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RISK AND THE VALUE OF INFORMATION IN IRREVERSIBLE DECISIONS

ABSTRACT. The analysis of the nexus between the value of information and risk is examined for sequential decisions with different degrees of future commitment, as e.g. environmental decisions. We find that in the linear case a riskier environment in general will increase the value of information. This result will be extended in the separable case to decreasing and increasing stochastic returns to scale. An example shows the ambiguity in the general case.

KEY WORDS: Irreversibility, flexibility, value of information, risk.

1. INTRODUCTION

How much should a decision-maker, e.g. a government agency, invest in information acquisition prior to a decision? What is the connection of the information value with the underlying risk? These questions are especially important when decisions cannot easily be reversed since ex post inefficient choices may influence a long time period. This is of particular importance for environmental and real investment decisions.

The central concern of this paper is thus to investigate the relationship between risk and the value of information for irreversible decisions. Several authors (e.g. Gould, 1984; Hess, 1982; Murota, 1988) have analyzed the connection between the 'riskiness' of the returns of a project and the value of information in static decisions. We examine sequential decisions with different degrees of future commitment. Such decisions play an important role, especially in the context of the development of natural resources or real investment decisions if there are no well developed secondary markets. Moreover, many governmental policies require the utilization of environmental and land resources. Such policies are frequently implemented under a high degree of uncertainty and irreversibility.

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Consider, for example, public investments in its infrastructure, such as a new highway or a new building which require the development of a natural resource. The problem this poses is that the conversion is for all practical purposes irreversible. Moreover, at the initial stage, there may be uncertainty about the benefits of alternative uses of the environment. For instance, amenity services may be more appreciated in the future by the individuals than originally assumed. But also development can have risks. For instance, one may not know the extent to which the new highway will be used by the consumers. Thus, before deciding on that project, the public can examine how much should be invested in order to reduce the associated project risks. For instance, if such issues are decided in legislatures, committees are established or experts are consulted before a decision is taken. Then, it will be important to know if higher risk of the project justifies a higher investment in information acquisition.[1] In the example above, a greater variability of oil prices may lead to a higher risk in future demand for highway services. Does this imply that, before the project decision, more studies have to be done to investigate carefully the willingness to pay for potential highway users? This is the basic question studied in this paper.

Clearly, not all uncertainty can be removed because sometimes only the passage of time results in new information. Many authors have examined the optimal timing of actions if agents expect passively new information in the future about payoffs or availability of options (e.g. Arrow and Fisher, 1974; Bernanke, 1983; Jones and Ostroy, 1984; Henry, 1974; Freixas and Laffont, 1986; Pindyck, 1991; Viscusi, 1988). Ramani and Richard (1993) recently reexamined the relationship between irreversible decisions and anticipated information. In this paper, however, we analyze the decision to invest actively in information gathering at the beginning of the decision process.

We focus on the relationship between risk and the value of information in the conventional framework concerning irreversible and flexible decisions. The value of information is the natural measure how much one expects to gain from new information.

We derive a proposition which establishes a monotonical relationship between risk and the value of information for constant stochastic returns to scale. This result will be extended in the separable case to decreasing and increasing stochastic returns to scale. We also illustrate, however, the case and the reasons why the monotonical relationship can break down in the general nonlinear setting.

2. THE FRAMEWORK

We consider a decision problem with two periods. In the first period the decision maker chooses an initial action $x \in [0,1]$. The consequence is described by the payoff function:

$$g(x)$$

In the second period, there is again a choice. Let z be the control variable in the second period. The individual receives an additional payoff

$$f(z,\omega),$$

ω is an exogenous integer-valued random variable beyond the control of the decision-maker. We assume that $\omega \in \{1,\ldots,n\}$ with initial probabilities $\pi_1,\ldots,\pi_n$, i.e. the decision-maker's *a priori* judgment is described by the probability vector $\pi_1,\ldots,\pi_n$, denoted by π. The mean value of ω is denoted by $\overline{\omega}$. Typical realizations of the random variable are denoted by ω or j and the function f is expressed accordingly as $f(z,\omega)$ and $f(z,j)$.

