Whitehead and Pythagoras

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Abstract—While the appeal of scientific materialism has been weakened by developments in theoretical physics, chemistry and biology, Pythagoreanism still attracts the allegiance of leading scientists and mathematicians. It is this doctrine that process philosophers must confront if they are to successfully defend their metaphysics. Peirce, Bergson and Whitehead were acutely aware of the challenge of Pythagoreanism, and attempted to circumvent it. The problem addressed by each of these thinkers was how to account for the success of mathematical physics if the world consists of creative processes. In this paper I critically examine the nature of the challenge posed by Pythagoreanism to process philosophy and examine the efforts by process philosophers, particularly Whitehead, to overcome it, and offer some suggestions for advancing these efforts.

Keywords— Pythagoreanism, Pythagoras, philosophy of mathematics, Peirce, Bergson, Whitehead, Meyerson, Kauffman, process metaphysics.

In an anthology devoted to Nicholas Rescher, *After Whitehead: Rescher on Process Metaphysics,* Lieven Decock argued that substance philosophy no longer stands as a serious challenge to process philosophy; the real alternative to process philosophy comes from neo-Pythagoreanism.[1] While in response, Rescher dismissed this claim, it is clear that leading scientists are more likely to be committed to neo-Pythagoreanism than process philosophy. Roger Penrose, for instance, begins his massive exposition of the state of modern physics, *The Road to Reality: A Complete Guide to the Laws of the Universe,* with a defence of Pythagoreanism.[2] Many proponents of complexity theory see it not in relation to process philosophy but as the triumph of Pythagoreanism. Pythagoreanism underpins the quest by physicists for an ultimate 'theory of everything', that is, 'a single all-embracing picture of all the laws of nature from which the inevitability of all things seen must follow with unimpeachable logic.'[3] It seems to me that the challenge of

Pythagoreanism to process philosophy is greater than Rescher credits it.

Furthermore, to fully understand the importance of Charles Sanders Peirce, Henri Bergson and Alfred North Whitehead to the tradition of process philosophy it is necessary to properly appreciate this challenge. Scientific materialism was only one form of Pythagoreanism, and the demolition of this doctrine was not in itself enough to establish process philosophy as its necessary successor. The reason why Peirce and Whitehead, and to a lesser extent Bergson, are more important to the tradition of process thought than William James or John Dewey is that they understood the achievements and problems of the Pythagorean tradition and attempted to go beyond Pythagoreanism in a way that would do justice to its achievements.

How could this be done? To begin with, it was necessary to comprehend the achievements of Pythagoreanism, and then clarify its problems. To some extent the achievements of Pythagoreanism are more obvious; they are evident in the successes of mathematical physics. The advance of mathematical physics has been associated with clarity of thought and explanation unrivalled elsewhere. A major problem with Pythagoreanism is that it seems to render the conscious beings, which are advancing mathematical physics, unintelligible. How would it be possible for beings

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[1] Lieven Decock, 'The Taming of Change', in *After Whitehead: Rescher on Process Metaphysics,* Frankfurt: Ontos, 2004, pp.95-111.

[2] Roger Penrose, *The Road to Reality: A Complete Guide to the Laws of the Universe,* New York: Alfred A Knopf, 2005, p.5ff.

[3] John D. Barrow, *Theories of Everything: The Quest for Ultimate Explanation,* London: Vintage, 1992, p.1.

'governed in completely precise detail by mathematical principles', as Penrose put it,[4] to come to understand and know this and what it means? This problem manifests an even deeper problem, a problem which Peirce, Bergson and Whitehead were alive to: that the ideal of explanation to which Pythagoreanism is committed makes the evolution of any real variety, let alone the evolution of conscious beings, unintelligible. As Peirce put it:

> Is there such a thing in nature as increase in variety? Were things simpler, was variety less in the original nebula from which the solar system is supposed to have grown than it is now when the land and the sea swarms with animals and plant forms with their intricate anatomies and still more wonderful economies? It would seem as if there were an increase in variety, would it not? And yet mechanical law, which the scientific infallibilist tells us is the only agency of nature, mechanical law can never produce diversification. That is a mathematical truth – a proposition of analytic mechanics; and anybody can see without any algebraical apparatus that mechanical law out of like antecedents can only produce like consequents. It is the very idea of law.[5]

Bergson made the same point when he wrote that 'no complication of the mathematical with itself, however elaborate we may suppose it, can introduce an atom of novelty in the world.'[6] The problem is, as Émile Meyerson (almost certainly under the influence of Bergson[7]) pointed out, the model of explanation which modern science has embraced, involves showing that what appears to be variety or diversity is not really diversity at all but only the appearance of an underlying identity. Consequently, as he argued: 'scientific explanation actually ends up dissolving the external world into undifferentiated space.'[8] Elaborating on this, he wrote:

> [D]iversity in space is unquestionably an enigma for us, a ground for astonishment if not identical, at least very similar to that we discover in the case of diversity in time. As a consequence we cannot escape the conclusion that if our reasoning

is correct, the goal of explanations and theories is really to replace the infinitely diverse world around us by identity in time and space, which clearly can only be space itself[9]

Or, as G. Spencer Brown put it more dramatically:

> To *explain,* literally to lay *out* in a *plane* where particulars can be readily seen. Thus to *place* or *plan* in *flat* land, sacrificing other dimensions for the sake of appearance. Thus to *expound* or *put out* at the cost of ignoring the *reality* or *richness* of what is so put out. Thus to take a view away from its *prime reality* or *royalty,* or to gain knowledge and lose the kingdom[10.]

THE EMERGENCE OF THE PROBLEM

How did such a notion of explanation emerge? And what is its relation to mathematics?[11] The problem really became evident in Ancient Greece and emerged out of different strands of Anaximander's philosophy. It was Anaximander who originated the idea of the cosmos, and it was probably he who first deployed the term *kosmos* to characterize this.[12] It appears that Anaximander was also the first thinker to propose what in Ancient Greece became the standard form of explanation for the cosmos, first, postulating an undifferentiated unity, secondly, arguing that from this unity two opposite powers separated out to form the world order and thirdly, showing how these two opposites unite again to generate life. Anaximander postulated the *apeiron* or 'unlimited' as the all-enfolding and all-controlling, divine and immortal and indestructible source of the cosmos. Through limitation (*peirata)* of the unlimited the cosmos emerges, to begin with by generating the polar opposites: on the one hand, hot, dry, bright and rare; on the other, cold, damp, dark and dense.[13] It was the interaction between these that generated the diversity of the cosmos, the celestial bodies, meteorological phenomena, the sea and dry land, animal life and humans. In characterizing celestial bodies, Anaximander was the first mathematical physicist outside Babylon. Given the dimensions he postulated, he evidently

[4] Penrose, *The Road to Reality,* p.19.

[5] Charles Sanders Peirce, 'Synechism, Fallibilism, and Evolution' [1897] in *Philosophical Writings of Peirce,* ed. Justus Buchler, New York, Dover Publications, 1955, p.357.

[6] Henri Bergson, *Creative Evolution,* [1907] trans. Arthur Mitchell, Lanham: University Press of America, 1984, p.217.

[7] Émile Meyerson, *Identity & Reality,* [1908] trans. Kate Loewenberg, New York: Dover, 1962, p.13.

[8] Émile Meyerson, *Explanation in the Sciences,* [1921] trans. Mary-Alice and David A. Sipfle, Dordrecht, Kluwer Academic Publishers, 1991, p.1.

[9] Meyerson, *Explanation in the Sciences,* p.136f.

[10] G. Spencer Brown, *Laws of Form,* London, George Allen and Unwin, 1969, p.126n.

[11] The following history is a condensed version of my longer paper, 'Mathematics, Explanation and Reductionism: Exposing the Roots of the Egyptianism of European Civilization, *Cosmos and History,* 1:1, 2005, 54-89.

[12] See Charles H. Kahn, *Anaximander and the Origins of Greek Cosmology,* [1960] Indianapolis, Hackett, 1994, Appendix I.

[13] See Paul Seligman, *The Apeiron of Anaximander,* Athlone Press: The University of London, 1962, p.122.

believed that the universe was governed by simple mathematical ratios.[14] On this basis he argued that 'The earth is aloft, not dominated by anything; it remains in place because of the similar distance from all points [of the celestial circumference].'[15] This, as noted by Charles Kahn, involved a new form of mathematical reasoning. It is, as he put it, 'a general expression for the principle of symmetry or indifference. It is indeed the same notion which was glorified in modern time by Leibniz as his Principle of Sufficient Reason, according to which everything which is true or real implies a reason why it is so and not otherwise.'[16] But outside the celestial realm, and despite reference to the immortal *apeiron,* the *kosmos* was conceived by Anaximander as dynamic and historical. Here, mathematical reasoning had at most a very subordinate position. Life had evolved and was continuing to evolve. The first living beings had emerged in moisture and migrated to drier parts, and humans had evolved from fish.

