

## Representation of Time

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## Abstract

Gibbon, J., and Church, R.M.. 1990. Representation of time. *Cognition*, 37: 23-54.

*Memory representation for time was studied in two settings. First, an analysis of timing in a laboratory analog of a foraging situation revealed that departure times from a patchy resource followed a Weber Law-like property implied by scalar timing. A trial-by-trial analysis was then pursued in a similar but more structured experimental paradigm, the Peak procedure. Study of covariance structures in the data implicated scalar variance in the memory for time as well as in the decision process, but the correlation pattern ruled out multiple access to memory within a trial.*

Time present and time past  
Are both perhaps present in time future,  
And time future contained in time past.  
T.S. Eliot, *Burnt Norton*.

## Introduction

Eliot's words remind us that our current position on the arrow of time, fixed on the present instant, has been prefigured in the past, and in turn foreshadows what is to come. There is a sense in which the life of any living thing is continuously timed, from its genetically prescribed beginning into its programmed senescence.

On a smaller, one might say fractal, scale all movement is timed, some-

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times with exquisite precision, as when the batter starts his swing, the musician his downstroke, or the peregrine folds its wings and dives. These are but some more dramatic examples of prefiguring a future event with precisions that are extraordinary. In a broader sense, however, we continuously prefigure future events. Any movement entails a representation of a future event, its anticipated outcome, and of course no system of the body is ever entirely at rest.

Much temporal anticipation is rhythmic: activity/rest cycles, respiration cycles, heartbeats, even our speech patterns, reflect rhythms that may repeat endlessly, or sometimes at will. Once initiated, each shows a characteristic period and variability. Periodicity of this sort may be represented in our nervous system in the time constants of a collection of oscillating neurons. Most such rhythmic timing systems exhibit remarkable precision, but little flexibility in the range of times represented. Evolution has built them for essentially one time value, and they often execute this value with precisions of 1–2% or less. Several circadian rhythms, for example, are in this class, with low variability and an entrainment range of a few hours around 24 (cf. Aschoff, 1984).

Some rhythmic timing systems, however, can synchronize with a variety of temporal values in the environment, for example when a musician changes tempo, or when a horse adjusts its gait. Timing functions of this sort usually are in the seconds, or possibly minutes, range, and they may be begun at any arbitrary point in time, and reset arbitrarily. Such rhythms have received some study in humans (e.g., Wing, 1980; Wing & Kristofferson, 1973), and almost none in animals. These timing systems share features with the more rigid and precise biological oscillators mentioned above, as well as with the more arbitrary interval timers, described below.

This, most flexible, kind of timing allows arbitrary onsets for beginning of a timed interval, arbitrary records for the time of important events in the interval, and discriminations among past and present time intervals. Such a system is exemplified by the one that allows foragers, including human hunter-gatherers, to adjust their food-search strategies to changing, usually depleting, resources. The representation of times like these – intervals that may take on a variety of values requiring different sorts of behavioral adjustments, are the topic of this paper. These timing functions are wonderfully flexible: they may begin virtually at will, reset virtually at will, and time a broad range of target values, from seconds to minutes or perhaps even hours. Interval timing functions pay for their flexibility with imprecision. Evolution cannot tune them for a given target value, but must build in record-keeping systems and temporal readout systems, which can assume a range of values and forms.

Representation is going to mean for us here a rather abstract conception of some semi-permanent feature of these timing functions. We will not be so concrete as to specify neural mechanisms, nor so abstract as to specify complete and rigorous mathematical development. Rather, our representations will be memories lying somewhere between the cell and the theorem. We will ask what kind of mnemonic variability and subjective scale are implied by memory for arbitrary time intervals.

Perhaps unfortunately, we must approach these questions through still another construct, harboring its own vagaries, namely the performance, including especially a decision structure, that reveals properties of temporal memory. It will be a major aim, then, to partition variance we see in performance into components reflecting differing features of temporal processing. In particular, our interest centers on three sets of processes: (1) the mechanism whereby animals perceive the passage of time – the clock system; (2) processes whereby a given time value is marked, distinguished and recorded in memory; and finally (3) the way in which decisions based on these temporal memories are made. We will identify features of temporally organized performances which differentially reflect imprecision in these three basic components of timing – clock, memory, and decision.

### **An example: Foragers' giving-up times**

Let us be concrete. A starling in the springtime forages for food in fields round about its nest ("central place" foraging). The pressure to forage efficiently, and thereby collect the greatest yield per unit time, is extreme when birds are feeding their young. Evolution has tuned their timing system so that they can recognize productive as opposed to unproductive fields ("patches"), and also to recognize when such patches are depleted – when to search for richer pickings. The optimal time for such a new search may be shown to depend critically on how long it took to travel to the patch in the first place, as well as on the initial prey density, and the depletion rate (e.g., Krebs & Kacelnik, 1984).

A performance like this requires at least two different kinds of time records, that is, two representations. First, birds must know how long it takes to get to a given area from the nest site. To forage optimally, they should stop foraging in a given patch at different overall residence times, depending on how far away the path is from the nest. When the patch is distant, persistence in the patch pays better than when the patch is close. Intuitively, the investment in a longer travel must be made up by a greater yield. Second, they must have an appreciation of when a patch is no longer productive. The

decision to leave may depend on variability in the perception of prey density. If the patch is initially rich, and depletes fast, detection of depletion may be easier. Subjects may stay a shorter time after finding the last prey item in a rapidly depleting patch, than if the initial prey density was lean, and capture rates low.

Our example will examine an extreme case, the appreciation of a given prey density which has abruptly soured. We ask when birds "think" an expected prey item is no longer forthcoming. If rate of food capture is represented as a typical interval between feedings, at what point after that interval has been exceeded do subjects no longer expect food? We will attempt first to understand the mechanism for the simplest of these cases, strict periodicity, in which there is no variability in interfood delivery times.

The data we present are from the dissertation research of Danielle Brunner in collaboration with Alex Kacelnik and John Gibbon (Brunner, Kacelnik, & Gibbon, 1989; Kacelnik, Brunner, & Gibbon, 1988).

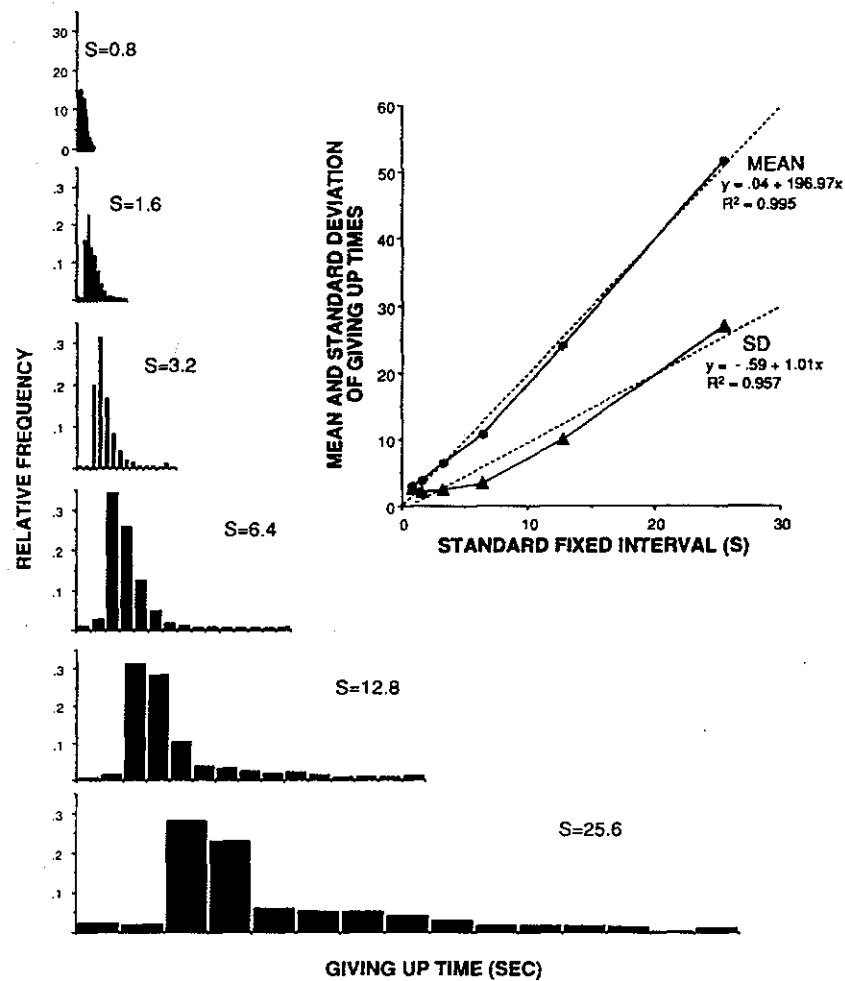
Brunner simplified the foraging problem to study it in the laboratory in the following manner: Starlings were required to fly a certain distance (comprising the "travel" or "search" time) by completing a number of flights between two perches in a large chamber. When the ratio was completed, a "patch key" was illuminated above a third perch, at which they could earn food on a given schedule. The schedule is the analog of the "patch". It is a repeating, periodic fixed-interval schedule, which at some point "dies". After arrival at the perch, the next feeding was scheduled a standard time ( $FI = S$ s), with probability  $p$ . After food occurred, the next feeding was scheduled at the same standard,  $S$  s later, again with probability  $p$ . If a feeding was missed, with probability  $q = 1 - p$ , then all subsequent feedings for that visit to the patch were cancelled. Thus a string of 0, 1, 2, 3, ...,  $n$  feedings occurred with a geometrically declining probability in any one patch visit.

