## CRITICAL DISCUSSION

# Against the Modal Argument 

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#### Abstract

The relationship between alethic modality and indeterminacy is yet to be clarified. A modal argument-an argument that appeals to alethic modalityagainst vague objects given by Joseph Moore offers a potential clarification of the relationship; it is proposed that there are cases for which the following holds: if it is indeterminate whether $\mathrm{A}=\mathrm{B}$ then it is possible that it is determinate that $A=B$. However, the argument faces three problems. The problems remove the argument's threat against vague objects and prompt a fuller scrutiny of Moore's proposed relationship between alethic modality and indeterminacy. Such a scrutiny offers valuable lessons concerning the justification for claims of indeterminate identity, appeals to identity principles in contexts involving both alethic modality and indeterminacy, and how to identify the form of Gareth Evans's argument against vague objects in other arguments.


Keywords Vague objects • Alethic modality • Problem of the many • Benacerraf's puzzle • Identity

## 1 Introduction

Joseph Moore has given what he calls a modal argument against vague objects, ${ }^{1}$ so called because some of its premises contain the alethic modal operators of necessity

[^0]and possibility and because the argument is a reductio of the assumption that it is indeterminate whether $A=C$ because $A$ is a vague object. ${ }^{2}$

The modal argument is designed for a specific context. So it is not an argument against all brands of vague objects-just those that some ${ }^{3}$ claim are found in the specified context. That context is situations in which it is indeterminate whether $A$ is identical to $B$, indeterminate whether $A$ is identical to $C$, and determinate that $B$ is distinct from $C .{ }^{4}$ Examples of such situations include instances of Peter Unger's Problem of the Many ${ }^{5}$ and Benacerraf's problem of what numbers are. ${ }^{6}$

After stating Moore's argument in Sect. 2 I present three counter-arguments in Sect. 3. The first counter-argument (Sect. 3.1) pertains to justification; Moore claims that the only justification for it being indeterminate whether $A=B$ is that it is possible that $A \neq B$. But a different justification is available. The second counterargument (Sect. 3.2) pertains to logic; Moore relies upon an implicit use of Leibniz's Law and its contrapositive ('identity principles'). But a crucial step in the argument in which these identity principles are used is fallacious, since it relies upon an illegitimate shift in the range of quantifiers. The third counter-argument (Sect 3.3) pertains to metaphysics; it applies my take on Jonathan Lowe's response to Evans's argument against vague objects (and the argument against noncontingent identity) to the modal argument. The three counter-arguments illuminate some valuable lessons that ought to be acknowledged if there is to be a feasible interaction between alethic modality and indeterminacy logic or modality.

I argue not for vague objects but on behalf of those that do, endorsing the first two counter-arguments, but remaining agnostic over the third. I present the third in order to make a dialectical point: that the modal argument is analogous to Evans's argument against vague objects (and to the argument against contingently identical objects). Moore claims that it is not but, since my take on Lowe's response applies to the modal argument, the modal argument has sufficient structural similarity to indicate that it is analogous.

I conclude that, contra Moore, the modal argument is no more persuasive an argument against vague objects than Evans's argument is against vague objects and than the argument against contingently identical objects is against the existence of objects which are contingently identical.

[^1]
## 2 The Modal Argument

We begin with two oversimplified examples. Consider Mount Kilimanjaro and Sparky, an electron hovering at the periphery of the mountain. Since Sparky is neither inside nor not outside the periphery, Sparky is questionably part of Kilimanjaro and it is indeterminate whether Sparky is part of Kilimanjaro. However, it is determinate that Sparky is part of a mountain-for the body of land that is composed by Sparky and Kilimanjaro without questionable parts fulfills the sufficient conditions for being a mountain. ${ }^{7}$ Let us call this body of land $\mathrm{K}^{+}$. There are other bodies of land very similar to $\mathrm{K}^{+}$in the vicinity of Kilimanjaro-in particular, there is a body of land not composed by Sparky and just composed by Kilimanjaro without questionable parts. Just like $\mathrm{K}^{+}$, this body of land also fulfills the sufficient conditions for being a mountain. Let us call this body of land $\mathrm{K}^{-}$.

Both $\mathrm{K}^{+}$and $\mathrm{K}^{-}$are equally good candidates for being Kilimanjaro. Yet Kilimanjaro cannot be both. For if it were then, by the transitivity of identity, $\mathrm{K}^{+}$ and $\mathrm{K}^{-}$are the same object. But this contravenes Leibniz's Law ${ }^{8}$ (and mereological extensionality) since $\mathrm{K}^{+}$and $\mathrm{K}^{-}$do not share all their properties (and do not share all their parts). In addition, it contravenes the law of non-contradiction since Sparky would be and not be a part of Kilimanjaro. So Kilimanjaro cannot be both $\mathrm{K}^{+}$and $\mathrm{K}^{-}$.

Nor could Kilimanjaro be neither. For $\mathrm{K}^{+}$and $\mathrm{K}^{-}$are not the only candidates for being Kilimanjaro-there are many other candidates, all of which are located in the vicinity of Kilimanjaro. Every candidate differs from another by at least one part. Some candidates differ more than others. All other candidates are equal to $\mathrm{K}^{+}$and $\mathrm{K}^{-}$in respect of $\mathrm{K}^{+}$,s and $\mathrm{K}^{-}$'s candidacy for being Kilimanjaro. Hence if $\mathrm{K}^{+}$and $\mathrm{K}^{-}$are not Kilimanjaro, then neither are the other candidates. But $\mathrm{K}^{+}, \mathrm{K}^{-}$, and the other candidates exhaust the potential candidates for being Kilimanjaro. So if none of them were Kilimanjaro, then there is no mountain in the vicinity of Kilimanjaro and Kilimanjaro is not a mountain. Then either the geomorphologists are wrong or it is not the case that Kilimanjaro exists. So Kilimanjaro cannot be neither $\mathrm{K}^{+}$nor $\mathrm{K}^{-}$. Let us call this case the Problem of the Many case.

Now consider Benacerraf's puzzle concerning the identity of the natural numbers. ${ }^{9}$ Consider the number 2. Is it the Zermelo ordinal $\{\{\varnothing\}\}$ or the von Neumann ordinal $\{\varnothing,\{\varnothing\}\}$ ? There is no further information that would determine one ordinal over the other. Both are equally good candidates.

Yet 2 cannot be both. For if it were then, by the transitivity of identity, we would be committed to $\{\{\varnothing\}\}=\{\varnothing,\{\varnothing\}\}$. And this contravenes the axiom of extensionality since $\{\{\varnothing\}\}$ and $\{\varnothing,\{\varnothing\}\}$ have different elements. In addition, it contravenes the law of non-contradiction since 2 would both be and not be a singleton. So 2 cannot be both $\{\{\varnothing\}\}$ and $\{\varnothing,\{\varnothing\}\}$.