Pythagoras and the Pythagoreans appear to have accepted the basic structure of Anaximander's cosmology, but radically extended the place accorded to mathematics. While the Pythagoreans built on the mathematics developed by the Egyptians and Babylonians and by Thales and Anaximander, according to Proclus, it was they who developed it as a systematic body of knowledge 'seeking its first principles in ultimate ideas, and investigating its theorems abstractly and in a purely intellectual way.'[17] While the basic principle and root of all things was taken to be number, the basic principle and root (*arche)* of number was taken to be the Monad or Unity. 'One' was not taken to be a number, but as the principle underlying number conceived as diversity, which in turn is the condition for achieving relations between diversity. That is, the triadic form of explanation of the cosmos was taken over from Anaximander, but was conceived in mathematical terms. As the historian of Pythagorean thought, Kenneth Sylvan Guthrie, put it, 'If One represents the principle of Unity from which all things arise, then Two, the Dyad, represents Duality, the beginning of multiplicity, the beginning of strife, yet also the possibility of *logos,* the relation of one thing to

another.'[18] In the movement from one to two to three to four we have a return to unity of the *tetractys* of the Decad (an equilateral triangle of ten points), which was taken to be perfect and to embrace the whole of nature. For instance One represents the point, Two represent the line, Three represents the surface, and Four the tetrahedron, the first three dimensional form, the Tetraktys representing this emerging multiplicity from unity as the unity of the Decad. So, as Theon of Smyrna put it, 'the Decad determines every number, including the nature of everything, of the even and the odd, of the mobile and immobile, of good and evil.'[19] The Decad was identified with *kosmos* (conceived as 'world-order'), essentially a mathematic harmony. There was no distinction between physical bodies and ideal mathematical constructions, or between rigid geometrical form and the vital processes of living things; numbers were thought to be separated by breathing in spirit and void out of the unlimited.[20] The Pythagoreans identified the unlimited with Even numbers and limit with the Odd and saw the world order as compounded of these elements. The Unlimited is indefinite and in need of Limit which is a definite boundary. The study of number was divided into four branches: arithmetic which deals with number in itself, geometry which deals with number in space, music or harmonics which deals with number in time, and astronomy which deals with number in space and time. With this doctrine the Pythagoreans were led to examine the relationships of numbers and geometrical forms as a means to investigate the entire cosmos, extending Anaximander's presupposition that everything that is true or real requires a 'sufficient reason' to explain why it is so and not otherwise. But they did not do this entirely consistently. Their ideas resemble numerology rather than modern mathematical physics, and this obscured the full implications of their views from being appreciated.

This quest by the Pythagoreans for rigorous explanation was taken up by Parmenides, who took the principle of sufficient reason to its logical conclusion. Focusing on what is intelligible he concluded that there is simply the One or unity of Being. There can be no development of diversity from the One and so no harmonizing of the diverse. Such would imply coming into being and ceasing to be. Since this assumes that we can know a prior state of not being, which is by definition not, and therefore unknowable, this is unintelligible, Parmenides argued. The only secure way to truth is

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[14] Kahn, *Anaximander and the Origins of Greek Cosmology,* p.96f.

[15] Quoted Kahn, *Anaximander and the Origins of Greek Cosmology*, p.76.

[16] Kahn, *Anaximander and the Origins of Greek Cosmology,* p.77.

[17] Cited by Marshall Clagett, *Greek Science in Antiquity,* 2nd ed. [1963], Princeton Junction, The Scholar's Bookshelf, 1988, p.36.

[18] Guthrie, *The Pythagorean Sourcebook* , p.21.

[19] Guthrie, *The Pythagorean Sourcebook,* p.171.

[20] Aristotle, *Physics,* 213 b22.

that which concerns what 'is', and this cannot come to be or perish, change or move, nor be subject to imperfection. So, from postulating more abstract entities and charting new notions of intelligibility in comprehending these abstract entities and their relations pioneered by Anaximander and developed by Pythagoras, Parmenides focused on this abstract realm of thought and what is intelligible and concluded that the world that we normally take to be reality is merely the opinion of men. This did not stop him going on to elaborate a whole cosmology, portraying the cosmos as developing out of the opposition between light and night, but at the same time he denigrated such accounts as nothing but inventions of the human mind. Parmenides defended his claims by showing the logical coherence of his ideas and that the alternatives to them led to contradictions. This was the origin of logic (as 'dialectic'), which was further developed by Parmenides' student, Zeno, who used it to show the incoherence of believing in plurality or motion. Parmenides' arguments were based on two assumptions: that logically true things and properties of real things coincide, and that any proposition is either true or false; there can be no third case.[21] Both of these assumptions came to be accepted and have pervaded thought ever since (although each assumption has been questioned).

SUBSEQUENT HISTORY OF PYTHAGOREANISM

Almost all subsequent thinking about the cosmos has involved efforts to embrace the rigor of Parmenides while evading his conclusions. With the revival of Pythagoreanism in the seventeenth century these conclusions were avoided through postulates of a creative deity or creative minds not subject to the laws governing the physical world, or in the case of Leibniz who seemed to have most clearly seen the problem, by postulating a deity who created an infinite diversity of monads which would unfold in harmony so that each monad would reflect in itself to different degrees this harmonious order. Kant dealt with the problem by arguing that the world as understood through mathematics is only the world of appearances, experience organized through imagination, the forms of intuition and the categories of the understanding, not reality as it is in itself. However, as these

postulates and evasions were discarded, the problem arose again to haunt natural philosophy.[22] Most scientists embraced the assumptions which led to Parmenides conclusions, usually without fully appreciating the significance of what they were doing. Process philosophers reacted against these conclusions and critically examined the assumptions that led to them. How did Bergson, Peirce and Whitehead deal with these issues?

BERGSON'S PROPOSED SOLUTION

Bergson was concerned to show the delusory nature of the achievements of mathematical physics, or more generally, the analytic intellect, attempting to do so in a way that would point to different way of knowing the world free from the distortions of this defective form of knowledge. The intellect, Bergson argued, freezes reality as extension in space in order to analyse it and reveal how to control it. It is within this impoverished experience of reality that mathematics finds mathematical order. Knowledge gained in this way is extremely important to humans in their efforts to control nature, but it only cognizes nature insofar as it can be controlled. But it does not and cannot grasp reality as such, as creative becoming. The cardinal sin of philosophers is to have confused the extension grasped by the intellect with durational creative becoming.[23] Even in the work in which he concentrated most fully on developing his cosmology, *Creative Evolution,* Bergson was primarily concerned to free experience of this creative becoming from spatializing intellectual knowledge and to show how the latter is a derivative and defective form of knowledge. In *Creative Evolution* Bergson characterized the seductive achievements of mathematics:

> When we consider the admirable order of mathematics, the perfect agreement of the objects it deals with, the immanent logic in numbers and figures, our certainty of always getting the same conclusion, however diverse and complex our reasonings on the same subject, we hesitate to see in properties apparently so positive a system of negations, the absence rather than the presence of a true reality. …. The complexity the intellect puts into its object by analysing it, the more complex is the order it finds there. And this order and this complexity necessarily appear to the intellect as a positive reality, since reality and intellectuality are turned in the same direction.

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[21] For an analysis of Parmenides' and Zeno's arguments, see George Kampis, *Self-Modifying Systems in Biology and Cognitive Science: A New Framework A New Framework for Dynamics, Information and Complexity,* Oxford, Pergamon Press, 1991, p.84ff.

[22] This is the theme of Barrow's book, *Theories of Everything.*
[23] Bergson, *Creative Evolution,* p.210 n.1.

This mathematical order and complexity, Bergson argued, while appearing to the intellect as a positive reality, is achieved through an inversion of the will and suppression of positive reality. This creates 'at once extension in space and the admirable order which mathematics finds there.'[24] So, 'the infinite complexity of the parts and their perfect coordination among themselves are created at one and the same time by an inversion which is, at bottom, an interruption, that is to say, a diminution of positive reality.'[25] Bergson argued that it is the function of the intellect to construe reality in this way, leading it to construe the world as a geometrical order. As he put it:

> All the operations of our intellect tend to geometry as to the goal where they find their perfect fulfilment. But geometry is necessarily prior to them (since these operations have not as their end to construct space and cannot do otherwise than take it as given) it is evident that it is a latent geometry, immanent in our idea of space, which is the main spring of our intellect and the cause of its working.[26]

Both deduction and induction presuppose this spatial intuition.