The patch could be reset only by leaving and again flying between the two travel perches. As soon as subjects left the patch perch, its key darkened. Interest centered on the properties of the giving-up time, defined as the time between the last feeding (or arrival at the patch) and departure to resume travel. Brunner studied these giving-up times for several weeks at six different fixed-interval values, ranging in log steps from  $S = 0.86$  s to  $S = 25.6$  s between feedings.

If subjects appreciated the size of the fixed-interval with perfect accuracy, then an optimal forager would leave the patch the moment it is clear that no more food is coming. And this should happen just after  $S$  s from the last feeding. Of course, a little variability in the representation of the time interval, and a tight criterion for leaving would result in occasional missed feedings when subjects in fact left before the next delivery time. Hence a conservative

strategy would be to wait somewhat longer than the expected feeding time to avoid missing a programmed feeding. And when the fixed-interval schedule is long, and memory for it is correspondingly more variable, one might expect the margin of safety, the duration of the overshoot experienced before leaving, also to be long.

Figure 1. *Relative frequency distributions of giving-up times at six different standard values. Inset shows mean and standard deviation of giving-up times as a function of the standard. The straight lines are regression fits to each data set.*

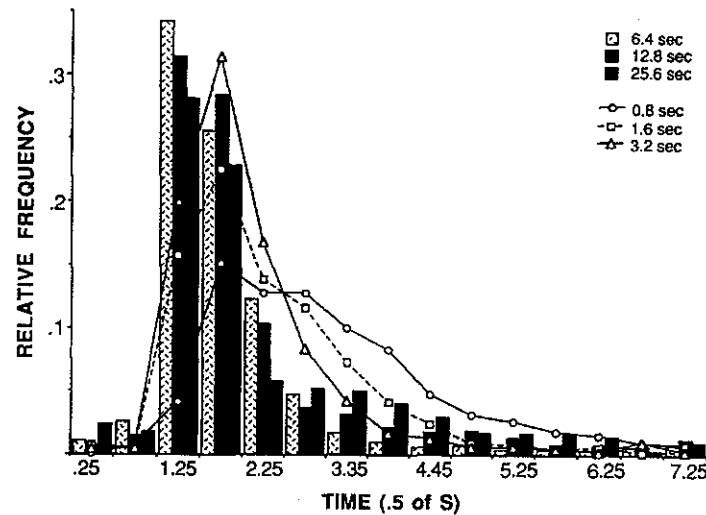


This is indeed what occurred. Figure 1 presents distributions of giving-up times pooled over 6 subjects at each of the 6 fixed-interval values. The scheduled feedings occur at the beginning of the third time bin, so that times to the left of this value (two left-most bars) are departures before subjects could "know" whether the next feeding was scheduled or not. The distributions have some skew, and are rather peaked with a modal giving-up time which moves further to the right of  $S$  as  $S$  grows. The distributions also flatten somewhat at the longer fixed-interval values. In the inset we plot the mean and standard deviation of these distributions as a function of the standard FI value. Both are relatively linear, especially at the longer time values. The very small intercept in the mean function makes it nearly proportional to real time. This means, most importantly, that birds are, on average, leaving the patch when a given proportion of time beyond the standard has been passed: they leave at about twice the standard time after the last feeding. The standard deviation function appears to level off as the FI value becomes smaller and smaller, perhaps approaching some minimum variance level for this timing system.

#### *Scalar timing: Giving-up times*

The distributions for the longer FI values, like many others in the literature, exhibit a property we have called the scalar property: they are approximately scale transforms, one of another. This property may be revealed by replotting these distributions as a function of the giving-up time divided by the fixed-interval standard. This is done in Figure 2. The three longest  $S$  values are shown with histogram bars, while the three shorter  $S$  values are indicated with connected points. Notice first that the proportion of "missed" opportunities, represented by the leftmost two categories, are low and roughly constant for the three long standards. The modal category for these distributions is the third, in which reinforcement is obtained. The decline on the right shows some skew, and is roughly comparable for these three standards. In contrast, the connected point distributions show a mode in the category to the right of reinforcement, and an increased spread with *decreasing* FI value. This is the reflection, noted above, of some minimal variance in temporal processing shown in the flat portion of the standard deviation curve in Figure 1. This irreducible component of variance has greater relative impact as scalar variance grows small.

Figure 2. Distributions of giving-up times (Figure 1) plotted in time relative to the standard.



### Memory variance

We have argued that these data reflect the scalar property, but it is not obvious just how giving-up times are tied to a scalar representation of the reinforcement interval. The simplest proposal for variance in subjective representation is shown in Figure 3. In the top panel, two distributions associated with two different standards are shown, with mean and standard deviation function below. The distributions may be thought of as errors in the representation that possess the scalar property in its simplest form: the proportion of errors below a given proportion of the interval is constant for any size interval – the result of scalar multiplication of the entire distribution. The standard deviation is proportional to the mean, so the coefficient of variation, the sensitivity of the system (usually represented by  $\gamma = \sigma/\mu$ ) is constant. These assumptions are discussed in considerable quantitative detail elsewhere (Church & Gibbon, 1982; Gibbon, 1981; Gibbon & Church, 1981).

Brunner has adapted this memory representation to a threshold model for giving-up times. In Figure 4 the left-hand ordinate shows subjective time increasing as a linear function of real time, and a memory distribution around the subjective representation of the standard is indicated there. The decision to give up is based on the relative discrepancy between the perceived subjective duration of the currently elapsing interval, and the memory for the rein-

Figure 3. Hypothetical distributions of subjective time associated with one and two units of real time. The distributions possess the scalar property with mean and standard deviation proportional to real time.

