Unitary quantum theories are incompatible with special relativity

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Abstract

It is shown that the combination of a unitary quantum theory and special relativity may lead to a contradiction when considering the statistics of certain measurement results in different Lorentz frames in a Gedankenexperiment. This result seems to imply that either unitary quantum theories are wrong or if a unitary quantum theory is right then there must exist a preferred Lorentz frame.

It has been debated whether quantum mechanics and special relativity are compatible. In 1964, based on the Einstein-Podolsky-Rosen (EPR) argument [1], Bell derived an important result that was later called Bell's theorem [2]. It states that certain predictions of quantum mechanics cannot be accounted for by any local realistic theory, and thus strongly suggests that quantum mechanics and special relativity are incompatible. However, there have been controversies on the underlying assumptions and deep implications of Bell's theorem [3-5]. Moreover, there are also unitary quantum theories (without wave function collapse) which claim that they can avoid the nonlocality consequence of Bell's theorem and are compatible with special relativity [6-11]. In this paper, I will give a simple proof of the incompatibility between unitary quantum theories and special relativity based on an analysis of a Wigner's friend variant of the EPR-Bohm experiment. The work can be regarded as a further development of Bell's theorem.

Consider a usual EPR-Bohm experiment. There are two observers Alice and Bob who are in their separate laboratories and share an EPR pair of spin 1/2 particles in the spin singlet state:

$$\frac{1}{\sqrt{2}}(|up\rangle_1 |down\rangle_2 - |down\rangle_1 |up\rangle_2). \tag{1}$$

Alice measures the spin of particle 1 at angle a, and Bob measures the spin of particle 2 at angle b. Each measurement result is $+1$ or -1, corresponding to spin up or spin down. Then we can calculate the probabilistic correlation function $E(a, b)$ for Alice's and Bob's measurement results according to the Born rule, which is $E(a, b) = -cos(a - b)$. In particular, in the EPR anti-correlation case of $b = a$, we have $E(a, b) = -1$, which means that when Alice's result is $+1$, Bob's result is -1, and vice versa.

Now consider a Wigner's friend variant of the EPR-Bohm experiment in which there is an additional superobserver who can undo a measurement [12-18]. The existence of such superobservers is permitted in principle by a unitary quantum theory. First, suppose in the laboratory frame (in which Alice's and Bob's laboratories are at rest), Alice first measures the spin of particle 1 at angle a and obtains her result, then the superobserver undoes Alice's measurement (which restores the states of Alice and the particles to their initial states, see [18]), and then Alice measures again the spin of particle 1 at angle a and obtains her second result, and then the superobserver undoes Alice's second measurement, and this process repeats a large number of times, and finally Bob measures the spin of particle 2 at angle $b = a$. According to the Born rule, the probability distribution of Alice's results is $P(+1) = 1/2$ and $P(-1) = 1/2$.

Next, suppose in the laboratory frame Bob first measures the spin of particle 2 at angle $b = a$, and then Alice measures the spin of particle 1 at angle a and obtains her result, and then the superobserver undoes Alice's measurement, and then Alice measures again the spin of particle 1 at angle a and obtains her second result, and then the superobserver undoes Alice's second measurement, and this process in Alice's side repeats a large number of times. In this case, according to the Born rule, the probability distribution of Alice's results is $P(+1) = 1$ and $P(-1) = 0$ (when Bob's result is -1) or $P(+1) = 0$ and $P(-1) = 1$ (when Bob's result is $+1$).

It can be seen that Bob's measurement influences the results of Alice's measurements (when assuming there is no superdeterminism). When Bob does not make a measurement before Alice's measurements, the probability distribution of Alice's results is $P(+1) = 1/2$ and $P(-1) = 1/2$, while when Bob makes a measurement before Alice's measurements, the probability distribution of Alice's results is a different one, which is either $P(+1) = 1$ and $P(-1) = 0$ or $P(+1) = 0$ and $P(-1) = 1$. When Bob's measurement is spacelike separated from Alice's measurements, this influence will be faster than light or even instantaneous, which is inconsistent with the spirit of special relatvity.

Note that the above analysis does not concern merely a theoretical probability distribution but an actual frequency. In the Gedankenexperiments, Alice can repeat her measurement many times, and she will either obtain the same result each time, or spin up and spin down with roughly equal frequency - even though she is unable to remember or report these statistics.

Indeed, at the end of these experiments, all of Alice's measurement results except the last one (observed in some inertial frames) are erased by the superobserver. As thus, the statistics of Alice's results can only be calculated from a theory, and it cannot be found by the experiments. This is consistent with the no-signaling theorem.