Through exposing how the intellect works and pervades almost every aspect of our thinking, Bergson sought to reveal an alternative way of understanding nature, the path he claimed should be pursued by philosophy. It is through intuition that we appreciate the reality of durational becoming, and Bergson argued that the intuition of duration is prior to and the condition of the kind of thinking associated with the intellect. Through the use of intuition, he argued, philosophy can 'turn the mind homeward' to 'coincide with the living principle whence it emanates', thereby to 'study becoming in general'. This is 'the true evolutionism and consequently the true continuation of science.'[27] Bergson's concern was to develop an appreciation of the kind of order associated with becoming as durational. Duration is not infinitely divisible; existence is durational, with different processes existing over different durations. Duration implies a reality in which the past in some sense really exists and can be intuited as that which no longer acts but is still intimately related to the present, while the future is not yet created. Bergson frequently invoked the example of music to illustrate this connection of the indestructible past to the present and its movement into the future, but also pointed out that listening to a word or a sentence also manifests this integral relation between sounds or words that have already been uttered and the present.[28] On this basis nature was portrayed as creative, continually generating novelty not predictable from antecedent states of the universe. While initially focussing on the mind, Bergson later strove to develop a cosmology based on appreciation of this durational becoming, a world of creative evolution.

PEIRCE'S PROPOSED SOLUTION

Mathematics, Peirce argued, is the science which draws necessary conclusions from exclusively hypothetical states of things.[29] An hypothesis is 'a proposition imagined to be strictly true of an ideal state of things'.[30] Necessary conclusions are drawn by mathematicians through the use of diagrams which function as an analogy to such hypotheses. As he put it:

> [Mathematical] deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, or experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts.[31]

Peirce rejected the idea of mathematics as the science of pure space, as De Morgan had proposed, or as the science of pure time, as Rowan Hamilton had proposed.[32] While if an hypothesis turned out to be true of an actual state of affairs then conclusions drawn from it would be necessary, it can never be known with certainty (or apodictically) that the hypothesis is true of an actual state of affairs. In other words, Peirce, following to some extent Kant, rejected the Parmenidean assumption that logically true things and the properties of real things coincide. Having rejected this, Peirce gave a central place to non-deductive inference, what he called 'ampliative inference' (to emphasise its creative nature), involving not only induction, but also 'abduction', speculative

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[24] Bergson, *Creative Evolution,* p.210.
[25] Bergson, *Creative Evolution,* p.210.
[26] Bergson, *Creative Evolution,* p.210.
[27] Bergson, *Creative Evolution,* p.369f.

[28] Henri Bergson, *The Creative Mind*, Totowa: Littlefield, Adams and Co., 1975, p.74.
[29] Peirce, 'The Nature of Mathematics, in *Philosophical Writings of Peirce,* ed. Justus Buchler, New York: Dover, 1955, p.137 & 140.
[30] Charles Sanders Peirce, 'The Nature of Mathematics', p.137.
[31] Charles Sanders Peirce, 'On the Algebra of Logic' in *The Essential Peirce, Vol.1,* p.227.
[32] Peirce, 'The Nature of Mathematics, p.136.

thinking, in the logic of inquiry.[33] That is, he gave a place to creative imagination in the development of knowledge. He defended the use of analogy as well as hypotheses in such inference and justified the use of non-mathematical terms which cannot be precisely defined to characterized reality. These are the objective or real 'vagues'.[34]

Since the world quite obviously has evolved enormous variety, Peirce rejected Pythagoreanism. He argued that the fact that aspects of the world can be understood through mathematically expressed laws should not be merely accepted but must be accounted for – as the result of evolution from a chaotic world. Peirce defended a form of objective idealism, suggesting that nature is originally mind; or, as he put it 'a chaos of unpersonalized feeling.'[35] As mind it was prone to develop habits of action (equivalent to the 'limiting' of Anaximander). Matter is the outcome of the mind developing inveterate habits (becoming more limited in its freedom) and is thus nothing but effete mind.[36] As nature developed habits it also became possible to interpret these habits as signs, and on this basis, make predictions. Peirce characterized the sign most generally as 'anything which is so determined by something else, called its Object, and so determines an effect upon a person, which effect I call its Interpretant, that the latter is thereby mediately determined by the former.' He quickly corrected this definition, however, writing to Lady Welby, 'My insertion of "upon a person" is a sop to Cerebrus, because I despair of making my own broader conception understood.'[37] For Peirce, signs and their interpretation pervade nature. The universe, he wrote, 'is perfused with signs, if it is not composed exclusively of signs.'[38] However, this too is misleading. For there to be semiosis, there must first be 'chance' and second 'Brute reaction' without which semiosis would have nothing on

which to operate.[39] Semiosis as the production and interpretation of signs, has evolved from a world of chance and brute reaction. As Peirce conceived it, semiosis is triadic, and thus lends itself to forming sequences and networks of semiosis, with interpretants becoming increasingly complex. Matter, the outcome of mind developing inveterate habits, although always involving some element of chance and associated with the development of ever more complex and creative forms of semiosis, can now be interpreted through mathematically expressible law (although even here actuality will involve an element of chance and so will not entirely conform to the necessity of mathematical deductions). This interpretation of matter through mathematics is simply a further development of semiosis, made possible by the tendency of nature to develop habits, but as such, is subordinate to the interpretation of all this through Peirce's characterization of nature, matter and semiosis. Peirce only sketched his cosmology on this basis, proclaiming that 'Before this can be accepted it must show itself capable of explaining the tridimensionality of space, the laws of motion, and the general characteristics of the universe, with mathematical clearness and precision; for no less should be demanded of every philosophy.'[40] The clarity of mathematics was still exalted, Pythagoreanism was accorded appropriate recognition, but it was subordinated to a form of process philosophy.

WHITEHEAD'S PROPOSED SOLUTION

It is more difficult to interpret Whitehead's proposed solution to this problem than Bergson's or Peirce's. Whitehead was more fundamentally influenced by the Pythagorean tradition, and the speculative turn in his philosophy in which he embraced the central place Peirce had allotted to 'ampliative inference' associated with abduction, which Whitehead referred to as 'speculative thought', came relatively late in his career. To some extent, Whitehead was responding to Bergson, and indirectly to Peirce (who influenced both James and Dewey while contributing to the development of symbolic logic). Whitehead was attempting to develop a form of process philosophy that could not be charged with anti-intellectualism. As he wrote in the 'Preface' to *Process and Reality,* 'I am also greatly indebted to Bergson, William James, and

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[33] Charles Sanders Peirce, 'The Doctrine of Necessity Examined' in *The Essential Peirce: Selected Philosophical Writings, Volume 1 (1867-1893),* Bloomington: Indiana University Press, 1992, p.300.

[34] See Peirce, 'Issues of Pragmatism' in *The Essential Peirce: Selected Philosophical Writings, Vol.2 (1893-1913)* ed. The Peirce Edition Project, Bloomington: Indiana University Press, 1998, p.350f.

[35] Peirce, 'The Architecture of Theories' in *The Essential Peirce, Vol.1,* p.297.

[36] Peirce, 'The Law of Mind', in *Philosophical Writings of Peirce, Vol.1,* p.136, and 'Man's Glassy Essence' in *The Essential Peirce, Vol.1,* p.348f.

[37] Peirce, 'Excerpts from letters to Lady Welby', *The Essential Peirce, Vol.2 (1893-1913),* p.478.

[38] Charles Sanders Peirce, *Collected Works,* Cambridge, Mass.: Harvard University Press, 1931-58, 5.448n.

[39] Peirce, 'Reasoning and the Logic of Things, Lecture 8,' *Collected Works,* 6.202.

[40] Peirce, 'The Architecture of Theories' in *The Essential Peirce, Vol.1,* p.296.

John Dewey. One of my preoccupations has been to rescue their type of thought from the charge of anti-intellectualism, which rightly or wrongly has been associated with it.'[41] This concern to avoid anti-intellectualism was partly responsible for a more favourable attitude towards the Pythagorean tradition, and his alliance to the Pythagoreans was affirmed in what was one of his last publications, 'Mathematics and the Good' which he contributed to the Library of Living Philosophers tradition devoted to his work: *The Philosophy of Alfred North Whitehead*.[42] Even more than Peirce's, his philosophy manifests the quest to both affirm the achievements of the Pythagoreans while going beyond it to give a central place to creative becoming. His proposed solution works at several levels.

