during the motion. Since T_U is the minimum time interval in the discrete space-time, which means that the duration of any change cannot be shorter than T_U, the motion with a speed larger than L_U / T_U will be prohibited. Therefore, there is a maximum speed in discrete space-time, which equals to the ratio of the minimum length and minimum time interval.

There are some clues of the discreteness of space-time in modern physics. For instance, the appearance of infinity in quantum field theory and singularity in general relativity suggests that space-time may be not continuous but discrete. Moreover, it has been widely argued that the proper combination of quantum theory and general relativity also suggests the discreteness of space-time, and the minimum time interval is $T_U \equiv 2T_P$ and the minimum length is $L_U \equiv 2L_P$, where $T_P = (\frac{G\hbar}{c^5})^{1/2}$, $L_P = (\frac{G\hbar}{c^3})^{1/2}$ are respectively the Planck time and the Planck length (see, e.g. Garay 1995 for a review). For example, the minimum (observable) length can be derived from the following generalized uncertainty principle (Adler and Santiago 1999):

$$\Delta x = \Delta x_{QM} + \Delta x_{GR} \geq \frac{\hbar}{2\Delta p} + \frac{2L_p{}^2 \Delta p}{\hbar} \qquad (19)$$

Then according to the above analysis, the speed of light *c*, which is equal to the ratio of the

[2] In this paper we only consider the motion of the mass center of an object or a particle, which can be described by a material point. For simplicity, we always say the motion of an object or a particle.

minimum length and minimum time interval, is the maximum speed in discrete space-time.

Now I will further argue that the maximum speed *c* is invariant in all inertial frames in discrete space-time. According to the principle of relativity, the discrete character of space and time, in particular the minimum time interval T_U and the minimum length L_U, should be the same in all inertial frames. If the minimum sizes of space and time are different in different inertial frames, then there will exist a preferred Lorentz frame. This contradicts the principle of relativity. Thus $c \equiv L_U / T_U$ will be the maximum speed in any inertial frame (see also Rindler 1977; 1991). Next, let's analyze the transformation of *c* in different inertial frames. Suppose an object moves with the maximum speed *c* in an inertial frame S. Since *c* is the maximum speed in any inertial frame, the speed of the object can only be equal to or smaller than *c* in another inertial frame S'. If its speed in the frame S', denoted by c', is smaller than *c,* then due to the continuity of the velocity transformation function, there must exist a speed larger than c' and a speed smaller than c' that correspond to the same speed in the frame S. This means that when the object moves with a certain speed in the frame S, its speed in the frame S' will have two possible values. This is impossible. Therefore, if an object moves with the maximum speed *c* in one inertial frame, it will also move with the same speed *c* in other inertial frames. In short, the maximum speed *c* is invariant in all inertial frames.

So far so good. However, it seems that there is a problem in the above argument that the discreteness of space-time requires the existence of a maximum speed. In fact, if motion is essentially continuous, we can similarly argue that the motion with a speed smaller than the maximum speed will also be prohibited in discrete space-time. Suppose an object moves with a speed smaller than the maximum speed L_U / T_U. Then it will move less than L_U during T_U. But L_U is the minimum length in discrete space-time, thus this is impossible. Therefore, objects can only move with the maximum speed in discrete space-time if motion is essentially continuous. This result obviously contradicts experience. An object can move with a speed smaller than the maximum speed *c* in reality[3]. Certainly, this contradiction can be used to favor continuous motion and disfavor discrete space-time. However, on the one hand, it is widely accepted that the assumption of continuous motion of particles is inconsistent with quantum theory, the most fundamental theory of nature (for a recent argument see Gao 2017). Moreover, the assumption also has serious drawbacks within the framework of classical mechanics (see, e.g. Arntzenius 2000); On the other hand, the discreteness of space-time has strong support from the combination of quantum theory and general relativity. Therefore, the above contradiction may actually indicate that the discreteness of space-time further disfavors the assumption of continuous motion.

