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Alexander of Aphrodisias' Solution to the Puzzle of the Two Modal *Barbaras* : a Semantic Approach*

Aristotle's exposition of the two modal *Barbaras* in *Prior Analytics*, A, 9 has been puzzling for centuries. How can a LX-L *Barbara* be a valid syllogism, given that the hypothetical XL-L *Barbara* is an invalid formula ? As is well known, many logicians attempted to answer this question, and some claimed that Aristotle's thesis was inconsistent. However things may be with respect to the Stagirite's own thought, it is worthwhile to consider the series of answers which has been given to this puzzle ; and in this history — which is in some sense the history of the reception of Aristotle's logic — a preeminent role was played by Alexander of Aphrodisias, the Interpreter *par excellence* of Aristotle's texts in Late Antiquity. How could Alexander make sense of Aristotle's controversial claim ? Could the commentator avoid the objections that ancient logicians moved against it ?

The attempt to answer these questions brings to light at the same time many aspects of Alexander's presentation of Aristotle's syllogistic.

In this paper I present certain aspects of Alexander's semantics for modal syllogistic, which emerge from his commentary on Aristotle's *Prior Analytics*. My claim is twofold : I maintain that (i) propositions which figure as premisses and conclusions of modal syllogisms express a connection between subject and predicate which may or may not vary in time, due to the particular modal operator of the proposition ; (ii) this temporal aspect is the counterpart of an analysis of terms, which are arranged in a certain modal proposition according to the kind of praedicabile which figures as a subject and as a predicate of the proposition. I will take into account the definition of contingency proposition, which may provide an ideal test case in order to verify my claim. My aim is to show that, endorsing the semantics which I ascribe to him, the commentator is able to provide a quite reasonable solution to the puzzle of the two modal syllogistic connections in *Barbara*.

My arguments will appear clearer after a brief presentation of Alexander's core ideas about modal syllogistic.

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I. The structure of Alexander's modal syllogistic

Alexander considers modal syllogistic as an extension of assertoric syllogistic. While assertoric syllogistic deals with assertoric propositions, modal syllogistic considers as premisses and conclusions both modal and non-modal propositions. Alexander had grouped all propositions which can figure as premisses and conclusions of a syllogism into three kinds, depending on the time in which it is true to say that the predicate holds of the subject. We therefore have :

a) necessity propositions, if the predicate always holds of the subject ;
b) contingency propositions (according to more general meaning of 'contingency'), if the predicate not-always holds of the subject. Among them we find
  b.1) predicative non modal propositions, if the predicate now holds of the subject ;
  b.2) genuine contingency propositions, if the predicate not-now holds of the subject[1].

As modal propositions are a set of propositions larger than that of predicative non modal propositions, but obtained from the latter by means of the addition of the modal operator, so also modal syllogisms are an extended class of syllogisms, obtained from the class of non modal syllogisms. They differ from non modal syllogisms in the same way as modal propositions differ from non modal predicative propositions, according to Alexander[2]. As a consequence if there would be principles of non modal syllogistic, by means of some additional devices it would be possible to build up modal syllogistic. But are there such principles or is syllogistic only a class of valid inferences, without any order among them ? As is well known, Aristotle introduced various methods of validation for syllogisms, namely use of the rules of conversion, *reductio ad impossibile* and ἔκθεσις. The reduction of an incomplete syllogism, namely a syllogism either of the second or of the third figure, to a complete syllogism of the first figure uses conversions of premisses ; this method may suggest that there is a certain order at least among syllogistic figures. But why are syllogisms in the first figure complete and more fundamental than syllogisms which can be reduced to first figure syllogisms ? In other words : are there more basic principles in virtue of which we may build up (assertoric) syllogistic ? Aristotle says that a syllogism is complete because the necessity of the inference is *evident*. But the concept of 'evidence' is rather obscure in Aristotle's thought : when he appeals to 'evident' principles, we are not told how we can grasp them : we only know that they cannot be derived by means of demonstration. In the same way

[1] Cf. *in Pr. An.*, pp. 25, lin. 24 - 26, lin. 22.
[2] Cf. *in Pr. An.*, p. 119, lin. 9-19.

incomplete syllogisms may be derived from complete ones by means of conversion, but the concept of 'evident necessity' of the inference is not particularly clear. Indeed there must be something more basic, in virtue of which first figure syllogisms were chosen as 'complete' by the Stagirite : he was aware that it could be possible to reduce first figure syllogisms to the other figures too (cf. *Pr. An.*, A, 7 and A, 45). Why did he choose these four (and not others) as complete ?

