The Wave Function and Particle Ontology

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Abstract

In quantum mechanics, the wave function of a N-body system is a mathematical function defined in a 3N-dimensional configuration space. We argue that wave function realism implies particle ontology when assuming: (1) the wave function of a N-body system describes N physical entities; (2) each triple of the 3N coordinates of a point in configuration space that relates to one physical entity represents a point in ordinary three-dimensional space. Moreover, the motion of particles is random and discontinuous.

In quantum mechanics, the wave function of a N-body system is a mathematical function defined in a 3N-dimensional configuration space. We assume that (1) the wave function is a representation of the physical state of a single system; (2) the wave function of a N-body system describes N physical entities, which have respective masses and charges as indicated by the mass and charge parameters in the Schrödinger equation for the system; (3) each triple of the 3N coordinates of a point in configuration space that relates to one physical entity represents a point in an ordinary three-dimensional space. The first assumption is usually called wave function realism. It is a common assumption in the realistic alternatives to quantum mechanics, and it is also supported by some arguments (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Pusey, Barrett and Rudolph 2012; Gao 2013a, 2013b). The other two assumptions seem obvious when considering the many-body Schrödinger equation and its Galilean invariance, and they are also supported by some arguments (Monton 2002; Lewis 2004). A direct consequence of these assumptions is that the N physical entities described by the wave function of a N-body system exist in the region of space where the wave function is not zero, and do not exist in the region of space where the wave function is zero. In the following, we will analyze the ontological meaning of the wave function based on these assumptions.

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For simplicity, we consider a two-body system whose wave function is defined in a six-dimensional configuration space. We first suppose the wave function of the system is localized in one position $(x_1, y_1, z_1, x_2, y_2, z_2)$ in the configuration space of the system at a given instant. This wave function can be decomposed into a product of two wave functions which are localized in positions (x_1, y_1, z_1) and (x_2, y_2, z_2) in our ordinary three-dimensional space, respectively. According to the above assumptions, this wave function describes two independent physical entities, which are localized in positions (x_1, y_1, z_1) and (x_2, y_2, z_2) in our three-dimensional space, respectively, and which have respective masses, say m_1 and m_2 (as well as charges Q_1 and Q_2 etc), respectively.

Now suppose the wave function of the two-body system is localized in two positions $(x_1, y_1, z_1, x_2, y_2, z_2)$ and $(x_1^{'}, y_1^{'}, z_1^{'}, x_2^{'}, y_2^{'}, z_2^{'})$ in the configuration space of the system at a given instant[1]. According to the above analysis, the wave function of the two-body system being localized in position $(x_1, y_1, z_1, x_2, y_2, z_2)$ in configuration space means that physical entity 1 with mass m_1 and charge Q_1 exists in position (x_1, y_1, z_1) in three-dimensional space, and physical entity 2 with mass m_2 and charge Q_2 exists in position (x_2, y_2, z_2) in three-dimensional space. Similarly, the wave function of the two-body system being localized in position $(x_1^{'}, y_1^{'}, z_1^{'}, x_2^{'}, y_2^{'}, z_2^{'})$ in configuration space means that physical entity 1 exists in position $(x_1^{'}, y_1^{'}, z_1^{'})$ in three-dimensional space, and physical entity 2 exists in position $(x_2^{'}, y_2^{'}, z_2^{'})$ in three-dimensional space. Moreover, according to the above consequence of the three assumptions, the wave function of the two-body system being localized in both positions $(x_1, y_1, z_1, x_2, y_2, z_2)$ and $(x_1^{'}, y_1^{'}, z_1^{'}, x_2^{'}, y_2^{'}, z_2^{'})$ in configuration space means that the above two situations both exist in reality. The question is: In what form?

An obvious existent form is that physical entity 1 exists in both positions (x_1, y_1, z_1) and $(x_1^{'}, y_1^{'}, z_1^{'})$, and physical entity 2 exists in both positions (x_2, y_2, z_2) and $(x_2^{'}, y_2^{'}, z_2^{'})$. However, the above consequence also requires that the physical entities described by their wave function do not exist in the region of space where the wave function is zero. Therefore, when physical entity 1 exists in (x_1, y_1, z_1), physical entity 2 cannot exist in $(x_2^{'}, y_2^{'}, z_2^{'})$, and when physical entity 1 exists in $(x_1^{'}, y_1^{'}, z_1^{'})$, physical entity 2 cannot exist in (x_2, y_2, z_2), or vice versa. In other words, the wave function that describes this existent form should be localized in four positions $(x_1, y_1, z_1, x_2, y_2, z_2)$, $(x_1^{'}, y_1^{'}, z_1^{'}, x_2^{'}, y_2^{'}, z_2^{'})$, $(x_1, y_1, z_1, x_2^{'}, y_2^{'}, z_2^{'})$, and $(x_1^{'}, y_1^{'}, z_1^{'}, x_2, y_2, z_2)$ in the configuration space of the system. Therefore, the above existent form, which seems to be the only possible form, is not possible.