Notwithstanding the consistency with the no-signaling theorem, unitary quantum theories and special relativity are incompatible. Here is a rigorous proof. Suppose in the laboratory frame Alice and the superobserver first make their series of measurements and then Bob makes his measurement. Then Alice will obtain two different results, spin up and spin down, with roughly equal frequency. Then when Bob's measurement is spacelike separated from Alice's and the superobserver's measurements, the following time order of events in another inertial frame is permitted by special relativity.[1] In the frame, Bob first makes his measurement, and then Alice and the superobserver make their series of measurements. Then Alice will obtain the same result each time, either spin up or spin down. Since the results of the same measurement observed in two Lorentz frames should be the same, we have derived a contradiction.

A similar contradiction can also be derived from an analysis of single measurement results. Consider again the EPR anti-correlation case of $b = a$. Suppose in the laboratory frame, Alice first measures the spin of particle 1 at angle a and obtains her result, then the superobserver undoes Alice's measurement, which restores the states of Alice and the particles to their initial states, and finally Bob measures the spin of particle 2 at angle $b = a$ and obtains his result. When Bob's measurement is spacelike separated from the superobserver's reset operation, the following time order of events in another Lorentz frame is permitted by special relativity. In this frame, Alice first measures the spin of particle 1 at angle a and obtains her result, then Bob measures the spin of particle 2 at angle $b = a$ and obtains his result, and finally the superobserver undoes Alice's measurement. According to the Born rule, in this frame, when the result of Alice's measurement is $+1$, the result of Bob's measurement must be -1 with certainty. On the other hand, in the laboratory frame, when the result of Alice's measurement is $+1$, the result of Bob's measurement cannot be -1 with certainty. The reason is that if the result of Bob's measurement is -1 with certainty, then if Alice makes her second measurement her result will be $+1$ with certainty, which further means that if Alice and the superobserver repeat their measurements a large number of times, Alice's results will be all $+1$, which violates the Born rule.[2] Thus we also have a contradiction.

[1] Note that when the distance between Alice's and Bob's laboratories is very large and the duration between Alice's and the superobserver's measurements and Bob's measurement is very short, the relative velocity between this inertial frame and the laboratory frame may be close to zero.

[2] Strictly speaking, when the result of Alice's measurement is $+1$, it is possible that the

It can be seen that the above contradiction results from the combination of a unitary quantum theory and special relativity. A unitary quantum theory permits the existence of the superobserver's reset operations due to the universal validity of the Schrödinger equation and gives the Born rule for the measurement results of Alice and Bob, and special relativity permits the change of time order of events such as these measurements in different Lorentz frames. It is these two elements in combination that lead to the contradiction in the above Gedankenexperiments. Note that the unitary quantum theory used here is composed of the universal Schrödinger equation and the Born rule, which is the core of quantum mechanics. Moreover, the analysis concerns only its predictions for the measurement results, and thus the validity of the analysis does not depend on any further interpretation of quantum mechanics beyond its core, such as the ontological status and meaning of the wave function and how to solve the measurement problem etc.

Avoiding the above contradiction requires that either unitary quantum theories are wrong or special relativity is wrong. If unitary quantum theories are wrong, e.g. in some collapse theories in which the collapse of the wave function is time irreversible, then the superobserver who can undo a measurement will not exist, and the above contradiction can be avoided.[3] On the other hand, if a unitary quantum theory is right, then special relativity must be violated. Concretely speaking, in a unitary quantum theory there must exist a preferred Lorentz frame, in which the time order of events is real and the predictions of the theory are true, while in other Lorentz frames the time order of events may be apparent and the predictions of the theory such as the Born rule may be not true.[4] Moreover, in a unitary quantum theory there are faster than light influences induced by measurements, and the preferred Lorentz frame can be defined as the Lorentz frame in which these influences happen instantaneously.

Finally, I will briefly discuss the possible implications of the above result for several unitary quantum theories. The first one is hidden-variable theories such as Bohm's theory and the modal interpretation. It has been shown that in these theories special relativity is violated and a preferred Lorentz frame exists at the assumed ontological level [21,22]. For example, in Bohm's theory, the joint distributions given by the Born rule for position measurements cannot in general agree with the distributions of the actual

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result of Bob's measurement is -1 with certainty, but the probability of this possibility is zero according to the Born rule. This means that if Alice and the superobserver repeat their measurements, the probability of Alice's results being all $+1$ is zero (when the times of measurements approach infinity), which is in accordance with the Born rule.

[3] It seems that there is also a nonzero probability that a measurement can be undone in collapse theories with tails. Then the above analysis may be also valid even for these collapse theories.

[4] Similar results have also been obtained in [19,20].

Bohmian particle positions in all Lorentz frames. The above result further shows that a preferred Lorentz frame must also exist in these theories when considering only actually observed measurement results.

