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ANALYSIS OF INTERTEMPORAL CHOICE:
A NEW FRAMEWORK AND EXPERIMENTAL RESULTS

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Abstract

This paper reports the results of a series of experiments examining intertemporal choice. The paper makes three contributions: First, it presents a new analytic device, the *intertemporal choice triangle*, which is analogous to the Marschak-Machina choice triangle used in the analysis of choice under risk. Second, we have developed a *new experimental design* based on the intertemporal choice triangle which allows subjects greater flexibility in making choices, and which allows the researcher to make more subtle inferences, than are possible with designs previously employed. Subjects are able to create their most-preferred outcome in each choice situation by choosing a constrained linear combination of two extreme options. Third, our results show that while subjects do not typically maximize present value, they are significantly influenced by present value considerations. We refer to this finding as *present value-seeking behavior*. We find only weak evidence of several previously documented intertemporal choice anomalies in our framework.

Keywords: Intertemporal choice, present value maximization.
JEL Classifications: C91, D90

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1. Introduction

Many important decisions in life concern allocating economic resources over time: choice of education and career, the purchase of a house or other investment. Many individuals in our society face decisions about how to allocation annuity contributions as they face retirement. Such decisions will become even more pervasive if recent suggestions to privatize the social security system become reality. For decisions about allocating income over time, the economic theory of present value maximization provides clear predictions: conditional on knowing an individual's (constant) discount rate, one should choose the alternative that maximizes the present value of income.

Experimental studies, such as those by Ben-Zion, Rapoport, and Yagil (1989), Ainslie (1975), Loewenstein and Sicherman (1991), Thaler (1981), Gigliotti and Sopher (1997a), and many of the contributions in Loewenstein and Elster (1992) illustrate behavior that is inconsistent with present value maximization. This paper makes three further contributions to the literature about rational intertemporal choice, with special reference to the theory that individuals will choose among streams of income by selecting the stream with the highest present value. First, it presents a new analytic device, the *intertemporal choice triangle*, which is analogous to the Marschak-Machina choice triangle used in studying the theory of choice under risk. Second, we have developed a *new experimental design* based on the intertemporal choice triangle which allows subjects greater flexibility in making choices, and which allows the researcher to make more subtle inferences, than are possible with designs previously employed. Subjects are able to create their most-preferred outcome in each choice situation by choosing a constrained linear combination of two income streams. Third, our results show that while subjects do not typically maximize present value, they are significantly influenced by present value considerations. We refer to this finding as *present value-seeking behavior*.

We now briefly discuss each of these contributions in turn.

The Intertemporal Choice Triangle

The Marschak-Machina (MM) probability triangle has been put to extensive use in the analysis of choice under risk (Machina (1987)). It is an elegant and useful tool, which gives an intuitive and clear picture of how expected utility theory should function, and how individuals may act in violation of it. In the probability triangle framework, shown in Figure 1, any point on the boundary or interior of the triangle represents a lottery over three prizes. The probabilities of the large and small prizes, p_l and p_s respectively, are measured from 0 to 1 on the vertical and horizontal sides of the triangle, respectively. The probability of the middle-sized prize is expressed as the residual $p_m=1-p_l-p_s$. Constant expected utility contours in the triangle are straight parallel lines whose slopes are determined by the ratio scale of an individual's utility function. Constant expected value contours are also linear, making it easy to determine an individuals' risk aversion or risk preference; a risk averse individual would have expected utility contours flatter than expected value contours, a risk neutral individual would have expected utility contours with the same slope as expected value contours, and a risk preferring individual would have expected utility contours that are steeper than expected value contours.

The probability triangle is based on the assumption that probabilities change in a constrained fashion within the triangle. For example, moving from the origin along a ray towards the hypotenuse, the probability of winning the middle prize rises, and the probability of winning the large and small prizes change in fixed proportions. The utility of dollar outcomes does not change unless the dollar outcomes change. Many experimental studies using the MM triangle have shown that subjects may have variable levels of risk aversion as the probability of winning the middle prize changes, and that simple expected utility theory cannot explain this phenomenon.

A similar triangle-based analysis can be done in intertemporal choice theory, as shown in Figure 2. We assume three payout dates, t_0, t_1, t_2, which may or may not be equally spaced. The legs of the triangle represent not probabilities, but the amount of money received in a given period. The vertical leg measures the amount of money received in the first of three given payout dates, and the horizontal leg measures the amount of money received in the last of the three payout dates. These values range from 0 to π, where the latter represents the total dollars available over all three periods, and $\pi_0+\pi_1+\pi_2 = \pi$, analogous to the sum $p_1+p_2+p_3=1$ in the MM triangle. As described below, constant present value contours within the triangle will be linear, with the slope value dependent on the spacing of payments. If the payout dates are equally spaced, then the slope of the constant present value contour will equal the discount factor for the middle payout period, $0<d_{t1}<1$.

New Experimental Design

Other experimental studies of intertemporal choice have typically asked subjects questions of the following sort: "State the amount of money to be received one year from today that you would be just as happy to receive as you would be to receive $100 today." That is, subjects are typically asked to make an indifference statement, and they are asked to compare income streams with payments in only one of two periods. These indifference statements are then interpreted in various ways in order to test hypotheses about intertemporal choice. For example, the implicit discount rate subjects are using may be calculated, under the null hypothesis of present value maximization. Comparisons between the discount rates in different situations may be made to test for equality of the discount rate, a basic consistency requirement.

Our design, which uses the intertemporal choice triangle as the underlying analytical device for interpreting subject choices, effectively asks subjects questions of the following sort: "Suppose you have the choice between (i) receiving $100 in 4 months, (ii) receiving $50 today and $50 in 1 year, or (iii) receiving any weighted average of the income streams in (i) and (ii). Which income stream would you most prefer to receive?" That is, subjects are asked to pick the income stream they most prefer from a large set of possible income streams, and they are asked to compare income streams that, in general, involve payments in three different periods. In terms of the intertemporal choice triangle, subjects are asked to choose their most preferred income stream from a chord lying in the triangle.

For example, consider the chord running from the origin to the hypotenuse with slope 1 in Figure 2. Our experimental design asks subjects to choose the point on this line that they would most prefer to receive. Using a computer program, subjects move a cursor along a range that redistributes the total dollars available across the three designated payout dates in a constrained fashion, i.e., funds may be shifted completely to the middle payout date, or distributed proportionately to the first and third payout date. As mentioned above, for any given discount rate, constant present value contours will be a family of positively sloped, parallel straight lines in the choice triangle.

If the choice questions in the experiment consist of asking which points on parallel chords within a triangle the subject most prefers, then the subject should always choose the lower end of a chord, as illustrated, or the upper end, at the hypotenuse. Choices at other than the ends of the chords could be rationalized in at least three ways:

a. Constant present value contours that coincide with the choice sets, a set of measure zero (possible, but not probable).

b. Choices consistent with present value maximization in the presence of capital market constraints (end points need not be chosen).

c. Choices consistent with some preference structure that does not seek maximum present value in income (e.g., some generalization of present value maximization).

The advantages of our design are several. First, it does not require that a comparison between two choices be made in order to draw conclusions about present value maximization, as is the case with other designs that have been used. A single response can be interpreted as consistent or inconsistent with present value maximization. Second, analysis of joint choice patterns allow distinction among strict present value maximization, present value "seeking" behavior, and "anti" present value maximization behavior. Previous designs only allowed one to infer that behavior was or was not consistent with present value maximization.