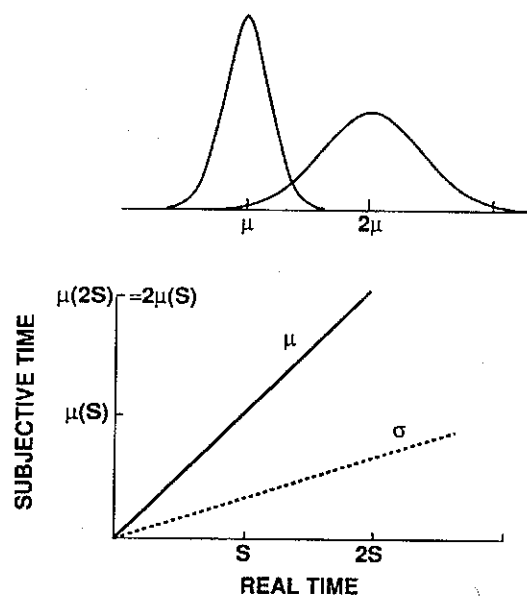
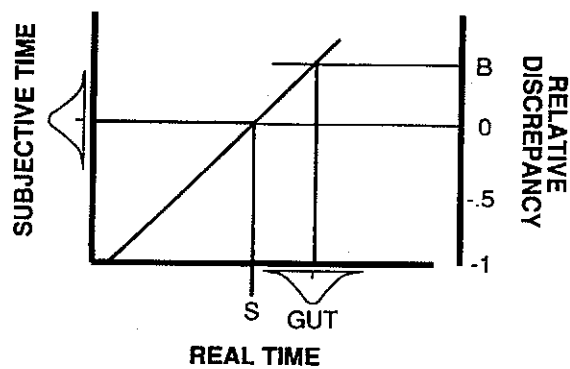


Figure 4. Threshold model for giving up time distributions associated with variable memory and fixed threshold (adapted from Brunner et al., 1989).



forced standard duration. This is the difference between the current and remembered times, divided by the remembered time. Subjects leave the patch when the elapsed time since the last feeding exceeds some fraction beyond the expected time to the next feeding. Formally, the giving up time rule is:

$$(t - s)/s \geq b, \quad (1)$$

where  $t$  is the current time since the last feeding,  $s$  is the remembered time to the next feeding, the subjective value corresponding to  $S$  in real time, and  $b$  is the threshold. (We adopt the convention that lower-case letters represent random variables and upper-case letters fixed values. For example, in the figure the threshold is represented at the fixed value,  $B$ .)

The relative discrepancy between the expected delivery time and the perceived current time is shown on the right-hand ordinate of the figure. This measure begins at  $-1$ , when the time since the last feeding is zero, and increases through zero when the time since the last feeding equals the remembered time to the next feeding. A departure decision occurs when the relative discrepancy crosses some positive threshold level beyond the expected time of food.

It is readily shown that a giving-up rule based on relative discrepancy will generate superposition of the giving-up time distributions as long as memory variance for the standard is scalar. It is also true, however, that a scalar memory representation alone is not sufficient to produce superposition. The rule for generating giving-up times must be based on relative rather than absolute discrepancy. If the proximity between current and remembered time is not normed by remembered time, the scalar property is violated. For a larger standard, an absolute discrepancy rule would result in a mean giving-up time relatively closer to  $S$ , but with a concomitant increase in variance so that a larger proportion of giving-up times would occur before  $S$  had elapsed. That is, such a system would produce a great many underestimates or "too short" errors as  $S$  is increased. In fact, subjects keep the proportion of these errors about the same, and lengthen the mean giving-up time linearly with the size of the standard.

An absolute discrepancy rule would be unlikely in any case, since it would have to entail an extremely small margin of error above  $S$  to accommodate the range of standards used here, and not involve missing more than one reinforcer. An alternative way to say this is that if subjects are guided by the amount of potential reinforcement missed, then the way to keep this at a minimum is to use a relative rule. Such a rule ensures that missing only one feeding when the standard is large does not result in missing many when the standard is small.

*Comparator variance*

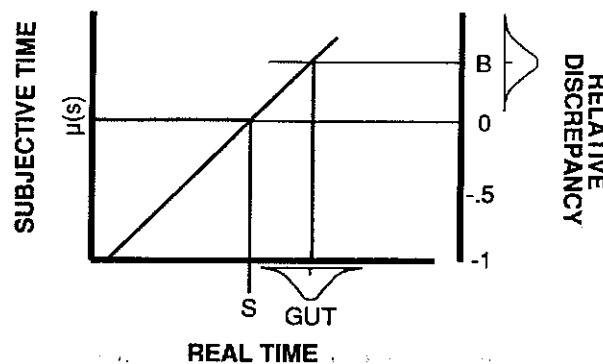
We argue, then, here as previously, that a relative comparator rule (equation (1)) is required for performance of this sort. Subjects must remember what the appropriate food delivery time is, and compare that memory with the current elapsed time in a manner that permits a given proportional, rather than absolute, discrepancy to be detected. Equivalently,

$$t \geq (1 + b)s. \quad (2)$$

In this form it is clear that when there is no variance in the appreciation of the current time ( $t \equiv T$ ), scalar variance in memory for  $S$  would directly translate (via the proportional constant,  $1 + b$ ) into scalar variance in the giving-up time distribution. This is illustrated in Figure 4. Varying the memory shifts the relative discrepancy (right ordinate) zero point, and thus also the time at which the threshold is met.

In the above form (2), however, it is clear that variable giving-up times might be produced by another source as well. Variance in the threshold,  $b$ , since it multiplies  $S$ , also would generate scalar giving-up time variance. This is illustrated in Figure 5, which depicts no variance in the memory for the target time, but variance in the threshold,  $b$ , around a mean,  $B$ .

Figure 5. *Distribution of giving-up times associated with threshold variance, but no memory variance.*



### Current time variance

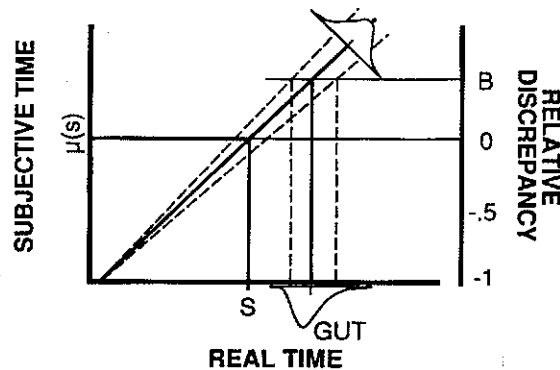
It is equally clear from equation (2) that one might ascribe all variability in giving-up time to the percept of the current time,  $t$ , rather than memory or threshold. For such a mechanism to be realistic, one would require that the translation of a reinforced current time into memory for the standard include an averaging mechanism, so that the noisy percept would vary from trial to trial, while the remembered (mean) value would not.

Figure 6 shows a system in which the accumulation of subjective time as a function of real time drifts from trial to trial. The representation of the remembered reinforced time,  $S$ , is now fixed on the subjective scale [ $\mu(S)$ ] and the threshold is fixed on the discrepancy scale ( $B$ ). Variability in the giving-up time distribution on the abscissa is induced solely by changes in the rate of accumulation of subjective time. Examples in which the current time appreciates faster than usual (higher slope line), or slower than usual (lower slope line), are indicated in the figure, and would correspond to early or late giving-up times in the abscissa distribution.

### Skew

This process translates symmetry in variability in the appreciation of current time (in this example normality in the rate of the clock) into asymmetry in the giving-up time distributions represented on the abscissa (an inverse Gaussian form). It is imported to recognize that the scalar property is pre-

Figure 6. Distribution of giving-up times associated with accumulator rate variance. Note the induced skew.



served in a system like this, even though skew is introduced into the performance measure. When rate varies as shown in Figure 6, more variation occurs for long times than for short times, but the scalar property evident in Figure 2 is still satisfied.

Since the data show some skew, there is a question whether they are better described by clock rate variance than memory or threshold variance. In general, we will argue that discriminations based on distribution shape alone are not strong inferences. There are at least three ways in which skew may be induced. First, rate may indeed vary symmetrically, inducing skew as in Figure 6. Second, as is evident from Figures 4 and 5, the shape of a memory-variance-only or threshold-variance-only distribution is translated directly into giving-up times, hence skew in the memory or in the threshold distributions also would translate into skew in the giving-up time distribution.

While it is often attractive to assume the action of many factors contributing to variability in laying down memories or in establishing a threshold, and hence normality at some level in these processes, it is equally likely that sub-components of these systems multiply rather than add, which brings us to a third reason for the weakness of inferences based on skew.