[^2]Nor could 2 be neither. For current mathematical practice strongly favours the view that numbers are sets. And there are other set-theoretical construals of numbers that fulfill the sufficient conditions for being the numbers laid down by current mathematical practice. If we discount these Z and VN ordinals as numbers then we discount other equally plausible set-theoretical construals of numbers, such one where 1 is $\{\{\varnothing\}\}$ and 2 is $\{\{\{\{\varnothing\}\}\}$ or one where 1 is $\{\varnothing,\{\varnothing\}\}$ and 2 is $\{\varnothing,\{\varnothing,\{\varnothing\}\}\}$. Since all these exhaust the set-theoretical construals of numbers, numbers would not be sets. So then either current mathematical practice is wrong or numbers do not exist. ${ }^{10}$ So 2 cannot be neither $\{\{\varnothing\}\}$ nor $\{\varnothing,\{\varnothing\}\}$. Let us call this case the Benacerraf case. ${ }^{11}$

There are several methods of response to the Problem of the Many and Benacerraf cases. Many respondents concede that the first case illustrates that it is indeterminate whether Kilimanjaro is $\mathrm{K}^{+}$and that it is indeterminate whether Kilimanjaro is $\mathrm{K}^{-}$and concede that the second case illustrates that it is indeterminate whether 2 is $\{\{\varnothing\}\}$ and that it is indeterminate whether 2 is $\{\varnothing,\{\varnothing\}\} .{ }^{12}$ For the Problem of the Many case, it is not determinate that Kilimanjaro is $\mathrm{K}^{+}$and it is not determinate that Kilimanjaro is not $\mathrm{K}^{+}$. So it is indeterminate whether Kilimanjaro is $\mathrm{K}^{+}$. Additionally, it is not determinate that Kilimanjaro is $\mathrm{K}^{-}$and it is not determinate that Kilimanjaro is not $\mathrm{K}^{-}$. So it is also indeterminate whether Kilimanjaro is $\mathrm{K}^{-}$. For the Benacerraf case, it is not determinate that 2 is $\{\{\varnothing\}\}$ and it is not determinate that 2 is not $\{\{\varnothing\}\}$. So it is indeterminate whether 2 is $\{\{\varnothing\}\}$. Also, it is not determinate that 2 is $\{\varnothing,\{\varnothing\}\}$ and it is not determinate that 2 is not $\{\varnothing,\{\varnothing\}\}$. So it is also indeterminate whether 2 is $\{\varnothing,\{\varnothing\}\} .{ }^{13}$

Indeterminate identity provides an obvious comparison between the two cases, revealing a salient similarity. In both cases there is an object $A$ (Kilimanjaro, 2) which is indeterminately identical to two other objects $B$ and $C\left(\mathrm{~K}^{+},\{\{\varnothing\}\}\right.$ and $\left.\mathrm{K}^{-},\{\varnothing,\{\varnothing\}\}\right)$. That there is such a similarity is a theoretical asset for it justifies an economy of theory; only one solution need apply.

In both cases it is determinate that $B$ is distinct from $C$. Indeed, under assumption, it is necessary that $B$ is distinct from $C$. In the Benacerraf case this is obvious if we assume that, since sets are abstract objects, if a pure set exists then it is necessary that the pure set exists. If $\{\{\varnothing\}\}$ and $\{\varnothing,\{\varnothing\}\}$ exist in every possible world then each retain their elements across worlds. So in every possible world they are distinct from each other. That it is necessary that $B$ and $C$ are distinct in the Problem of the Many case is more controversial and I will not argue for that here. ${ }^{14}$ Let us assume their necessary distinctness in accordance with Moore.

[^3]Fig. 1 Problem of the many structure


Fig. 2 Benacerraf puzzle structure

For expository purposes I give three diagrams below to display the similarities between the two problems where I use ' $\boldsymbol{\nabla}$ ' for 'it is indeterminate whether' and ' $\boldsymbol{\nabla}$ ' for 'it is determinate that'.

Both cases present a problem: which is $A ; B$ or $C$ ? One response is to claim that $A$ is a vague object. The postulation of vague objects has had its objectors in the past, most prominently Gareth Evans ${ }^{15}$ and Nathan Salmon, ${ }^{16}$ but until recently no objector has specifically criticised the postulation of vague objects in order to respond to the sort of cases we have been considering. Joseph Moore presents a modal argument that is specifically designed against the postulation of vague objects as a response to these cases. He claims that

The modal argument against vague objects is different from those of Evans, Salmon, and others, and to my mind more convincing.

Moore (2008: 15).
If Moore is right then the postulator of vague objects needs a new defence of his position against the modal argument in order to retain the tenability of his position.

I show that there are two problems with the modal argument and thus that the postulator need not defend. Even if he did, then the application of a third problem reveals that his defence would not need to be new. For, since the third problem reveals a close structural similarity between the modal argument and Evans's argument against vague objects and the argument against contingent identity, a response from the postulator of vague objects to the latter two arguments is a response to the former argument.

Let us begin our examination of the modal argument by acknowledging those assumptions that Moore explicitly makes and by providing some justification for them.

[^4]
### 2.1 Assumptions

We have already seen two of Moore's central assumptions for the modal argument;
(Necessary Distinctness) $B$ and $C$ are necessarily distinct.
We saw some justification for this above. So too did we for:
(Democracy) $B$ and $C$ are equally suitable candidates for $A$.
(Democracy) is self-evident from examination of the two cases; the different properties that $\mathrm{K}^{+}(\{\{\varnothing\}\})$ and $\mathrm{K}^{-}(\{\varnothing,\{\varnothing\}\})$ instantiate makes no difference as to which is more appropriate to be Kilimanjaro (2). The next assumption is a plausible assessment of the truth-value of ' $A=B$ ' and ' $A=C$ ';
(Lack of Det. T-Value) The statements ' $A=B$ ' and ' $A=C$ ' lack determinate truth-value.

We have already noted that it is indeterminate whether $A=B$ and that it is indeterminate whether $A=C$. In accordance with the assumptions of the modal argument, we assume that $A$ is a vague object. If the vague object $A$ is responsible for the aforementioned indeterminate identities between $A$ and $B$ and $A$ and $C$ then it is plausible that it prevents an ascription of truth to ' $A=B$ ' and to ' $A=C$ ' and plausible that it prevents an ascription of falsity to ' $A=B$ ' and to ' $A=C$ '. (If a non-vague world enables ascriptions of truth and of falsity, then a vague world prevents them.) If ' $A=B$ ' and ' $A=C$ ' are true then they have a determinate truthvalue. If ' $A=B$ ' and ' $A=C$ ' are false then they have a determinate truth-value. Truth and falsity exhaust the determinate truth-values. So ' $A=B$ ' and ' $A=C$ ' lack determinate truth-value.