It seems that there is a contradiction here. However, there is still another

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[1] This is a so-called entangled state, which can be generated from a non-entangled state by the time evolution of the system. The existence of entangled states has also been confirmed by experiments.

possibility. It can be seen that the contradiction only requires that the above two situations cannot exist at the same time at a single instant. As we will show below, however, they may exist "at the same time" during an arbitrarily short time interval. Concretely speaking, the situation in which physical entity 1 is in (x_1, y_1, z_1) and physical entity 2 is in (x_2, y_2, z_2) exists in one part of continuous time, and the situation in which physical entity 1 is in $(x_1^{'}, y_1^{'}, z_1^{'})$ and physical entity 2 is in $(x_2^{'}, y_2^{'}, z_2^{'})$ exists in the other part. The restriction is that the temporal part in which each situation exists cannot be a continuous time interval during an arbitrarily short time interval; otherwise the wave function describing the state in the time interval will be not the original superposition of two branches, but one of the branches, according to the above consequence. This means that the set of the instants when each situation exists is not a continuous set but a discontinuous, dense set. At some discontinuous instants, physical entity 1 with mass m_1 and charge Q_1 exists in position (x_1, y_1, z_1), and physical entity 2 with mass m_2 and charge Q_2 exists in position (x_2, y_2, z_2), and at other discontinuous instants, physical entity 1 exists in position $(x_1^{'}, y_1^{'}, z_1^{'})$, and physical entity 2 exists in position $(x_2^{'}, y_2^{'}, z_2^{'})$. By this way of time division, the above two situations exist "at the same time" during an arbitrarily short time interval (or during an infinitesimal time interval).

This way of time division also implies a strange picture of motion for the involved physical entities. It is as follows. Physical entity 1 with mass m_1 and charge Q_1 jumps discontinuously between positions (x_1, y_1, z_1) and $(x_1^{'}, y_1^{'}, z_1^{'})$, and physical entity 2 with mass m_2 and charge Q_2 jumps discontinuously between positions (x_2, y_2, z_2) and $(x_2^{'}, y_2^{'}, z_2^{'})$. Moreover, they jump in a precisely simultaneous way. When physical entity 1 jumps from position (x_1, y_1, z_1) to position $(x_1^{'}, y_1^{'}, z_1^{'})$, physical entity 2 must jump from position (x_2, y_2, z_2) to position $(x_2^{'}, y_2^{'}, z_2^{'})$, or vice versa. In the limit situation where position (x_2, y_2, z_2) is the same as position $(x_2^{'}, y_2^{'}, z_2^{'})$, physical entities 1 and 2 are no longer entangled, while physical entity 1 with mass m_1 and charge Q_1 still jumps discontinuously between positions (x_1, y_1, z_1) and $(x_1^{'}, y_1^{'}, z_1^{'})$. This means that the picture of discontinuous motion also exists for one-body systems. Since quantum mechanics does not provide further information about the positions of physical entities at each instant, the discontinuous motion described by the theory is also essentially random.

The above analysis can be extended to an arbitrary entangled wave function for an N-body system. Since each physical entity is only in one position in space at each instant, it may well be called particle. Here the concept of particle is used in its usual sense. A particle is a small localized object with mass and charge etc, and it is only in one position in space at an instant. Moreover, the motion of these particles is not continuous but discontinuous and random in nature, and especially, the motion of entangled particles is precisely simultaneous. By a more detailed mathematical analysis of random discontinuous motion of particles (Gao 2013a), it can be further argued that the wave function of a N-body system provides a description of the state of random discontinuous motion of N particles[2], and in particular, the modulus squared of the wave function gives the probability density that the particles appear in certain positions in space.

To sum up, we have argued that wave function realism, along with two seemingly obvious assumptions: (1) the wave function of a N-body system describes N physical entities; (2) each triple of the 3N coordinates of a point in configuration space that relates to one physical entity represents a point in ordinary three-dimensional space, imply that the physical entities described by the wave function are particles, and these particles move in a discontinuous and random way.

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[2] At a deeper level, it may represent the property of the particles that determines their random discontinuous motion. For a many-particle system in an entangled state, this property is possessed by the whole system.