Present-Value Seeking Behavior

Our results imply that our subjects typically do not maximize present value, but are nonetheless influenced by present value considerations. We call this behavior "present-value seeking behavior." In the first experiment, we document the basic finding that a majority of subjects make choices inconsistent with present-value maximization in the presence of no capital market constraints. In a second experiment, we find some evidence that subjects act as if they feel constrained by an inability to borrow against future income. When explicitly told that they will not be able to borrow from future income, a larger proportion of subjects are consistent with present value maximization for smaller than for larger amounts of money. In other words, the smaller the total change in income, the less likely it is that one's optimal expenditure stream will be constrained by the inability to borrow. In terms of the possible rationalizations for non-present value maximizing behavior mentioned above, Case (a) is not a serious contender, but case (b) has some explanatory power. Case (c) is more open-ended, and difficult to test for, without specific hypotheses. A third experiment attempts to replicate the anomalies observed in previous experiments with our new framework, by analyzing choices over two-period income streams. We find generally weak evidence for these anomalies, with the exception of a "gain/loss" asymmetry.

The rest of the paper is organized as follows. Section 2 contains a more formal development of the intertemporal choice triangle. The basic experimental design for Experiments 1 and 2 is contained in section 3. Section 4 contains analysis and discussion of the empirical results from Experiments 1 and 2. Section 5 contains discussion of the design and empirical results of Experiment 3. Section 6 contains conclusions. An appendix contains instructions used in the experiment.

2. The Intertemporal Choice Triangle

Consider a (T+1)-period income stream S, denoted by S=[π_0, π_1, ... ,π_T], with discount rate δ. Since total payments, π, are fixed, we can set

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for some i. For a three-period income stream we can illustrate and analyze income streams in a triangle. In this case, the present value of an income stream S is

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Note that t_0 is the amount of time until the initial payment, and t_1 and t_2 are the additional time spans from the initial payment until the subsequent payments. Taking the total derivative and setting equal to zero, we can derive the following useful expression:

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This is the (constant) slope of a constant-present-value contour in an intertemporal choice triangle. Note that this slope does not depend on the time until the initial payment, t_0. If $t_1 = t_2/2$ (i.e., the payments are equally spaced in time), then the expression reduces to

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If $t_1>t_2/2$ (that is, there is a longer delay from the first payment until the middle payment than from the middle payment to the last payment), then

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so the slope is flatter than when the payments are equally spaced. Similarly, if $t_1<t_2/2$ (that is, there is a shorter delay from the first payment to middle payment than from the middle payment to the last payment), then

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so the slope is steeper than when the payments are equally spaced.

Figure 2 illustrates the implications of the above analysis. Consider the chord running from the origin to the hypotenuse with slope 1. A typical choice question in the experiment amounts to asking subjects which point on this line they would most prefer to receive. A subject whose preferences over alternative income streams is described by the present value maximization model will have constant preference contours similar to the straight dashed line in the figure. For each triangle there will be a family of such contours, all parallel to one another. If the choice questions in the experiment consist of asking which points on parallel chords in the triangle the subject most prefers, then the subject should always choose the lower end of a chord, as illustrated, or the upper end, at the hypotenuse.

Some researchers have proposed a hyperbolic discounting function as an alternative to the constant discount rate (Loewenstein and Prelec, (1992)). Augmenting the basic present value equation with such a discount function generates the same prediction for choice within a given intertemporal choice triangle as the standard present value model. Specifically, the hyperbolic discount function is written as: $\delta(t) = (1 + \alpha t)^{-\alpha/\beta}$. The parameter α measures the degree of departure from constant discounting. In the limit, as α approaches zero, the discount function approaches $e^{-\beta t}$, the standard constant discount rate. Using the hyperbolic discounting function, the slope of constant hyperbolically discounted present value contours are given by:

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For a given triangle, this is a constant, and thus, just as for standard present value maximization, choice should be at either end of a chord in the triangle, and not in the middle. Since the slope depends on t_2, however, there may be switching from one end to the other of a chord if all time periods are shifted by a common amount.

Choices at other than the ends of the chords could be rationalized as resulting from (i) constant present value contours that coincide with the choice sets, (ii) capital market constraints or (iii) choices consistent with some other, non-present value maximization, preference structure, such as illustrated by the curved dotted line in the figure.

The above analysis can be generalized further by considering the behavior of individuals who rank income streams on the basis of discounted and *transformed* payments. Such generalizations of present value maximization include discounted utility theory, and intertemporal analogies of prospect theory (Loewenstein and Prelec (1992)) or rank-dependent expected utility theory (Gigliotti and Sopher (1997b)). Any such approach requires an appropriate interpretation of the transformation function. For example, discounted utility, as typically applied in labor economics or macroeconomics, requires the interpretation that consumption occurs when payments occur. This could be a sensible if the payments are in perishable goods that must be immediately consumed or if for some reason money payments must be immediately spent or lost.

We think a better approach is to think of a transformation of payments in much the same way that certain generalizations of expected utility theory transform probabilities. That is, the transformation function is a technical device that, depending on its properties, has certain implications for intertemporal choice. Such transformation functions will not always have an intuitive interpretation, which is a strike against them, but they may have explanatory value nonetheless. We refer to models in which payments are transformed as *Generalized Present Value* models, and will denote a transformation of payment π_t by $f(\pi_t)$ (rather than by $u(\pi_t)$) to emphasize the agnostic interpretation of the function. We briefly develop comparative static implications of *Transformed Present Value* (discounted utility, for true believers) below, to illustrate the fact that alternative models can be analyzed in the triangle framework. In a separate paper (Gigliotti and Sopher (1997b)) we develop and test *Time-Ordered Present Value*, which employs a decumulative transformation of payments, analogous to anticipated or rank-dependent utility.

The Transformed Present Value for a three-period stream of income S is

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where $f(\pi_i)$ is a transformation of π_i dollars. Note that we are implicitly assuming that f(0) = 0, since there may be intervening periods with payments of zero. Taking the total derivative of the above expression and setting equal to zero, we can derive an expression for the slope of the constant transformed present value contours in the intertemporal choice triangle:

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where, as before, $\pi_1 = \pi - \pi_0 - \pi_2$. Note that the slope of a constant transformed present value contour now depends on the shape of the transformation function as well as on the discount rate. Moreover, since, in general, the slope of the transformation function changes with the size of the payment, the slope of the constant transformed present value contours will change as one moves around in the triangle. Consider, for example, movements in a northeasterly direction along a chord with slope of 1 in the triangle. As one moves in this way, π_0 and π_2 increase and π_1 decreases. If f is strictly concave, then $f'(\pi_0)$ and $f'(\pi_2)$ decrease and $f'(\pi_1)$ increases. Thus, the numerator in the above expression is getting larger and the denominator is getting smaller, so the slope is getting steeper as we move up a chord, to the northeast. That is, the constant transformed present value contours bow towards the southeast in the triangle when the transformation function is strictly concave. This opens the possibility of interior choices being optimal, unlike the strict present value or the hyperbolic present value models. More specifically, the condition for an interior optimum, given a choice set in the triangle which is a chord with slope equal to one, is given by

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The comparative statics of shifting the middle time period are similar to those for the standard present value model. If $t_1 = t_2/2$ (equally spaced payments), then

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If $t_1 > t_2/2$, the slope of a constant TPV contour will be less than the above expression, and if $t_1 < t_2/2$ the slope will be greater than the above expression. In any case, an interior optimum is possible in this case.

Transformed present value is just one generalization of the present value model, and we have only discussed technical issues of what the model implies about choice in the triangle. We do not develop the results here, but it can be shown that *Hyperbolic Transformed Present Value* will allow interior chord choices to be optimal. We defer further discussion of conceptual problems with applying transformed present value, or any other generalization, to section 6.