There are two further assumptions:
(Parsimony) Ceteris paribus, $A$ is of the same kind as $B$ and $C .{ }^{17}$
(Referential Determinacy) Naming expression ' $A$ ' determinately (and rigidly) has a sole referent; ' $B$ ' determinately has a sole referent, and so does ' $C$ '.
(Parsimony) is disguised in our talk of equally good candidates for being $A$ and sufficient conditions for being $A . \mathrm{K}^{+}$and $\mathrm{K}^{-}$are equally good candidates for being Kilimanjaro because they both fulfill the sufficient conditions for being Kilimanjaro. They both fulfill the sufficient conditions because Kilimanjaro is the same kind of entity as $\mathrm{K}^{+}$and $\mathrm{K}^{-} ; \mathrm{K}^{+}$and $\mathrm{K}^{-}$are each a collection of mountain-stuff-rock and dirt-in the vicinity of Kilimanjaro and Kilimanjaro is a collection of mountainstuff in its own vicinity. $\{\{\varnothing\}\}$ and $\{\varnothing,\{\varnothing\}\}$ are equally good candidates for being 2 because they both fulfill the sufficient conditions for being 2 . They both fulfill the sufficient conditions because 2 is (strongly suggested by mathematics to be) the same kind of entity as $\{\{\varnothing\}\}$ and $\{\varnothing,\{\varnothing\}\} ;\{\{\varnothing\}\}$ and $\{\varnothing,\{\varnothing\}\}$ are each a set and a second successor and 2 is (strongly suggested by mathematics to be) a set and a second successor. ${ }^{18}$

[^5]There are two purposes of assuming (Referential Determinacy). One is to ensure that the assumption that $A$ is a vague object does not get conflated with the claim that it is indeterminate whether ' $A$ ' refers to the non-vague $B$ or $C$. It is determinate whether a name refers if the name refers to a unique object. It is indeterminate whether a name refers if the name fails to refer to a unique object and the name is not empty. A conflation needs to be avoided in order to preclude the results of the modal argument from influencing whether there is indeterminacy of reference and to preclude any response to the argument that would appeal to referential indeterminacy. The modal argument is not against the indeterminacy of reference. Nor is it against the claim that vague objects and indeterminate reference are incompatible. ${ }^{19}$ It is against the claim that the ontological nature of the vague object $A$ is solely responsible for it being indeterminate whether $A=B$ and for it being indeterminate whether $A=C$.

The second purpose of assuming (Referential Determinacy) is also to avoid a conflation. It is to ensure that the assumption that ' $A$ ' determinately refers to the vague object $A$ does not get conflated with the claim that ' $A$ ' does not rigidly refer to the non-vague $B$ or $C$. A name is rigid if it refers to the same object in all possible worlds in which it exists. Otherwise a name is non-rigid. To assume that $A$ is a vague object is not to assume that ' $A$ ' is a non-rigid designator; ' 2 ' is rigid whether or not 2 is vague. The modal argument is not against non-rigid designators. Nor is it against the claim that vague objects and non-rigid designators are incompatible.

We now have sufficient information to examine the modal argument. I reconstruct the argument in two stages and stay as faithful to Moore's presentation as clarity permits. The only alterations involve adding the notions of determinacy and indeterminacy to parts of the argument in order to maximise explicitness.

I state the argument in its entirety in a premise-conclusion form as a reductio ad absurdum but will formalise a section of the second stage when I come to criticise it in Sect. 3.2. First, though, let us begin by considering a stripped-down rendering of the modal argument;
(P1) $A$ is a vague object, it is indeterminate whether $A=B$, and it is indeterminate whether $A=C$.
(P2) If it is indeterminate whether $A=B$ then it is possible that $A=B$ and that $A$ and $B$ can exist without $C$.
(AC) It is possible that $A=B$ and that $A$ and $B$ exist without $C$.
[modus ponens on (P2) with (P1)]
(ND) It is necessary that $B \neq C$.
[Necessary Distinctness]
(PN) If it is possible that $A=B$ and $A$ and $B$ exist without $C$ then it is necessary that it is determinate that $A \neq C$. [Leibniz's Law on (AC) with (ND)]
$\left(^{*}\right)$ It is determinate whether $A \neq C$ and it is indeterminate whether $A=C$. [modus ponens on (PN) with (AC) and Conj. Intro. on conjunct of (P1)]

[^6]Justification for some of the moves in the stripped-down argument above are missing. These are supplied in the next two subsections where the argument is divided into two stages (in accordance with Moore's presentation). The first stage involves the moves from ( P 1 ) (the response of the postulator of vague objects to the Problem of the Many and Benacerraf cases) to (AC); the most pertinent information here is the justification for (P2). The second stage involves the moves from (ND) to $(*)$; the most pertinent information here is the move from (PN) to $\left({ }^{*}\right) .{ }^{20}$

Let us now examine the first stage of the argument. I use ' $\square$ ' and ' $\diamond$ ' in the usual manner as operators that express the alethic modal notions of necessity and possibility respectively.

### 2.2 First Stage

(P1) $A$ is a vague object, it is indeterminate whether $A=B$, and it is indeterminate whether $A=C$.
(2) If it is not possible that it is determinate that $A=B$ then we have no justification for the claim that it is indeterminate whether $A=B$.
(3) We have justification for the claim that it is indeterminate whether $A=B$.
[from Parsimony and Democracy]
(4) It is possible that it is determinate that $A=B$.
[modus tollens on (2) with (3)]
(5) Either it is possible that $A$ and $B$ exist without $C$ or it is not possible.
[from the Law of Excluded Middle]
(6) If it isn't possible that $A$ and $B$ exist without $C$ then it is necessary that if $A$ and $B$ exist then so does $C$.
$[$ from $\neg \diamond \neg \phi \equiv \square \phi]$
(7) If it is necessary that if $A$ and $B$ exist then so does $C$ then we can make no sense of the claim that if $C$ didn't exist then $A$ would be $B .^{21}$
(8) If we can make no sense of the claim that if $C$ didn't exist then $A$ would be $B$ then we have no justification for the claim that it is indeterminate whether $A=B$.
(9) If it isn't possible that $A$ and $B$ exist without $C$ then we have no justification for the claim that it is indeterminate whether $A=B$.
[Hypothetical Syllogism on (6), (7), and (8)]
(10) It is possible that $A$ and $B$ exist without $C$. [modus tollens on (9) with (3)]
(P2) If it is indeterminate whether $A=B$ then it is possible that $A=B$ and that $A$ and $B$ can exist without $C$.
[cond. intro. on conjunct of (P1) and the conj. intro of (4) and (10)]

[^7](AC) It is possible that $A=B$ and that $A$ and $B$ exist without $C$.
[modus ponens on (P2) with (P1)]
(AC) is an auxiliary premise for the following second stage of the argument.

### 2.3 Second Stage

(ND) It is necessary that $B \neq C$.
[Necessary Distinctness]
(11) It is possible that (it is determinate that $A=B$ ) and that $A$ and $B$ exist without $C$ and $B \neq C$. [Conj. Intro. on (AC) and with $a \square \phi \rightarrow \diamond \phi$ on (ND)]
(PN) If it is possible that $A=B$ and $A$ and $B$ exist without $C$ then it is necessary that it is determinate whether $A \neq C$. [Leibniz's Law on (AC) with (ND)]
(12) It is necessary that it is determinate whether $A \neq C$.
[modus ponens on (PN) with (11)]
(*) It is actual that it is indeterminate whether $A=C$ and determinate whether $A=C$.
[Conj. Intro on conjunct of (P1) and (12)]
As it stands, the contradiction in (*) isn't derivable from (P1) until we assume that (P1) is true in the actual world. Both the Problem of the Many case and the Benacerraf case hold in the actual world, so (P1) is true in the actual world and $\left({ }^{*}\right)$ is derivable. If the postulator of vague objects wants to retain the claim that it is indeterminate whether $A=C$ and abide by Moore's five assumptions then he needs to deny at least one of the premises or inferences in order to block the argument.