To begin with, he treated mathematics as abstraction. Whitehead did not denigrate abstraction. He praised it and what has been achieved by it, particularly the abstractions of mathematics; but he decried the tendency to take abstractions as concrete reality, ignoring the level of abstraction involved and ignoring the background concrete unity of experience. This is the 'fallacy of misplaced concreteness', a fallacy which characterises the world-view of scientific materialists who take colourless atoms moving in space to be real and dismiss as unreal the sensible world around us. To highlight the limitations of the abstractions of the scientific materialists, in which matter is construed in a way that is amenable to mathematical treatment, Whitehead pointed out that this is incapable of evolution; but this does not mean that there is no evolution. It means that the abstractions of the scientific materialists are too limited to grasp this evolution. Whitehead analysed what is involved in abstracting. He argued that abstractions must always be understood in relation to the broader context from which they are made, ultimately, the broader context of the entire universe. Most importantly, Whitehead strove to develop more adequate abstractions. He attempted to elaborate a cosmology which gave a place to beautiful sunsets and provided an account of creative evolution while simultaneously showing how abstraction is both possible and an essential feature of life, and how there are beings who can both appreciate the beauty of a sunset while being capable of making and understanding the

abstractions of mathematics. He did not do each of these sequentially, however; all Whitehead's ideas on mathematics, abstraction, the nature of the cosmos of life and human life presupposed each other and were developed together. Still, it is necessary to begin somewhere, and the best starting point is Whitehead's characterization abstraction, and particularly, abstraction associated with mathematics.

According to Whitehead, 'The growth of consciousness is the uprise of abstractions. It is the growth of emphasis. The totality is characterised by a selection from its details. … Thus a fortunate use of abstractions is of the essence of upward evolution.'[43] When we abstract, 'we necessarily introduce the notion of potentiality' since we are considering what might be apart from what might be together.[44] The highest form of abstraction grasps the 'eternal objects', the pure possibilities or 'forms of definiteness', such as colours, numbers, relations and patterns. While Whitehead's conception of these is equivalent in some ways to Platonic forms, Whitehead distanced himself from Platonism by firstly abandoning the notion of a 'realm' of eternal objects (replacing 'realm' with 'multiplicity' – a problematic term for Whitehead since he denied it referred to any reality) and then in his last major work, *Modes of Thought,* he apparently abandoned the notion of 'eternal objects' and replaced it with equivalents such as 'potentialities of definiteness'.[45]

As a process of thought, abstraction is a disjoining of entities which are in fact joined, and an abstraction is an entity considered apart from some of the roles it has in the actual world. If abstraction is not to destroy its 'massive basis for survival', it must not deny 'a preservative instinct aiming at the renewal of connection, which is the reverse of abstraction.'[46] This preservative instinct is characterized by 'the sense of realities behind abstractions' in which 'the sense of process is always present': 'There is the process of abstraction arising from the concrete totality of value experience, and this process points back to its origin.'[47] Whitehead was concerned to emphasise the importance of this background experience. He noted in 'Mathematics and the Good':

[41] Alfred North Whitehead, *Process and Reality,* Corrected Edition, New York: The Free Press, 1978, p.xii.

[42] Alfred North Whitehead, 'Mathematics and the Good', *The Philosophy of Alfred North Whitehead, The library of Living Philosophers Volume III,* 2nd ed. ed. Paul Arthur Schilpp, La Salle: Open Court, 1951, pp.666-681.

[43] Alfred North Whitehead, *Modes of Thought,* [1938] New York: The Free Press, 1968, p.123.

[44] Whitehead, *Modes of Thought,* p.99.

[45] See William Christian, *An Interpretation of Whitehead's Metaphysics,* Westport: Greenwood Press, 1959, p.195ff.

[46] Whitehead, *Modes of Thought,* p.123f.

[47] Whitehead, *Modes of Thought,* p.124.

The notion of the complete self-sufficiency of any item of finite knowledge is the fundamental error of dogmatism. Every such item derives its truth, and its very meaning, from its unanalysed relevance to the background which is the unbounded Universe. Not even the simplest notion of arithmetic escapes this inescapable condition for existence.[48]

Consciousness, however, does not rest content with the 'dumb sense of importance behind the veil', as Whitehead characterized this unanalysed background. It seeks the essential connections 'within the apparent isolation of abstracted details.'[49]

The development of mathematics and its application in science is part of the effort to go beyond mere abstractions to grasp these connections. Whitehead concurred with Bergson in holding that mathematics can never fully grasp concrete reality, but believed that more could be achieved through mathematics than Bergson had allowed. Mathematics, Whitehead argued, is the study of objective eternal objects: forms, relations and most importantly, patterns. It is only in special branches of mathematics that quantity and number are the dominant themes, and quantity by itself only provides a very crude understanding of nature. '[B]eyond all questions of quantity', Whitehead proclaimed, 'there lie questions of pattern, which are essential for the understanding of nature.'[50] Mathematics is a 'general science' for the investigation of 'patterns of connectedness, in abstraction from the particular relata and the particular modes of connection'.[51] It is 'the most powerful technique for the understanding of pattern, and for the relationships between patterns.'[52] Pattern, for Whitehead, 'involves a concept of different modes of togetherness'.[53] Following his criticism of the idea that abstractions could ever completely abstract from any possible context, Whitehead argued against the idea that mathematics deals in tautologies. When we say 'twice three is six', we are not saying that these two sides of the equation mean the same thing, but that two threes is a fluent process which become six as the completed fact. Even the statement 'six equals six' need not be a mere tautology; it could mean that six is a specific form of combination which issues in six as a datum for further process. So, for Whitehead, 'mathematics is concerned with certain forms of process issuing into forms which are components of further process.'[54] The alternative, he complained 'is the reduction of the universe to a barren tautological absolute, with a dream of life and motion.'[55]

While mathematics can illuminate patterns, including temporal patterns as modes of togetherness, it is still one-sided. There are aspects of reality that can never be grasped thought mathematics. As Whitehead complained of science (understood as the effort to understand the world through mathematics):

Science can find no individual enjoyment in nature: Science can find no aim in nature: Science can find no creativity in nature; it finds mere rules of succession. … The reason for this blindness of physical science lies in the fact that such science only deals with half the evidence provided by human experience. It divides the seamless coat – or, to change the metaphor into a happier form, it examines the coat, which is superficial, and neglects the body which is fundamental.'[56]

Pythagorean science must be complemented by other perspectives on reality: art, ethics, logic, religion, myths and philosophy.

If there is one perspective that does aim at complete comprehensiveness it is philosophy. The goal of philosophy is to construct schemes that can interpret all items of experience. It aims at 'full comprehensiveness'. One of Whitehead's major legacies was to have elaborated a comprehensive metaphysics which did aim to provide interpretations of all items of experience. This metaphysical theory is complex. It was continually evolving through Whitehead's work and has been interpreted in different ways by different commentators. So, it cannot be described here except in the most schematic way.

For Whitehead, the ultimate existents (actual entities) are actual occasions, processes of concrescence which integrate or 'ingress' (that is, make ingredients) through the act of prehending the forms of definiteness of (or eternal objects realized in) past actual occasions (dative and subjective ingressions) and eternal objects as such (conceptual ingression). The most basic eternal objects are

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[48] Alfred North Whitehead, 'Mathematics and the Good', *The Philosophy of Alfred North Whitehead,* ed. Paul Arthur Schilpp, 2nd ed. La Salle: Open Court, 1951 p.670.

[49] Whitehead, *Modes of Thought,* p.124.

[50] Whitehead, *Modes of Thought,* p.143.

[51] Whitehead, *Adventures of Ideas,* p.258ff & 196ff.

[52] Alfred North Whitehead, 'Mathematics and the Good', *The Philosophy of Alfred North Whitehead,* ed. Paul Arthur Schilpp, 2nd ed. La Salle: Open Court, 1951 p.678.

[53] Whitehead, *Modes of Thought,* p.143.

[54] Whitehead, *Modes of Thought,* p.92.

[55] Whitehead, *Modes of Thought,* p.93.

[56] Whitehead, *Modes of Thought,* p.154.

contrasts, which are more basic than pattern, but these are the precondition for more complex forms of definiteness, including number, relations and patterns. Having postulated a plurality of actual entities which are by their very nature related to other actual entities, entities which come into being and perish, Whitehead built into the very foundations of his cosmology unity, multiplicity and creativity, his 'categories of the ultimate' reflecting his commitment to giving a place to these: 'creativity', 'many' and 'one'. Actual occasions form a nexus when they prehend each other. A nexus forms a society when:

> (i) there is a common element of form illustrated in the definiteness of each of its included actual entities, and (ii) this common element of form arises in each member of the nexus by reason of the conditions imposed upon it by its prehensions of some other members of the nexus, and (iii) these prehensions impose that condition of reproduction by reason of their inclusion of positive feelings of the common form.[57]

The common form (or 'complex eternal object') is the defining characteristic of that society. When the genetic relatedness of actual occasions orders these serially, the society is a 'personal order'.