If the actual motion is essentially discontinuous and continuous motion is merely its average

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[3] It can be conceived that a free object moves with *c* during some time, and stays still during other time. Then its average speed can be smaller than *c*, and thus the motion can be consistent with the existing experience. However, the change of the speed of the free object during such motion can hardly be explained. In addition, the motion will contain some kind of unnatural randomness (e.g. during each time the speed of the free object will assume *c* or 0 in a random way), which has no logical basis.

display[4], then the apparent continuous motion with a speed smaller than the maximum speed $c \equiv L_U / T_U$ will not be prohibited in discrete space-time. The reason is that an object undergoing such motion may not move less than L_U during T_U, as its motion is discontinuous and it can move a distance larger than L_U during T_U in a discontinuous way. Moreover, since the direction of each discontinuous movement may be forward and backward, the average velocity of the object can still be smaller than the maximum speed. However, the average velocity of the object cannot be larger than the maximum speed c, or else we can detect a time interval shorter than T_U by measuring the average moving distance of the object. This is prohibited in discrete space-time. Therefore, although the motion of objects is discontinuous, the apparent continuous motion with a speed larger than c is also prohibited, and there is still a maximum speed c in discrete space-time[5].

Since time interval and space interval are primary physical quantities, while speed, which is defined as the ratio of space interval and time interval, is a secondary physical quantity, it is understandable that the properties of the characteristic speed *c* can be further explained by the properties of space and time. As I have argued above, the maximum and constancy of *c* may result from the discreteness of space-time. By comparison, if space and time are continuous, then no characteristic space and time sizes exist, and thus it seems unnatural that there exists a characteristic speed. On the other hand, if my argument is right, then the existence of a maximum and invariant speed *c* will be a firm (and maybe the first) experimental evidence of discrete space-time, in which the ratio of the minimum length L_U and minimum time interval T_U is *c*.

In conclusion, it seems that the discreteness of space-time may explain the existence of an invariant and maximum speed, and thus it may be the postulate that determines the finiteness of the invariant speed in the theory of relativity without light. In this way, the discreteness of space-time might not only reveal the meaning of *c*, but also provide a deeper logical foundation for special relativity.

4. Further discussions

If space and time are indeed discrete, then the theory of relativity should be defined in discrete space-time. Relativity in discrete space-time will be based on two postulates: (1) the principle of relativity; and (2) the constancy of the minimum size of discrete space-time, which states that the minimum time interval T_U and the minimum length L_U are invariant in all inertial frames. The theory may be regarded as a more complete theory of relativity without light. According to the

[4] If discontinuous motion happens in extremely short space and time intervals, a large number of minute discontinuous motions can generate the average display of continuous motion. For a detail analysis of discontinuous motion, see Gao (2013, 2017).

[5] For a microscopic particle moving in vacuum, its average velocity can be defined as the group speed of its wave function. Note that the group speed of photons can be larger than *c* in some special media. This does not contradict the discreteness of space-time. What the discreteness of space-time really limits is the speed of any (apparently continuous) causal influence, which cannot be larger than *c*. The speed of discontinuous causal influence such as quantum nonlocality may be larger than *c* (see, e.g. Gao 2004).

above analysis, special relativity can be derived from the theory of relativity in discrete space-time, as the constancy of the minimum size of discrete space-time leads to the constancy of the speed of light $c \equiv L_U / T_U$. In this meaning, Galileo's relativity is a theory of relativity in continuous space and time, while Einstein's relativity is a theory of relativity in discrete space-time.

It should be noted that some variants of relativity in discrete space-time has already appeared in the literature (see Hagar 2009 for a general discussion). For example, doubly special relativity assumes two invariant scales, the speed of light c and a minimum length λ (Amelino-Camelia 2000, 2004; Kowalski-Glikman 2005), while triply special relativity assumes three invariant scales, the speed of light c, a mass κ and a length R (Kowalski-Glikman and Smolin 2004). In these theories, the classical Minkowski space-time is replaced by a quantum space-time, such as κ-Minkowski noncommutative space-time etc. Although these theories still have problems (e.g. energy-momentum conservation problem and composition problem) due to their extreme nonlinearity (Amelino-Camelia 2004), they may be some in-between points along the road to a complete theory of quantum gravity (Amelino-Camelia and Smolin 2009). Moreover, if the constancy of the speed of light is really a result of the discreteness of space-time, then it should not be an independent assumption, while a minimum time interval, together with a minimum length, should be the only two invariant scales in a fundamental physical theory.