Note that two worlds are crucial in Moore's argument. There is the actual world and the world that exists in virtue of making (AC) true. Moore claims that in the latter world the postulator of vague objects will find it true that
it is determinate that $A=B$ and that $A$ and $B$ exist without $C$.
Let us call this world $w_{1}$. Moore claims that it is determinate that $A=B$ and that $B \neq C$ in $w_{1}$.

Let us now consider criticism of the modal argument.

## 3 Criticism

Moore considers three potential objections to his argument;

- To deny (2); that our justification for the claim that it is indeterminate whether $A=B$ is that it is possible that $A=B$.
- To claim that there is an equivocation between epistemic and metaphysical possibility.
- To find fault with the inferences involving alethic modality in the second stage of the argument.

Moore responds to all three potential objections;

- Moore grants that (2) may not have general applicability to all situations in which it is indeterminate whether $x=y$, but challenges an objector to give an
alternative justification for the claim that it is indeterminate whether $A=B$ in the context of the Problem of the Many case and the Benacerraf case. ${ }^{22}$
- Moore demonstrates a distinction between epistemic and metaphysical possibility and contrasts which modal sentences of the latter the postulator of vague objects endorses with which modal sentences of the former the postulator does not endorse. ${ }^{23}$
- Moore employs a counterpart-theoretic analysis of $A, B$, and $C$ to closely examine those inferences that involve alethic modality which may be contested. He concludes that such an analysis conflicts with it being indeterminate whether $A=C$. ${ }^{24}$

I present a counter-argument that rejects (2) by meeting Moore's challenge to provide alternative justification (Sect. 3.1). Unlike the justification that Moore appeals to, the justification I provide does not appeal to modality. I also present a counter-argument that rejects (PN) due to a fallacious shift in the domain of the universal quantifier in Leibniz's Law (Sect. 3.2). Finally, I show that there is a metaphysical counter-argument available to the postulator of vague objects. This also blocks (PN) for a different reason. A similar objection has been given before in the literature by Jonathan Lowe ${ }^{25}$-I offer a comparison which sheds light on the putative novelty of the modal argument (Sect. 3.3).

### 3.1 Against (2)

Moore provides two criteria that must be met in order to successfully refute the premise that if it is not possible that it is determinate that $A=B$ then we have no justification for the claim that it is indeterminate whether $A=B$ :

One might challenge stage one of the argument by pointing out that we do not always infer possibility from indeterminacy. This is a reasonable challenge, but in order for it to work, we need some reason for thinking that the inference doesn't hold in the type of case I'm considering-for resisting my argument that the reasons (a mix of Parsimony and Democracy) for holding that it is indeterminate whether A is B are also reasons for holding that their determinate identity is possible.

Since the indeterminacy here is putatively metaphysical, we need a metaphysical reason, and not a semantic or epistemic reason, for holding that it is not determinately possible that A and B are identical.

Moore (2008: 9) (my capitalisation).

[^8]I note a concern with Moore's proposed justification and then propose more appropriate justification. With his twofold criteria in mind let us recall:
(2) If is not possible that it is determinate that $A=B$ then we have no justification for the claim that it is indeterminate whether $A=B$.

If (2) is true then it results in the following threat that the notions of indetermincy and contingency collapse. ${ }^{26}$ It is possible that $A$ has a property which $B$ does not have and that $A$ and $B$ exist. For example, it is possible that Kilimanjaro has Sparky as a part, that $\mathrm{K}^{-}$does not have Sparky as a part, and that Kilimanjaro and $\mathrm{K}^{-}$exist. And, for example, it is possible that 2 has $\varnothing$ as a member and that $\{\{\varnothing\}\}$ does not have $\varnothing$ as a member, and that 2 and $\{\{\varnothing\}\}$ exist. In such a world in which there is this possibility it is determinate that $A \neq B$. This is the case if it is indeterminate whether $A=B$. So it is true that:
(I) If it is indeterminate whether $A=B$ then it is possible that it is determinate that $A \neq B$.

Since we do have justification for the claim that it is indeterminate whether $A=B,{ }^{27} \mathrm{I}$ ) and a modus tollens on (2) yields:
(II) If it is indeteminate whether $A=B$ then it is possible that it is determinate that $A=B$ and it is possible that it is determinate that $A \neq B$.

But now we face the threat that claims of the form: 'it is indeterminate whether $\phi$ ' (where ' $\phi$ ' contains ' $=$ ') collapse into claims of the form: 'it is contingent whether $\phi$ ' (where ' $\phi$ ' contains ' $=$ '). On the canonical conception of determinacy and indeterminacy (fn. 4), there is an analogous logical relationship between 'it is indeterminate whether', 'it is indeterminate that', 'it is indeterminate that not' and the alethic modal 'it is contingent whether', 'it is possible that', 'it is possible that not'. If we abide by this canonical conception and are forced to accept II) then indeterminate identity threatens to collapse into contingent identity.

This has two undesirable consequences: it conflicts with potential indeterminate non-contingencies (such as the continuum hypothesis, whether all projective sets admit of projective uniformization, and Goldbach's conjecture) and it runs the risk that determinacy notions collapse into alethic modal notions. The consequences are undesirable because for the former, there is no independent reason to deny that there are indeterminate non-contingencies and for the latter, an absence of a fact of the matter is not the same as the possibility that that fact obtains and the possibility that that fact does not obtain. So either Moore's justification requires an abandonment of the canonical conception of determinacy and indeterminacy or we are owed an account that prevents a collapse from indeterminate identity to contingent identity.

There is an alternative to the justificatory possibility expressed in (2) which is both available to the postulator of vague objects and immune to the threat of indeterminate identity collapsing into contingent identity. Consider again the Problem of the Many and Benacerraf case with respect to $A^{\prime} \mathrm{s}$ and $B^{\prime}$ s properties.

[^9]For the Problem of the Many case consider Sparky. Sparky is part of $\mathrm{K}^{+}$. But it is not determinately part of Kilimanjaro. So it is indeterminate whether Kilimanjaro and $\mathrm{K}^{+}$share Sparky as a part. For the Benacerraf case consider the property of being a singleton. $\{\{\varnothing\}\}$ is a singleton. But it is not determinate whether 2 is a singleton. So it is indeterminate whether 2 and $\{\{\varnothing\}\}$ are both singletons. This motivates us to claim, for both cases, that it is indeterminate whether $A=B$. Since it is indeterminate whether $A$ and $B$ fail to share all their properties, it is indeterminate whether $A$ and $B$ are identical. ${ }^{28}$

Our judgement of the instantiation of $A$ 's and $B$ 's properties plays an important explanatory role concerning the identity of $A$ and $B$ since it applies to relevant counterfactual situations. If it were not indeterminate whether $A$ and $B$ both share all their properties, then it would be determinate whether they do. Then it would either be determinate that $A=B$ or determinate that $A \neq B$. Either way, we would not claim that it is indeterminate whether $A=B$. So, the claim that it is determinate whether $A$ and $B$ share all their properties justifies the claim that it is determinate whether $A=B$, just as the claim that it is indeterminate whether $A$ and $B$ share all their properties justifies the claim that it is indeterminate whether $A=B$.