The third reason is that it is also readily demonstrated that symmetric normals acting multiplicatively generate a skewed product random variable (cf. Gibbon, Church, & Meck, 1984). Therefore, were *both* memory and decision variance operating in the giving-up time performance, the right side of equation (2) requires that these act multiplicatively, thus inducing some skew in the resultant giving-up time distribution. To compound this problem, note that multiplicative variance with right skew in either or both of the variates alone is enhanced in the product. For all of these reasons skew in a performance variable such as that seen in Figure 2, and indeed in many latency phenomena from a variety of literatures, is not yet diagnostic of an identifiable source.

#### *Psychophysical problem*

We arrive then at a classical psychophysical conundrum: which of several sources of influence contribute to performance, and to what extent? We have identified three such sources. We have noted that they produce somewhat differently shaped performance distributions, but we are unable to discriminate on that basis alone how each contributes.

We develop below a technique which discriminates much more powerfully the components of variance in performance based on remembered time. The technique follows in spirit the advance in classical psychophysics provided by obtaining latency as well as probability correct measures. In our context we will provide two measures quite different from latency and probability, but

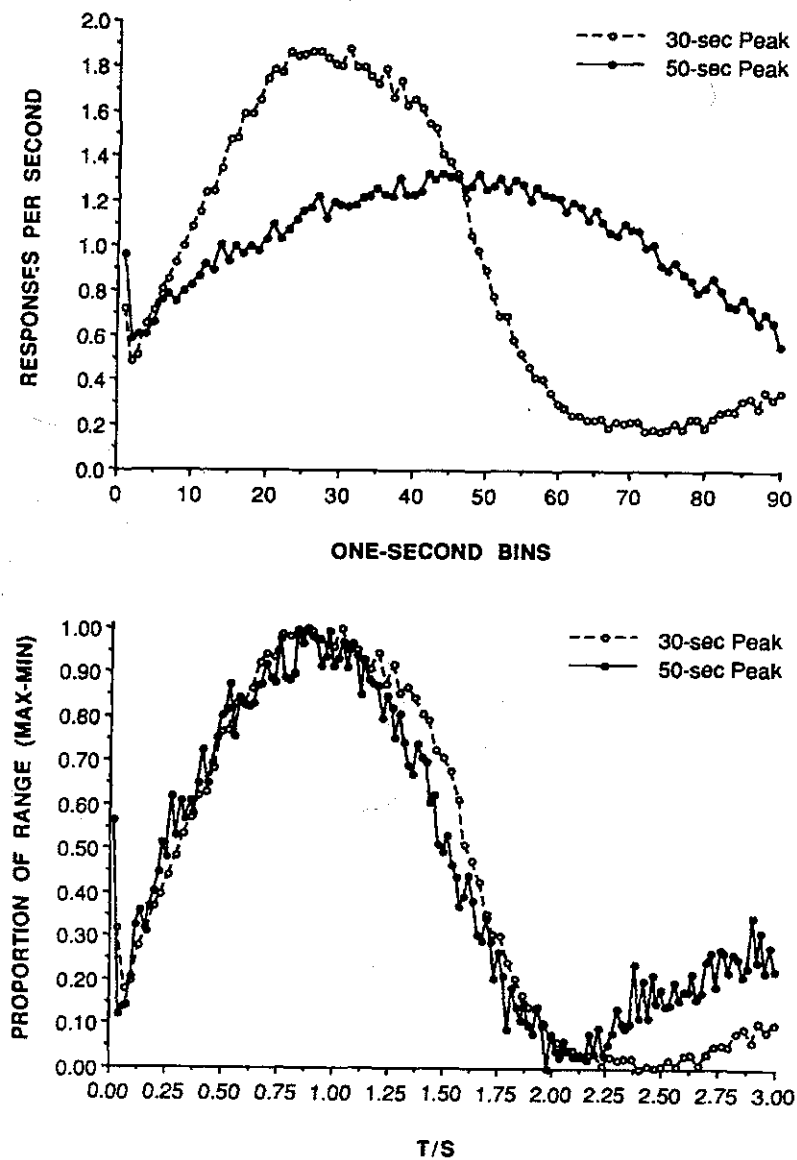
which share the feature that different patterns of these measures implicate different sources of control. The technique requires a trial-by-trial analysis of an alternative timing procedure, the peak procedure. This procedure has received considerable attention in other contexts, and models for peak performance provided the basis of Brunner's adaptation to giving-up times.

We will see that, just as in the analog of foraging, we can identify a giving-up time. We can also identify a "start" time based on expectation of food in the same trial. We will develop tests of trial-by-trial patterns expected when the primary components of variance are either response (comparator) based, or clock or memory based.

### **The peak procedure: Individual trial analysis**

The peak procedure is an extension of an operant fixed-interval schedule to discrete trial, partial reinforcement. It was devised by Catania (1970), and has since been used by Roberts (1981) and others to identify temporal expectation on both sides of a remembered time for food. The technique is simple and elegant. Discrete trials begin with the onset of a cue and terminate sometimes with response-produced food at a given criterion, standard time,  $S$ . On other trials, no food is given (peak trials), and the trial signal simply remains present for a long time beyond the food time. This is our reference procedure, and some typical data pooled over four pigeons is shown in Figure 7. The trial signal was the onset of a white key light, food was the delivery of a few seconds of grain, and the peak trials, when no food was given, lasted a minimum of  $3S$ , plus a random time averaging an additional  $S$  s. The data shown here were collected from peak trials from the last 4 days of two 3-week determinations at different standards,  $S = 30$  and  $S = 50$ . Responding rises to a peak near the criterion time for both functions, and declines in a roughly symmetric manner beyond this time. There is a small rise in both peak functions toward the end of the trial which probably reflects anticipation of its termination and the onset of the next trial (Church, Miller, Meck, & Gibbon, submitted). In the bottom panel the scalar property is shown for these data, with superposition of the peak functions when plotted as proportions of the mean. Again, the scalar property (a form of Weber's law in this setting) will be seen to force strong constraints on the kind of variance in percept and memory, and the kind of response rule used to generate performances of this sort.

Figure 7. Pooled peak functions for four subjects studied at 30 s and 50 s peak reinforcement times. The upper panel shows absolute rate functions, the lower panel shows the same functions as proportions of the reinforcement time,  $T/S$ . The lower ordinate has been scaled min. to max.

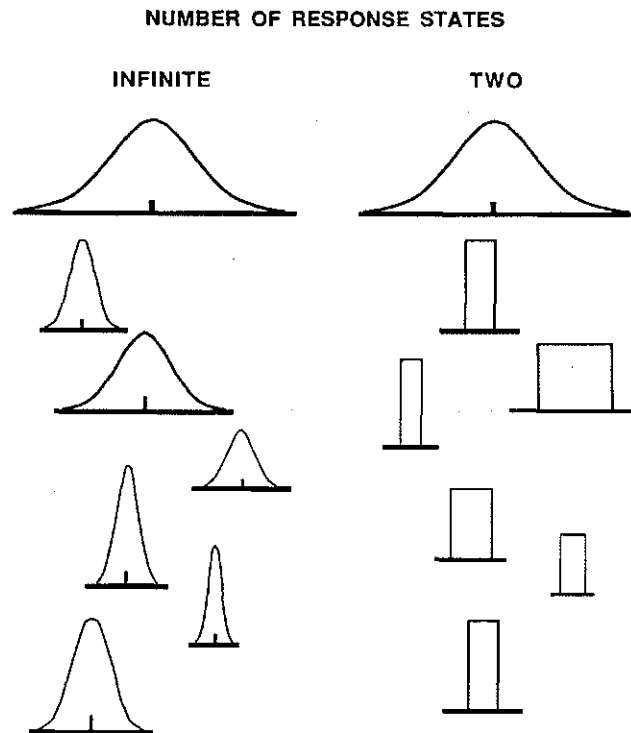


*Two states: Break-run-break*

The smooth, bell-shaped peak functions we see in this procedure might be achieved in at least two different ways: either with a smooth acceleration and deceleration on every trial, or with a discrete, two-state process, in which responding abruptly changes from a low rate to a high constant rate for a period of time around the time of reinforcement, and then abruptly falls to a low rate again. With variable locations for the high rate on different trials, averaging these discrete, two-state, individual trial functions would produce a smooth curve. This idea is an extension of Schneider's early seminal "break-run" analysis of fixed-interval performance (Schneider, 1969) to a break-run-break pattern appropriate to the peak procedure.