References

Adler, R. J. and Santiago, D. I. (1999). On gravity and the uncertainty principle. *Modern Physics Letters A* 14, 1371-1378

Amelino-Camelia, G. (2000). Relativity in space-times with short-distance structure governed by an observer-independent (Planckian) length scale. arXiv:gr-qc/0012051. *Int. J. Mod. Phys. D* 11 (2002) 35-60.

Amelino-Camelia, G. (2004). Some encouraging and some cautionary remarks on doubly special relativity in quantum gravity. arXiv:gr-qc/0402092.

Amelino-Camelia, G. and Smolin, L. (2009). Prospects for constraining quantum gravity dispersion with near term observations. /xxx.arxiv.org/abs/0906.3731S.

Arntzenius, F. (2000). Are there really instantaneous velocities? *The Monist*, 83(2), 187-208.

Berzi, V. and Gorini, V. (1969). Reciprocity principle and the Lorentz transformations. *J. Math. Phys.*, 10:1518–24.

Brown, H. (2005). *Physical Relativity: Space-time structure from a dynamical perspective*. Oxford: Clarendon Press.

Budden, T. (1997). A star in the Minkowskian sky: Anisotropic special relativity. *Studies in History and Philosophy of Modern Physics* 28(3), 325–361.

Certik, O. (2007). Simple derivation of the special theory of relativity without the speed of light axiom. arXiv: 0710.3398.

Coleman, B. (2003). A dual first-postulate basis for special relativity. *Eur. J. Phys.* 24(3):301–313.

de Broglie, L. and Vigier, J. P. (1972). Photon mass and new experimental results on longitudinal displacements of laser beams near total reflection. *Phys. Rev. Lett*. 28, 1001-1004.

Dixon, W. G. (1940). *Special Relativity: The Foundation of Macroscopic Physics*. Cambridge:

Cambridge University Press.
Edwards, W. (1963). Special relativity in anisotropie space. *Am. J. Phys.* 31, 482-489
Einstein, A. (1935). An elementary derivation of the equivalence of mass and energy. *Bulletin of the American Mathematical Society*. 41, 223-230.
Feigenbaum, M. J. (2008). The theory of relativity - Galileo’s child. arXiv: 0806.1234.
Field, J. H. (1997). A new kinematical derivation of the Lorentz transformation and the particle description of light. *Helv. Phys. Acta.*, 70, 542.
Frank, P. and Rothe, H. (1911). Über die Transformation der Raumzeitkoordinaten von ruhenden auf bewegte Systeme. *Ann. Physik* 34, 825-853.
Frank, P. and Rothe, H. (1912). Zur Herleitung der Lorentz transformation. *Phys. Zeits.* 13, 750-753.
Gannett, J. W. (2007). Nothing but relativity, redux. *Eur. J. Phys.* 28, 1145-1150.
Gao, S. (2004). Quantum collapse, consciousness and superluminal communication, Foundations of Physics Letters, 17, 167-182.
Gao, S. (2013). A discrete model of energy-conserved wavefunction collapse, Proceedings of the Royal Society A 469, 20120526.
Gao, S. (2017). The Meaning of the Wave Function: In Search of the Ontology of Quantum Mechanics. Cambridge: Cambridge University Press.
Garay, L. J. (1995). Quantum gravity and minimum length. *International Journal of Modern Physics A* 10, 145.
Gorini, V and Zecca, A. (1970). Isotropy of space. *J. Math. Phys.*, 11:2226–30.
Hagar, A. (2009). Minimal Length in Quantum Gravity and the Fate of Lorentz Invariance. *Studies in the History and Philosophy of Modern Physics* 40, 259-267.
Ignatowski, W. V. (1910). Einige allgemeine Bemerkungen zum Relativitätsprinzip. *Phys. Zeits.* 11, 972-976.
Ignatowski, W. V. (1911a). Eine Bemerkung zu meiner Arbeit ‘Einige allgemeine Bemerkungen zum Relativitätsprinzip’. *Phys. Zeits.* 12, 779.
Ignatowski, W. V. (1911b). Das Relativitätsprinzip. *Arch. Math. Phys. Lpz.* 17, 1-24, and 18, 17-41.
Kaluza, T. (1924) Zur Relativitätstheorie, *Phys. Z.* 25, 604–606.
Kowalski-Glikman, J. (2005). Introduction to doubly special relativity. *Lect. Notes Phys.*, 669, 131-159.
Kowalski-Glikman, J. and Smolin, L. (2004). Triply special relativity. *Phys. Rev. D* 70, 065020.
Lalan, V. (1937) Sur les postulats qui sont à la base des cinématiques. *Bull. Soc. Math. France*, 65, 83-99.
Lee, A. R. and Kalotas, T. M. (1975). Lorentz transformations from the first postulate. *Am. J. Phys.* 43, 434-437.
Lévy-Leblond, J. M. (1976). One more derivation of the Lorentz transformation. *Am. J. Phys.* 44, 271-277.
Malament, D. (1977). Causal theories of time and the conventionality of simultaneity. *Noûs* 11, 293-300.
Mermin, N. D. (1984). Relativity without light. *Am. J. Phys.* 52, 119-124.
Miller, A.I. (1981). *Albert Einstein’s Special Theory of Relativity: Emergence (1905) and Early Interpretation (1905–1911)*. New York: Addison-Wesley.