This justification is more appropriate than Moore's proposed justification since it does not rely upon alethic modality and it complements our pre-theoretical gloss on indeterminacy as absence of fact. We claim that it is indeterminate whether $A=B$ because there is no fact of the matter whether $A$ and $B$ share all their properties. Hence justification of the indeterminacy expressed in (2) does not require a recourse to alethic modality. So the alternative justification bypasses any commitments which pose the threat of a collapse of indeterminate identity into contingent identity.

In addition, the alternative justification meets Moore's twofold criteria: we have just seen how the justification applies to the Problem of the Many and Benacerraf cases, and that the justification proposed is metaphysical since it concerns the metaphysical relationship between property instantiation and identity. Hence this justification is highly suitable for the postulator of vague objects to adopt.

Let us now turn to the second counter-argument.

### 3.2 Against (PN)

Recall the second stage of the argument. Here Moore argues that in $w_{1}, A=B$ and it is necessary that $A \neq C$. This requires:
(PN) If it is possible that $A=B$ and $A$ and $B$ exist without $C$ then it is necessary that it is determinate whether $A \neq C$. [Leibniz's Law on (AC) with (ND)]

[^10]Moore informs us that (PN) relies upon Leibniz's Law. ${ }^{29}$ But more than one identity principle is required if the inference is permissible. The derivation of (PN) requires:
(a) It is necessary that $B \neq C$
[Necessary Distinctness (ND)]
(b) It is possible that $A=B$
[ a conjunct of (AC)]
(c) It is necessary that it is determinate whether $A \neq C$
[consequent of (PN)] and two instances of identity principles;
(Ib) $x=y \rightarrow \forall \Phi[\Phi(x) \equiv \Phi(y)]$
(Leibniz's Law)
(Ic) $\neg[\forall \Phi(\Phi(x) \equiv \Phi(y))] \rightarrow(x \neq y)$
(Diversity of the Dissimilar)
An instance of (Ib) is required for the following inference: since $A$ is identical to $B$ in $w_{1}, A$ and $B$ share all their properties in $w_{1}$. So since the property of being necessarily distinct from $C$ is instantiated by $B$ in $w_{1}, A$ also instantiates that property in $w_{1}$.

An instance of (Ic) is then required in order to distinguish $A$ from $C$ via the follow inference: since $A$ has the property of being necessarily distinct from $C$ in $w_{1}$, and since $C$ does not have that property in any world, it is necessary that $A$ is distinct from $C$.

Before we proceed let it be noted that Moore analyses (PN) as obtained by the principle of the substitutivity of identity. ${ }^{30}$ This would employ a schematic version of Leibniz's Law as opposed to invoking universal quantification over formulae as in Ib ) and Ic). The following counter-argument also applies if the principle of the substitutivity of identity is preferred (This can be seen by making a simple modification to the argument below: in place of talk of the domain of quantification and properties within it, think instead of all schematic instances of the principle of the substitutivity of identity and the expression of those properties in its schematic instances.) I have adopted property abstraction notation as an expository aid. Accordingly I reconstruct this section of the argument as follows:
(i) $\square B \neq C$
(ii) $\square \hat{x}[\square(x \neq C)] B$
(iii) $\diamond A=B$
(iv) $\diamond \forall \Phi[\Phi(A) \equiv \Phi(B)]$
(v) $\diamond \hat{x}[\square(x \neq C)] A$
(vi) $\square \neg \hat{x}[\square(x \neq C)] C$
(vii) $\square A \neq C$
(from Necessary Distinctness)
[Property Abstraction on $i$ )]
[a conjunct of (AC)]
[modus ponens on Ib) from iii)]
[from (ii) and (iv)]
[Property Abstraction on the trivial $\square \neg \square(C \neq C)$ ] [modus ponens on Ic) from (v) and (vi)]

The above argument contains a fallacy in the move from (ii) and (iv) to (v). The move is only permissible if the domain of the universal quantifier in (iv) includes the property $\hat{x}(\square x \neq C)$ for this is the property ascribed to $A$ in (v). (Such a property is a 'modal property'.) Since the domain of the universal quantifier in (iv) must include $\hat{x}(\square x \neq C)$, on pain of placing a restriction on the domain, it must also include properties expressed in the form ' $[\hat{x}(\square x=C)]$ '. That is, if it is permitted

[^11]that the domain includes the property of being necessarily distinct from an object, then the domain must also include properties of being necessarily identical to an object, unless good reason can be provided why such properties should not be included. There may be such a good reason if one thinks that there is no such thing as necessary identity. ${ }^{31}$ However, let us set aside this response for now, and first see how the objection works. (I will return to address this point in fn. 32.)

If $\hat{x}(\square x=A)$ is included in the domain then this is a property instantiated by $A$ in $w_{1}$ and not instantiated by $B$ in $w_{1}$. Hence in this case $A$ and $B$ would fail to share all their properties in $w_{1}$ and a modus ponens on Ic) yields that $A \neq B$ in $w_{1}$. This contradicts the claim expressed in iii) that $A=B$ in $w_{1}$. Note that iii) is contradicted, not (P1) - the central claim made by the postulator of vague objects. Hence the postulator of vague objects retains (P1) and blocks the reductio. ${ }^{32}$

The point can be illustrated in other contexts. Consider a comparison between this alethic modal situation and an analogous temporal modal situation. Instead of possible worlds, think of times. Bob (think A) has two legs. In the future, Bob loses his legs. Always, biped Bob (think B) and legless Bob (think $C$ ) are distinct (Bob cannot have legs that he does not have). There is a time in the future in which Bob is identical to legless Bob. But it is incorrect to conclude from this and that always biped Bob and legless Bob are distinct, that always Bob and biped Bob are distinct.

The diagnosis of the fallacy does not entail that ii) is false. $B$ does have the property $\hat{x}(\square x \neq C)$ (on pain of contradicting the assumption of (Necessary Distinctness)). Instead, diagnosis of the fallacy entails that certain modal properties cannot witness instances of identity principles. Those properties that cannot witness are those that do not determine the identities that are under consideration. For example, $B$ 's self-identity in $w_{1}$ is not determined by $\hat{x}(\square x=A)$. Rather $B$ 's selfidentity in $w_{1}$ is determined only by those non-modal properties that $B$ instantiates in $w_{1}$. The identity of an individual in a world is determined by the properties it has in that world, not by properties it has in other worlds. Hence only the properties instantiated by an individual in a world can witness those identity principles which govern over that individual's identity (with itself) and distinctness (with other individuals) in that world.