The two possibilities are sketched in Figure 8. Scalar timing theory, from

Figure 8. *Diagram of two possible alternatives generating smooth peak functions. In the left column the peak function is an average of smooth functions on individual trials. In the right-hand column it is an average of step functions on individual trials that vary in location and spread.*



its earliest analysis of this situation (Gibbon, 1977), assumed that the system worked in the two-state manner, but until now we had not tested the implications of this idea.

The smooth curves at the top of the figure might result from smaller, variably located, smooth rate functions as shown on the left, or from variations in start and stop times for a middle portion of the trial, representing a high, constant rate, as shown on the right. The two-state assumption is precisely the kind of assumption embodied in the giving-up time models (Figures 4–6) in that a discrete threshold crossing is associated with a “stop,” or giving-up time in the patch, when food is no longer expected. What is added for the peak procedure is that prior to each stop time there is a “start” time as well, when proximity is close enough to the expected time of food to warrant increased responding.

We present below a break–run–break analysis which distinguishes between these two kinds of mechanisms.<sup>1</sup> The analysis is applied to our illustrative data set from the four birds shown in Figure 7.

A least-squares regression program was developed to fit three horizontal line segments to the data from each trial, an extension of Schneider's (1969) “break–run” analysis. It permits us to average rate functions from individual trials when they are lined up so that either the start or stop break points coincide. Rates averaged forward and backward from the break point, both for the transition from low to high (start) and again from high to low (stop), are shown in the top panels of Figure 9. In the upper left panel the start times were lined up before taking the average; in the upper right, the stop times were lined up. The overall peak function is shown for reference in both panels.

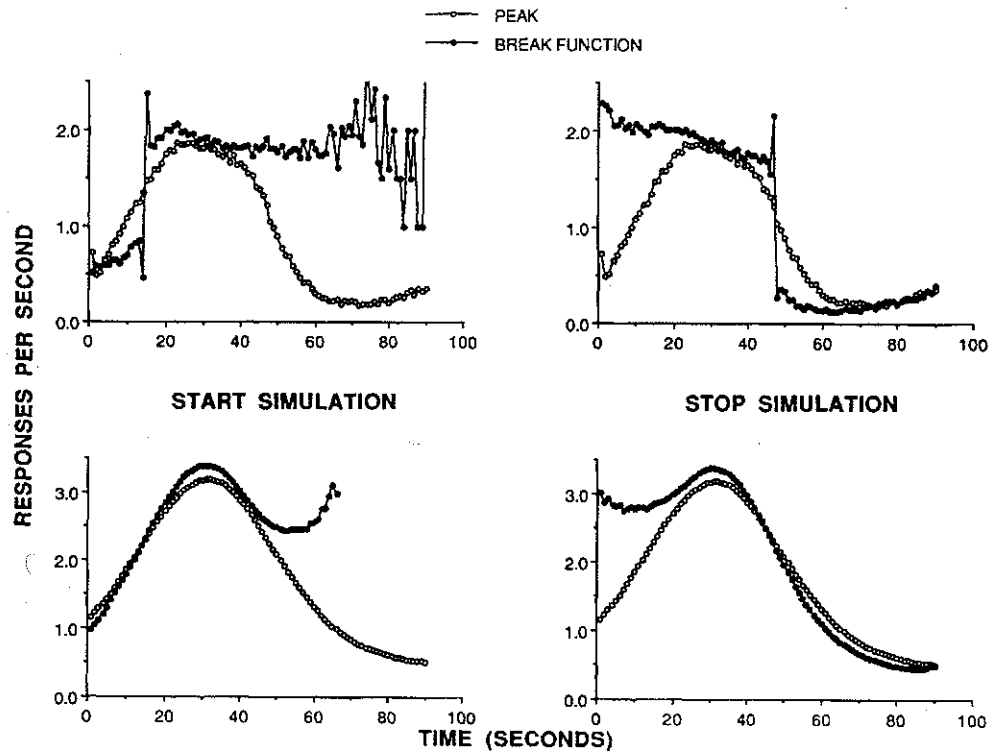
The position of the break points were set at their average position across trials. They may be seen to lie near the middle of the rising and falling wings of the average peak function. (The program tends to identify a large difference between adjacent time bins as a break point, and this accounts for the especially low and especially high values just before and just after the high state is entered, and vice versa when it is left.) Once responding has begun at a good clip, however, it remains roughly constant for a period of time.<sup>2</sup> The stop break function in the right-hand panel shows the reverse pattern.

These data in the upper two panels of Figure 9 are to be contrasted with

<sup>1</sup>A more complete analysis, including idiosyncratic subject differences and more quantitative detail, is in preparation.

<sup>2</sup>The start break function on the left becomes ragged at the right edge since very few high states last this long, and so the number of observations entering into the average is low for long times. The reverse is true for the stop break function on the right.

Figure 9. Mean break functions for the start of a two-state process (upper left), and the end, or stop, of the two-state process (upper right). They are positioned at the mean break position for these data. The lower two panels show continuous function simulations of these break patterns using Gaussian curve fits for individual trials. See text for details.



an extreme control analysis shown below. The control simulates smooth trial-by-trial curves like those on the left of Figure 8. The break functions in the lower panels were obtained using the fitting program on artificial "data" curves, which themselves were smooth, bell-shaped functions fit to the real trial-by-trial data. For each real data trial, we first fit a three-parameter bell-shaped curve to the data: a Gaussian error curve with mean, variance, and level as free parameters. We then subjected the Gaussian, not the data, to our break-run-break regression program. Thus this analysis asks whether the break point fitting program, when operating upon smooth continuous functions, produces a different form from that seen in the data.

The results are shown below the real data. The forward and backward average break functions are those with the rising right and left tails. The smooth bell-shaped functions are the overall averages of the Gaussian curves, comparable to the peak functions that would have been obtained if the data themselves were smooth Gaussian curves.

The simulation break functions share some features with the real data, but are clearly different in shape. The start break function shows a smooth rise with no abrupt break, to a peaked middle portion of the curve, unlike the real data. On the other hand, like the real data, beyond the peak the break function falls less steeply than the underlying average peak function. The late rise in the tail is due to the fact that the real data, upon which the Gaussian curves were based, showed considerable variability in location and spread of the high rate portion of each trial. Gaussians with very broad spread contribute predominantly in the right tail of the start break functions. The stop break functions show these same features in mirror image.

To a first approximation, we argue that the break-run-break pattern in the real data shows a two-state character not matched by our continuous simulation. A question currently under study is the degree to which the continuous alternative might approach the flat character of the data as noise is added to the instantaneous levels around a smooth, bell-shaped function.<sup>3</sup>

For our present purposes we will regard Figure 9 as justifying the following examination of patterns in start and stop times obtained through this analysis. We will see that the constraints these patterns place on several different models permit strong inferences about sources of variability contributing to the data.

#### *Scalar timing: Break-run-break*

As with giving-up, or stop times, we may define a start time for the peak procedure as that time in the trial at which the (positive) discrepancy crosses threshold before the target time is reached. The stop time is the analogous value, defined just as for giving-up times on the far side of the standard. This defines an absolute value discrepancy rule,

$$|(t - s)/s| \geq b \quad (3)$$

as a generalization of equation (1). The absolute value of the relative discrepancy function has the value 1.0 at the beginning of the trial, decreases linearly

<sup>3</sup>In an extreme case, in which the added noise swamps any underlying form in the bell-shaped curves, the break-run-break regression should fit horizontal line segments which do not differ in height, since the break points should then be a random sample from the time continuum.