This cosmology provides a place for the 'eternal objects' of mathematics, most importantly patterns, while allowing that there are other kinds of eternal objects that are also ingredients in actual entities, and gives a place to decision and real creative emergence. Insofar as actual occasions prehend a common form of definiteness they create an order that can be comprehended through mathematics. Serial ordering requires an appreciation of mathematics understood dynamically. An eternal object chosen can be a possibility never before actualized. So, there can be real creativity in the universe. Some of this creativity will involve prehending new patterns, which can then be described mathematically after they have come into being.

EVALUATING WHITEHEAD'S SOLUTION

How successful are these proposed solutions? To begin with, we need to consider Whitehead's proposed solution, since to some extent this draws on ideas developed by Peirce and Bergson and as such is an effort to go beyond them, and Whitehead's proposal is by far the most fully worked out. Evaluating Whitehead's ideas in this

regard on the basis of my very schematic account of his ideas is problematic to say the least. Despite this, I think that something useful can be said on this basis, and in fact, I will court caricature by attempting to illuminate Whitehead's philosophy by showing its applicability in practice. I will do this in order to suggest limitations to Whitehead's philosophy. Rather than evaluating the ideas of Peirce and Bergson independently, I will then show how they need to be taken more seriously in order to overcome limitations in Whitehead's philosophy.

Supposing we consider the addition of five and six to make eleven. We could consider this in relation to five people getting together with six other people to form a cricket team. In terms of Whitehead's metaphysics, this can be characterized in terms of the prehensions of high grade actual occasions within each of these individuals, themselves functioning the way they do because they are within an immense number of personal societies of actual occasions which make up the bodies of the individuals and their natural and social environments. These high grade actual occasions (or rather, personal societies of them) then form themselves through the common ingression of this form of definiteness (along with a range of other eternal objects, physical, subjective and conceptual, some mathematical, some not) into the 'pattern of togetherness' of a cricket team. The addition of five and six and the interpretation of this event through this summation would then provide a one-sided but within limits a valid and important description of the event. As eleven members they are then able to ingress more complex forms of definiteness, for instance, associated with fielding. Such a pattern is not merely eleven players in a spatial arrangement but also involves complex interactions with each other and the opposing team. It is possible that in the effort to win, entirely new patterns might be envisaged and implemented, that is, ingressed, patterns which have always been possibilities (i.e. eternal objects) but have never before been actualized, and these could require new mathematical descriptions.

While this example exemplifies Whitehead's characterization of how eternal objects come to ingress in personal societies of actual occasions which are then mathematically describable, it also raises questions about the applicability of Whitehead's proposals to other cases. Suppose we consider the much more complex mathematics of catastrophe theory, a development of differential geometry. This was developed by René Thom and was largely inspired by a biologist strongly influenced by Whitehead, C.H. Waddington, to characterize epigenesis, the differentiation of cells

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[57] Whitehead, *Process and Reality*, p.34.

and generation of form in the development of organisms. Under the influence of Whitehead, Waddington had characterized this development as a form of concrescence which involves self-stabilizing paths of development which, at times became unstable, leading to situations where different paths could be and sometimes are taken. Thom showed how these paths, their stability and instability, and sudden changes of path could be modelled mathematically. Apart from illicitly extending the notion of concrescence from actual occasions to personal societies of actual occasions (that is, illicitly from the perspective of Whitehead's categories), the problem here is the nature of the conceptual prehensions of the actual occasions of these personal societies. Can we really believe that high grade actual occasions within the nuclei of cells are able to prehend and choose to ingress the complex mathematical patterns (including their operations) described by catastrophe theory? Could we account for cancerous growths destroying this whole process through Whitehead's concepts? How would we characterize a stray piece of shrapnel knocking out the developing brain of an Iraqi child through conceptual prehensions?

The difficulty in conceiving the actualization of mathematical forms through their ingression by personal societies of actual occasions can be highlighted by considering the higher grade actual occasions associated with human consciousness. Suppose we consider the efforts of Brian Arthur to use the mathematics associated with complexity theory (involving the use of non-linear equations) to characterize economic processes. Arthur's primary concern has been with 'increasing returns' generated by positive feedback loops which augment the dominance of products, actors and regions within the economy (although implicitly increasing returns imply there are concomitant self-reinforcing patterns of decreasing returns). Arthur's work destroyed the assumption of mainstream economists that free markets will lead to an equilibrium with the optimum allocation of scarce resources to satisfying unlimited wants. While Arthur considered products and individual actors, his original interest was in regions, to account for the observations of Jane Jacobs on how cities took off to generate increasingly complex patterns of economic activity.[58] The point of Jacobs' analysis was that such take off was and is not entirely predictable and is not the result of design by any of the actors, but a by-product of individual decisions by vast numbers of actors with no thought for the economy of the city as a whole. So, the process studied by Arthur could hardly be characterized entirely as the prehension of the patterns of togetherness studied by complexity theorists by high grade actual occasions, since a feature of these patterns is that they are the outcome of intentions of people who do not intend, and in fact are not even aware of, the patterns they are actualizing (although later actors might take cognizance in their decisions of the patterns that had emerged). If this is the case with many patterns generated by humans, it is likely to be even more common in the pre- and proto-biotic world.

Of course economists can influence governments of cities and countries, and in this way the patterns of complexity theory could be ingressed by the conceptual prehensions of high grade actual occasions within their governors and the economists who advise them. In fact 'protectionists', that is politicians who promoted tariff and other barriers to trade in order to develop national economies, could be said to have had some sense of the 'patterns of togetherness' which would generate the self-generating complexity studied by economists such as Arthur. In these cases we could perhaps think of the high grade actual occasions within them prehending these patterns before the nature of these patterns were clarified by mathematicians. However, such politicians pre-existed complexity theory. Present day politicians are much more likely to be guided in their economic policy decisions by mainstream neo-classical economists with no knowledge of or interest in Arthur's work. In the case of Australia, the consequent removal of tariff barriers and other means to promote a national economy has led to a downward spiral of deindustrialisation, trade deficits, asset inflation and massive increases in net national and personal debt along with huge and still growing disparities of wealth and income, a downward spiral disguised by treating revenue generated by selling off public assets and natural and accumulated social resources as income. As in Argentina which followed similar economic policies, the eventual outcome of this is likely to be chaos. In this, the 'pattern of togetherness' of a downward spiral of the economy is a by-product of the conceptual prehensions of the economy within the high grade actual occasions of economists and politicians intending, through their policies, patterns quite different from the patterns which are likely to unfold in the near future. There have been even higher grade actual occasions within radical economists and process philosophers such as

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[58] See 'Positive Feedback in the Economy' in W. Brian Arthur, *Increasing Returns and Path Dependence in the Economy,* Ann Arbor: The University of Michigan Press, 1994, chap.1.

myself promoting different public policies, policies which would preserve the national economy and its natural environment, but these have had no influence on policies enacted. Which patterns processes prehend and attempt to realize and which patterns are sidelined can only be understood in terms of the power relations and power struggles between component processes prehending different patterns, something to which Whitehead paid little attention.

Despite some illumination of the relationship between mathematics and actuality, it seems that Whitehead's philosophy is inadequate to all items of experience.

AUGMENTING WHITEHEAD

As noted, Whitehead never thought of his work as completed; his philosophy was continually evolving. To some extent he was trying to go beyond Bergson and the pragmatists influenced by Peirce, but he was more influenced by James than either of these two philosophers, and did not incorporate into his own work some of Bergson's and Peirce's most important insights. To some extent this was deliberate; Whitehead was concerned to show that the analytical intellect could grasp more of the reality of becoming than Bergson believed; in other cases Whitehead has not been aware of or ignored ideas developed by these thinkers. Furthermore, there were other thinkers in the tradition of process philosophy whose insights Whitehead was barely aware of. Briefly, I will suggest some ways in which Whitehead's philosophy could be augmented by appropriating some of the insights of other process philosophers, particularly of Bergson and Peirce, to make more plausible his analysis of the achievements and limitations of mathematics. In the light of these ideas, I believe Whitehead's notion of 'concrescence' and 'prehension' should be revised. The notion of 'eternal object' should also be revised, or possibly abandoned (as possibly Whitehead did abandon it).[59]

To begin with, a general point should be made in relation to Whitehead's metaphysics. One of the features of the whole tradition of process thought, from Anaximander onwards (including Peirce, and to a lesser extent Bergson), has been the view that order in the world has in some sense emerged from a background of disorder, flux or chaos. Anaximander characterized the cosmos as

developing through the limiting of the unlimited, and emphasised the precarious nature of what emerged in this way, characterizing its existence as an 'injustice' that eventually would have to be paid for. Even the Pythagoreans accepted the dichotomy between the limited and the unlimited. Heraclitus, to some extent defending Anaximander against later philosophers, characterized the cosmos as in perpetual motion and emphasised the central place within it of strife and conflict. It is only through a balance between opposites that the existence of anything is maintained, and nothing is permanent except this principle, Heraclitus claimed. As noted, Peirce also assumed that necessity in the world arose from chaos and chance through limitation. Recently, it has been argued in process physics that it is necessary to postulate an 'intrinsic randomness' or 'self-referential noise' to generate a self-organising relational information system, sufficiently rich that self-referencing is possible.[60] Whitehead in his concern to avoid the charge of anti-intellectualism developed a cosmology which gave only a derivative and thereby relatively small place to disorder and conflict.[61] The problematic nature of this becomes evident when efforts are made to interpret reality through his metaphysics, as I have attempted here. This limitation, I suggest, should be borne in mind as relevant to all other limitations of Whitehead's metaphysics and efforts to overcome these.

