On the Annual and Diurnal Variations of the Anomalous Acceleration of Pioneer 10

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An apparent anomalous acceleration of about 8×10^{-8} cm/s^2 (directed towards the Sun) has been detected in the Doppler residuals of Pioneer 10 and 11. A considerable amount of effort has been made in searching for a conclusive origin of this apparent acceleration, however, without success till date. Detailed study of the data has revealed that an annual and a daily variation of the data exist and these can be interpreted as the fluctuating components of the apparent acceleration superimposed on the steady anomalous acceleration. Since these components are definitely related to the Earths motion an explanation has been found for these annual and diurnal fluctuations. The doppler effects due to the motions of the Earth are already incorporated in the model; there should thus be no residual redshift present in the results. It has been shown that the excess redshift of the signal between the Earth and Pioneer 10 due to inertial induction can manifest itself as the apparent acceleration of the spacecraft. It has been shown that the annual and the diurnal components can be accounted for by the excess redshifts due to inertial induction. Both the magnitude and the temporal phase match with the observation.

Introduction

Existence of an apparent anomalous acceleration, as revealed by the analysis of the Doppler data, of Pioneer 10 and 11, is well known by now [1, 2]. The magnitude of this apparent acceleration for both the spacecrafts is of the order of 8×10^{-8} cm/s^2 and it is directed towards the sum. The magnitude is also reasonably independent of the distance from the Sun beyond 20 AU. Considerable effort has been made in order to identify the source of this apparent acceleration but till now no conclusive result has been published. In more recent publications [2, 3] it has been shown that the apparent acceleration has periodic components (varying both annually and daily) superimposed on to the steady acceleration. The amplitude of the annual oscillatory term is approximately 1.6×10^{-8} cm/s^2 and that of the daily fluctuations is of the order of 0.03×10^{-8} cm/s^2 or a little less [3].

Though still there is no clue to the cause of the unmodelled acceleration directed towards the Sun it is clear that the annual and daily fluctuating components of the unmodelled acceleration must be linked to the orbital and spin motions of the Earth. This paper attempts to explain these fluctuating components due to the motions of the Earth and excess redshifts of the signal caused by velocity dependent inertial induction [4-6].

Excess Redshift of Signal due to Inertial Induction

A theory of inertial induction, based on an extension of Mach's principle, has been proposed and applied to a number of astronomical, astrophysical and cosmological phenomena with remarkable success. Most of these already exist in published literature [4,7-11]. According to this theory an object is subjected to a drag when moving with
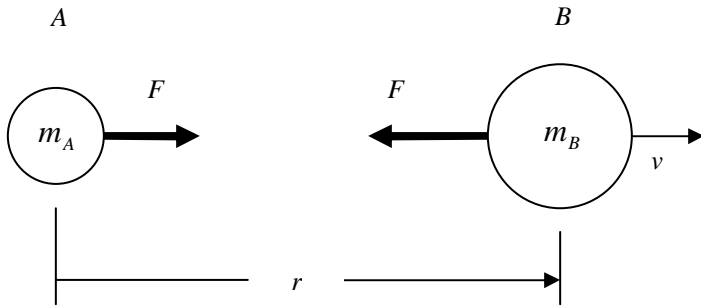
Fig. 1

respect to another body. Figure 1 shows a simple situation when a body B moves with respect to another body A with a speed v as indicated. A drag force F acts on B as it moves away from the body A and the magnitude of F is given by the following relation [6]:

$$F = \frac{Gm_A m_B}{c^2 r^2} v^2 \qquad (1)$$

where G is the gravitational constant, m_A and m_B are the masses of the objects A and B, respectively, c is the speed of light and r is the instantaneous distance between the two bodies. When B approaches A then also the force will oppose v. When the body B is a light photon

$$v = c \qquad (2a)$$

and $$m_B = h\nu / c^2 \qquad (2b)$$

where h is the Planck's constant and ν is the frequency of the photon. Using (2a) and (2b) in (1)

$$F = Gm_A \frac{h\nu}{c^2 r^2} \qquad (3)$$

When the photon moves a distance dr the drop in energy of the photon is given by

$$dE = -F.dr = -\frac{Gm_A h}{c^2 r^2} v dr \tag{4}$$

Again $dE = hdv$. Substituting this in the L.H.S. of (4) one gets

$$-hdv = \frac{Gm_A hv}{c^2 r^2} dr$$

or

$$\frac{dv}{v} = -\frac{Gm_A}{c^2} . \frac{dr}{r^2}$$

Solving the above equation one obtains

$$\ln v = \frac{Gm_A}{c^2} \frac{1}{r} + D \tag{5}$$

If at $r = r_E$ $v = v_0$, then

$$\ln v_0 = \frac{Gm_A}{c^2} + D \tag{6}$$

where r_E is the distance of the observing station from the body A.