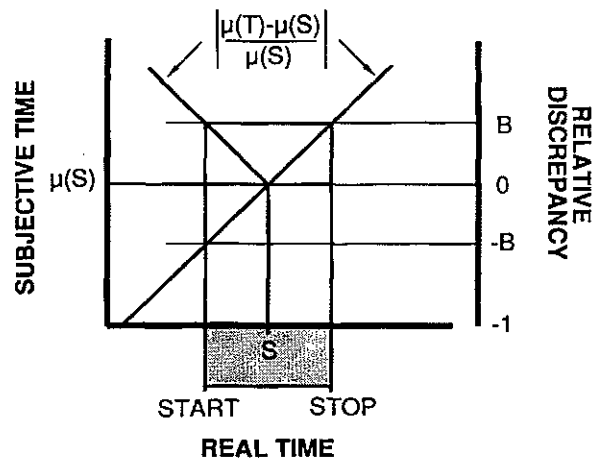
to zero as time in the trial approaches the reinforced time, and increases linearly beyond this time. The subject starts responding at a high rate when the discrepancy function falls below the threshold, and stops when it rises above the threshold on the far side of the reinforced time. That is, the high rate continues as long as the proximity of the current percept,  $t$ , to the target, remembered time,  $s$ , is less than the threshold fraction,  $b$ , of the target time.

#### *No variance*

The absolute value rule is illustrated in Figure 10, which shows the start and stop times associated with fixed values of memory and threshold, and no variance in the accumulation of clock time. The threshold crossing for the start time is shown both for the positive discrepancy,  $B$ , on the positive relative discrepancy function at the top, and for a negative threshold,  $-B$ , on the linear accumulator function for current time. No variance in the timing process results in a step function for response rate beginning at the same start time and ending at the same stop time on every trial (shown on the abscissa).

The smooth, bell-shaped peak functions we observe, of course, require variance in start and stop times, resulting in a variety of step functions which,

Figure 10. Threshold model for peak functions indicating start and stop times associated with the same threshold,  $B$ , and no variance.



when averaged, produce a smooth curve. However, we have seen that almost any source of variance is compatible with the average data. We will show below that this is not true for covariance patterns of trial-by-trial start and stop times.

The variance sources to be analyzed are just those already considered: memory or clock, and threshold. However, we now analyze cases in which different sources contribute variability simultaneously, and also successively, for the start and stop decision separately. For example, variability might be present in the representation of the target time over successive trials, but remain constant, though in error, within a trial (one sample case). Alternatively, memory might be accessed separately for the start and again for the stop decision within a trial (two-sample case). But the same is true for var-

Table 1. *Nine models*

		Threshold		
		No variance	Variance	
		0	1	2
Memory or clock	No variance	0	1 (0,0)	3 (0,1)
	Variance	1	2 (1,0)	4 (1,1)
		2	5 (2,0)	6 (2,1)
				7 (0,2)
				8 (1,2)
				9 (2,2)

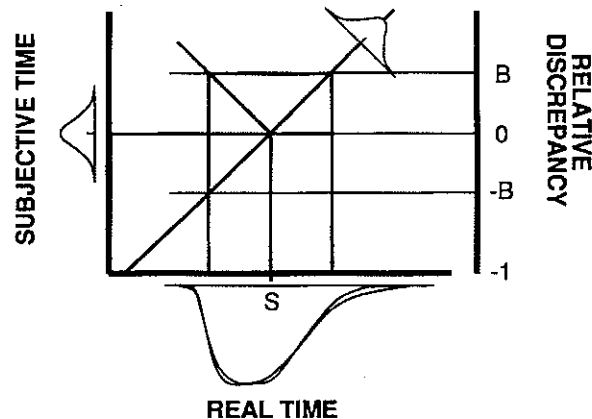
iance in threshold, so that combined with the fixed, no variance (zero sample) case, there are nine combinations, shown in Table 1. While the evaluation of nine different models may seem at first blush a formidable task, we will see that the strength of the correlational analysis is such as to render most of the inferences transparent.

The nine cells are numbered in the order in which the models will be considered. The first, with no variance from either source,  $1(0,0)$ , is the trivial case just considered (Figure 10), disqualified on the basis of the bell-shaped peak function. The other cases all involve one or more samples of random variables and hence all pass the simple test of the form of the peak function. We begin with memory or clock variance.

*2(1,0). Memory or clock variance, no threshold variance*

In Figure 11, we reproduce the schematic of Figure 10, but now indicate the kind of peak function produced by introduction of variability in either the memory (left ordinate), or the rate of accumulation of current time (slope). Trials in which memory overestimates the target time result in broader step functions than for underestimates, with the result that peak functions for memory variance show some right skew. But skew is induced by current time (slope) variance also, just as with giving-up times in our earlier analysis. In the figure the two functions lying nearly on top of each other on the lower ordinate represent the predicted shape of the peak func-

Figure 11. *Hypothetical peak functions. The two functions indicated on the abscissa reflect the predictions associated with normal variance in either the memory system (ordinate) or the rate of accumulation (diagonal function). Note the similarity between the functions.*



tion implied by variance of equal magnitude operating in either memory or current clock time alone. They have roughly the character of the data (Figure 7), with some skew, and again we will not attempt a discrimination based on shape.

It will be convenient in what follows to assume that the perceived accumulation of subjective time is veridical, proportional to real time, and examine the consequences of memory variance in the trial-by-trial pattern. It should be remembered that while we will speak of memory variance, we mean memory or clock variance, since these have the same trial-by-trial pattern. However, we will be able to draw a strong distinction between variance of any stripe in the memory (or clock) versus variance in threshold or decision processes. The distinctions will be forged by examining the trial-by-trial covariance pattern implicated by these two sources of variability. These patterns are independent of distribution shape.

Over a set of trials, then, start and stop times distribute, with means, variances, and covariances. It is useful to look at the variance/covariance patterns between start and stop, and also two derived measures, the "spread", the duration of the high state between start and stop, and the "middle", the arithmetic center of the high state. Covariance patterns amongst these four descriptors are quantitatively, and, we hope to illustrate intuitively, diagnostic for relative contribution to performance.

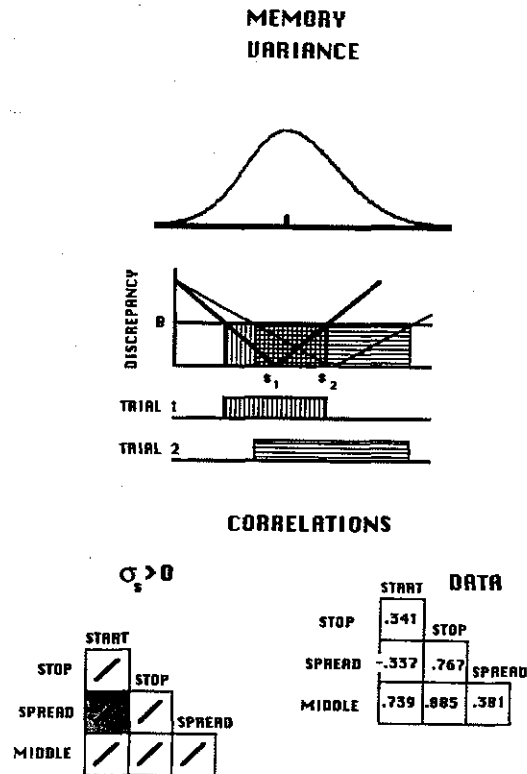
If noise in memory for the target reinforced time were the only contributor to variability, a pair of trials might look like those shown in the discrepancy diagram in Figure 12. (Recall that we are assuming proportionality in the clock, and hence variability in remembered time results in variance in the target value at which the discrepancy on the ordinate in Figure 12 is zero.) In this example, trial 1 has a somewhat shorter target time than trial two. Notice that this results in an earlier start time, and earlier stop time, and hence a shorter duration of responding on this trial as well.