Mitavalsky, V. (1966) Special relativity without the postulate of constancy of light. *Am. J. Phys.* 34 (1966) 825.
Pal, P. B. (2003). Nothing but relativity. *Eur. J. Phys.* 24, 315-319.
Pars, L. A. (1921). The Lorentz transformation. *Philosophical Magazine* 42, 249-258.
Pauli, W. (1921). Theory of Relativity, *Encyclopedia of Mathematics*, V19. Leipzig: Teubner.
Proca, A. (1936). Sur la théorie ondulatoire des électrons positifs et négatifs. *Le Journal de Physique et le Radium* 7, 347-353.
Resnick, R. (1968). *Introduction to Special Relativity*. New York: John Wiley & Sons.
Rindler, W. (1977). *Essential Relativity* (2nd ed.). New York: Springer-Verlag.
Rindler, W. (1991). *Introduction to Special Relativity*. Oxford: Oxford University Press.
Salecker, H. and Wigner, E. P. (1958). Quantum limitations of the measurement of space-time distances. *Physical Review* 109, 571-577.
Schwartz, H. M. (1984). Deduction of the general Lorentz transformations from a set of necessary assumptions. *Am. J. Phys.* 52, 346-350.
Schwartz, H. M. (1985). A simple new approach to the deduction of the Lorentz transformations. *Am. J. Phys.* 53, 1007-1008.
Sen, A. (1994). How Galileo could have derived the special theory of relativity. *Am. J. Phys.* 62, 157-162.
Silagadze, Z. K. (2007). Relativity without tears. arXiv:0708.0929. *Acta Phys. Polon. B* 39, 811-885 (2008).
Singh, S. (1986). Lorentz transformations in Mermin's relativity without light. *Am. J. Phys.* 54, 183-184.
Sonego, S and Pin, M. (2005). Deriving relativistic momentum and energy. *Eur. J. Phys.*, 26(1):33–45.
Srivastava, A. M. (1981). Invariant speed in special relativity. *Am. J. Phys.* 49, 504-505.
Stachel, J. (1995). History of relativity. In L. M. Brown, A. Pais, and B. Pippard, (eds.) *Twentieth Century Physics*, *vol.1*, pp. 249–356. New York: American Institute of Physics.
Terletskii, Y. P., (1968). *Paradoxes in the Theory of Relativity*. New York: Plenum Press.
Torretti, R. (1983). *Relativity and Geometry*. Oxford: Pergamon Press, 1983.
Weinstock, R. (1965). New approach to special relativity. *Am. J. Phys.* 33, 540-545.
Winnie, J. (1970). Special relativity without one-way velocity assumptions, Part I; Part II. *Philosophy of Science*, 37, 81–99, 223–238.