An irreversible or environmental decision reduces for a long time the available options whereas a flexible decision leaves open future options. A more precise definition of different degrees of future commitment can be stated as follows (see Kreps, 1979, and Jones and Ostroy, 1984): an initial action x_1 is more flexible (less irreversible) than another x_2 if all future actions are available from x_1 at a 'cost' that does not exceed that of x_2. This definition leads to a partial ordering of initial actions. As discussed in the introduction, the choice of options with different degrees of irreversibility (or flexibility) is a component of a wide range of economic decisions, especially environmental decisions, widely discussed in the literature.

We define a most important flexibility (irreversibility) ordering on the initial actions. The available options in the second period are restricted by:

$$z \in [x,1]$$

Therefore, by our definition, x_1 is more flexible than x_2 if $x_1 \leqslant x_2$ whereas x_2 is more irreversible than x_1. Such flexibility or irreversibility structures frequently occur in decisions concerning development or preservation of natural resources (e.g. Hanemann, 1989). But many real investment decisions, too, are subject to the same irreversibility constraints if there are no well developed secondary markets.

The problem is economically relevant only when $g(x)$ is increasing with x, and $f(z, \omega)$ decreasing in z for some ω, so that there are opportunity costs if the decision-maker initially chooses a flexible position.

The information structure is assumed to be time dependent, that is, the realization of the stochastic variable becomes known before the choice in the second period has to be made. The basic question is how much the decision-maker gains if when he could resolve the uncertainty at the beginning of the decision.

We assume risk neutrality. This assumption is especially important for studying governmental decisions. Then, without any information gathering, the decision-maker faces the following maximization problem:

$$\text{(1)} \qquad \max_{x \in [0,1]} g(x) + \sum_{\omega=1}^{n} \pi_\omega \max_{z \in [x,1]} f(z, \omega).$$

We denote by Z^*_ω (depending on the realization of the state of the nature) and X^* the maximizers of the above decision problem, which are assumed to be unique throughout the paper for all relevant probability vectors π. Note that X^* and Z^*_ω depend on π.

We analyze the situation in which the decision-maker has an information source in the first period from which he could obtain perfect information about the true state of the nature before the project decision has to be taken. Of course, the comparison of perfect with no information is a rather extreme case and only made for tractability. However, the results are also applicable to partial information.

We proceed in the following way. We investigate in detail the (gross) value of information. Then the value of information must be compared to the information costs in order to decide whether or not to acquire information at the beginning of the decision process. The latter, obvious comparison is not discussed further in this paper.

Assuming that the decision-maker could obtain the true state ω, he would choose x and z to maximize:

$$(2) \qquad \max_{x\in[0,1]} g(x) + \max_{z\in[x,1]} f(z,\omega).$$

We denote by X°_{ω} and Z°_{ω} the maximizers of this second decision problem, which again are assumed to be unique.

Further, we assume that the functions $g(x)$ and $f(z,\omega)$ are continuous on [0,1] for given ω. The value of information $V(\pi)$ (e.g. Gould, 1974) or the value of inquiry in the terminology of Marschak and Radner (1972) is defined as a function V: $R^n \to R$ for all admissible probability vectors π:

$$(3) \qquad V(\pi) = \sum \pi_\omega\{g(X^{\circ}_{\omega}) + f(Z^{\circ}_{\omega},\omega) - [g(X^*(\pi)) + f(Z^*_{\omega}(\pi),\omega)]\}.$$

The benefits the decision-maker expects from information is the difference between the expected payoff if information is used and the expected payoff under ignorance. It is equal to the willingness to pay for perfect information. We focus on the relationship between risk and the defined value of information.

3. CHARACTERISTICS OF $V(\pi)$

We first mention some obvious characteristics about $V(\pi)$. The proofs are straightforward and therefore omitted.

LEMMA 1. *$V(\pi)$ is continuous on the set of probability vectors π and $V(\pi) \geqslant 0$ for all π.*

Also, if $\pi_\omega = 1$ for some ω, then $V(\pi) = 0$.