Bergson's most important insights pertain to the nature of duration. The order associated with processes themselves is essentially a durational order in which extension is a subordinate aspect, Bergson argued. Physical time is not infinitely divisible. To some extent Whitehead incorporated this insight into his philosophy, but not all aspects of it. Bergson argued for a 'pulsational' theory of time, not an atomic theory of time which surreptitiously reintroduces instants at the beginning and end of each temporal atom.[62] Secondly, Bergson argued that not only does nature consist of processes which are essentially durational, but that there are different minimum durations, with longer durational processes being made up of overlapping shorter durational processes.[63] As in the relationship of a melody to

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[59] On this, see Code, 'On Whitehead's Almost Comprehensive Naturalism', p.25ff.

[60] See Reginald T. Cahill and Valerie V. Dvoeglazov, *Process Physics: From Informational Theory to Quantum Space and Matter,* Hauppauge NY: Nova Science Publications, 2005.

[61] See Whitehead, *Process and Reality,* p.91.

[62] On this, see Milič Čapek, *Bergson and Modern Physics,* New York: Reidel, p.205.

[63] See Čapek, *Bergson and Modern Physics,* p.159. See also Pete A. Y. Gunther, 'Bergson, Mathematics, and Creativity', *Process Studies,* 28:3-4, Fall-Winter 1999, 271ff.

the notes which make it up, supervening longer durational processes should be seen as genuinely emergent, having characteristics and dynamics and an order not entirely explicable in terms of their originating conditions and the shorter durational processes of which they are composed. Bergson could allow for a great variety of interacting processes on this basis, including hierarchical order.

Such ideas have been recently put forward again by hierarchy theorists, notably by Howard Pattee, Timothy Allen and Stanley Salthe, among others, who have argued that emergence is associated with new constraints emerging which are not in the initial conditions.[64] While developed without reference to pre-twentieth century thought (or to Bergson), this conception of nature revives Anaximander's conception of cosmos as having formed through the limiting of the unlimited (an idea also taken up further developed by Schelling at the end of the eighteenth century).[65] Along with the notion of different minimum durations, or different process rates, this has enabled Pattee, Allen and Salthe to clarify the nature of both emergence and hierarchical ordering in nature. Treating time as pulsational rather than atomic and treating causation as essentially a matter of constraining,[66] overcomes a number of difficulties in Whitehead's philosophy, but then requires a rethinking of the nature of concrescence.

On the basis of this pulsational, multi-levelled view of time and conception of causation as constraining, I have suggested elsewhere that actual occasions and societies of actual occasions be treated as ontologically on a par, without any need to explain entirely societies of actual occasions though concrescence of their component actual occasions.[67] If the different ontological status granted to actual occasions and societies of actual occasions is abandoned, if we allow that societies of actual occasions are also actual entities, then in terms of Whitehead's philosophy we should identify actual entities with 'organisms'. Alternatively, following von Bertalanffy we could speak of 'systems', which is very close to the notion of 'organism' without the connotations of referring only to biological entities. The problem with both these notions is that they presuppose too much coherence in actual entities to deal with marginal cases. They do not give a place to chaos and actual entities close to chaos. Even Dorothy Emmet's notion of 'things-in-process' implies too much coherence or definiteness.[68] Can a language or a culture be characterized as an 'organism' a 'system' or a 'thing-in-process'? To acknowledge both the dependence of actual entities on their environments and components while at the same time their partial autonomy from the conditions of their emergence (that is, as immanent causes of their own becoming), I suggest that we speak merely of 'patterns-in-process' or simply 'processes' which in their most elementary form can be nothing more than enduring centres of action (or patterns of such centres), centres which are not merely the effects of other processes. The notion of organisms or systems should then be reserved for more organized processes.

It is then necessary to see all processes as in relation not only to past processes and to future possibilities, but also in relation to co-becoming processes which support or facilitate, threaten or hinder their becoming. This means that it is necessary to accord different powers to processes to maintain themselves in existence and flourish, and to augment or to undermine and even destroy other processes (as Nietzsche, following Heraclitus and Boscovich, had argued, and which Aleksandr Bogdanov made central to his 'tektology', the general theory of organization which was the precursor to systems theory). Some interactions between processes might result in destruction or radical modification of the processes involved, while in other cases the interaction might generate enduring patterns of interaction the outcome of which is the coming into being of emergent processes with their own autonomous dynamics which then alter by constraining the processes which gave rise to them. In such cases it might be necessary to allow for the possibility of ongoing communication between these interacting processes. In these cases, even where there is emergence of new processes, some degree of contingency and even chaos is added to the world. Along with such contingency there is a kind of

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[64] For a short account of hierarchy theory, see Valerie Ahl & T.F.H. Allen, *Hierarchy Theory: A Vision, Vocabulary, and Epistemology,* New York: Columbia University Press, 1996. For more comprehensive studies, see T.F.H. Allen and Thomas B. Starr, *Hierarchy: Perspectives for Ecological Complexity,* Chicago and London: The University of Chicago Press, 1982, and Stanley N. Salthe, *Evolving Hierarchical Systems,* New York: Columbia University Press, 1985.

[65] F.W.J. Schelling, *First Outline of a System of the Philosophy of Nature,* [1799] trans. Keith R. Peterson, N.Y.: State University of New York Press, 2004, p.17ff.

[66] For a more developed defence of the notion of causation as constraining, see Alicia Juarrero, *Dynamics in Action: Intentional Behaviour as a Complex System,* Cambridge, Mass. MIT Press, 1999, Chap. 9.

[67] Arran Gare, 'Process Philosophy and the Emergent Theory of Mind: Whitehead, Lloyd Morgan and Schelling', *Concrescence: The Australasian Journal of Process Thought,* 3, 2002.

[68] Dorothy Emmet, *The Passage of Nature,* Houndmills: Macmillan, 1992, chap.5.

order produced in the world associated with the relationships of power between processes which is derivative from and can only be understood through and in relationship to processes without this being prehended by the interacting processes producing this order. Spatial form, for instance, as the order of potential for causal independence and interaction between processes could be generated (at least initially) by processes without this being prehended as such by them. The form of an unplanned city might be generated in this way, as might the form of a crystal. That is, larger patterns can and frequently do emerge of which component processes are not only blind, but do not have any feeling for whatsoever, at least initially.

Peirce's and Whitehead's metaphysics, although developed independently, have much in common, and the limitations of each are illuminated by the other, while each provides resources for the other to overcome these limitations.[69] The aspect of Peirce's philosophy which in my view is most important as a means to revise Whitehead's philosophy, particularly his notion of 'prehension', is his work on semiotics, particularly as this came to be appreciated after Peirce's metaphysical writings came to be taken more seriously.[70] Central to all semiosis is that it involves interpretation which could be mistaken and involves hypothetical thinking or 'abduction'. Semiosis is triadic (a sign, an object and an interpretant), allowing for the possibility of each interpretant itself becoming a sign in turn to be interpreted, possibly creatively, generating a new interpretant. This triadic relation makes possible endless semiosis, generating webs of semiosis and the development of more and more complex kinds of semiosis. It is important to appreciate that an interpretant in semiosis need not be a thought. It can be growth of a particular kind. In his study of 'phytosemiotics' (semiosis in plants), deploying ideas from von Uexküll and Peirce (or at least thinkers influenced by Peirce), Kalevi Kull characterized a vegetative sign system as 'the system that is responsible for the genesis of multicellular biological form, the whole morphology of the body.'[71] The biological form

taken by the organism is essentially an interpretant, the outcome of interpreting its internal (including DNA) and external environment. While being the dominant form of semiosis in plants, this vegetative semiosis occurs in all multicellular organisms. Beyond such vegetative semiosis is semiosis where the interpretant is moving or acting in a certain way. Only with highly developed forms of consciousness do we have semiosis independent of growth and action, and such semiosis presupposes the other forms of semiosis. Peirce's analysis of semiosis provides a better basis than Whitehead's metaphysics for understanding the creativity in nature, particular in relation to the primordial chaos of nature and to on-going interaction between co-existing processes.