3. Experimental Design

Design of Experiment 1

We will illustrate the design by examining Triangle I in Experiment 1, shown in Figure 3. Each subject is asked to allocate the total dollar payout across three periods in a constrained fashion. For example, on Chord b, the subject can choose to allocate all \$100 to two extremes, or to some linear combination of those extremes. The subject can allocate all \$100 to Period 2, or can transfer dollars from Period 2 to Today and Period 10 equally, reaching the extreme of \$50 Today, Zero in period 2, and \$50 in Period 10.

We will define an allocation as an ordered triple, (Payout Today, Payout in Middle period, Payout in End period). For Triangle I, the periods are Today, 2 weeks from today, and 10 weeks from today. A subject who maximizes present value when facing the choices on chord b in triangle I must choose either the allocation (0,100,0) or (50,0,50). In other words, the subject will choose Point X, the origin of the triangle, or point H, on the hypotenuse. Any choice between Point X and Point H will violate present value maximization for any positive discount rate. Both Point X and Point H can represent present value maximization for appropriates rates of discount.

In Experiment 1, we compared the choices subjects made when the middle payout period was pushed into the future. For example, Triangle I' has the same dollar payout as Triangle I, and the same Chords, but the middle period is 8 weeks from today, not 2 weeks from today. By comparing choices for Triangle I and I', we can investigate whether or not the shift of the middle period affects decisions. In particular, a subject maximizing present value in Triangle I should still do so in Triangle I'.

Table 1a illustrates all possible choices by subjects on Chord b in Triangle I and Triangle I'. For each choice, subjects are categorized as "near point H," "middle," or "near point X," where "near" means that at least a 90% weight is placed on the indicated income stream. Thus, for example, "near point H" means that the subject chose either the allocation (45,10,45) or (50,0,50) or somewhere in between. Using the convention that the letter in a cell is the name of the cell, Cell **a** in Table 1a is the number of subjects choosing "near point H" in both triangle I and I'. An entry in Cell **i** means that a subject chose an allocation at or between (0,100,0) and (5,90,5) in both triangle I and I'. An entry in Cell **g** means that a subject chose an allocation "near point X" in Triangle I and "near point H" in Triangle I'.

----[table 1a about here]----

For a subject who maximizes present value (allowing a little tolerance from the extremes), only one of cells a, i, and g is permissible. An entry in any other cell would imply a violation of present value maximization. In particular, an entry in Cell **c** would indicate that the subject had acted directly contrary to present value maximization. It is easy to see that such a choice pattern violates present value maximization, since the present value of point H does not change as the middle period is shifted into the future, while the present value of point X clearly does decline. Switching from point H to point X as the middle period is pushed into the future implies either a choice of a strictly lower present value income stream, or a mistake on the original question, where point H was chosen.

In our experiment, the modal choice pattern on any given comparison is in Cell **e** ("middle" in both triangles) clearly a failure to maximize present value. The next largest response pattern is in the present value maximizing cells, a, i, and g, summed together. These results show that subjects do not, in general, maximize present value. The small number of responses in cell **c** indicates that subjects do not act directly contrary to present value maximization. For example, in Experiment 1, for Chord b, we had the following results: 11 of 41 subjects chose at or near Point H, allocation (50,0,50), 9 of 41 subjects chose at or near Point X[1], allocation (0,100,0), and 21 of 41 subjects chose an allocation between (0,100,0) and (50,0,50). In this case, a majority of subjects did not maximize present value. Similarly, for all other chords in Triangle I, subjects who choose an allocation at one axis or the hypotenuse may be

[1]"Near point X" and "near point H" means that the subject chose a mixture with at least 90% weight on the indicated allocation. The vast majority of choices shown as "near" a point in Table 1b were, in fact, at the point.

maximizing present value, but subjects choosing on the interior of a chord are not maximizing present value.

----[table 1b about here]----

The results of Experiment 1, Chord b in Triangle I' are as follows. 16 of 41 subjects chose at or near Point H, (50,0,50), 2 of 41 subjects chose at or near Point X, (0,100,0), and 23 of 41 subjects chose between Point H and Point X. The analogue to Table 1a for Chord b, experiment 1 is given in Table 1b. As can be seen from the table, the majority of subjects did not maximize present value in either triangle. In addition, a number of subjects, 18 to be precise, did not move their positions appreciably near to Point H or Point X. They remained in the middle of Chord b.

These results imply little support for strong present value maximization on Chord b in either triangle. But it is important to note that when we compare the choices subjects made between Triangle I and Triangle I', we find present value-seeking behavior, and we do not find significant responses that directly contradict present value maximizing behavior, i.e., Cell **c** in Table 1b has very few responses.

Similar results are obtained when we examine the choices our subjects made when faced with the other chords in Triangle I and Triangle I' and when subjects face choices in Triangle II and II' and Triangle III and III'. Triangle III and III' have the same time-payout structure as Triangles I and I' respectively, but the payments in III and III' are doubled, to $40. Triangles II and II' have the same total payments as I and I', but the time-payout structure is different: Triangle II has payments in 2, 4 and 12 weeks, and Triangle II' has payments in 2, 10 and 12 weeks. The full design is summarized in Figure 4.

Design of Experiment 2

The design of Experiment 2 is similar to that of Experiment 1. For each of 2 different total payoff levels ($1 million and $1 thousand) there are questions corresponding to Triangles I, I', II and II' as in Experiment 1. The difference is that all of these questions are hypothetical, and the time periods are now in years instead of weeks. The proportionate spacing of time periods is the same as in Experiment 1. For a third total payoff level of $20, there are questions corresponding to Triangles I and I' in Experiment 1. These payoffs are (potentially) real dollar payoffs, and were included mainly as a way of compensating subjects, but also for comparison with Experiment 1. The only other difference from Experiment 1 is that subjects were explicitly told in Experiment 2 that they should make their decisions under the assumption that they could not borrow money from future payments, that they could not bequeath future payments to another person, and that all payments are after tax. The motivation for including these additional instructions comes from the results of Experiment 1, and are discussed in Section 4. Discussion of the design and results of Experiment 3 is deferred until after the discussion of the results of Experiments 1 and 2.

Method

The experiment was conducted by computer. Subjects were recruited from economics classes at Rutgers University. Subject arrived at an appointed time and were seated at individual computer terminals. When all subject were present, instructions were read aloud and questions answered. The instructions are included as an appendix to the paper. Subjects were then free to work through the questions at their own pace. There were twenty questions in Experiment 1, eighteen of which are analyzed in the paper. There were 32 questions in Experiment 2, 30 of which are analyzed in the paper. The first two questions in each experiment were warm-up questions which gave subjects choices among strictly increasing or decreasing income streams.

Payments in Experiment 1 were denominated in a currency called the "Rutgers Dollar," which had a known exchange rate to real US Dollars of 5 to 1. Thus, the questions in Experiment 1 presented subjects with payments totalling either 100 or 200 Rutgers Dollars. The questions in Experiment 2 presented subjects with payments totalling $1 million, $1 thousand, or $20 dollars, and the payments were simply denominated in U.S. Dollars. At the end of Experiment 1, one of the income streams chosen was randomly selected and paid to the subject (delayed payments were mailed to subjects at the specified times). In Experiment 2, one of the $20 dollar questions was randomly selected and paid to the subject. There were a total of 41 subjects in Experiment 1 and 27 subjects in Experiment 2.