LEMMA 2. *If $Z^{\circ}_{\omega} \neq Z^{\circ}_{j}$ for some $\omega, j, \omega \neq j$, then there exists a π with nonzero probability in at least two states such that $V(\pi) > 0$.*

This lemma illustrates the trivial observation that, as soon as the outcomes of the decision depend on the information, information will be valued positively.

4. THE LINEAR CASE

We examine in detail the linear case, which is also commonly known as constant stochastic returns to scale, i.e. $f(z,\omega) = \omega f(z)$. This will be the reference case. $f(z)$ is assumed to be monotonically decreasing since otherwise the decision problem would be trivial.

Note that, in the linear case, the maximization problem (1) depends only on the expected value $\overline{\omega}$ of the probability distribution, so that the arguments in X^* and Z^*_ω can be replaced by $\overline{\omega}$. The following lemma provides the first step of the analysis.

LEMMA 3. *For $\omega > j$ we have: $Z^\circ_\omega \leqslant Z^\circ_j$.*[2]

Proof. Let us assume that $Z^\circ_\omega > Z^\circ_j$. Since $X^\circ_\omega, X^\circ_j$ are assumed to be unique optimizers, we get:

$$g(X^\circ_j) + jf(Z^\circ_j) > g(X^\circ_\omega) + jf(Z^\circ_\omega).$$

Thus:

$$\begin{aligned} g(X^\circ_j) + jf(Z^\circ_j) + (\omega - j)f(Z^\circ_\omega) &> g(X^\circ_\omega) + jf(Z^\circ_\omega) \\ &+(\omega - j)f(Z^\circ_\omega) = g(X^\circ_\omega) + \omega f(Z^\circ_\omega). \end{aligned}$$

Because of our assumption $Z^\circ_\omega > Z^\circ_j$ and the fact that f is decreasing, we get $f(Z^\circ_\omega) \leqslant f(Z^\circ_j)$ which implies:

$$g(X^\circ_j) + \omega f(Z^\circ_j) > g(X^\circ_\omega) + \omega f(Z^\circ_\omega).$$

This is a contradiction to the assumption that X°_ω and Z°_ω are optimizers. This completes the proof. ∎

Now we are prepared to prove the first main result.

THEOREM 1. *Assume that $f(z, \omega) = \omega f(z)$. Moreover, assume that g is monotonically increasing in x, f is monotonically decreasing in z, and that all optimizers are unique. Then a riskier probability distribution never lowers, but in general increases the value of information.*

Proof. We consider the concept of risk suggested by Rothschild and Stiglitz (1970), which is applied to random variables that have the same mean. From the three possible equivalent definitions we analyze the effect of a mean preserving spread on the value of information.

Consider two probability distributions π^1 and π^2 and suppose $\pi^1_\omega = \pi^2_\omega$ for all but four ω, say $\omega_1, \omega_2, \omega_3$, and ω_4 where $\omega_k < \omega_{k+1}$ for $k = 1, 2, 3$.

We define:

$$\delta\pi_k = \pi^2_{\omega_k} - \pi^1_{\omega_k}$$

Then if

$$\delta\pi_1 = -\delta\pi_2 \geqslant 0, \quad \delta\pi_4 = -\delta\pi_3 \geqslant 0 \quad \text{and}$$

$$\sum_{k=1}^{4} \delta\pi_k \omega_k = 0$$

π^2 differs from π^1 by single mean preserving spread, or equivalently, π^2 has more weight in the tails than π^1.