This is shown graphically by the vertical and horizontal hatching for these two trials below the discrepancy diagram. The correlation pattern is immediately obvious. Start and stop times should be positively correlated, as should start time and spread. Since the relative discrepancy crosses threshold later for long target times than for short times, the system acts just as though there were in fact two different target times, and scalar timing theory requires that spread be proportional to target time. This is the scalar property that we see so dramatically when studying different target times experimentally across different conditions (e.g., Figure 7). Now we are in a position to ask whether this property holds *within* trials. When subjects overestimate the reinforced time, do they show a broader duration of the high state than when they underestimate? Below the diagram a qualitative schematic of correlations

between the start, stop, spread, and middle are shown. If memory is the only source of variance, all of these correlations should be positive. In particular, the start and the spread should be positively correlated as described above.

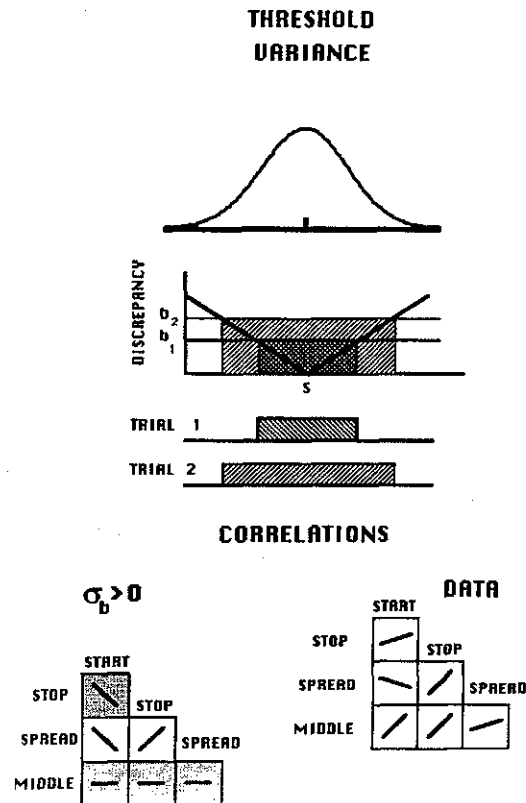
Correlations computed from the bird data (Figures 7 and 9) are presented next to this pattern, and we see that the correlations between all measures except start and spread are positive, as memory variance would imply. However, the start and spread measures are negatively correlated. (The box for

Figure 12. Example of two trials associated with two different samples from a variable memory. The spread is greater in trial two with a larger memory sample. The lower left panel shows linear diagrams for the predicted correlations between start, stop, spread, and middle. The corresponding real correlations from the data set of Figure 9 are shown on the right. Note the discrepancy between the start, spread prediction (positive) and the data ( $-0.337$ ).



this prediction is shaded, to indicate the discrepancy with the data pattern.) This means, intuitively, that subjects compensate for a late start by an early stop. It is as if they "knew" that they were closer than usual to the target time when they started their high state and used a strict criterion to stop. This is in fact the correlation expected for these two measures from threshold variance, studied next.

Figure 13. Example of two trials associated with two different samples from a variable threshold. The lower left panel shows linear diagrams for the predicted correlations. The data pattern is schematized on the right with three levels of slope, indicating high, moderate, or zero correlations. Note that without memory variance there is no correlation predicted between the middle and the other variables, and that the start and spread correlation is negative, as required by the data. However, the start and stop correlation is also negative, contrary to the data.



### 3(0,1). No memory variance, threshold variance

The pattern for threshold variance is shown in Figure 13. Again, two trials are indicated, produced by two different threshold levels on the relative discrepancy axis. Here the discrepancy function does not change location from trial to trial, and is shown decreasing toward zero at the mean remembered time,  $S$ , and increasing proportionally on the other side. Variance in threshold produces a pattern in which the center on all trials is located at the mean, but start and stop times are perfectly *negatively* correlated, with a late start inducing an early stop, and vice versa. Thus on trial 1, a tight threshold is adopted, and the response state is correspondingly short, while on trial 2, the threshold is more conservative, and responding begins earlier and ends later. The correlation pattern shown in diagram form below has zero correlation between the middle (constant) and the other measures, while start and stop, and start and spread, are negatively correlated. Spread and stop are the only positively correlated measures.

The data pattern is shown schematically next to the predicted pattern diagram. It reveals discrepancies in most of the measures (shaded boxes in the predicted pattern), with the important exception of the key negative correlation found in the data between start and spread. This correlation is an important, and new, index of timing patterns in the peak procedure, and we see that it implies some contribution from variation in the decision process to respond. The negative correlation at some level has been found for nearly every subject analyzed with our break-run-break program (more than 40) to date. The ubiquity of this negative correlation is important not only for its contribution to our thinking about threshold variance, but also because it helps to rule out some alternative timing processes considered later.

On the other hand, while a negative correlation between start and spread is implied by threshold variance – a late start predicts a short period of responding – one should also expect a negative correlation between start and stop – a late start predicts an early stop. The data we have analyzed are equally clear in showing a positive correlation between start and stop – a late start predicts a *late* stop – which is the pattern expected from memory variance.

We are then faced with the question of the relative control of the correlation pattern by two sources of variation, memory or clock on the one hand, and decision strategy on the other. The relative contribution of both variance sources dictates the degree to which the quantitative features of the actual data pattern may be accommodated. In fact, for some subjects there may be some strain between the data pattern and the theoretical account, even when both sources of variation are allowed, but with one sample per trial for each.

*4(1,1). Memory and threshold variance*

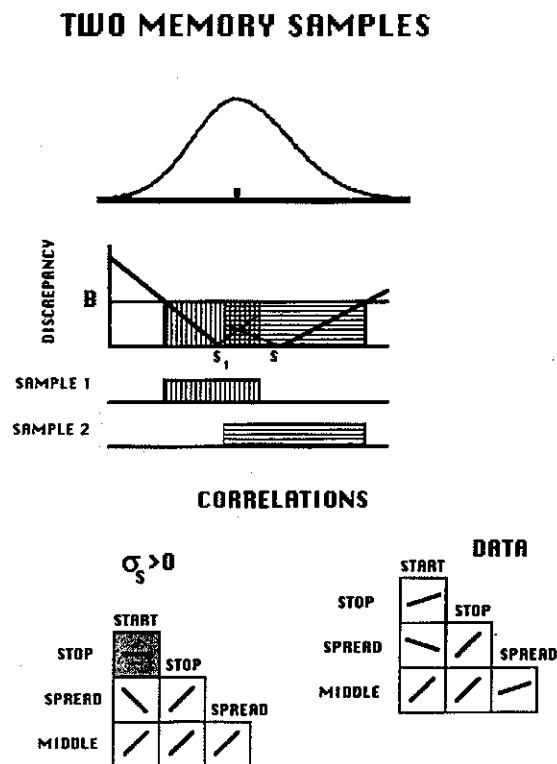
The data requires *both* the strong negative correlation between start and spread, and the positive correlation between start and stop. It may be shown that if subjects make but one decision (sample) about the proximity required for responding, and another decision about what the target time is on each trial, then the positive correlation between start and stop and negative correlation between start and spread directly trade. That is, if there is large variation in memory the correlation between start and stop is high positive and that between start and spread is at best low positive. Whereas if there is a large contribution of variance from threshold, the correlation between start and spread is high negative, and that between start and stop is at best low negative. Thus while group correlation patterns may be consonant with the pattern expected from one-sample variance from memory and threshold, individual subject's patterns may occasionally show some strain for this model, with unusually high positive correlations between start and stop, and start and middle – the pattern expected from memory variance – and unusually high negative correlations between start and spread – the pattern expected from threshold variance.

The models considered thus far assume but one random sample per trial, from memory or threshold or both, so that variation is at the level of successive trials. We consider now models assuming different samples associated with each response decision, to start and to stop responding fast.

*5(2,0). Two memory samples, threshold constant*

Consider first the pattern generated by two independent samples from memory, one for starting and another for stopping. The situation is shown schematically in Figure 14. The start decision is based on sample one, and the stop decision on sample two. With independent samples, any correlation between the two decisions is abolished. However, a negative correlation between start and spread is induced. In the figure, the target time associated with starting is shorter than usual, and it is clear that sample two, via regression to the mean, induces a longer spread than would be produced by sample one (and vice versa for a late start) so that this model fits the moderate negative correlation seen in the data between these two measures. The predicted correlation patterns are contrasted with the data pattern below, and it is clear that they are similar, except for the disqualifying exception of the zero correlation between start and stop.