The program is Windows-based and written in Visual Basic. For each question, the program would initially display a point of a chord in one of the triangles. The income stream was displayed in both tabular format and in a graphical, bar chart, format. The subject could use a mouse to move to the right or left on a bar at the bottom of the screen to change the income stream displayed. Movements to the right and left corresponded to moving up and down a chord in a triangle. The subject could, at any time, select and confirm the currently displayed income stream. In

Experiment 1, the initial point displayed was always the midpoint of the chord. In Experiment 2, the initial point displayed was always either the "axis" end of the chord or the "hypotenuse" end of the chord for a given subject. A statistical test showed no difference in the distribution of choices made in each group, so no further reference to this issue is made below.

4. Empirical Results

Experiment 1

All analysis is in terms of *normalized choices*, by which we mean a number between zero and one indicating the proportionate distance of the chosen point on a chord from the hypotenuse of a triangle. Smaller numbers indicate choices closer to the hypotenuse. In terms of Tables 1a and 1b, small normalized choices ($<.1$) are thus "Near H" and large normalized choices ($>.9$) are "Near X," that is, near the axis, while normalized choices between .1 and .9 are "middle" choices.

The main statistical test used in the analysis is the Fleiss-Everitt Ordered Cells test for 3 x 3 contingency tables (Fleiss (1973), Fleiss and Everitt (1971)). As discussed above in section 3 in relation to Table 1a, we are primarily interested in determining consistency in present-value maximization in pair-wise choice comparisons. Referring again to Table 1a, the Fleiss-Everitt Ordered Cells Test is designed to detect systematic movements in a particular direction. The test statistic, written in terms of the cell entries in Table 1a, is as follows: $FS = [(b-c) + 2(c-g) + (f-h)]^2/[(b+d) + 2(c+g) + (f+h)]$. The statistic is chi-squared with one degree of freedom[2]. A large chi-squared value rejects the null hypothesis of no systematic difference in favor of the alternative hypothesis of a systematic shift in the direction predicted by present value maximization or present-value seeking behavior. Note that $d>b$ and $h>f$ imply present-value seeking behavior, but not strict present value maximization. g is the number of subjects who move from the origin to the hypotenuse on the chord, which is consistent with present value maximization. d is the number of subjects who move from the middle of the chord to the hypothenuse, while h is the number who move from the origin to the middle

[2]The statistic has one degree of freedom because present value maximization predicts a specific direction of change. Without theoretical guidance (i.e., if one were simply looking for any kind of change), the statistic would have two degrees of freedom.

of the chord. The latter two movements are "in the right direction," or "present value seeking."

----[table 2 about here]----

Table 2 summarizes the results of Experiment 1. The average dollar amount chosen on each of 3 chords in each of 6 triangles, along with the average normalized choice, are shown. Table 3 summarizes Experiment 1 in terms of pairwise choice patterns over all chords in primed versus unprimed triangles. The table shows the percentages of choice patterns falling into each of the nine possible cells implied by the 3x3 categorization required for the Fleiss-Everitt test. The largest percentages of choices occur in the middle of the table, implying that the largest number of choices on each chord occur in the middle of the chord. As noted in Section 3 above, the percentages show a pattern much like that observed in Table 1b for chord B in Triangle I. Below the table are shown the Fleiss-Everitt Ordered Cell test statistics on a chord-by-chord basis, which largely confirms what the total percentages suggest. Eight of the nine comparisons indicate a systematic shift in the direction predicted by present value maximization, significant at the 10% level. Thus, what systematic movement that is observed tends to be in the direction of present value maximization. On the other hand, more than 40% of choices were in the middle of the chords on both choices, which is a startling rejection of strict present value maximization.

----[table 3 about here]----

Some studies have found evidence of negative discounting (e.g., Loewenstein and Sicherman (1991)), but we find little evidence of this sort. Some *individuals* do shift choices opposite to the direction that a positive discount rate would imply, but they are a minority. The Fleiss-Everitt tests reported in Table 3 shows that choices are systematically biased towards, and not away from, present value maximization.

We find little evidence of the widely-documented phenomenon that Loewenstein and Prelec (1992) refer to as the "common difference effect" (see Thaler (1981)) in our data. The common difference effect is the observation that displacing all payments by a common length of time into the future leads to less discounting. Comparisons of Triangles I and II, and of I' and II', are the relevant choices to consider. The results of Wilcoxon signed-rank tests reported at the bottom of Table 3 show that there is no systematic difference between such choices. Tested on a chord-by-chord basis, there is no difference in the distribution of the normalized choices between triangles where all payments are displaced by a common amount (2 weeks) in 4 of 6 possible cases. In two cases, which show significant differences at the 10% level, the implied effects are in opposite directions, suggesting no overall systematic effect. In our questions, taking more money up front also means *delaying* part of the second payment to the third period as well, and this seems to make all the difference.

We also find insignificant magnitude effects, unlike previous studies (again, see Loewenstein and Prelec (1992)). The magnitude effect is the observation that large monetary amounts suffer less discounting than small dollar amounts. Comparisons of Triangles I and III, and of I' and III' are the relevant choices to consider. Tested on a chord-by-chord basis, there is no difference in the distribution of the normalized choices between triangles where all payments are doubled in 6 of 6 possible cases.

Experiment 2

Why would so many subjects, while not behaving in a manner directly contrary to present value maximization, fail to maximize present value in such a simple, stark setting? Experiment 2 was designed to test one hypothesis, that subjects feel constrained to make intertemporal income choices that best fit with their intertemporal expenditure desires. An inability to borrow against future earnings, an inability to control oneself, or an inability to imagine the possibilities of moving income to different periods are all possible reasons for such behavior. The instructions for Experiment 2 included a statement that

> First, all payments are after-tax payments. Second, you cannot borrow against future payments. Third, no one but you can receive the payments. For example, you cannot designate another individual to receive your payments in the event that you would not live long enough to receive a payment. Finally, all payments will be corrected for inflation.

----[table 4 about here]----

----[table 5 about here]----

----[table 6 about here]----

Tables 4 to 6 contain summary results of choices in the Million Dollar, Thousand Dollar, and Twenty Dollar total payment questions. The average dollar amount chosen on each of 3 chords in each of 10 triangles, along with the average normalized choice, are shown. Table 7 contains the results of Experiment 2 organized in terms of the basic present value comparison (primed vs. unprimed triangles). The matrix shows the percentage of choices that fall in each of the nine possible cells. As for Experiment 1, the percentages show a pattern much like that observed in Table 1b for chord b in Triangle I, and the Fleiss-Everitt Ordered Cells test statistics on a chord-by-chord basis, again largely support present value seeking behavior. Five of the six comparisons for the Million and Thousand Dollar questions, and two of the three comparisons for the Twenty Dollar questions, indicate a systematic shift in the direction predicted by present value maximization, significant at the 5% level.

----[table 7 about here]----

Another interesting pattern evident in Table 7 is the growth in the percentage of choices consistent with present value maximization as the total payments are reduced (18% in the Million Dollar questions, 30% in the Thousand Dollar questions, 55% in the Twenty Dollar questions). This is consistent with present value maximization with borrowing constraints. A million dollar injection of funds into one's lifetime income stream will alter one's desired expenditure path more than a thousand dollar injection of funds would and, in particular, the highest present value stream available could be suboptimal, in terms of the feasible expenditure paths permitted. An injection of $20 over a 10 week period should be trivial, from a lifetime optimal consumption path perspective, and one should clearly chose the highest present value stream. Interestingly, the choice patterns for the same Twenty dollar questions in Experiment 1 were very different, with only about 40% of choices consistent with present value maximization. It seems, in fact, that the mere suggestion in the instructions that intertemporal shifting of payments is possible lead to different behavior by some proportion of subjects in the Twenty dollar questions in Experiment 2. This suggests that there is some "inability to imagine" expenditure patterns other than the income streams that are offered.