Because the means are not affected by mean preserving spreads, X°_ω, Z°_ω, X^*, and Z^*_ω are the same for the two probability distributions π^1 and π^2. Therefore, the difference between the corresponding information values is given by:

$$\begin{aligned} V(\pi^2) - V(\pi^1) \; = \; & \Sigma\delta\pi_\omega\{g(X^\circ_\omega) + \omega f(Z^\circ_\omega) - g(X^*) \\ & -\omega f(Z^*_\omega)\}. \end{aligned}$$

We have to show that the above expression is not negative. Since X^* and Z^*_ω only depend on $\overline{\omega}$, expected returns under ignorance are the same for both π^1 and π^2. Thus, after rearranging terms we obtain:

$$\begin{aligned} V(\pi^2) - V(\pi^1) \; = \; & \delta\pi_1\{g(X^\circ_{\omega_1}) - g(X^\circ_{\omega_2}) + \omega_1 f(Z^\circ_{\omega_1}) \\ & -\omega_2 f(Z^\circ_{\omega_2})\} + \delta\pi_4\{g(X^\circ_{\omega_4}) - g(X^\circ_{\omega_3}) \\ & +\omega_4 f(Z^\circ_{\omega_4}) - \omega_3 f(Z^\circ_{\omega_3})\}. \end{aligned}$$

Because $X^\circ_{\omega_1}$ and $X^\circ_{\omega_4}$ are optimizers we obtain:

$$g(X^\circ_{\omega_1}) - g(X^\circ_{\omega_2}) + \omega_1 f(Z^\circ_{\omega_1}) \geqslant \omega_1 f(Z^\circ_{\omega_2}) \quad \text{and}$$

$$g(X^\circ_{\omega_4}) - g(X^\circ_{\omega_3}) + \omega_4 f(Z^\circ_{\omega_4}) \geqslant \omega_4 f(Z^\circ_{\omega_3}).$$

Therefore, it follows:

$$\begin{aligned} & V(\pi^2) - V(\pi^1) \geqslant \delta\pi_1(\omega_1 - \omega_2) f(Z^\circ_{\omega_2}) \\ & \quad + \delta\pi_4(\omega_4 - \omega_3) f(Z^\circ_{\omega_3}). \end{aligned}$$

The second condition which defines a mean preserving spread allows us to combine the two terms:

$$V(\pi^2) - V(\pi^1) \geqslant \delta\pi_4(\omega_4 - \omega_3)\{f(Z^\circ_{\omega_3}) - f(Z^\circ_{\omega_2})\}.$$

Because of Lemma 3 we conclude:

$$V(\pi^2) - V(\pi^1) \geqslant 0.$$ ■

5. EXTENSIONS

In the preceding section we showed that for constant stochastic returns we have a monotonical relationship between risk and the value of information. In this section we first demonstrate how the result can be extended.

We provide sufficient conditions that increasing risk does increase the value of information. We consider now the following specification:

$$(4) \qquad f(z,\omega) = h(\omega)f(z).$$

Let us consider again two probability distributions π^1 and π^2 which differ by a mean preserving spread. Let us define the following value:

$$\omega_*^2 = \max\{\omega \in \{1,\ldots,n\} | h(\omega) \leqslant \sum \pi_\omega^2 h(\omega)\}.$$

Hence, ω_*^2 is the highest possible realization of the stochastic variable ω, such that $h(\omega_*^2)$ is not greater than the expected value of $h(\omega)(= \sum \pi_\omega^2 h(\omega))$ under π^2. Then, we obtain:

THEOREM 2. *Assumptions:*
$f(z,\omega) = h(\omega)f(z)$;
$h(\omega)$ is monotonically increasing;
$g(x)$ is monotonically increasing;
$f(z)$ is monotonically decreasing;
all optimizers are unique.
Let us consider two probability distributions π^1 and π^2 which differ by a mean preserving spread in $\omega_1, \omega_2, \omega_3$, and ω_4. Then the following holds:

(i) *If $\omega_2 \leqslant \omega_*^2 \leqslant \omega_3$ then the increase of riskiness raises the value of information.*[3]
(ii) *If $\omega_2 \leqslant \omega_3 \leqslant \omega_*^2$ and $h(\omega)$ is concave in ω then the increase of the risk leads to greater value of information.*
(iii) *If $\omega_*^2 \leqslant \omega_2 \leqslant \omega_3$ and $h(\omega)$ is convex in ω then the increase of the riskiness raises the value of information.*