Modern scholars have proposed different answers to this question.

Jonathan Barnes has recently pointed out that Aristotle's syllogistic is indeed a *quasi*-axiomatic system and that it would easily fit the Aristotelian requirement for a science : it has some basic rules, namely principles like the so-called *dictum de omni et de nullo*, the rules of conversion, the rules for the *reductio ad impossibile*, and an object of inquiry, namely sound inferences. Now the soundness of the first figure syllogisms is easily proved by means of the *dictum de omni et de nullo*, without any reference to other basic principles. These syllogisms, and in particular the first two, to which *Darii* and *Ferio* may be reduced, can be considered quasi-axioms of the system[3]. All of these syllogisms are complete, since their validity is evident in virtue of the *dictum*. The others need auxiliary proofs in order to show that they are valid. Alexander proposed the same explanation when he pointed out that first-figure syllogisms are complete because their validity can be shown only with reference to the *dictum de omni et de nullo* (cf. *in Pr. An.*, p. 61, lin. 3-5 : πάντες δὲ οἱ προειρημένοι τέλειοι· πάντες γὰρ τῷ κατὰ παντὸς μόνῳ ἢ κατὰ μηδενὸς προσχρώμενοι, ὅ ἐστι κείμενον, φανερὰν τὴν συναγωγὴν ἔχουσιν).

Other scholars such as Joseph Bochenski thought that syllogistic was an axiomatic system and that Aristotle chose among many possible sets of axioms in order to build it up in an arbitrary way[4]. There is no more basic principle which explains the perfection of first figure syllogisms : they are complete only in so far as they are intuitively evident[5]. This explanation seems far from the core ideas of Alexander, whose main

[3] See J. Barnes, *Truth, etc.*, Oxford University Press, Oxford 2007, pp. 366-367 : « so Aristotle's syllogistic has the structure of an axiomatized deductive science : even if, as Aristotle urges, it is not in itself a genuine science, it makes a good job of imitating one. The theorems, or quasi-theorems, of syllogistic include all the imperfect syllogisms which Aristotle considers and also the two particular perfect syllogisms […]. Aristotle's predicative syllogistic is, or can be reconstructed as, an axiomatized deductive system the axiom (or quasi-axioms) of which are two syllogistic forms, certain principles of conversion and of subordination, a principle of reduction to the impossible, and a rule of exposition or ecthesis. And the theorems (or quasi-theorems) are certain derived principles of conversion and subordination — and an infinite number of syllogisms ».

[4] See J. Bochenski, *Formale Logik*, Verlag Karl Aber, Freiburg - München 1956, p. 86 : « die Syllogistik ist das erste uns bekannte axiomatische System, oder, genauer gesagt, die erste Klasse von solchen System : denn Aristoteles hat sie in mehrere Weisen axiomatisiert ».

[5] Cf. Bochenski, *Formale Logik*, p. 87 : « dies kann aber nur bedeuten : vollendete Syllogismen sind intuitiv evident ».

intuition is that syllogistic has the structure of and can be exposed as an Aristotelian science. From Aristotle's texts it is difficult to establish whether syllogistic can be accounted a science or not. Alexander, in his commentary on the *Prior Analytics*, develops many arguments against the Stoics' claim that logic (and therefore syllogistic) is a *part* of philosophy and maintains that it is rather an *instrument* of philosophy. But his arguments do not rule out the possibility that logic, and in particular syllogistic, is a science, although there is not such an explicit claim in Alexander's texts. We are faced instead with an exposition of syllogistic which is strictly connected to the structure of science : Alexander searches always for the principles and the rules from which it is possible to establish the validity of sound syllogisms and to rule out unsound syllogisms, and for each figure he finds properties which define it (concerning for example the quantity or the quality of premisses, the position of the middle term etc.). This account is not particularly original in its content, but what is original is the way in which Alexander thought it could be expressed. Even though we cannot be certain that he thought that syllogistic is a science, it is likely that for Alexander it can be expanded in terms of an Aristotelian science.