It is worth noting that the above proof may have a loophole in deterministic nonlocal hidden-variable theories such as Bohm's theory. In these theories, when the superobserver's reset operation is an exact time-reversal,[5] it will also restore the exact initial values of all hidden variables such as the initial positions of all the Bohmian particles. Then, in the above Gedanken-experiments, the results of Alice's measurements will be all the same, independently of Bob's measurement[6], and thus the above result cannot be derived. Interestingly, however, a further analysis of the invariance of the results of Alice's and Bob's spacelike separated measurements in different Lorentz frames also shows that these theories are incompatible with special relativity and require the existence of a preferred Lorentz frame [27].

Another well-known unitary quantum theory is Everett's theory or the many-worlds theory. It is widely thought that the many-worlds theory is consistent with special relativity, and the consistency is a major advantage of the theory. Then, does the above result also hold true in the many-worlds theory? I think the answer may be positive. The reason is that the derivations of the contradiction are based only on the Schrödinger equation and the Born rule, and the predictions of the many-worlds theory for the statistics of Alice's results in all worlds are the same as the above. For example, if Bob makes his measurement at last, the statistics of Alice's results will be 50% and 50% for spin up and spin down in each world. While if Bob first makes his measurement, Alice's results will be all spin up in one world, and all spin down in the other world.[7] This means that the contradiction will exist in each world in the many-worlds theory.

It can be seen that the usual way of the many-worlds theory to avoid Bell's theorem is not available here, since the first derivation concerns only the statistics of Alice's own results, and no correlation between Alice's results and Bob's result is involved in the derivation. Moreover, it has been shown that in the EPR anti-correlation case or parallel case, unlike the non-parallel cases, the many-worlds theory does not require that a joint measurement should be performed, comparing the results from Alice and

[5]In general, the superobserver's reset operation is not necessarily an exact time-reversal, and it only needs to restore the initial state of Alice and the initial spin states of the particles.

[6]When Bob makes his measurement at last, the statistics of Alice's results will violate the Born rule, and the violation may exist in all Lorentz frames. Although this is understandable in a deterministic hidden-variable theory and does not contradict experience, it suggests that an indeterministic hidden-variable theory is more consistent with quantum mechanics [24-26].

[7]Note that in Wallace's (2012) formulation of the many-worlds theory the number of the emergent worlds after a measurement is not definite due to the imperfectness of decoherence. This does not influence the analysis here.

Bob, which can only take place in the overlap of the future light cones of the measurements of Alice and Bob, in order to make definite spin and outcome states from one side definite relative to definite spin and outcome states from the other[8]; rather, following the two local measurements of Alice and Bob, from the point of view of one side, the states of the systems in the other side already correspond to a definite, perfectly anti-correlated, measurement result [6]. Thus, the second derivation of the contradiction, which is based on an analysis of single measurement results, is also valid in the many-worlds theory, since it concerns only the EPR anti-correlation case.

How about other unitary quantum theories such as consistent histories [7], retrocausal theories [8], relational quantum mechanics [9], quantum perspectivalism [10], QBism [11], and superdeterminism? It seems that they will give the same predictions for the statistics of Alice's results as above (otherwise they will violate the Born rule), and thus the (first) derivation of the contradiction is still valid in these theories. In particular, since the derivation concerns only the statistics of the results obtained by Alice herself, it seems that even relationalism and QBism cannot avoid the contradiction either.

The only way to avoid the contradiction seems to insist that the statistics of Alice's results does not exist or cannot be defined. If there is no such a thing, then the analysis based on it will be invalid. Indeed, even though Alice can repeat her measurement many times, she will not be able to remember or report the statistics of these results in a unitary quantum theory. But this is only an impossibility of testing certain predictions of a theory at the empirical level,[9] while what we consider here is whether the predictions of two theories are compatible at the theoretical level. After all, the statistics of the results of Alice's repeated measurements in each Lorentz frame can be properly defined and also precisely predicted by a unitary quantum theory and special relativity.[10] The result I have derived above is just that the combination of these two theories will lead to a contradiction when considering their predictions for the statistics of Alice's results in different Lorentz frames.

To sum up, I have argued that the combination of a unitary quantum theory and special relativity may lead to a contradiction when considering

[8]According to the reasoning, in the non-parallel cases, we can only think of the correlations between measurement results on the two sides of the experiment actually obtaining in the overlap of the future light-cones of the measurement events, and thus there will be no violation of special relativity [6].

[9]This does raise an interesting issue in philosophy of science.