Figure 14. Correlation pattern expected with two memory samples, one for start, and one for stop. The data pattern is reproduced on the lower right, and the predicted correlation patterns are shown in diagram form on the left. Note the zero correlation between start and stop in the predicted pattern, contrary to the data.



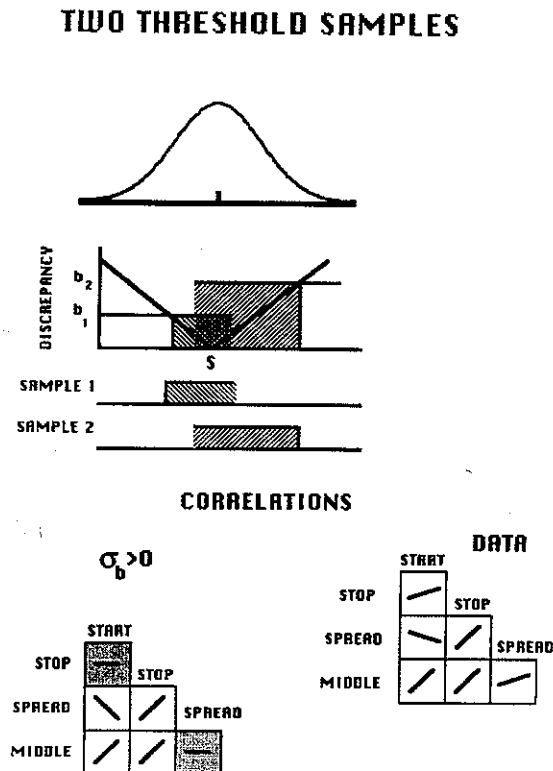
**6(2,1). Two memory samples, one threshold sample**

Adding variation in threshold level from trial to trial does not make the start, stop correlation positive. In fact, adding threshold variance turns that correlation somewhat negative. The threshold pattern in Figure 13, when added to two memory samples, exacerbates rather than alleviates the discrepancy with the positive start, stop correlation. Thus this modification does not accommodate the data pattern, and must be rejected.

7(0,2). Memory constant, two threshold samples

The alternative multiple sampling possibility, multiple thresholds, is schematized in Figure 15. Independence between a start and stop threshold results, as in the memory case, in a zero correlation between start and stop. However, the correlation between start and spread remains negative for the two-threshold model, for much the same reason that it is negative in the two-memory sample model. Regression toward the mean for the second sample ensures that unusually early (or late) starts are associated with stops closer to the average, hence with long (or short) spreads.

Figure 15. Correlation pattern associated with two threshold samples, one for start, and one for stop. The data pattern, as in previous figures, is on the right, and the predicted, two-sample pattern on the left. Note that, again, start and stop become uncorrelated with two samples.



This model must be rejected for the same reason as the two-memory sample model. It renders the start, stop correlation zero, while in the data this correlation is consistently positive. Another less critical feature perhaps of this correlation pattern is that the middle and spread become uncorrelated as well, and in the data these measures also remain somewhat positively correlated.

*8(1,2). One memory sample, two threshold samples*

A modification that is not disqualified, and indeed fits all of the data we have collected, is one in which two thresholds are assumed, one for starting and one for stopping, but only one memory sample is used per trial. Variation in this memory induces a positive coupling between start and stop, as expected from the memory pattern in Figure 12, and yet the negative correlation between start and spread is maintained, although, under some parameter choices, attenuated. This model suggests that the level of proximity which is good enough to start responding may not be the same level as that associated with stopping, and this accords with other qualitative features of the data. For example, it is usually the case that a sharp start onset time for the high state is learned earlier than a sharp stop time. The result is that early in training subjects show considerably more skew in their peak functions than late in training.

A second feature that this model accommodates is the fact that the start time is often closer to the target-reinforced time than is the stop. While two thresholds are not required to accommodate this kind of a finding, they do so quite naturally, and they may accord with differential costs for late starts versus late stops. Starting late simply means that reinforcement sometimes does not occur quite as soon as it might otherwise have done. Stopping early, however, might be more costly in that reinforcement could be missed altogether if no more responding occurs on a to-be-reinforced trial.

*9(2,2). Two memory samples, two threshold samples*

For completeness, we note that a model in which there are start and stop memory samples, and start and stop threshold samples, is also ruled out by our data. It is clear from the above analysis that a system in which the task is completely redefined for starting and again for stopping eliminates the correlation between start and stop, as do the two memory (5) or two-threshold (7) sample models.

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## Conclusion

This completes our conceptual analysis of covariance patterns associated with memory and decision variables underlying timing performance. We have shown that strong inferences eliminating several kinds of models may be made by examining the correlation pattern between start and stop in a two-state analysis of performance in the peak procedure. The key features of the data pattern that discriminate amongst several models are:

- (1) a positive correlation between start and stop times;
- (2) a negative correlation between start and spread times, that is, between the time that the high state begins and its duration; and
- (3) a positive correlation between the middle of the high state and its duration.

Of the nine possible scalar timing models, this data pattern disqualifies all but two: those with one memory sample and either one- or two-threshold samples.

Our analysis is summarized in Table 2. Successive rows correspond to the numbering in the cells of Table 1, in order of increasing number of random variables per trial. Models 4 and 8, with one memory sample, are the only survivors of the stringent test imposed by the three key data correlations listed above. The positive correlation between start and stop requires some

Table 2.

No. of independent random samples	Model no. (Table 1)	Models		Disqualified (X), or not (✓)	Data
		Memory/clock	Threshold		Start, stop, spread disqualification
0, no variance	1(0,0)	Fixed: $\mu(S)$	Fixed: $B$	X	$\sigma^2(\text{start}) > 0$ $\sigma^2(\text{stop}) > 0$
1 sample	2(1,0)	Variable: $s$	Fixed: $B$	X	$\rho(\text{start, spread}) < 0$
	3(0,1)	Fixed: $\mu(S)$	Variable: $b$	X	$\rho(\text{start, stop}) > 0$
2 samples	4(1,1)	Variable: $s$	Variable: $b$	✓(?)	$\rho(\text{start, stop}) \rightarrow +1$ , $\rho(\text{start, spread}) \rightarrow -1$
	5(2,0)	Variable: $s_1, s_2$	Fixed: $B$	X	$\rho(\text{start, stop}) > 0$
3 samples	6(0,2)	Fixed: $\mu(S)$	Variable: $b_1, b_2$	X	$\rho(\text{start, stop}) > 0$
	7(2,1)	Variable: $s_1, s_2$	Variable: $b$	X	$\rho(\text{start, stop}) > 0$
	8(1,2)	Variable: $s$	Variable: $b_1, b_2$	✓	NONE
4 samples	9(2,2)	Variable: $s_1, s_2$	Variable: $b_1, b_2$	X	$\rho(\text{start, stop}) > 0$

variance in memory for the target time, but this memory cannot be assessed independently for the start and stop decision.

Threshold variance is required as well, to accommodate the negative correlation between start and spread. This finding is new, and especially noteworthy as it is in direct opposition to the scalar property seen across different reinforcement time conditions. When reinforcement time is changed, the scalar increase in memory variance dominates the spread of the average peak function. But within a single reinforcement time condition, memory and threshold variance act in opposition: sufficient threshold variability induces a negative correlation between start and spread, while still permitting a positive correlation between start and stop.

The flexibility of the surviving models is of course bought at some cost, since additional assumptions ineluctably lead to additional parameters. In the three-sample case (model 8), we require different mean start thresholds and stop thresholds, although not different variances necessarily. On the other hand, such flexibility we believe is cheap enough considering the power with which the account excludes some otherwise reasonable alternatives, such as the two memory sample models. Indeed, it suggests to us that a considerable advance in our thinking about temporal memory will be effected by procedures which experimentally explore changing correlation patterns associated with changing task demands that differentially manipulate memory and decision variables.

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