As in Experiment 1, we find little evidence of common difference or magnitude effects. Tests for the common difference effect, based on comparisons between the relevant triangles within both the million dollar and the thousand dollar treatments, are reported immediately below the Fleiss-Everitt tests for each treatment in Table 7. The hypothesis of no change in the distribution of choices is not rejected by a Wilcoxon Signed Rank test in 6 out of 6 cases for the Million dollar triangles, and in 5 of 6 cases at the 10% level, and 6 of 6 cases at the 5% level for the Thousand Dollar triangles. The magnitude effect, which is tested with chord-by-chord comparisons between comparable triangles in the million and thousand dollar treatments, is not much in evidence either. The results are reported at the bottom of Table 7. The hypothesis of no change in the distribution of choices is not rejected in 11 of 12 cases at the 5% level. Note that this is in spite of the fact that the magnitudes are much different in Experiment 2: $100 vs $10,000, a squaring of the smaller amount, rather than just a doubling of the smaller amount as in Experiment 1.

5. Design and Results for Experiment 3

A number of intertemporal choice anomalies have been documented in previous studies, but we have found little evidence in support of these results in our first two experiments. Experiment 3 is intended to provide a more direct comparison with previous studies of intertemporal choice. To put things in context, Experiments 1 and 2 were mainly concerned with pair-wise comparisons of choices, each of which was a "most preferred " choice made along a single chord within a given intertemporal choice triangle, with each triangle representing the same total payoffs. Previous studies have used pair-wise comparisons of choices, each of which is an "open-ended" indifference statement of a monetary sum, to be received at some later date, that would be just as desirable as some given amount of money at an earlier date.

Put in terms of our triangle framework, previous studies have asked subjects to consider a specific income stream in one triangle ($100 today, for example), and to select another specific income stream ($X in one year, for example), that would be just as desirable. In other words, subjects are constrained as to which period the income is to be received in, but they are allowed freedom to state the value of X. Our approach is to give subjects a choice between either of two specific income streams, or to choose any convex combination of the two streams. Thus, as in Experiments 1 and 2, subjects are choosing their most-preferred income stream from a pre-specified menu, rather than making an indifference statement. For example, one question asks subject to choose between receiving $100 today, or $110 in one year. As in Experiments 1 and 2, there is clear interpretation of any choice that might be made in this situation: If the subject chooses at either extreme, there is no apparent violation of present value maximization, while any choice in the middle is a violation.

This framework will allow us to test for present value maximization by comparing a question like the one above to a second question, in which a subject is given a choice between $100 today and $180 (for example) in one year. There is, again, a clear interpretation of any choice that might be made in this situation. Moreover, there is a clear prediction that can be made for the joint choice made in the two questions. Specifically, there are only three (out of nine possible) types of joint choices that are consistent with present value maximization. Thus, as in Experiments 1 and 2, we can use the Fleiss-Everitt Ordered Cells statistic to test for present-value behavior in Experiment 3. Table 8 shows how we can classify choices, as for Experiments 1 and 2, into "Earlier," "Middle" and "Later" choices, according to whether the weight placed on the larger amount of money to be received in a year is more than .9, between .9 and .1, or less than .1. The cells consistent with present value maximization vary depending upon whether the payments are gains or losses. When the payments are gains, then cells a, c and i are consistent with present value maximization. When the payments are losses, then cells a, g, and i are consistent with present value maximization.

----[table 8 about here]----

We will also be able to test for the existence of some other previously documented anomalies. Besides the "common difference effect" and the "magnitude effect" already considered in Experiments 1 and 2, the anomalies we will test for are the "gain/loss asymmetry" and the "delay/speed-up asymmetry." The gain/loss asymmetry is the finding that gains are discounted differently than losses. Specifically, losses tend to be discounted less, implying that individuals prefer to make payments that are owed earlier than would be predicted, based on choices made over gains. The delay/speed-up asymmetry is a framing effect that has been observed by Loewenstein (1988). The amount of money one would be willing to accept to have a given payment delayed for some period of time should be the same as the amount of money one would be willing to pay to receive a delayed payment earlier. As for the tests of the common difference and magnitude effects in Experiments 1 and 2, we will use the Wilcoxon Signed-Rank tests, which is designed to detect more general changes in the distribution of choices than the Fleiss-Everitt test. In general, one is more likely to find a significant change with the signed-rank test, though there are changes that would be detected by the Fleiss-Everitt test that the signed-rank test would not detect[3].

----[table 9 about here]----

The design and results of Experiment 3 are summarized in Table 9. There are 12 "sets" of questions, with each set containing up to three related choice questions. The general format of the questions within a set is that a subject is given a choice between a given amount of money (stake) to be received in an early period and a "low," a

[3]For example, any change within the set of "middle choices" would not be detected by the Fleiss-Everitt test. On the other hand, a shift from the middle to one extreme might be detected by the Fleiss-Everitt test, and not by the signed-rank test. We have conducted both tests for all the anomalies and note the few differences observed in the text. The tables report only the signed-rank test results.

"medium" and a "high" alternative amount of money to be received in a later period. The alternatives are 10%, 80%, and 150% greater, respectively, than the stake to be received earlier. The time between the earlier and later periods is 2 years in sets 1-10 (the hypothetical questions) and 2 weeks in sets 11 and 12 (the real money questions). Sets differ in (i) the magnitude of the stake, (ii) whether the stake is a gain or a loss, and (iii) whether the early period is "today" or "5 periods into the future." Table 9 provides this information about each set, as well as the average dollar income streams chosen for each of the alternatives.

----[table 10 about here]----

Table 10 contains results of the basic test for present value maximizing behavior. The matrices show the percentages of joint choice patterns in pairwise comparisons (the low and the medium alternative questions, and the medium and the high alternative questions). In general, there is a systematic tendency for choices to move in a present value seeking direction in the low/medium joint choices, especially for gains. There is no significant systematic movement in the medium/high joint choices, indicating that those who are going to switch to the later payment (for gains) or the earlier payment (for losses) have already done so for the medium alternative, and thus there is no further change for the high alternative. Only 15-20% of choices are consistent with strict present value maximization, and the largest percentage of choices tend to be in the middle cell in all comparisons (about 1/3 of choices).

Tables 11-14 contain results of tests for anomalies previously documented in other experiments. In all of these Tables we present a summary of the joint choices made as in the previous tables, but we report results of the more general test, the Wilcoxon signed-rank test. Since an anomaly is, by definition, something that is at variance with standard theory and thus, unpredictable, we choose not to use the Fleiss-Everitt test, the results of which provide an interpretation in terms of present-value maximization. The signed-rank test is designed to detect unspecified changes, and thus we cannot say *a priori* what we expect, aside from the direction of the change expected (based on the previous observations of the anomaly).

----[table 11 about here]----

Table 11 contains results of testing for the common difference effect on gains and losses separately. The "earlier" and "later" income streams are those that start in period 0 and those that start in period 5, respectively. The tests, based on a pair-wise comparison of the relevant sets, alternative by alternative, are reported below the summary matrix of choice percentages. None of the 7 comparisons for gains or the 6 comparisons for losses show a significant difference at the 10% level. One of the gain comparisons (set 1 vs. set 2, alternative b) was significant at the 10% level using the Fleiss-Everitt test.

----[table 12 about here]----

Table 12 contains results of testing for the magnitude effect on gains and losses separately. The "smaller" and "larger" income streams are those for which the initial stake is $100 and $10,000, respectively. The tests, based on a pair-wise comparison of the relevant sets, alternative by alternative, are reported below the summary matrix of choice percentages. Two of the six comparisons for gains and one of the 6 comparisons for losses show a significant difference at the 5% level. The Fleiss-Everitt test were exactly consistent with the signed-rank tests.