Thus, from (5) and (6) the following relation is obtained.

$$\ln \frac{v_0}{v} = \frac{Gm_A}{c^2} \left(\frac{1}{r_E} - \frac{1}{r} \right). \tag{7}$$

Anomalous Redshift of Pioneer Signal due to Inertial Induction:

Figure 2 shows the path of Pioneer 10, the Sun and the Earth projected onto the ecliptic plane. As the heliocentric latitude of Pioneer 10 is around 3° only (once it crosses the distance of 50AU) its component of velocity normal to the ecliptic plane is very small. Furthermore r being so large compared to the Sun- Earth distance for an approximate analysis the Sun and the Earth may be treated as almost coincident.

As given by (1) on page 9 of Ref [2] a change in frequency $\Delta v \ (= v_0 - v)$ can be expressed as follows:

$$\Delta v = v_0 \frac{1}{c} \frac{d(2r)}{dt} = 2 \ v_0 \ \frac{v_{ap}}{c} \tag{8}$$

where $2r$ represents the overall optical distance traversed by a photon in both directions, and v_{ap} is the apparent velocity of the spacecraft that can produce the same frequency shift by Doppler effect. Differentiating both sides of (8)
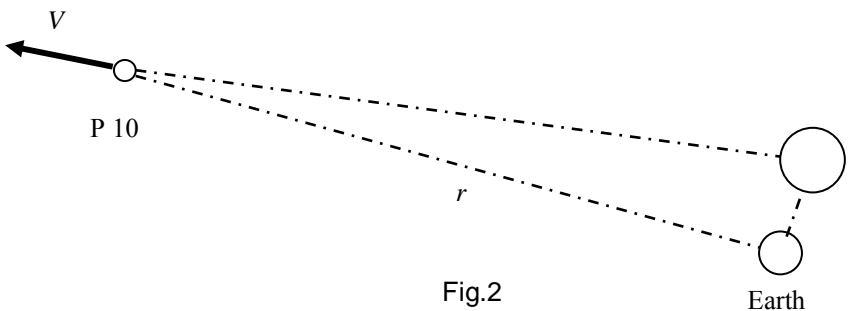

Fig.2

$$\frac{d}{dt}(\Delta v) = \frac{2v_0}{C}a_{ap} \tag{9}$$

where a_{ap} is the apparent acceleration due to the variation in frequency shift with time. When all contributions to the change in r due to the motions of the Earth and the spacecraft are taken into account and the resulting theoretical frequency shift is subtracted from the observed values, ideally the residual should be zero. Any residual implies either some unknown motion or some unaccounted source of frequency shift other than the doppler effect. To consider the effect of the proposed inertial induction drag on signals let (7) be considered. (Effect of the Sun is much larger than those due to the cosmic drag [4-6] and, therefore, only the effect due to the Sun is taken into account). Equation (7) can be re written as follows:

$$\exp\left[\frac{Gm_A}{c^2}\left(\frac{1}{r_E} - \frac{1}{r}\right)\right] = \frac{v_0}{v}$$

or,

$$\exp\left[-\frac{Gm_A}{c^2}\left(\frac{1}{r_E} - \frac{1}{r}\right)\right] = \frac{v}{v_0} = \frac{v_0 - \Delta v}{v_0} = 1 - \frac{\Delta v}{v_0} \tag{10}$$

When the above relation is used for the Sun- Earth- Pioneer 10 combine m_A is replaced by m_S, i.e. the mass of the Sun, $r_E = 1\,\text{AU}$ and *r* presents the distance of the spacecraft from the Sun (> 40 AU for the period of observation under consideration). For these values

$$\frac{Gm_S}{c^2}\left(\frac{1}{r_E} - \frac{1}{r}\right) \ll 1$$

and (10) can be written as follows:

$$1 - \frac{Gm_S}{c^2}\left(\frac{1}{r_E} - \frac{1}{r}\right) \approx 1 - \frac{\Delta\nu}{\nu_0}$$