This basic assumption is crucial in order to understand how Alexander presents modal syllogistic. In his commentary, modal syllogistic is a sort of extension of non-modal predicative syllogistic, with the introduction of a modal version of the *dictum de omni et de nullo*, which is the basic principle in virtue of which it is possible to show the validity of sound modal syllogisms[6].

[6] Scholars do not agree in interpreting the so-called *dictum de omni et nullo*. Two interpretations have mainly been proposed, which I will label 'orthodox version' and 'heterodox version', following Jonathan Barnes (cf. his *Truth, etc.*, pp. 386-412). The orthodox version states that if 'A is said of all B', then 'there is no individual of which B is said that is not A' ; the heterodox version says on the other hand that if 'A is said of all B', then 'there is no term — of the kind of A and B — of which B is said which is not A'. In the first case we introduce a new predication in the *dictum*, namely a kind term is predicated of individuals. In the other case we are faced only with kind terms predicated of other kind terms. Barnes argues in his book that the first interpretation must be preferred, because (i) 'Aristotle's Greek can hardly be construed in the way demanded by the heterodox *dictum*' and (ii) 'the *dictum* is meant to offer a definition of 'of every' and 'of no', but it is hard to think that the *definiens* of 'of every' will also include an instance of the predication 'of every' — and this would be the case, if the *dictum* were heterodox (cf. Barnes, *Truth, etc.*, p. 412). Other scholars have instead defended the heterodox version : among them was the late Michael Frede, who attended the John Locke lectures in which Barnes was explaining his reconstruction of ancient logic and during the discussion inspired Barnes to discuss this version of the *dictum* (cf. B. Morrison, *Aristotle, etc.*, « Phronesis », 53, 2008, pp. 212-213). The heterodox version presents many advantages : it allows us, for example, to construct a semantic interpretation of Aristotle's modal syllogistic which rejects all the syllogistic connections rejected by the philosopher and to validate all modal syllogisms he considers valid (cf. M. Malink, *A reconstruction of Aristotle's modal syllogistic*, « History and Philosophy of Logic », 27, 2006, pp. 95-141 ; Malink himself stresses that this heterodox interpretation of the *dictum* is preferable, because it is open to the possibility of a 'mereological' account of the relations among terms : see on this topic M. Malink, *ΤΩΙ vs. ΤΩΝ in Prior Analytics I, 1-22*, « Classical Quarterly », 58, 2008, pp. 519-536, especially p. 523). All in all, the issue is rather

Aristotle explicitly presented the *dictum* only in *Pr. An.*, A, 1, 24b28-30 : λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι, ὅταν μηδὲν ᾖ λαβεῖν [τοῦ ὑποκειμένου] καθ' οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως. There is no reference to a modal version of the *dictum*, which seems to work only for non-modal predicative propositions. However, in *Pr. An.*, A, 8, 30a2-3 Aristotle appears to make reference to a possible extension of the *dictum* to modal syllogistic : καὶ τὸ ἐν ὅλῳ εἶναι καὶ τὸ κατὰ παντὸς ὁμοίως ἀποδώσομεν. Aristotle does not explain what such an extended version of the *dictum* would say, but according to Alexander it means that if A is said of all B necessarily, then it is not possible to take one of the Bs of which A is not said necessarily. Alexander believes that there is a translation for modal propositions in virtue of the *dictum*, as in the case of non-modal propositions.