----[table 13 about here]----

Table 13 contains results of testing for the gain/loss asymmetry. Choices of "gain" and "loss" income streams of the same dollar magnitude are compared. The tests are reported below the summary matrix of choice percentages. Eight of 12 comparisons are significant at the 10% level. The Fleiss-Everitt test were almost precisely complementary to the signed-rank tests, indicating a significant difference at the 5% level for three of the four cases that were insignificant using the signed-rank test, but showing no significance for the eight cases that were significant using the signed-rank test. Only the Set 3 vs Set 7, Alternative A comparison is insignificant for both tests. Thus, there is substantial evidence of a gain/loss asymmetry in our data.

----[table 14 about here]----

Table 14 contains results for the delay/speedup asymmetry. To test for this anomaly, we made use of the feature of our program that allows specification of which income stream is initially displayed to the subject. In Experiment 3 the later stream was always displayed first, except in 2 sets of questions, sets 9 and 10, which are otherwise identical to sets 1 and 2. Thus, in sets 1 and 2, subjects started by viewing the later stream, and had to experience a loss of part of the later payment in order to receive an earlier, smaller increment. In sets 9 and 10, subjects started by viewing the earlier stream, and had to experience the loss of part of the earlier payment in order to receive a later, larger increment. The questions clearly are identical, and should yield the same answer, if subjects are not influenced by the presentation of the question. Clearly, this is not such a strong "frame" as Loewenstein's questions provided, and the fact that subjects in our experiment could move back and forth, checking out difference streams, allows a dilution of the initial frame to occur. The test results indicate, indeed, no sign of a difference for any of the 6 possible comparisons, for either the signed rank tests reported at the bottom of Table 14, or for the (unreported) Fleiss-Everitt tests.

6. Conclusions

In this paper we have developed a new tool of analysis which can be used to investigate intertemporal decision making. The intertemporal choice triangle, modelled on the probability triangle of Marschak and Machina, allows the analyst to construct experimental controls with great precision. A subject acting to maximize present value will have to choose a well-defined location in the triangle. The intertemporal choice triangle also permitted the construction of simple questions, easily understood by subjects, that gave each subject great freedom to decide just what distribution of payments was most preferred. Using simple graphs, a subject could easily see the possible pay-off streams he or she could choose. The questions we asked permitted a range of answers, so subjects were not forced into expressing a preference between two extreme choices. Subjects could pick, with great latitude, the stream of payments that suited them best.

When permitted to choose in this fashion, we found that, in most cases, subjects did not choose streams of income that could be the maximum present value available, for any positive constant discount rate. Instead, they chose streams of payments that were more dispersed over various periods than the extreme options were. But when subjects' choices in various time and payment configurations, i.e., between different intertemporal choice triangles, were compared, a meaningful and significant result was observed. Although subjects may not always maximize present value in their decisions, they do, systematically, move towards higher present value outcomes, rather than away from them. We have called this *present value-seeking* behavior.

We found too that subjects in responding to questions in our framework do not exhibit most of the well-known anomalies of choice that have been observed by other researchers. Common difference, magnitude and delay/speed-up effects, were either non-existent, or very weak. The gain/loss asymmetry is apparent in our data. This does not mean that such anomalies are not important, but it does appear that they are not robust phenomena. A provisional conjecture that might be made is that there are reference point effects that are less salient in our framework, due to the flexibility that subjects have in surveying the menu of possible choices. The fact that the gain/loss asymmetry is significant in our data is consistent with this: there is nothing in our framework that makes a negative payoff seem less bad than it would seem in any other framework.

It seems that generalizations of present value maximization, similar in structure to generalized expected utility theory in the analysis of choice under risk, can be used to rationalize much of the behavior we have observed. It must be noted, though, that our intertemporal choice triangle suggests a specific way of translating generalized expected utility models into generalizations of present value maximization. Discount rates will not play the role of probability values, nor will a transformation of payment per period be the analogue of the utility of lottery outcomes. It is the total payments over time that can be normalized to permit the creation of the intertemporal choice triangle. Discount rates are more like the utility of lottery outcomes in this analogy, with the distribution of payments over time playing the role of probabilities. The brief exploration of the transformed present value model in section 2 of this paper is one such generalization, in which the payments are transformed. We report on another generalization, a translation of rank-dependent utility into the intertemporal choice framework, in another paper (Gigliotti and Sopher (1997b)).

It is important, though, to ask what we are trying to prove when engaged in experimental studies of choice under risk, or choice over time, and when we develop new theories to explain observations. It is as easy to find violations of present value maximization as it is to find violations of the independence axiom of expected utility theory. Expected utility, though it has clearly been falsified in a strict sense, is still around and has not been replaced by any one alternative to this date. Perhaps the reason for this is that it still has a lot of appeal as a normative theory, and a good normative theory is frequently going to be descriptive of how some people behave. Similarly, the idea that one should maximize the present value of intertemporal choice alternatives has a lot of normative appeal, though it is clearly not descriptive of how many people behave. A more constructive program of research than simple falsification is to investigate just how individuals behave, and then attempt to understand how useful and meaningful our theories will be in explaining this behavior. From our experiments, for example, one might conclude that present value maximization is not the only type of rational behavior individuals will exhibit, but that rational individuals will tend towards it. What makes this type of conclusion possible is the fact that our design permitted a clear interpretation of any given choice, and the data generated by our design provided a basis for making a constructive inference--that subject behavior could be generally described as present value seeking--rather than just the negative implication that most subjects do not maximize present value. Given our complex and sophisticated world of experience, this may be the best that we can expect of a theory.

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TABLE 1A
CATEGORIZATION OF CHOICES: EXPERIMENTS 1 AND 2

Triangle I' / Triangle I	Near Point H	Middle	Near Point X
Near Point H	a	b	c
Middle	d	e	f
Near Point X	g	h	i

TABLE 1B
CATEGORIZATION OF CHOICES: EXPERIMENT 1, TRIANGLES I and I', CHORD B

Triangle I' / Triangle I	Near Point X	Middle	Near Point X
Near Point H	6	4	1
Middle	3	18	0
Near Point X	7	1	1

TABLE 2

AVERAGE DOLLAR AND NORMALIZED CHOICES
EXPERIMENT 1

Triangle (I,II,...), Chord (a,b,c), Time (wks.) Stake (RU$) (1 US$ = 5 RU$)	Dollar Choices in Early Period	Dollar Choices in Middle Period	Dollar Choices in Late Period	Normalized Choice (N=41)
Ia, (0,2,10), RU$100	63.1	23.8	13.1	.48
Ib, (0,2,10), RU$100	28.8	42.4	28.8	.42
Ic, (0,2,10), RU$100	11.8	26.4	61.8	.53
IIa, (2,4,12), RU$100	63.1	23.7	13.1	.47
IIb, (2,4,12), RU$100	27.9	44.2	27.9	.44
IIc, (2,4,12), RU$100	11.1	27.9	61.1	.56
IIIa, (0,2,10), RU$200	127.2	45.5	27.2	.46
IIIb, (0,2,10), RU$200	62.7	74.5	62.7	.37
IIIc, (0,2,10), RU$200	25.7	48.5	125.7	.49
I'a, (0,8,10), RU$100	65.1	19.8	15.1	.40
I'b, (0,8,10), RU$100	39.9	20.2	39.9	.20
I'c, (0,8,10), RU$100	19.4	11.3	69.4	.23
II'a, (2,10,12), RU$100	66.3	17.5	16.3	.35
II'b, (2,10,12), RU$100	34.4	31.2	34.4	.31
II'c, (2,10,12), RU$100	19.9	10.3	69.9	.21
III'a, (0,8,10), RU$200	132.4	35.2	32.4	.35
III'b, (0,8,10), RU$200	81.3	37.4	81.3	.19
III'c, (0,8,10), RU$200	40.2	19.5	140.2	.20