Proof. The difference between the values of information can be decomposed as follows:

$$\begin{aligned}
V(\pi^2) - V(\pi^1) = & \sum_{\omega=1}^{n} \pi_\omega^2 \{g(X_\omega^0) + h(\omega)f(Z_\omega^0) - g(X(\pi^2)) \\
& -h(\omega)f(Z(\pi^2))\} \\
& - \sum_{\omega=1}^{n} \pi_\omega^1 \{g(X_\omega^0) + h(\omega)f(Z_\omega^0) \\
& -g(X(\pi^1)) - h(\omega)f(Z(\pi^1))\}. \\
= & \sum_{\omega=1}^{n} \delta\pi_\omega [g(X_\omega^0) + h(\omega)f(Z_\omega^0)] \\
& + \sum_{\omega=1}^{n} \pi_\omega^1 [g(X(\pi^1)) + f(Z(\pi^1))h(\omega) \\
& -g(X(\pi^2)) - f(Z(\pi^2))h(\omega)] \\
& + \sum_{\omega=1}^{n} \pi_\omega^2 [(-g(X(\pi^2)) - f(Z(\pi^2))h(\omega)] \\
& +\pi_\omega^1 g(X(\pi^2)) + \pi_\omega^1 f(Z(\pi^2))h(\omega) \\
= & \sum_{\omega=1}^{n} \delta\pi_\omega [g(X_\omega^0) + h(\omega)f(Z_\omega^0)] \qquad (A) \\
& + \sum_{\omega=1}^{n} \pi_\omega^1 [g(X(\pi^1)) + f(Z(\pi^1))h(\omega) \\
& -g(X(\pi^2)) - f(Z(\pi^2))h(\omega)] \qquad (B) \\
& + \sum_{\omega=1}^{n} \delta\pi_\omega [-g(X(\pi^2)) - f(Z(\pi^2))h(\omega)]. \qquad (C)
\end{aligned}$$

Note that $\delta\pi_\omega = 0$ except for $\omega_1, \omega_2, \omega_3$, and ω_4.

We have to show that $V(\pi^2) - V(\pi^1) \geqslant 0$. Therefore we examine the terms A, B, and C.

Term B

$$\sum_{\omega=1}^{n} \pi_\omega^1 [g(X(\pi^1)) + f(Z(\pi^1))h(\omega) - g(X(\pi^2)) - f(Z(\pi^2))h(\omega)]$$

$$= g(X(\pi^1)) + \sum_{\omega=1}^{n} \pi_\omega^1 f(Z(\pi^1))h(\omega)$$
$$- \left\{ g(X(\pi^2)) + \sum_{\omega=1}^{n} \pi_\omega^1 f(Z(\pi^2))h(\omega) \right\} \geqslant 0,$$

because $X(\pi^1), Z(\pi^1)$ are optimizers.

Term C

$$\sum_{\omega=1}^{n} \delta\pi_\omega [-g(X(\pi^2)) - f(Z(\pi^2))h(\omega)]$$
$$= -f(Z(\pi^2)) \sum_{\omega=1}^{n} \delta\pi_\omega h(\omega), \quad \text{because}$$
$$\delta\pi_1 = -\delta\pi_2, \quad \delta\pi_4 = -\delta\pi_3.$$

Term A

$$\sum_{\omega=1}^{n} \delta\pi_\omega [g(X_\omega^0) + f(Z_\omega^0)h(\omega)]$$
$$= \delta\pi_1 [g(X_1^0 - g(X_2^0) + h(\omega_1)f(Z_1^0) - h(\omega_2)f(Z_2^0)]$$
$$+ \delta\pi_4 [g(X_4^0) - g(X_3^0) + h(\omega_4)f(Z_4^0) - h(\omega_3)f(Z_3^0)].$$