A universal affirmative proposition such as

(a) A is said of all B

controversial. More recently, Paolo Crivelli has cautiously argued in favor of the orthodox interpretation of the *dictum* (cf. P. Crivelli, *Aristotle's logic*, in C. Shields ed., *Oxford Handbook of Aristotle*, Oxford University Press, Oxford 2012, pp. 113-149). I am inclined to agree with him, mainly because I do not see any plausible solution to the second objection which Barnes raises against the heterodox version. Both Morison and Malink, in their attempt to solve this objection, suggest that Aristotle is merely presenting some properties of the *dictum de omni et de nullo*, but not properly proposing a definition (cf. Morrison, *Aristotle, etc.*, p. 214 ; M. Malink, *A non extensional notion of conversion in the* Organon, « Oxford Studies in Ancient Philosophy », 37, 2009, pp. 105-141, esp. pp. 116-117). This answer to Barnes's remark would have been plausible, if Aristotle had not said, at the very beginning of *Pr. An.*, A, 1, that he first wants to define what propositions, syllogisms, terms, perfection and the *dictum de omni* are (cf. *Pr. An.*, A, 1, 24a11-15). After that presentation, the Stagirite clearly gives a definition of proposition (cf. *Pr. An.*, A, 1, 24a16-17) and of syllogism (cf. *Pr. An.*, A, 1, 24b18-20). Hence, it is reasonable to suppose, with Barnes, that Aristotle's elucidations on the *dictum de omni et de nullo* (cf. *Pr. An.*, A, 1, 24b28-30) are true definitions too. However things might seem with respect to Aristotle's own opinions, it is worth noting that Alexander's strategy too appears to be very similar to that of modern supporters of the 'heterodox version' of the *dictum de omni*. Whilst in the case of proposition (cf. *in Pr. An.*, p. 10, lin. 15), syllogism (cf. *in Pr. An.*, p. 16, lin. 24-31) and perfection (cf. *in An. Pr.*, p. 23, lin. 17-18) Alexander clearly says that Aristotle is expounding their definitions, the commentator remarks that as far as the *dictum* is concerned, the Stagirite « first makes clear those things and teaches us what is to be in a whole and what is the *dictum de omni* » (πρῶτον ταῦτα γνώριμα ποιεῖ, καὶ διδάσκει ἡμᾶς, τί μέν ἐστι τὸ ἐν ὅλῳ εἶναι καὶ τὸ κατὰ παντός, *in Pr. An.*, p. 24, lin. 23-24). Alexander does not explicitly speak of definitions : according to him, Aristotle is only presenting a clarification of what he thinks that the relation 'to be in a whole' and the *dictum de omni* are. And this is consistent with both readings of the *dictum de omni et de nullo*. — At this stage of the analysis I would only remark that for Alexander the *dictum* — however it can be interpreted — allows us to translate standard propositions into prosleptic ones and thus to give justifications of the validity of complete syllogisms. This idea was not entirely new. It is highly probable that traditional Aristotelianism before Alexander, as in the case of Boethos of Sidon, held that all syllogisms were valid and complete, because all of them were justified on the basis of the *dictum de omni et de nullo* : on this topic I take the liberty to refer to L. Gili, *Boeto di Sidone e Alessandro di Afrodisia intorno alla sillogistica aristotelica*, « Rheinisches Museum für Philologie », 154/3-4, 2011, pp. 375-397.

can be translated, in virtue of the standard *dictum de omni*, into a prosleptic proposition, namely

(a') A is said of all of which B is said.

In the same way a universal affirmative necessity proposition like

(b) A is said necessarily of all B

can be translated into prosleptic modal propositions like

(b') A is said necessarily of all of which B is said,

(b") A is said necessarily of all of which B is said necessarily.

This translation-thesis, which is crucial for Alexander's account of syllogistic, can be used in order to prove the validity of a sound syllogism. If we consider a non-modal syllogism like *Barbara*, we have

(1) A is said of all B
(2) B is said of all C
Therefore (3) A is said of all C.

How can we prove that (3) follows from (1) and (2)? If we translate the major premiss into a prosleptic premiss, this is quite easy. We will have that

(1') A is said of all of which B is said.

Since C is one of the terms of which B is said (premiss 2), (1') says also that A is said of C. With the *dictum de nullo* it is possible to prove the validity of *Celarent*. From *Barbara* and *Celarent* it is possible to prove all the valid inferences of non-modal syllogistic.