TABLE 3
PRESENT-VALUE MOVERS
EXPERIMENT 1

Late Middle Period Streams

	Choices	Near H	Middle	Near X
Early Middle Period Streams	Near H	17.88%	5.15%	0.81%
	Middle	6.78%	41.73%	0.81%
	Near X	15.45%	3.79%	7.59%

Fleiss-Everitt Ordered Cell Test
X^2 statistic (p-value)

	Comparison	Chord a	Chord b	Chord c
Present	**I vs. I':**	2.0 (.16)	3.6 (.06)	9.3 (.00)
Value	**II vs. II':**	3.9 (.05)	3.2 (.07)	14.1 (.00)
Max	**III vs. III':**	4.5 (.03)	4.8 (.03)	10.0 (.00)

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

	Comparison	Chord a	Chord b	Chord c
Common	**I vs. II:**	0.5 (.60)	0.8 (.45)	-0.7 (.46)
Difference:	**I' vs. II':**	1.7 (.09)	-1.9 (.06)	0.4 (.69)
Magnitude:	**I vs. III:**	0.7 (.46)	0.9 (.39)	-0.3 (.97)
	I' vs. III':	0.7 (.48)	-0.1 (.92)	-0.3 (.98)

TABLE 4
AVERAGE DOLLAR AND NORMALIZED CHOICES
EXPERIMENT 2: MILLION DOLLAR STAKES

Triangle (I,II,..) Chord (a,b,c) Time (years)	Dollar Choices in Early Period	Dollar Choices in Middle Period	Dollar Choices in Late Period	Normalized Choice (N=27)
Ia, (0,10,50)	608,807.5	282,385.0	108,807.5	.55
Ib, (0,10,50)	225,528.2	548,943.5	225,528.2	.56
Ic, (0,10,50)	89,855.8	320,288.5	589,855.8	.64
IIa, (10,20,60)	612,071.8	275,856.3	112,071.8	.43
IIb, (10,20,60)	286,073.6	427,852.8	286,073.6	.55
IIc, (10,20,60)	86,181.6	327,636.8	586,181.6	.66
I'a, (0,40,50)	679,454.8	141,090.4	179,454.8	.19
I'b, (0,40,50)	406,086.7	187,826.6	406,086.7	.28
I'c, (0,40,50)	179,328.2	141,343.7	679,328.2	.28
II'a, (10,50,60)	688,899.5	122,201.0	188,899.5	.17
II'b, (10,50,60)	414,790.0	170,419.9	414,790.0	.24
II'c, (10,50,60)	190,459.0	119,082.0	690.459.0	.24

TABLE 5
AVERAGE DOLLAR AND NORMALIZED CHOICES
EXPERIMENT 2: THOUSAND DOLLAR STAKES

Triangle (I,II,...) Chord (a,b,c) Time (years)	Dollar Choices in Early Period	Dollar Choices in Middle Period	Dollar Choices in Late Period	Normalized Choice (N=27)
Ia, (0,10,50)	629.5	241.0	129.5	.38
Ib, (0,10,50)	309.4	381.2	309.4	.48
Ic, (0,10,50)	104.2	291.5	604.2	.58
IIa, (10,20,60)	600.3	299.4	100.3	.58
IIb, (10,20,60)	209.6	580.7	209.6	.60
IIc, (10,20,60)	87.7	324.6	587.7	.65
I'a, (0,40,50)	680.6	138.7	180.6	.26
I'b, (0,40,50)	369.7	260.6	369.7	.28
I'c, (0,40,50)	209.6	80.9	709.6	.16
II'a, (10,50,60)	691.4	117.1	191.4	.18
II'b, (10,50,60)	410.4	179.1	410.4	.23
II'c, (10,50,60)	209.4	81.3	709.4	.16

TABLE 6
AVERAGE DOLLAR AND NORMALIZED CHOICES
EXPERIMENT 2: TWENTY DOLLAR STAKES

Triangle (I,I') Chord (a,b,c) Time (weeks)	Dollar Choices in Early Period	Dollar Choices in Middle Period	Dollar Choices in Late Period	Normalized Choice (N=27)
Ia, (0,2,10)	11.7	6.6	1.7	.66
Ib, (0,2,10)	4.5	10.9	4.5	.55
Ic, (0,2,10)	2.1	5.7	12.1	.57
I'a, (0,8,10)	13.5	3.1	3.5	.31
I'b, (0,8,10)	7.3	5.3	7.3	.27
I'c, (0,8,10)	3.7	2.6	13.7	.39

TABLE 7a
PRESENT-VALUE MOVERS: EXPERIMENT 2

MILLION DOLLAR STAKES

Late Middle Period Streams

	Choices	Near H	Middle	Near X
Early Middle	Near H	4.32%	12.35%	0.62%
Period	Middle	23.46%	28.40%	0.62%
Streams	Near X	12.35%	16.67%	1.23%

Fleiss-Everitt Ordered Cell Test
X^2 statistic (p-value)

	Comparison	Chord a	Chord b	Chord c
Present Value Max	**Im vs. Im':**	6.8 (.01)	9.8 (.01)	7.0 (.01)
	IIm vs. IIm':	7.1 (.01)	1.3 (.26)	10.8 (.00)

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

	Comparison	Chord a	Chord b	Chord c
Common Difference:	**Im vs. IIm:**	1.6 (.12)	-0.2 (.82)	-0.2 (.83)
	Im' vs. IIm':	0.3 (.78)	0.5 (.65)	0.5 (.61)

THOUSAND DOLLAR STAKES

Late Middle Period Streams

	Choices	Near H	Middle	Near X
Early Middle	Near H	9.88%	10.49%	0.00%
Period	Middle	18.52%	20.99%	3.70%
Streams	Near X	20.37%	16.05%	0.00%

Fleiss-Everitt Ordered Cell Test
X^2 statistic (p-value)

	Comparison	Chord a	Chord b	Chord c
Present Value Max	**It vs. It':**	4.5 (.03)	2.7 (.10)	8.4 (.00)
	IIt vs. IIt':	8.3 (.00)	10.8 (.00)	12.9 (.00)

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

	Comparison	Chord a	Chord b	Chord c
Common Difference:	**It vs. IIt:**	-1.7 (.09)	-1.0 (.31)	-0.8 (.43)
	It' vs. IIt':	0.7 (.46)	-0.4 (.67)	0.0 (1.0)

	Comparison	Chord a	Chord b	Chord c
Magnitude:	**Im vs. It:**	1.6 (.11)	0.9 (.36)	0.5 (.59)

IIm vs. IIt:	-1.2 (.22)	-0.7 (.49)	0.3 (.76)
Im' vs. It':	-0.7 (.54)	0.4 (.70)	2.2 (.03)
IIm' vs. IIt':	0.3 (.74)	0.5 (.62)	1.1 (.27)

TABLE 7b
PRESENT-VALUE MOVERS: EXPERIMENT 2

TWENTY DOLLAR STAKES

Late Middle Period Streams

Early Middle Period Streams	Choices	Near H	Middle	Near X
	Near H	16.05%	4.94%	7.41%
	Middle	13.58%	3.70%	4.94%
	Near X	32.10%	9.88%	7.41%

Fleiss-Everitt Ordered Cell Test

X^2 statistic (p-value)

	Comparison	Chord a	Chord b	Chord c
Present Value Max	**I vs. I':**	5.4 (.02)	5.1 (.02)	6.4 (.01)