The diagnosis of the fallacy does not entail that it is inconsistent that it is possible that $A=B$ and it is indeterminate whether $A=B$. Rather, the diagnosis of the fallacy entails that the inference from (ii) and (iv) to (v) is illicit. Whether or not it is inconsistent that it is possible that $A=B$ and it is indeterminate whether $A=B$ depends on something else, not the inference from (ii) and (iv) to (v). And this is despite the fact that the fallacy is diagnosed to be with the inference because the inference leads to a contradiction between (v) and (iii). Since the inference is illicit, whether or not (v) and (iii) are non-contradictory or consistent

[^12]depends on something else. Only if the inference were legitimate would (v) and (iii) be contradictory. ${ }^{33}$

Let us now consider a third counter-example to Moore's argument. For this, let us start by considering a comparison between the modal argument and two related arguments.

### 3.3 Comparison With the Evans Argument Against Vague Objects and With the Argument Against Contingent Identity (Also Against (PN))

As a basis of comparison for the modal argument, consider Evans's argument against vague objects. The first premise is the assumption that it is indeterminate whether $a=b$. Recall that the negation of 'it is indeterminate whether' is logically equivalent to 'it is determinate whether'.
(E1) $\quad \mathbf{\nabla}(a=b)$
(E1) can be understood as an attribution of the property $\hat{x}[\mathbf{\nabla}(x=a)]$ to $b$. So we also have the truth of:

$$
\begin{equation*}
\hat{x}[\mathbf{V}(x=a)] b \tag{E2}
\end{equation*}
$$

But we also have the self-evident truth of:
(E3) $\neg \mathbf{\nabla}(a=a)$
which can also be understood as the non-ascription of a property, only this time to $a$. So we get the truth of:

$$
\begin{equation*}
\hat{x}[\mathbf{\nabla}(x=a)] a \tag{E4}
\end{equation*}
$$

But now, by Leibniz's Law, we get from (E2) and (E4) the truth of:
(E5) $\neg(a=b)$
If $\neg(a=b)$ entails that it is determinate whether $\neg(a=b)^{34}$ then the original premise of (E1) is contradicted.

Jonathan Lowe has offered what I take to be the following response to Evans's argument against vague objects. ${ }^{35}$ The success of the argument depends upon whether it is determinate that the property $\hat{x}[\mathbf{\nabla}(x=a)]$ (the property of being indeterminately identical to $a$ ) is distinct from all properties that might determine

[^13]that $a$ and $b$ are the same. If it not determinate then, since it is appealed to in order to distinguish $a$ from $b$, the argument begs the question that $a$ is distinct from $b$ (E5). The identity of the property $\hat{x}[\mathbf{V}(x=a)]$ is crucial in the proof since this is the property that is appealed to in the move from (E4) to (E5) in order to distinguish $a$ from $b$. If the appeal is inadmissible, then the move from (E4) to (E5) cannot be made and so a contradiction cannot be derived.

All of $a$ 's properties might determine that $a$ and $b$ are the same. So the move from from (E4) to (E5) is only admissible if the property $\hat{x}[\mathbf{V}(x=a)]$ is distinct from all properties that $a$ has. If the property $\hat{x}[\mathbf{\nabla}(x=a)]$ were not distinct from all properties had by $a$ then it is illegitimate to claim that $a$ does not have the property $\hat{x}[\mathbf{V}(x=a)]$. On such an eventuality, the move to (E4) is illegitimate.

The argument is unsuccessful because there is a property that $a$ has such that it is not determinate that it is distinct from the property $\hat{x}[\mathbf{\nabla}(x=a)]$; namely, the property $\hat{x}[\mathbf{\nabla}(x=b)]$. That it is not determinate that $\hat{x}[\mathbf{V}(x=a)]$ is distinct from $\hat{x}[\mathbf{\nabla}(x=b)]$ follows from the assumption of (E1) that $\boldsymbol{\nabla}(a=b)$. To assume that it is indeterminate whether $a=b$ is to also assume that it is not determinate that the property of being indeterminately identical to $a$ is distinct from the property of being indeterminately identical to $b .^{36}$ Hence the move from (E4) to (E5) is inadmissible because it requires the assumption that $\hat{x}[\mathbf{\nabla}(x=a)]$ is distinct from $\hat{x}[\mathbf{V}(x=b)]$. Yet under the assumption of (E1), it is not determinate that $\hat{x}[\mathbf{\nabla}(x=a)]$ is distinct from $\hat{x}[\mathbf{\nabla}(x=b)]$. Thus the contradiction required for the reductio cannot be derived.

Let us compare Evans's argument with the argument against contingent identity. The first premise of the argument is the assumption that it is contingent whether $c=d$. We use the operator ' $\nabla$ ' for 'it is contingent whether'. The negation of 'it is contingent whether' is non-contingency; it is non-contingent whether $\phi$ iff it is necessary that $\phi$ or it is necessary that $\neg \phi$.
(C1) $\quad \nabla(c=d)$
( C 1 ) can be understood as an attribution of the property $\hat{x}[\nabla(x=c)]$ to $d$. So we also have the truth of:

$$
\begin{equation*}
\hat{x}[\nabla(x=c)] d \tag{C2}
\end{equation*}
$$

But we also have the self-evident truth of:
(C3) $\neg \nabla(c=c)$
which can also be understood as the non-attribution of a property, only this time to $c$. So we get the truth of:

$$
\begin{equation*}
\hat{x}[\nabla(x=c)] c \tag{C4}
\end{equation*}
$$

[^14]But now, by Leibniz's Law, we get from (C2) and (C4) the truth of:
(C5) $\neg(c=d)$
If $\neg(c=d)$ entails that it is non-contingent whether $\neg(c=d)$ then a contradiction with (E1) is derivable and this contradicts the original premise of (C1).

My take on Lowe's response to Evans's argument can also be given to the above argument against contingent identity. The move from (C4) to (C5) can be blocked by claiming that the success of the argument depends upon whether it is necessary that the property $\hat{x}[\nabla(x=c)]$ (the property of being contingently identical to $c$ ), expressed in the lambda extract of ( C 4 ) and appealed to in order to distinguish $c$ from $d$, is distinct from all other properties that might determine whether $c$ and $d$ are contingently distinct. In particular, the argument is only successful if it is necessary that $\hat{x}[\nabla(x=c)]$ is distinct from all other properties that $c$ has.

All of $c$ 's properties might determine that $c$ and $d$ are the same. The argument is unsuccessful because $c$ has the property $\hat{x}[\nabla(x=d)]$ which is not necessarily distinct from the property $\hat{x}[\nabla(x=c)]$. Hence the move from (C4) to (C5) is illegitimate because it assumes that $\hat{x}[\nabla(x=c)]$ is necessarily distinct from $\hat{x}[\nabla(x=d)]$. Yet under the assumption of (C1), it is not the case that $\hat{x}[\nabla(x=c)]$ is necessarily distinct from $\hat{x}[\nabla(x=d)]$. Thus the contradiction required for the reductio cannot be derived.

Strong structural similarities between Evans's argument and the argument against contingent identity are apparent. This is nothing new. But Moore claims that the modal argument is different. ${ }^{37}$ However, my take on Lowe's response is also a counter-argument to the modal argument.