[10]In addition, it is worth pointing out that whether the statistics of Alice's results exists or not (when she cannot remember or report the statistics) does not depend on any quantum theory, and it is purely a classical issue. Thus, if the existence of the statistics is denied, then the denial will also change our current understandings of classical mechanics. It seems, at least to me, that the change will lead to a very limited worldview or even solipsism (see also [27]).

the statistics of certain measurement results in different Lorentz frames. This result seems to imply that either unitary quantum theories are wrong or if a unitary quantum theory is right then there must exist a preferred Lorentz frame.

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References

[1] Einstein, A., B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? Physical Review 47, 777 (1935).

[2] Bell, J. S. On the Einstein-Podolsky-Rosen paradox. Physics, 1, 195-200 (1964).

[3] Maudlin, T. Quantum Non-Locality and Relativity: Metaphysical Intimations of Modern Physics, 3rd edn. Oxford: Blackwell (2011).

[4] Bell, M and S. Gao (eds.) Quantum Nonlocality and Reality: 50 Years of Bells theorem. Cambridge: Cambridge University Press (2016).

[5] Bertlmann, R. and Zeilinger, A. (eds.) Quantum [Un]Speakables II: Half a Century of Bell's Theorem. Springer International Publishing, Cham (2017).

[6] Brown, H. and Timpson C. Bell on Bell's theorem: The changing face of nonlocality. In Bell, M and S. Gao (eds.) Quantum Nonlocality and Reality: 50 Years of Bells theorem. Cambridge: Cambridge University Press (2016). pp. 91-123.

[7] Griffiths, R. B. Quantum locality. Found. Phys. 41, 705-733 (2011).

[8] Price H. and Wharton K. Dispelling the Quantum Spooks: A Clue That Einstein Missed?. In: Bouton C., Huneman P. (eds) Time of Nature and the Nature of Time. Boston Studies in the Philosophy and History of Science, vol 326. Springer, Cham. (2017). pp 123-137.

[9] Rovelli, C. and Smerlak, M. Relational EPR, Foundations of Physics, 37, 427-45 (2007).

[10] Dieks, D. Quantum Mechanics and Perspectivalism. arXiv:1801.09307 (2018).

[11] Fuchs, C., Mermin, N. D. and Schack, R. An introduction to QBism with an application to the locality of quantum mechanics. Am. J. Phys. 82, 749-54 (2014).

[12] Wigner, E. Remarks on the mind-body question. In: Good, I. (ed.) The Scientist Speculates. Heinemann, London (1961).

[13] Deutsch, D. Quantum theory as a universal physical theory. Int. J. Theor. Phys. 24, 141 (1985).

[14] Brukner, C. On the quantum measurement problem (2015). https://ar-xiv.org/abs/1507.05255. In: Bertlmann, R., Zeilinger, A. (eds.) Quantum [Un]speakables II, pp. 95-117. Springer International, Cham (2017).

[15] Frauchiger, D. and Renner, R. Single-world interpretations of quantum theory cannot be self-consistent. https://arxiv.org/abs/1604.07422. (2016).

[16] Pusey, M. Is QBism 80% complete, or 20%, talk given at the Information-Theoretic Interpretations of Quantum Mechanics workshop, Western University, Canada. (2016).

[17] Frauchiger, D. and Renner, R. Quantum theory cannot consistently describe the use of itself. Nat. Commun. 9, 3711 (2018).

[18] Healey, R. Quantum Theory and the Limits of Objectivity. Found. Phys. 48, 1568 (2018).

[19] Lazarovici, D. and Hubert, M. How Quantum Mechanics can consistently describe the use of itself. arXiv:1809.08070 (2018).

[20] Leegwater, G. When Greenberger, Horne and Zeilinger meet Wigner's Friend. arXiv:1811.02442 (2018).

[21] Berndl, K., D́urr, D., Goldstein, S., and Zanghi, N. Nonlocality, Lorentz invariance, and Bohmian quantum theory. Phys. Rev. A 53 , 2062-73 (1996).

[22] Myrvold, W. C. Modal interpretations and relativity. Foundations of Physics, 32, 1773-84 (2002).

[23] Bell, J. S. Beables for quantum field theory. Physics Reports 137, 49-54 (1986).

[24] Vink, J. C. Quantum mechanics in terms of discrete beables. Physical Review A 48, 1808 (1993).

[25] Sudbery, A. Single-world theory of the extended Wigner's friend experiment. Foundations of Physics, 47, 658-69 (2017).

[26] Gisin, N. Impossibility of covariant deterministic nonlocal hidden-variable extensions of quantum theory. Phys. Rev. A 83, 020102(R) (2011).

[27] Norsen, T. Quantum Solipsism and Nonlocality. In Bell, M and S. Gao (eds.) Quantum Nonlocality and Reality: 50 Years of Bells theorem. Cambridge: Cambridge University Press (2016). pp. 204-237.