TABLE 8
DATA INTERPRETATION: EXPERIMENT 3

$100 now vs $180 in 2 Years $100 now vs $110 in 2 Years	Earlier	Middle	Later
Earlier	a	b	c
Middle	d	e	f
Later	g	h	i

TABLE 9

AVERAGE DOLLAR CHOICES
EXPERIMENT 3

Set Number	**Stakes ($) Gain or loss (+ or -) Early, late periods**	Low Alternative Choice (Stakes+10%)	Medium Alternative Choice (Stakes+80%)	High Alternative Choice (Stakes+150%)
1	$100, +, 0,2 yrs	(76.3, 26.1)	(54.7, 81.5)	(43.5, 141.2)
2	$100, +, 5,7 yrs	(69.2, 33.9)	(43.0, 102.5)	(40.5, 148.8)
3	$10,000, +, 0,2 yrs	(5333.3, 5133.3)	(3098.3, 12423.0)	(3045.7, 17385.8)
4	$10,000, +, 5,7 yrs	(6027.3, 4369.9)	(3269.4, 12115.0)	(3057.0, 17357.6)
5	$100, -, 0,2 yrs	(61.6, 42.3)	(68.7, 56.4)	(73.8, 65.5)
6	$100, -, 5,7 yrs	(57.1, 47.2)	(68.9, 56.0)	(75.9, 60.2)
7	$10,000, -, 0,2 yrs	(4159.0, 6425.1)	(6592.1, 6134.2)	(7020.6, 7448.4)
8	$10,000, -, 5,7 yrs	(4357.0, 6207.3)	(6991.5, 5415.3)	(5333.3, 6629.7)
9	$100, +, 0,2 yrs, reversed	(84.5, 17.1)	(57.9, 75.8)	(55.2, 112.1)
10	$10,000, +, 0,2 yrs, reversed	(6806.4, 3512.9)	(3517.3, 11668.8)	(3602.4, 15994.0)
11	$20, +, 0,2 wks	(17.2, 2.9)		
12	$20, +, 5,7 wks	(15.3, 4.7)		

TABLE 10
PRESENT-VALUE MOVERS: EXPERIMENT 3

GAINS

Medium Alternative

	Choices	Earlier	Middle	Later
Low	Earlier	5.00%	20.83%	11.67%
Alternative	Middle	5.83%	33.33%	17.50%
	Later	0.00%	4.17%	1.67%

High Alternative

	Choices	Earlier	Middle	Later
Medium	Earlier	1.67%	6.67%	2.50%
Alternative	Middle	3.33%	35.00%	20.00%
	Later	0.00%	16.67%	14.17%

Fleiss-Everitt Ordered Cell Test
X^2 statistic (p-value)
Set 1: 7.0 (.01)
Set 2: 10.5 (.00)
Set 3: 6.0 (.01)
Set 4: 10.7 (.00)

Fleiss-Everitt Ordered Cell Test
X^2 statistic (p-value)
Set 1: 3.2 (.07)
Set 2: 0.5 (.82)
Set 3: 0.7 (.41)
Set 4: 0.3 (.62)

LOSSES

Medium Alternative

	Choices	Earlier	Middle	Later
Low	Earlier	11.67%	8.33%	0.00%
Alternative	Middle	19.17%	31.67%	9.17%
	Later	6.67%	12.50%	0.83%

High Alternative

	Choices	Earlier	Middle	Later
Medium	Earlier	11.67%	21.67%	4.17%
Alternative	Middle	24.17%	22.50%	5.83%
	Later	5.83%	4.17%	0.00%

Fleiss-Everitt Ordered Cell Test

X^2 statistic (p-value)	
Set 5:	0.7 (.39)
Set 6:	2.1 (.14)
Set 7:	5.6 (.02)
Set 8:	5.8 (.02)

Fleiss-Everitt Ordered Cell Test

X^2 statistic (p-value)	
Set 5:	0.1 (.73)
Set 6:	0.0 (.88)
Set 7:	0.1 (.80)
Set 8:	0.0 (.83)

TABLE 11
COMMON DIFFERENCE EFFECT: EXPERIMENT 3

GAINS

Later Streams

	Choices	Earlier	Middle	Later
Earlier	Earlier	7.22%	9.44%	2.78%
Streams	Middle	8.89%	32.22%	16.67%
	Middle	0.00%	16.11%	6.67%

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

Comparison	**Low**	**Medium**	**High**
Set 1 vs. Set 2:	-1.0 (.34)	-1.1 (.28)	-0.4 (.70)
Set 3 vs. Set 4:	1.0 (.33)	0.3 (.98)	0.1 (.92)
Set 11 vs. Set 12:	-1.4 (.17)		

LOSSES

Later Streams

	Choices	Earlier	Middle	Later
Earlier	Earlier	14.44%	15.00%	4.44%
Streams	Middle	15.00%	31.11%	8.33%
	Later	2.78%	6.67%	2.22%

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

Comparison	**Low**	**Medium**	**High**
Set 5 vs. Set 6:	-0.3 (.77)	-0.0 (.98)	0.1 (.90)
Set 7 vs. Set 8:	0.6 (.54)	0.6 (.54)	0.6 (.57)

TABLE 12
MAGNITUDE EFFECT: EXPERIMENT 3

GAINS

Larger Streams

	Choices	Earlier	Middle	Later
Smaller	Earlier	4.44%	16.11%	2.78%
Streams	Middle	5.55%	33.89%	13.33%
	Later	0.00%	12.77%	8.89%

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

Comparison	**Low**	**Medium**	**High**
Set 1 vs. Set 3:	-2.4 (.02)	-2.5 (.01)	-1.3 (.18)
Set 2 vs. Set 4:	-1.5 (.14)	-0.9 (.37)	-1.0 (.30)

LOSSES

Larger Streams

	Choices	Earlier	Middle	Later
Smaller	Earlier	10.00%	32.22%	2.78%
Streams	Middle	8.33%	25.56%	6.11%
	Later	2.78%	9.44%	2.78%

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

Comparison	**Low**	**Medium**	**High**
Set 5 vs. Set 7:	-2.6 (.01)	-0.5 (.60)	-0.3 (.75)
Set 6 vs. Set 8:	-1.6 (.12)	-0.3 (.78)	-0.5 (.59)

TABLE 13
GAIN/LOSS: EXPERIMENT 3

Loss Streams

	Choices	Earlier	Middle	Later
Gain	Earlier	4.72%	9.44%	3.61%
Streams	Middle	19.17%	30.56%	8.06%
	Later	9.17%	13.61%	1.67%

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

Comparison	**Low**	**Medium**	**High**
Set 1 vs. Set 5:	-1.4 (.15)	1.1 (.26)	2.7 (.01)
Set 2 vs. Set 6:	-1.0 (.30)	2.3 (.02)	3.1 (.00)
Set 3 vs. Set 7:	-1.3 (.19)	3.9 (.00)	4.3 (.00)
Set 4 vs. Set 8:	-1.7 (.09)	3.6 (.00)	3.8 (.00)

TABLE 14
DELAY/SPEED-UP ASYMMETRY: EXPERIMENT 3

Start with Early Stream

	Choices	Earlier	Middle	Later
Start with	Earlier	11.11%	6.11%	2.22%
Late	Middle	16.11%	26.67%	15.00%
Stream	Later	6.11%	10.56%	6.11%

Wilcoxon Matched Pairs Signed Ranks Tests
z statistic (p-value)

Comparison	**Low**	**Medium**	**High**
Set 1 vs. Set 9:	0.9 (.35)	0.3 (.78)	1.3 (.20)
Set 3 vs. Set 10:	2.0 (.05)	0.3 (.76)	0.6 (.55)