It is plain that the extension of the *dictum* to modal propositions is able to provide a proof of the validity of syllogisms such as *Barbara* LLL and *Celarent* LLL. But it is more complex matter to understand which solution can be proposed for the case of *Barbara* LXL. As is well known, Aristotle maintains that a *Barbara* LXL syllogism is valid, but a *Barbara* XLL formula is invalid. Most logicians considered this assumption puzzling and either rejected the validity of *Barbara* LXL (Theophrastus) or they considered *Barbara* XLL sound too (Łukasiewicz[7]). Only in recent times have scholars

[7] Cf. J. Łukasiewicz, *Aristotle's syllogistic from the standpoint of modern formal logic*, Clarendon Press, Oxford 1958, pp. 184-185. Hence Aristotle's syllogistic, according to Łukasiewicz, « looks like a logical

such as Richard Patterson and Paul Thom tried to make sense of Aristotle's thesis, presenting a particular semantics for modal propositions, which could validate Aristotle's controversial thesis.

Let us first consider Alexander's attempt to present a solution to the problem. We will then examine the semantics he provides for modal propositions and we will see that his way out is quite similar to modern attempts to justify Aristotle's claim.

II. How the dictum works

Albrecht Becker first tried to explain Aristotle's modal syllogistic with the help of a formalization in terms of first order modal logic[8]. This formalization allows us to make a sharp distinction between de dicto and de re modality. Becker's claim was that Aristotle failed to provide a consistent modal logic, because while his laws of conversion for modal premisses must be interpreted as qualified by a de dicto modality, many modal syllogisms are sound only if the modality of the premisses is de re. I do not want to suggest that Becker's claim was right, but I think that his presentation can help us to understand the puzzle which arises from Aristotle's modal syllogistic.

According to Becker's reconstruction, a universal affirmative proposition can be expressed in this way :

(c) $\forall x\,(Bx \rightarrow Ax)$.

A necessity universal affirmative proposition can therefore be expressed in many ways, depending on the position of the modal operator.

If we want to consider a Barbara LXL syllogism valid, we can provide a de re reading of the modal operator. Thus

(1) $\forall x\,(Bx \rightarrow \Box(Ax))$
(2) $\forall x\,(Cx \rightarrow Bx)$
Therefore (3) $\forall x\,(Cx \rightarrow \Box(Ax))$.

It is easy to show that such a reading rules out a syllogistic connection like *Barbara* XLL. The problem is that such a de re reading of the modal operator does not validate the rules of conversion of modal premisses which Aristotle expands in *Pr. An.*, A, 3.

exercise which in spite of its seeming subtlety is full of careless mistakes and does not have any useful application to scientific knowledge » (*ibid.*, p. 181).

[8] See A. Becker, *Die aristotelische Theorie der Möglichkeitsschlüsse. Eine logisch-philologische Untersuchung der Kapitel 13-22 von Aristoteles'* Analytica Priora, Juncken und Dünnhaupt Verlag, Berlin 1933.

The first of these rules concerns universal negative necessity premisses : ἡ μὲν γὰρ καθόλου στερητικὴ καθόλου ἀντιστρέφει, τῶν δὲ καταφατικῶν ἑκατέρα κατὰ μέρος. εἰ μὲν γὰρ ἀνάγκη τὸ A τῷ B μηδενὶ ὑπάρχει, ἀνάγκη καὶ τὸ B τῷ A μηδενὶ ὑπάρχειν (*Pr. An.*, A, 3, 25a28-31).

If we translate the rule according to Becker's proposal, we will have the following statement :

(d) $\forall x\,(Bx \rightarrow \Box\neg(Ax)) \rightarrow \forall x\,(Ax \rightarrow \Box\neg(Bx))$.

This rule has a de re reading of the modal operator and is unsound. According to Becker, the only way of making sense of Aristotle's rule is to consider a de dicto reading of the modal operator. Thus, for example,

(d') $\Box(\forall x\,(Bx \rightarrow \neg Ax)) \rightarrow \Box(\forall x\,(Ax \rightarrow \neg Bx))$

is sound and can be derived from the non modal rule of conversion, by means of the rule of necessitation and with the distribution of the necessity operator with respect to the conditional[9].

In this way Becker thought it was possible to show that Aristotle held conflicting assumptions when he wrote his modal syllogistic. Since then Aristotle's modal syllogistic and in particular the case of the two *Barbaras* has been a puzzle which many scholars have tried to solve.