Or, for the frequency shift for the forward and return journey of the signal,

$$\frac{\Delta\nu}{\nu_0} \approx 2\frac{Gm_S}{c^2}\left(\frac{1}{r_E} - \frac{1}{r}\right) \tag{11}$$

Again it should be kept in mind that as $r_E \ll r$ an approximate evaluation of the effect of inertial induction can be made treating the Sun and the Earth to be at the same location without introducing any significant error. Now differentiating both sides of (11) with respect to time the following relation is obtained:

$$\frac{d}{dt}(\Delta\nu) \approx 2\nu_0 \frac{Gm_S}{c^2 r^2} \cdot \frac{dr}{dt} \tag{12}$$

dr/dt being the rate of change of the distance between the observer (i.e., the Earth) and the spacecraft it can be expressed as follows:

$$\frac{dr}{dt} \approx V - v\sin\theta - v_d\sin\phi \tag{13}$$

where v_d is the surface velocity near equator due to the Earth's daily rotation (the tilt of the Earth's axis of rotation has been ignored), v is the orbital speed of the Earth, V is the speed of Pioneer 10 and the angles θ and ϕ are as indicated in Fig. 3. Equating $\frac{d}{dt}(\Delta\nu)$ from (9) and (12) and using (13)

$$\frac{2\nu_0}{c}a_{ap} \approx 2\nu_0 \frac{Gm_S}{c^2 r^2} (V - v\sin\theta - v_d\sin\phi)$$
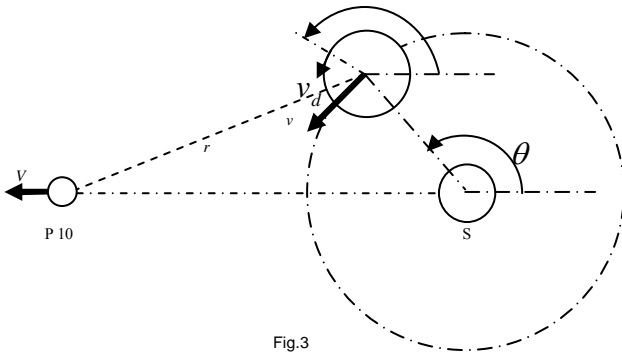
Fig.3

Thus, an unmodelled apparent acceleration is produced by the inertial induction effect and it can be expressed as follows:

$$a_{ap} \approx \frac{Gm_s}{cr^2}V - \frac{Gm_s}{cr^2}v\sin\theta - \frac{Gm_s}{cr^2}v_d\sin\phi \qquad (14)$$

The magnitudes of the unmodelled apparent acceleration's steady and fluctuating components can be estimated. The estimates using $V = 1.22 \times 10^4$ m/s, $v = 3 \times 10^4$ m/s and $v_d = 4.65 \times 10^2$ m/s are shown in Table 1 and are compared with the observation.

Table 1

	Year	1990	1994	1997
	r	**48 AU**	**59 AU**	**69 AU**
Apparent Acceleration Magnitude (annually fluctuating component)	Theoretical	2.6×10^{-8} cm/s²	1.7×10^{-8} cm/s²	1.25×10^{-8} cm/s²
	Observation	1.6×10^{-8} cm/s²		
Apparent Acceleration Magnitude (daily fluctuating component)	Theoretical	0.04×10^{-8} cm/s²	0.026×10^{-8} cm/s²	0.02×10^{-8} cm/s²
	Observation	0.03×10^{-8} cm/s²		
Apparent Acceleration Magnitude (steady component)	Theoretical	1.05×10^{-8} cm/s²	0.6×10^{-8} cm/s²	0.5×10^{-8} cm/s²
	Observation	-8×10^{-8} cm/s²		

From Table 1 it is quite clear that the inertial induction effect cannot explain the steady part of the observed apparent acceleration

of Pioneer 10. The magnitude due to this effect is an order of magnitude less than the observed value. On the other hand the annual and daily fluctuating components are in reasonable agreement with the observed values. What needs to be done is to check whether the dates of occurrence of high and low values of the annually fluctuating component match with the observation or not. That problem is taken up in the next section.