Because X_1^0 and Z_1^0 are optimizers we have

$$g(X_1^0) - g(X_2^0) + h(\omega_1)f(Z_1^0) \geqslant h(\omega_1)f(Z_2^0)$$

and in the same way we have

$$g(X_4^0) - g(X_3^0) + h(\omega_4)f(Z_4^0) \geqslant h(\omega_4)f(Z_3^0).$$

By definition of a mean preserving spread

$$\delta\pi_1 = \delta\pi_4 \frac{\omega_4 - \omega_3}{\omega_2 - \omega_1}.$$

This leads to:

$$\sum_{\omega=1}^{n} \delta\pi_\omega [g(X_\omega^0) + h(\omega)f(Z_\omega^0)]$$
$$\geqslant \delta\pi_4 \frac{\omega_4 - \omega_3}{\omega_2 - \omega_1} [h(\omega_1)f(Z_2^0) - h(\omega_2)f(Z_2^0)]$$

$$
\begin{aligned}
&+ \delta\pi_4[h(\omega_4)f(Z_3^0) - h(\omega_3)f(Z_3^0)] \\
&= \delta\pi_4(\omega_4 - \omega_3)\left[f(Z_3^0)\frac{h(\omega_4) - h(\omega_3)}{\omega_4 - \omega_3}\right. \\
&\left. -f(Z_2^0)\frac{h(\omega_2) - h(\omega_1)}{\omega_2 - \omega_1}\right].
\end{aligned}
$$

We combine Terms A and C:

$$
\begin{aligned}
C + A \geqslant \delta\pi_4(\omega_4 - \omega_3)&\left\{(f(Z_3^0) - f(Z(\pi^2))\frac{h(\omega_4) - h(\omega_3)}{\omega_4 - \omega_3}\right. \\
&\left. +(f(Z(\pi^2) - f(Z_2^0))\frac{h(\omega_2) - h(\omega_1)}{\omega_2 - \omega_1}\right\}.
\end{aligned}
$$

If $\omega_2 > \omega_*^2$ then $h(\omega_2) \geqslant h(\omega_*^2)$ and therefore we can apply Lemma 3 to say $f(Z_2^0) \geqslant f(Z(\pi^2)$. If $\omega_2 < \omega_*^2$ then $f(Z(\pi^2) \geqslant f(Z_2^0)$.

To examine the expression $C + A$ we have to distinguish three cases

$$
\begin{array}{lll}
(i) & \omega_2 \leqslant \omega_*^2 \leqslant \omega_3 & (f(Z_3^0) \geqslant f(Z(\pi^2) \geqslant f(Z_2^0)) \\
(ii) & \omega_2 \leqslant \omega_3 \leqslant \omega_*^2 & (f(Z(\pi^2)) \geqslant f(Z_3^0) \geqslant f(Z_2^0)) \\
(iii) & \omega_*^2 \leqslant \omega_2 \leqslant \omega_3 & (f(Z_3^0) \geqslant f(Z_2^0) \geqslant f(Z(\pi^2)).
\end{array}
$$

In case (i) we have

$$f(Z_3^0) \geqslant f(Z(\pi^2))$$

$$f(Z(\pi^2) \geqslant f(Z_2^0)$$

and hence both expressions of $C+A$ are positive. Therefore $C + A \geqslant 0$.

In case (ii) we have

$$f(Z(\pi^2) - f(Z_2^0) \geqslant f(Z(\pi^2) - f(Z_3^0).$$

Due to the concavity of h we also have

$$\frac{h(\omega_2) - h(\omega_1)}{\omega_2 - \omega_1} \geqslant \frac{h(\omega_4) - h(\omega_3)}{\omega_4 - \omega_3}$$

and thus $C + A \geqslant 0$.

In case (iii) we obtain

$$f(Z_3^0) - f(Z(\pi^2)) \geqslant f(Z_2^0) - f(Z(\pi^2)).$$

Due to the convexity of h we have

$$\frac{h(\omega_4) - h(\omega_3)}{\omega_4 - \omega_3} \geqslant \frac{h(\omega_2) - h(\omega_1)}{\omega_2 - \omega_1}$$

and hence again $C + A \geqslant 0$. ■

6. A COUNTER EXAMPLE

In this section we show how the monotonical relationship between the riskiness and the value of information can break down. Consider the following example with only four possible states of the world and corresponding probabilities $\pi_1, \pi_2, \pi_3, \pi_4$.