Recall that in Sect 3.2 I argued against Moore by demonstrating that a section of the second part of his argument requires an erroneous use of identity principles since it requires a shift in the domain of the universal quantifier. We saw here that the success of the modal argument crucially relies upon, inter alia, ascribing the property of being necessarily distinct from $C([\hat{x}(\square x \neq C)])$ to $A$ (in the move to (vii)). My take on Lowe's response contends that the modal argument relies upon a question-begging assumption concerning distinctness between properties, just as Evans's argument and the argument against contingent identity does.

On my take on Lowe's response to the modal argument, fault is found with the move from (v) and (vi) to (vii);

$$
\begin{equation*}
\diamond \hat{x}[(\square x \neq C)] A \tag{v}
\end{equation*}
$$

[from (ii) and (iv)]
(vi) $\square \neg \hat{x}[\square x \neq C)] C$
(vii) $\square A \neq C$
[Property Abstraction on the trivial $\square \neg \square(C \neq C)$ ]
[modus ponens on Ic) from (v) and (vi)

The success of the move depends upon whether the property $\hat{x}[(\square x \neq C)]$ (the property of being necessarily distinct from $C$ ), expressed in the lambda extract of (vi) and appealed to in order to necessarily distinguish $A$ from $C$ (vii), is distinct from all other properties that may determine whether $A$ and $C$ are necessarily

[^15]distinct. In particular, the argument is only successful if $\hat{x}[\square(x \neq C)]$ is distinct from all other properties which, if they were instantiated by $A$, would conflict with:
(iii) $\diamond(A=B)$
[a conjunct of $(A C)]^{38}$
The argument is unsuccessful because it assumes that it is determinate that $\hat{x}[\diamond \nabla(x=A)]$ is not distinct from the property $\hat{x}[\square(x \neq C)]$. Yet under the assumption in (P1) that it is indeterminate whether $A=B$, it cannot be determinate that $\hat{x}[\diamond \nabla(x=A)]$ is not distinct from $\hat{x}[\square(x \neq C)]$. Hence the move from (v) and (vi) to (vii) is illegitimate. Thus the contradiction required for the reductio of the modal argument cannot be derived.

Objection: If it is necessary that $B \neq C$ (in $w_{1}$ ) and it is possible that $A=B$ (in $w_{1}$ ), then there is no indeterminacy in $w_{1}$. In $w_{1}, A=B$ and it is necessary that $B \neq C$, so it follows by Leibniz's Law that it is necessary that $A \neq C$. If we assume that the actual world is possible relative to this world, then we get the conclusion that it is actually the case that $B=C$ (which is what Moore's reductio requires). ${ }^{39}$

Response: The objection presupposes that a Lowean fallacy cannot be diagnosed if there is no indeterminacy in $w_{1}$. Yet such a diagnosis does not depend on whether there is indeterminacy in $w_{1}$. The diagnosis depends upon whether the property that is inferred as being shared by $A$ and $B$ in $w_{1}$ (being necessarily distinct from C $(\hat{x}[\square(x \neq C)]))$ is determinately distinct from other properties that may determine whether A and C are necessarily distinct. If the property is determinately distinct then the Leibniz's Law inference is permissible since it would not conflict with A being necessarily distinct from C. However, the property is not determinately distinct; it is indeterminate whether it is distinct and this is due to the primary assumption that it is indeterminate whether $A=B$ (in this world). For the property of being necessarily distinct from C is not determinately distinct from the property of being possibly indeterminately identical to A .

I do not claim that my take on Lowe's response is a solution to Evans's argument against vague objects, to the argument against contingent identity, or to the modal argument. But my take on the response is applicable to all three since, for each argument, it demonstrates the requirement of an illegitimate assumption concerning distinctness of properties. It offers a diagnosis of the same mistake in each dialectic and explains why the mistake should not be made.

## 4 Conclusion

We have seen two counter-arguments and one potential counter-argument to Moore's modal argument against vague objects. The counter-argument pertaining to justification of the claim that it is indeterminate whether $A=B$ can only be surmounted if the alternative justification that it is indeterminate whether $A$ and $B$

[^16]share all their properties is unwarranted. The counter-argument pertaining to the shift in quantification can only be surmounted if it is proved that the move from (ii) and (iv) to (v) does not rely upon identity principles.

The counter-argument of my take on Lowe's response has a prominent objection: that for each property and for each of the following pairs:

$$
\begin{aligned}
& \hat{x}[\nabla(x=a)] \text { and } \hat{x}[\nabla(x=b)] \\
& \hat{x}[\nabla(x=c)] \text { and } \hat{x}[\nabla(x=d)] \\
& \hat{x}[\square(x \neq C)] \text { and } \hat{x}[\diamond \nabla(x=A)],
\end{aligned}
$$

there is a way to distinguish each property in a pair from the other property in that pair; and that this way does not contravene the assumptions of:

$$
\begin{aligned}
& \nabla(a=b), \\
& \nabla(c=d), \text { and } \\
& \nabla(A=C) .
\end{aligned}
$$

If we widened the contexts of the arguments then there is potential for revealing the way. For the contingent identity and modal argument instances, appeal could be made to objects other than $c, d$ and $A, B, C$ that instantiate one property $(\hat{x}[\nabla(x=c)]$ or $\hat{x}[\square(x \neq a)])$ and not the other $(\hat{x}[\nabla(x=d)]$ or $\hat{x}[\diamond \nabla(x=a)])$. It is, however, less obvious how such a way could be found for the context of the Evans argument (to find an object which instantiates $\hat{x}[\mathbf{V}(x=a)]$ but not $\hat{x}[\mathbf{V}(x=$ b)] or vice versa). ${ }^{40}$

That my take on Lowe's response applies to all three arguments and that the same method would have to be employed in order to rebut it illustrates the close structural similarity that all three arguments share. I conclude that the modal argument is subject to two counter-arguments against (2) from Sect. 3.1 and against (PN) from Sect. 3.2 and that, even if they could be surmounted, the evidence in Sect. 3.3 demonstrates that the modal argument is a camouflaged variant of the Evans argument and the argument against contingent identity. Its apparent novelty is in fact disguised familiarity. Only one solution need apply.

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${ }^{40}$ This is nothing new; Lowe states this (2005: 300-301).

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[^0]:    ${ }^{1}$ See Moore (2008).
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[^1]:    ${ }^{2}$ Throughout, I freely interchange the identity predicate 'is' for ' $=$ ' and the negation of the identity predicate 'distinct' for ' $\neq$ '.
    ${ }^{3}$ For example, Parsons (2000) and Van Inwagen (1995).
    ${ }^{4}$ I adopt the canonical view of the 'that'/'whether' distinction for indeterminacy and determinacy; it is indeterminate whether $\phi$ iff it is not determinate that $\phi$ and not determinate that $\neg \phi$, it is indeterminate that $\phi$ iff it is not determinate that $\neg \phi$,
    it is determinate whether $\phi$ iff it is determinate that $\phi$ or determinate that $\neg \phi$ (iff it is not indeterminate whether $\phi$ ), and
    it is determinate that $\phi$ iff (it is determinate whether $\phi$ ) and $(\phi)$.
    ${ }^{5}$ See Unger (1980).
    ${ }^{6}$ See Benacerraf (1965).