What is crucial for our intent is that even in Antiquity philosophers were aware of such a problem. The first who claimed that Aristotle's modal syllogistic needed a reformulation was his own pupil Theophrastus. The philosopher of Eresus thought that both *Barbaras* were invalid, because, according to him, in order to obtain a necessity conclusion it would have been necessary to have two necessity premisses. We do not know whether Theophrastus was brought to such a criticism of his master because he sought the difficulties of modal syllogistic as it is presented in *Prior Analytics*. My impression is that Theophrastus did not want to develop an alternative logic to Aristotle's (even though the philosopher of Eresus eventually did that). I think that Theophrastus tried to systematize Aristotle's system and to remain faithful to

[9] In Becker's formalization the rule of conversion for non modal universal negative proposition is the following one :

(e) $(\forall x\,(Bx \rightarrow \neg Ax)) \rightarrow (\forall x\,(Ax \rightarrow \neg Bx))$.

It has been proved that the rule is sound (cf. *Pr. An.*, A, 2, 25a15-19) and therefore we can put a necessity operator before the entire rule, by the rule of necessitation :

(f) $\Box(\forall x\,(Bx \rightarrow \neg Ax) \rightarrow \forall x\,(Ax \rightarrow \neg Bx))$.

The so-called axiom K (i.e. : '$\Box(a \rightarrow b) \rightarrow (\Box a \rightarrow \Box b)$') allows us to distribute the necessity operator, with respect to the conditional. From (f), by K, we obtain (d') and in this way we prove its soundness.

his master. It seems likely that Theophrastus considered logic as an instrument of epistemology ; since Aristotle himself says in the *Posterior Analytics* that if we want to derive a necessity conclusion we need two necessity premisses, his pupil may have thought that the introduction of such a principle in modal syllogistic was needed in order to make Aristotle's system consistent[10].

This picture seems to me more faithful to historical reality, since there is evidence that Theophrastus did not consider himself anything but an orthodox Aristotelian[11]. But however things are, Alexander thought that it was necessary to react against Theophrastus' objections and to establish the validity of Aristotle's claims in *Pr. An.*, A, 9.

Alexander wrote a booklet Περὶ τῆς κατὰ τὰς μίξεις διαφορᾶς Ἀριστοτήλους τε καὶ τῶν ἑταίρων αὐτοῦ (cf. *in Pr. An.*, p. 125, lin. 30-31), in which he presented the solution to this problem and solved the objections raised by Theophrastus. Unfortunately this small treatise is lost[12], but it is possible to have an idea of Alexander's solution from his brief presentation of it in his commentary. The passage in which the commentator expands Aristotle's explanation is as follows :

> αὐτὸς [namely : Aristotle] μὲν οὖν ἐχρήσατο τῷ κατὰ παντός. ἐπεὶ γὰρ τὸ Α κατὰ παντὸς τοῦ Β ἐξ ἀνάγκης, τὸ δὲ Γ ὑπὸ τὸ Β ἐστι καὶ τι τοῦ Β, εἴη ἂν καὶ κατὰ τοῦ Γ ἐξ ἀνάγκης· ὃ γὰρ κατὰ παντὸς τοῦ Β ἐξ ἀνάγκης, κἂν κατὰ τῶν ὑπο τὸ Β ἐξ ἀνάγκης κατηγοροῖτο, εἴ γε τὸ κατὰ παντός ἐστιν, "ὅταν μηδὲν ᾖ λαβεῖν τοῦ ὑποκειμένου καθ' οὗ τὸ κατηγορούμενον οὐ ῥηθήσεται"· τὸ δὲ Γ τι τῶν Β ἐστί. καὶ γὰρ τὸ κατὰ παντὸς ἐξ ἀνάγκης ὁμοίως λαμβάνεται, ὡς προεῖπεν ἐπὶ τῶν ἀναγκαίων εἰπὼν τό τε γὰρ στερητικὸν ὡσαύτως ἀντιστρέφει, καὶ τὸ ἐν ὅλῳ εἶναι καὶ τὸ κατὰ παντὸς ὁμοίως ἀποδώσομεν (*in Pr. An.*, pp. 125, lin. 33 - 126, lin. 8).

According to Alexander Aristotle uses the *dictum de omni* in order to validate *Barbara* LXL. He clearly makes reference to Aristotle's *Pr. An.*, A, 8, 30a2-3, in which is adumbrated an extension of the *dictum* to modal propositions. The syllogism whose validity we have to prove is the following :

[10] I argued in favor of this interpretation in my *Boeto di Sidone e Alessandro di Afrodisia intorno alla sillogistica aristotelica*, at pp. 388-397.