The payoff functions are given by:

$$g(x) = ax + b, \quad f_1(z) = az + b, \quad f_2(z) = e,$$
$$f_3(z) = 0 \quad \text{and} \quad f_4(z) = cz$$

with $a, b, e > 0, b > e, c < 0$ and $|c| > a$.

In the example there is uncertainty whether more development of a natural resource is preferred to less development in the future even if no irreversibility constraint were present. The fourth state may describe severe consequences from development, for instance, due to increasing recreation demand or due to decreasing usage value of a public infrastructure. The first state would imply that a complete development of the natural resource would be optimal. The maximizers under full information can be easily calculated:

$$X_1^\circ = X_2^\circ = X_3^\circ = 1,$$
$$Z_1^\circ = Z_2^\circ = Z_3^\circ = 1,$$
$$X_4^\circ = 0, \quad Z_4^\circ = 0.$$

Without information, the irreversibility condition is only binding in the fourth state. Thus, the maximization problem is given by:

$$\max_{x \in [0,1]} g(x) + \sum \pi_\omega \max_{z \in [x,1]} f(z, \omega) = ax + b + \pi_1(a + b) + \pi_2 e + \pi_4 cx.$$

We have to distinguish two cases:

If $a + \pi_4 c < 0$ we get $X^* = 0$ and $X^* = 1$ otherwise.

Thus one obtains

$$
\begin{aligned}
V(\pi) &= a\pi_1 + \pi_2 a + \pi_3 a = (1 - \pi_4)a \quad \text{for} \\
&\qquad a + \pi_4 c < 0 \quad \text{and} \\
&= \pi_4(-c - a) \text{ otherwise.}
\end{aligned}
$$

The value of information is solely a function of the probability in the fourth state. Graphically, we obtain Figure 1.

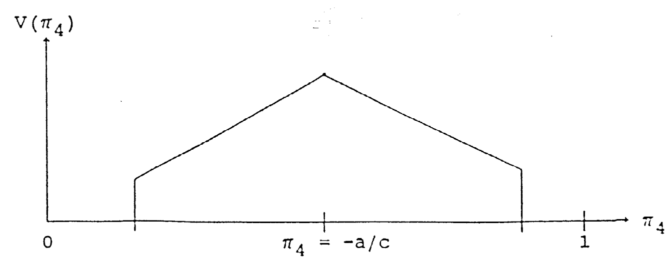

Fig. 1.

A mean preserving spread can only be achieved through an increase of π_4. But only for $\pi_4 < -a/c$ the value of information also increases, whereas for $\pi_4 > -a/c$ the value of information declines. This result follows because up to $-a/c$ an increase in π_4 is valuable since it allows to avoid an increasing loss in the fourth state. But afterwards the decision-maker changes his decision without information and the potential losses become smaller.

This analysis suggests for which cases it can be expected that the value of information would decrease if the riskiness of the random variable increases. If an increase of the risk induces a discontinuous change of the optimal choice under ignorance, the value of information also changes its properties because, with higher risk, it depends on completely different states and their related payoffs. Thus, at such points, the relationship between risk and the value of information may change.

7. CONCLUSION

Since the information value determines the investment in the acquisition of information before undertaking environmental projects, the relationship between the risk of the environment and the information value is important. If projects show constant returns to scale, the value of information is generally increasing with risk. This result can be extended if the return function can be separated in a stochastic and non-stochastic part. If, however, there is much uncertainty about the basic desirability of future developments of natural resources or real investments, the relationship may break down. That is, it holds only of a certain range of the riskiness.

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NOTES

[1] Note that we focus in this paper on normative decision rules and neglect political-economic considerations which play an important role in practical decision processes.

[2] Note that this also implies $X^\circ_\omega \leqslant X^\circ_j$ since $Z^\circ_\omega = X^\circ_\omega$.

[3] As before, the case that the value of information remains constant is always included.

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