If we are to speak of 'processes' as actual entities and allow some order which is generated without being in any sense 'prehended', if we allow that Peirce's notion of semiosis is superior in some ways to Whitehead's analysis of prehension, why should the notions of 'concresence' and 'prehension' be retained? The notion of concrescence assumes the validity of Bergson's claim that the pre-eminent order in the world is durational, while giving a place to diversity and the unification of this diversity in particular instances of becoming. It gives a place to proto-memory and proto-anticipation (perhaps involving proto-imagination) in the process of becoming and provides a basis for analysing both unity and diversity. It is important to retain this. The notion of prehension implies that causation is not merely a matter of something having an effect on something else, but that what is affected is actively responding to what is influencing it. This notion also should be retained. How then should the notions of 'concrescence' and 'prehension' be modified?

While 'limitation' (as self-limitation) has been given a place in concrescence by Whitehead to characterize the constraints of other actual occasions and decision between which possibilities to ingress,[72] the notion of 'process' outlined above suggests an even a greater place needs to be given to this concept. Limitation or constraint in concrescence of a process is the defining feature of its existence as an emergent entity. It is by virtue of self-constraining and constraining of its relationship to other processes that processes maintain themselves and their distinctive ordering; in fact, it is a major aspect of their ordering and facilitative of other ordering. The constraints imposed on and then embraced by individuals by language, for instance,

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[69] See for instance Murray Code, *Vagueness, Rationality, and the Lure of Logic,* New Jersey: Humanities Press, 1995.

[70] See Jesper Hoffmeyer, *Signs of Meaning in the Universe,* trans. Barbara J. Haveland, Bloomington: Indianan University Press, 1996. See also Torkild Leo Thellefsen, 'C.S. Peirce's Evolution Sign: an Analysis of Depth and Complexity within Peircean Sign Types and Peircean Evolution Theory', *S.E.E.D. (Semiotics, Evolution, Energy, and Development)* 1: 2 (December 2001), (no page numbers)

[71] Kalevi Kull, 'An Introduction to phytosemiotics: Semiotic botany and vegetative sign systems', *Sign System Studies,* 28 (2000), p.344.

[72] Whitehead, *Process and Reality,* p.164.

are central to the formation of humans as such and facilitate complex forms of communication and cooperation. In the case of compound individuals, such as human communities, the component processes are constrained and they become self-constraining, sustaining hierarchical ordering involving different process rates or 'durations'. It is in relation to this more complex notion of concrescence that the notion of prehension needs to be re-examined.

As noted, prehension is an aspect of the active response of processes to the influences upon it. If concrescence is more complex than Whitehead allowed, involving self-constraining, constraining of components and mutual constraining associated ongoing interaction (possibly communication) between processes, then prehension is also likely to be more complex. In particular, it is necessary to reconsider the place of 'forms of definiteness' in prehension. In many, if not most instances, concrescence does not involve prehensions of forms of definiteness, at least initially. Rather, definiteness emerges slowly through the concrescence. This is evident in the case of 'conceptual prehensions'. The architectural theorist Christopher Alexander has argued convincingly that it is precisely the effort by architects to specify (prehend) the final form of a building in all its detail that has generated the desolation of modern built-up environments. If buildings are to be produced which are alive and beautiful, it is necessary that these unfold through myriad small decisions by all those involved in the building, each of whom has developed a 'feel for the whole' so that the final form, grasped only vaguely to begin with, emerges through a series of structure preserving transformations.[73] And it is such structure preserving transformations that characterize all life, Alexander has argued. Prehensions of preceding or co-existent processes can involve grasping the definiteness of objects (as for instance of the bricks from which the building is constructed) but in other cases, for instance the broader natural environment and the processes unfolding within it, or efforts to develop a new idea or new way of thinking, what is involved is a much less focused feel for these with much less definiteness. It involves what Michael Polanyi characterized as the tacit knowledge of

'indwelling'[74] which might not be able to be made more definite because reality itself might be indefinite.

It is in rethinking prehension in the light of such complexity that Peirce's analysis of semiosis becomes relevant. While there are forms of prehension that are dyadic rather than triadic, and do not involve any kind of interpretation are in no sense forms of semiosis (in response to chance events and 'brute fact', for instance), many prehensions, particularly those associated with life, do involve interpretation and in fact are forms of semiosis as characterized by Peirce. Incorporating Peirce's analysis of semiosis (and the work of the semioticians influenced by Peirce) provides the basis for a much richer analysis of prehension than that offered by Whitehead (and, although I do not want to go into this here, the basis for overcoming problematic aspects of Whitehead's analysis of the relationship between present actual occasions and the past). The incorporation of hierarchy theory and semiosis into Whitehead's metaphysics can also be the basis for relating each of these to each other, and for clarifying the nature of each. Semiosis requires hierarchical order of processes characterized by different process rates to enable signs to endure with some stability relative to other processes for interpretation to be possible, and hierarchical order can be fruitfully analysed through the study of semiosis.[75] Allowing hierarchical ordering with different kinds of semiosis also allows for a better understanding of the nature of embodiment and what it means to be embodied.

RETHINKING THE PLACE OF MATHEMATICS IN A CREATIVE COSMOS

What difference does this revision of Whitehead's core concepts have for the way mathematics and its relation to the world is understood? As we have seen, there are two sides to Whitehead's answer to this. The first is associated with his conception of mathematics as abstraction, treating as disjoint what is in reality conjoined, and the second is associated with Whitehead's metaphysics according to which mathematical forms are instantiated though the conceptual prehensions of actual occasions. For Whitehead, these are consistent since the conceptual prehensions of mathematical forms in concrescence never occur in isolation but are

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[73] Christopher Alexander, *The Nature of Order: An Essay on the Art of Building and The Nature of the Universe: Book Two: The Process of Creating Life,* Berkeley: The Center for Environmental Structure, 2002, chap.3

[74] Michael Polanyi, 'Tacit Knowing', *Knowing and Being,* ed. Marjorie Grene, Chicago: Uni. of Chicago Press, 1969, Part Three.
[75] As Hoffmeyer has shown. See Hoffmeyer, *Signs of Meaning in the Universe,* chap.3.

conjoined with other prehensions to produce the rich, sensible world in which we find ourselves. What I have written above pertains mainly to Whitehead's metaphysics, which appears more problematic, but it also has implications for epistemology and suggests limitations to Whitehead's characterization of mathematics as abstraction.

The work of Peirce is most important here. Peirce would concur with Whitehead that mathematics involves abstraction and could have appreciated both Whitehead's argument that abstraction is essential to the evolution of life, and his criticism of the tendency to take abstractions for reality.[76] He might have agreed with Whitehead's efforts to reduce the level of abstraction of mathematics and to treat mathematical operations as dynamic. But Peirce appears to have more emphatically given a place to the imaginative creativity involved in developing any kind of abstraction, including mathematical abstractions. A study of the history of mathematics shows that like scientific and metaphysical theories, mathematics too is developed through the creative explication of analogies and metaphors. Giving a place to analogy and metaphor, Peirce's philosophy offers a greater appreciation of the creative input of mathematicians in construing the world mathematically. Despite Bergson's critical attitude towards mathematical thinking, his analysis of how the intellect works, particularly as this has been elaborated by Milic Čapek, provides support for such a characterization of mathematics. His study highlights what is really the creative work that generates what appears to be the rational, coherent, transparent world construed through mathematics. What Bergson has revealed is the extent to which mathematical and logical thinking is dominated by spatial metaphors and the creativity involved in explicating such metaphors, an aspect of his work that has been clarified by Čapek.[77] (That mathematics is the explication of metaphor has been supported recently by George Lakoff and Rafael E. Nunez.)[78]

It is when such creative work is interpreted through metaphysics that the significance of this difference between Peirce and Whitehead becomes fully manifest. To appreciate the importance of this, it is first necessary to acknowledge the importance of other contributions to process philosophy. As I suggested, it is necessary to acknowledge more chaos and conflict in the world than Whitehead was wont to allow. Acknowledging these implies that the world is less amenable to full comprehension than Whitehead believed. In particular, much of what has been successfully comprehended by mathematicians cannot be accounted for as due to the ingression into reality through the conceptual prehensions of actual occasions. Much of this mathematical order can be seen to be the by-product interactions and conflicts between processes that could not have been prehended by them, and in some cases, are the by-product of the inadequacy of what prehensions have been made.