Maxima and Minima of Annual Fluctuation

Figure 4 shows the positions of the Earth in its orbit when the rate of increase *dr/dt* will be minimum (at 1) and maximum (at 2). The angle made by the line *ES* with the line *PS* is 90° at both these positions. Approximately *PS/ES* is more than 40 for the period under consideration and angle β is very small. Since $v_d \ll v$, the diurnal fluctuation can be ignored while considering the maximum and
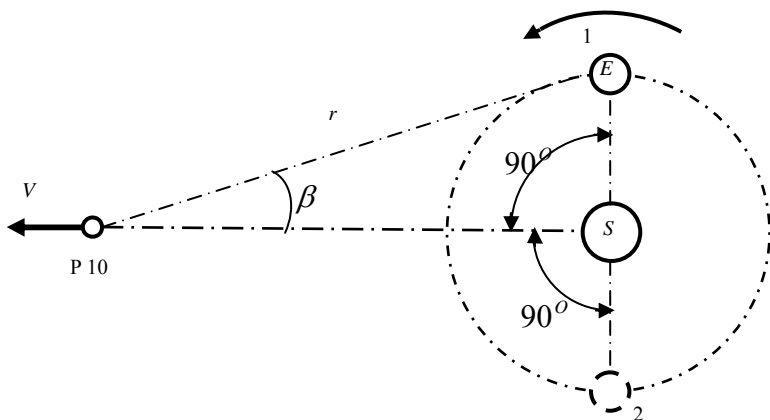

Fig.4

minimum values of *dr/dt*.

   To correlate the position 1 and 2 with the calendar dates it is necessary to compare the ecliptic longitude of Pioneer 10 with that of the Earth on a given date. Ecliptic longitude of Pioneer 10 slowly increases from 71° at a distance of 40 AU to 75.6° at 69 AU. Thus an average value of 73 will be considered. It is also known that on 1st January the ecliptic longitude of the Earth 99.8°. Figure 5 indicates the relative directions of the Earth and Pioneer 10 on 1st January. Location 2 corresponds to the maximum value of *dr/dt*, which is the maximum value of the apparent acceleration. It is 63.2° ahead of 1st January position which is equivalent to about 1/6th of a year, i.e. about 2 months. A careful examination indicates that the peaks in the

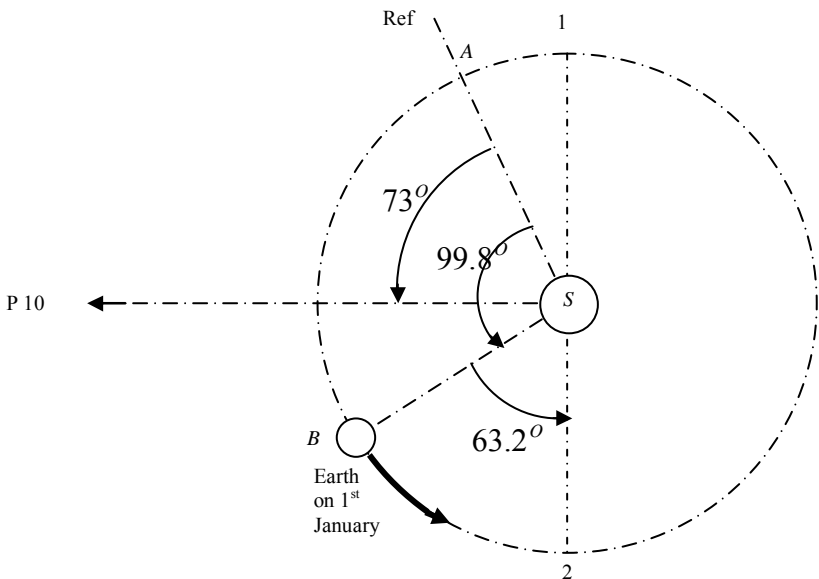

Fig.5

residual occur about 2 months after 1st January every year. There is also a faint suggestion of a gradual decrease of the amplitude over the years, though a reasonable quantitative analysis is difficult.

Hence it is seen that the estimates indicated in Table 1 match the observation and there is a faint indication of gradual decrease as expected from the theoretical calculations. Furthermore, the temporal phase of this annual variation also agrees well with the theoretical expectation. The steady part of the anomalous acceleration may have a separate origin. Thus the proposed model of inertial induction that has explained many observations appears to be able to explain the annual and diurnal fluctuations in the anomalous acceleration of Pioneer 10.

Concluding Remarks

The origin of the steady component of the anomalous acceleration of Pioneer 10 cannot be explained. However, the annual and daily fluctuating components of the anomalous acceleration must be linked with the annual and the daily motions of the Earth. It is found that when the hypothesis of Velocity Dependent Inertial Induction is applied the anomalous acceleration can be explained satisfactorily by the excess redshifts resulting from this phenomenon.

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