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REPLY TO VILKS

GIACOMO BONANNO
University of California, Davis

In his note Arnis Vilks (1994) raises two criticisms concerning my paper "The Logic of Rational Play in Extensive Games" (Bonanno, 1991). The author gives two examples: one to show that my logic "is inconsistent for some very simple games" and the other to show that the propositional representation of an arbitrary extensive game may fail to capture all the relevant details, with the consequence that two essentially different games might have the same (or tautologically equivalent) representation. Below I will argue that the second criticism is not well-founded, while there is an obvious and easy way of dealing with the first. I should like to stress, however, that at the root of Vilks's criticisms are insufficient details and an incomplete definition in my article. I am grateful to Arnis Vilks for prompting me to be more rigorous.

Vilks's second criticism is very important because it raises the following question: Is there a rigorous way of obtaining the logical description of an extensive game and in what sense is such a description "true"? In my article (Bonanno, 1991) I did not address this question and merely relied on the intuitive plausibility of the expressions that I used to describe the games that I analyzed. Prompted by Vilks's comment I have now addressed this issue (Bonanno, 1993) in a rigorous way. I first show how to obtain from an arbitrary extensive form a set of well-formed formulas (WFFs) of propositional calculus and then prove that those WFFs are true for the game. This is done by using a semantics for propositional calculus that differs from the standard one based on truth tables. As a by-product one can show that, contrary to what Arnis Vilks claims, the two games shown in figures 2 and 3 of his comment do not have tautologically equivalent descriptions. For a full account of this claim I have to refer the reader to my paper (Bonanno, 1993). Here I will give a heuristic counterargument to Vilks's criticism. The extensive forms of figures 2 and 3 have the same tree: They differ only in the assignment of players to the decision nodes. In moving from one to the

other, however, Vilks changes the labeling of the arcs of the tree. Without such a re-labeling, the description of the extensive form of figure 3 would become

$$(a_2 \vee b_2) \wedge \neg(a_2 \wedge b_2) \wedge (a_2 \Leftrightarrow c_1) \wedge (b_2 \Leftrightarrow d_1),$$

which is clearly not tautologically equivalent to the description of figure 2 (even the alphabet is different!). It is important to note that a labeling of the arcs of the tree is *not* part of the definition of extensive game (compare, for example, the definition given by Selten, 1975). Whenever one labels the arcs of the game-tree, one adds an element of arbitrariness. The labeling chosen by Vilks for the games of figures 2 and 3 leads to the conclusion that the two games have tautologically equivalent descriptions. The alternative labeling suggested above destroys this conclusion. Thus, one can raise the question of whether it is possible to define a rule (or algorithm) for obtaining the logical description of an arbitrary extensive game, having the property that the description is "true" for the extensive game and, furthermore, two "essentially different" games (such as the games of figures 2 and 3) are translated into WFFs that are not logically equivalent. In Bonanno (1993) I provide an affirmative answer to this question.

I now turn to the first criticism. Vilks's comment is correct because I was inattentive in my formulation of the notion of player-*i*-admissible hypothesis (Bonanno, 1991, p. 46). In its simplest form, a player-*i*-admissible hypothesis is a formula with the following structure

$$\Gamma \wedge (A_{i1} \vee A_{i2} \vee \ldots \vee A_{im})$$

and it is meant to represent the sentence "suppose that the game is as described by Γ and player i is at one of his decision nodes where he has to choose among actions $A_1, A_2, \ldots, A_m$." Thus Γ is the description of the game and $(A_{i1} \vee A_{i2} \vee \ldots \vee A_{im})$ is a description of the decision node in terms of the actions available there. I should have added the qualification that $A_1, A_2, \ldots, A_m$ are *all and only* the actions available at a decision node (of player i). With that natural proviso, Vilks's criticism no longer applies. In fact Vilks obtains an inconsistency by *"mixing" actions available at different decision nodes.* In fact, in his proof (which is given in the Appendix to his comment and refers to the game of figure 1), he eliminates actions e and f of player 2 by comparing them with action c (which is not available at the same node at which actions e and f are). Having eliminated both e and f, it follows from the description of the game that b of player 1 must be eliminated. But since a is inferior to b, action a of player 1 must be eliminated and, of course, eliminating both a and b contradicts the statement that either a or b must be chosen.

Of course, the proviso that $A_1, A_2, \ldots, A_m$ are all and only the actions available at a decision node raises the question of whether we can indeed associate with every decision node (or, in general, information set) of a player, say player i, a well-formed formula of propositional calculus of the type $(A_{i1} \vee A_{i2} \vee \ldots \vee A_{im})$, where each atomic sentence A_{ij} represents a choice available at that decision node. In Bonanno (1993) I show that this can be done unambiguously and, furthermore, that the WFFs so obtained are "true" for the game, in the precise sense that the game yields a topological model for those WFFs. Finally, I show that, within the standard truth-table semantics for propositional calculus, there is a one-to-one correspondence between the set of valuations that satisfy all those WFFs and the set of plays of the game.

I should like to conclude, however, by thanking Arnis Vilks for pointing out that my original definition of player-i-admissible hypothesis was incomplete and for showing the need for a rigorous approach to the problem of how to obtain the logical representation of an arbitrary extensive game.

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