Does acknowledging that some kind of mathematical order emerges in the world without any kind of prehension of this, at least initially, lead back to some form of limited Pythagoreanism in which the mathematical order discovered by mathematicians in the world is seen to have been already there before it was discovered? In this case, mathematics itself must be seen to be a realm of eternal truths embodied in the physical world which is discovered by mathematicians. Such a view of the world is difficult to reconcile with Peirce's and Bergson's appreciation of the creativity involved in the development of mathematics. We seem to be back with the old conflict between realism and constructivism which both Peirce and Whitehead sought to overcome. It is here that the fruitfulness of conceiving of prehension as semiosis reveals itself.

Both Peirce and Whitehead had appreciated how developments in logic liberated thought to deal with relations and the significance of this liberation. It was then no longer necessary to conceive relations as binary relations of substances to their attributes. Whitehead saw how this could free philosophers from the tendency to treat colours as in a beautiful sunset as either an attribute of physical objects or as an attribute of the mind. Allowing triadic relations enables us to avoid either position and treat the beautiful sunset in relation to both the perceived object and the perceiver. Peirce's theory of semiosis generalizes this insight so that all experience is understood in terms of such triadic relations. The development of more adequate comprehension of the world is a matter of creatively interpreting signs of objects to produce more adequate signs which can be the point of departure for further creative attempts at comprehension. The interpretation of physical or vegetative semiosis or the semiosis of animal activity through the theoretical elaborations of scientists and philosophers is just a continuation of the endless semiosis that pervades much, if not all, the cosmos, a process in which signs which are

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[76] For Peirce on abstraction, see Charles Sanders Peirce, 'On a New List of Cateories' in *The Essential Peirce, Vol.1,* p.2.

[77] Čapek, *Bergson and Modern Physics,* p.180ff.

[78] See George Lakoff and Rafael E. Nunez, *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being,* New York: Perseus, 2001.

themselves interpretants, are continually being reinterpreted, generating new signs which are then also interpreted again. The later interpretant can not only interpret signs but achieve a comprehension of the limits of what had been achieved by these earlier interpretants. In the case of humans, semiosis can involve interpreting earlier signs as interpretants and evaluating their adequacy or inadequacy as interpretants, showing why later interpretants provide more adequate interpretation of its object. Thus, in the mathematical interpretation of primitive processes, processes of blind assertion by physical processes or processes of growth in vegetation, can involve interpretation of these processes in relation to a spatial order of potentialities which has been generated without having been in any way prehended or interpreted by these more primitive processes, revealing at the same time the limited prehension of the primitive processes that generated this order and new possibilities for action or growth. The development of such mathematical interpretants is then an addition to this process of endless semiosis, an addition which is able to map out potentialities of which the interpretants of more primitive processes were and are oblivious.

But for all its achievements there is no reason to think of mathematics as the highest form of semiosis. It is possible that signs that are re-interpreted are in some way more profound interpretants than subsequent interpretants, even if subsequent interpretants have revealed new possibilities or achieved greater reflexivity. This is evident when people from literate cultures interpret people from oral cultures. Interpretants (for example the work of anthropologists) might reveal aspects of life in oral cultures of which their participants were blind, but there is often an appreciation by anthropologists that they can never capture the richness of the symbolic life of these cultures. Similarly, while ethologists and biosemioticians have made enormous advances in interpreting the sign systems and worlds of animals of all kinds, they are not in a position to fully appreciate what it is like to experience the world as a bee or a bat. If semiosis in literate cultures can be superior, it is because it can facilitate far greater efforts to interpret other cultures and other forms of life, and so can develop more reflexive forms of semiosis. This has been associated with two tendencies (sometimes combined, as in the case of Aristotle), both responses to the debilitating effects of cultural relativism that tends to follow this reflexivity, one to find some absolute foundation which stands above every culture, the other to humility about whatever claims to knowledge are

being made and a continual quest to engage with different points of view, expose taken for granted assumptions to overcome limited perspectives and to develop more comprehensive views of the world, views which incorporate the achievements of previous and rival ways of interpreting the world. The first tendency has been associated with the celebration of mathematical and logical reasoning, claiming that it provides absolute knowledge. In response to such claims, Bergson (among others) attempted to uphold an alternative absolute accessible through intuition. This is still a deviant form of the first tendency. Whitehead and Peirce exemplify the second kind of response.

While Whitehead tried to develop a comprehensive view of the world which would give a place to mathematics and logic and transcend the limitations of all previous efforts to achieve a comprehensive world-view, Peirce's notion of endless semiosis provides a better basis for upholding this second response coherently. Synthesising this with Whitehead's metaphysics, mathematics can be seen as a major advance in developing the means to interpret the world. While failing in its efforts to achieve apodicticity, it has facilitated great achievements in abstract thinking. But looked at from a semiotic perspective, it has to be seen as the elaboration of one set of analogies or metaphors which could not form the basis of a comprehensive view of the world. As Bergson and (following him) Čapek have revealed, mathematics is the elaboration of a spatial metaphor, and this metaphor is severely limited when it comes to appreciating duration and creative becoming, making sentient life unintelligible. If a comprehensive view of the world is to be achieved, it has to be subordinated to some other metaphor. Peirce, Bergson and Whitehead each offered and elaborated alternative root metaphors which were broader than and encompassed mathematical thinking.

In his effort to uphold mathematics and its achievements while appreciating its limitations, Whitehead rejected the extreme abstraction which reduces all mathematics to tautologies. Mathematical operations themselves are privileged over statements of truth, and even then the insight provided by these operations is regarded as one sided, unable to do full justice to the richness of the world. This way of thinking about mathematics is now evident in complexity theory where theorists watch computer images generated by non-linear equations. However, Whitehead's characterization of this as reducing the level of abstraction, while doing much to vindicate the potential of mathematics to be reconceived so that it is consistent with process metaphysics, fails to fully

account for what Whitehead was doing. In fact Whitehead had embraced a process view of the world and was reconceiving mathematics in terms of this process view, not merely avoiding excessive abstraction. This process view was not merely paying attention to concrete experience. If one examines carefully Whitehead's thinking, particularly in *Science and the Modern World,* it can be seen that Whitehead was guided in his thinking, like Bergson, by reflecting on music. He was first of all using mind as a metaphor for understanding the whole of nature, but his view of mind was underpinned by using music to clarify the nature of duration within experience, sometimes using music directly as a metaphor to characterize physical processes.[79] Mind interpreted through music functioned as a root metaphor to creatively redescribe the ultimate nature of reality, thereby reinterpreting all past thinking about the world, ourselves and our place in the world. Mathematics was reinterpreted by him through this new composite metaphor. Whitehead's work is then a prime illustration of the chain of semiosis where ampliative thinking through the use of metaphors enables past signs to be interpreted (or to generate an interpretant) in a way which is radically new, in this case the past signs and the new interpretant being interpretations of the entire cosmos, including effort to understand it through mathematics.

Once it is appreciated that this is what Whitehead was doing, and the major insights of Anaximander, Heraclitus, Schelling, Nietzsche, Bogdanov, Bergson and Peirce have been assimilated to Whitehead's metaphysics, then we can also see that Whitehead needed to leave Pythagoreanism behind more definitively than he did. It is a bias of mathematical physicists to think that all explanations must map out a configuration space of all possibilities in terms of which what is actual can be specified, and then what possibilities will be actualized under different circumstances ascertained. As Stuart Kauffman has pointed out, this assumption was shared by Newton, Einstein and Bohr.[80] Whitehead's early postulation of a realm of eternal objects is perhaps an expression of this tacit assumption. But with the creativity made possible through semiosis, mapping out all possibilities that may exist in the future is not only impossible but meaningless. 'Eternal objects', as Murray Code has argued, themselves emerge and

evolve (and so should not be characterized as eternal objects).[81] As in the case of particular processes, the concrescence of the universe as a whole (and life on earth) is opening up, creating and clarifying new possibilities, a process in which vague ideals are being given more definite form. The most adequate way to grasp the development of semiosis from its beginnings until the present, giving a place to chaos, conflict and vagueness as well as to what can be sharply defined, projecting a vague future which will be influenced by the creative work of semiosis, is not through mathematics but, as Kauffman argued, through stories.[82]

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[79] See for instance Alfred North Whitehead, *Science and the Modern World,* Cambridge: Cambridge University Press, 1932, p.166.

[80] Stuart Kauffman, 'Emergence and Story: Beyond Newton, Einstein and Bohr?' in *Investigations,* Oxford: Oxford University Press, 2000, pp.119-139.

[81] Murray Code, 'On Whitehead's Almost Comprehensive Naturalism' 31.1, 2002, 3-31.

[82] For a further defence of Kauffman's argument in this regard, see Anton Markoš, 'In the quest of novelty: Kauffman's biosphere and Lotman's semiosphere' in *Sign Systems Studies* 32: ½, 2004, pp.309-327.