[^2]:    ${ }^{7}$ It may be denied that unrestricted mereological composition holds in this instance. This is a response that rivals the positing of vague objects to account for the situation. Since Moore's argument is intended to motivate a rejection of the positing of vague objects and so requires the assumption that there are vague objects, let us ignore the rival responses.
    ${ }^{8}$ Sometimes known as the Indiscernibility of Identicals.
    ${ }^{9}$ See Benacerraf (1965).

[^3]:    ${ }^{10}$ Benacerraf's original conclusion was to bite the bullet in this fashion by claiming that numbers are not objects (1965). He later recanted; any old $\omega$-sequence would do for the natural numbers after all (Benacerraf 1996).
    ${ }^{11}$ Moore also claims that instances of the Ship of Theseus problem are analogous to the Problem of the Many case and Benacerraf case (2008: 2). For simplicity I ignore the Ship of Theseus problem and do not commit to whether it is analogous or not.
    ${ }^{12}$ The epistemicist response is a notable exception.
    ${ }^{13}$ This is merely application of the equivalences in fn. 4.
    ${ }^{14}$ Thank you to an anonymous referee who pointed out the controversial nature of this.

[^4]:    ${ }^{15}$ See Evans (1978).
    ${ }^{16}$ See Salmon (1981).

[^5]:    17 'Parsimony' is a potentially misleading name for the assumption, as admitted by Moore (2008: 3), since it does not concern shaving away the number of objects, just the number of kinds.
    ${ }^{18}$ On the assumption that the progression of the natural numbers begins at zero.

[^6]:    ${ }^{19}$ The Barnes-Williams theory of vague objects relies on their compatibility (Williams 2007; Barnes and Williams 2011; Barnes 2009). Since the theory rejects Referential Determinacy it is not subject to the modal argument.

[^7]:    ${ }^{20}$ I have altered the argument slightly from the original. In Moore's original version $\left({ }^{*}\right)$ does not explicitly contain 'it is determinate whether'. Without it may be though that $\left(^{*}\right)$ would not be a contradiction. But once it is acknowledged that the assumptions for the argument are assumed to be determinately true, $A \neq C$ is secured a determinate truth-value. For if Necessary Distinctness is determinately true then $B \neq C$ is determinately true. Hence the use of Leibniz's law to obtain (PN) will secure the determinate truth of $A \neq C$ just as long as we take it that Leibniz's Law is determinately true. I have added the redundant 'determinate' to sentences in the argument just to make the application of determinacy apparent.
    ${ }^{21}$ The claim that if $C$ didn't exist then $A$ would be $B$ is taken on intuition: consider if $C$ didn't exist yet $A$ and $B$ did - what prevents $A=B$ ?

[^8]:    ${ }^{22}$ See Moore (2008: 9).
    ${ }^{23}$ See Moore (2008: 10).
    ${ }^{24}$ See Moore (2008: 11-15).
    ${ }^{25}$ See Lowe (2005).

[^9]:    ${ }^{26}$ Thank you to an anonymous referee for diagnosing a fallacy in the initial rendering of this objection. ${ }_{27}$ A mix of Democracy and Parsimony, (Moore 2008: 9).

[^10]:    ${ }^{28}$ An appeal to it being indeterminate whether $A$ and $B$ share a property invites a version of Evans's argument against vague objects; if the property of indeterminately sharing a property with $A$ is had by $B$ (C) then it distinguishes $A$ from $B(C)$. There are several responses to Evans's argument (such as Lowe's, which we note in Sect. 3.3) that serve to preserve the appeal to $A$ and $B^{\prime} \mathrm{s}\left(C^{\prime}\right.$ s) properties in order to justify that it is indeterminate whether $A=B$ (indeterminate whether $A=C$ ). In any case, the modal argument disregards Evans's argument since if it did not then the reductio of the modal argument could be blocked by Evans's reductio. For if the latter is applied to ( P 1 ) then a contradiction from 'it is indeterminate whether $A=B^{\prime}$ and from 'it is indeterminate whether $A=C^{\prime}$ is derivable, thus blocking the modal argument at (P1).

[^11]:    ${ }^{29}$ See Moore (2008: 8).
    ${ }^{30}$ See Moore (2008: 11).

[^12]:    ${ }^{31}$ Thank you to an anonymous referee to pointing this out.
    ${ }^{32}$ If you think that there are no objects that are necessarily (self-)identical then you may think that $\hat{x}(\square x=A)$ cannot be instantiated since it does not exist. If so, then replace talk of $\hat{x}(\square x=A)$ with talk of $\hat{x}(\nabla x=B)$ (where ' $\nabla$ ' is 'it is contingent whether'). On this diagnosis of a fallacy, $A$ has the property (of being contingently identical to $B$ ) in $w_{1}$ whereas $B$ does not (since there is no possible world in which any object is distinct from itself).

[^13]:    ${ }^{33}$ Thank you to an anonymous referee for pointing out the need to make this clearer.
    ${ }^{34}$ This requires the assumption that it is determinate whether (E1). This needs to be assumed for every premise for otherwise the argument would be neither valid nor invalid (indeterminate truth is not sufficient to secure validity and not sufficient to secure invalidity). Hence it is legitimate to prefix (E5) with 'it is determinate whether' since each premise (E1)-(E4) needs to be prefixed with 'it is determinate whether' in order to ensure that the argument is either valid or invalid.
    ${ }^{35}$ See Lowe (2005). What follows is my take on Lowe's response since my explanation differs slightly; Lowe claims that it is indeterminate whether $\hat{x}[\mathbf{\nabla}(x=a)]$ is distinct from $\hat{x}[\mathbf{V}(x=b)]$ (Lowe 2005: 299) whereas I remain silent on this, instead I rely on the judgment that it is not determinate that $\hat{x}[\mathbf{V}(x=a)]$ is distinct from $\hat{x}[\mathbf{V}(x=b)]$. I do not know whether Lowe would endorse my interpretation of his response. For a defence of vague objects against Evans's argument and a defence against other arguments against vague objects see Barnes and Williams (2009).

[^14]:    ${ }^{36}$ For otherwise identity would not be an equivalence relation since it would not be be symmetric. If it is assumed that it is determinate that $\hat{x}[\mathbf{V}(x=a)]$ is distinct from $\hat{x}[\mathbf{V}(x=b)]$ then since it is determinate that $\hat{x}[\mathbf{V}(x=a)]$ is distinct from $\hat{x}[\mathbf{V}(x=b)], a$ has $\hat{x}[\mathbf{V}(x=b)]$ iff $b$ does not and $b$ has $\hat{x}[\mathbf{V}(x=a)]$ iff $a$ does not. So either it would be indeterminate whether $a$ is identical to $b$ but not indeterminate whether $b$ is identical to $a$, or it would be indeterminate whether $b$ is identical to $a$ but not indeterminate whether $a$ is identical to $b$.

[^15]:    ${ }^{37}$ I have omitted consideration of the Salmon argument (1981) for simplicity.

[^16]:    ${ }^{38}$ For
    (iv) $\diamond[\forall \Phi(\Phi(A) \equiv \Phi(B))] \quad$ [modus ponens on $I(b)$ from (iii)] depends upon (iii).
    ${ }^{39}$ This objection was provided by an anonymous referee.