[11] This thesis has been maintained, for example, by Marwan Rashed : cf. M. Rashed, *Essentialisme. Alexandre d'Aphrodise entre logique, physique et cosmologie*, de Gruyter, Berlin - New York 2007, pp. 6-16.

[12] A discussion of the sources, in which pieces of doctrine presumably expanded in this booklet has been transmitted, may be found in K. Flannery, *Ways into the logic of Alexander of Aphrodisias*, Brill, Leiden - New York - Köln 1995, pp. 53-108. Even though I appreciate Flannery's attempt to provide a reconstruction of Alexander's doctrine, as it was expanded in his lost treatise on mixed syllogisms, I think that we can obtain more information from Alexander's commentary on the *Prior Analytics* — surprisingly neglected by Flannery — in order to understand the commentator's thought on this difficult question. See on this topic L. Gili, *La sillogistica di Alessandro di Afrodisia. Sillogistica assertoria e sillogistica modale nel commento agli* Analitici Primi *di Aristotele*, Georg Olms Verlag, Hildesheim - Zürich - New York 2011 (Reihe : Spudasmata), pp. 219-237.

(1) A is said of all B of necessity
(2) B is said of all C
Therefore (3) A is said of all C of necessity.

As we have seen, the *dictum* allows a translation of the premisses into prosleptic propositions. In this case the major premiss

(1) A is said of all B of necessity

can be translated into

(1') A is said of necessity of all of which B is said.

This is only one of the two possible translations, as we have seen[13]. Since for (2) B is said of all C, it is possible to derive from this that A is said of necessity of all C[14].

Alexander's solution to the puzzle of the two *Barbara* is indeed very clever and in some ways echoes the subsequent explanation offered by Albrecht Becker, even though we do not find in Alexander's commentary such a distinction between *de dicto* and *de re* modalities.

Like Becker's solution, the one presented by Alexander is also puzzling with respect to the laws of conversion. We do not read, in his commentary on *Pr. An.*, A, 3, any reference to the principle of the *dictum* and therefore it is hard to understand if the *dictum* could be applied in the case of the conversions of modal premisses.

If we again consider the conversion of universal negative necessity propositions, we have the following rule :

(g) If (A is said of no B of necessity), then (B is said of no A of necessity).

Translating the rule according to the modal *dictum de nullo*, we are faced with two possible translations :

[13] The other possible translation is (1") 'A is said of necessity of all of which B is said of necessity'. But this possibility is not taken into account by Alexander, and his decision is quite reasonable. If (1") were the translation of (1), the syllogism would have been unsound.

[14] A similar explanation is possible if we directly refer to the possibility of translating standard propositions into prosleptic propositions, avoiding the use of the *dictum de omni* as a justification of such a translation. Although Alexander does not appear to think that this was Aristotle's presentation of the solution to the problem, we find in his commentary that such a way out is sound for the commentator of Aphrodisias : οἱ δέ γέ φασιν, ὅτι, εἰ ἡ λέγουσα τὸ κατὰ παντὸς τοῦ B ἡ αὐτή ἐστι τῇ λεγούσῃ, καθ' οὗ παντὸς τὸ B, κατ' ἐκείνου παντὸς τὸ A, ὡς καὶ αὐτὸς λέγει πολλάκις, ἔσται καὶ ἡ λέγουσα τὸ A κατὰ παντὸς τοῦ B ἐξ ἀνάγκης ἡ αὐτὴ τῇ λεγούσῃ, καθ' οὗ παντὸς τὸ B, κατὰ τούτου παντὸς ἐξ ἀνάγκης τὸ A. τοῦτο δὲ σημαινούσης τῆς ἀναγκαίας καθόλου πάντως ἀναγκαῖον γίνεται τὸ συμπέρασμα, κἂν ὑπάρχουσα ληφθῇ ἡ ἐλάττων (*in Pr. An.*, p. 126, lin. 23-28).

(h') If (A is said of necessity of none of the elements of which B is said), then (B is said of necessity of none of the elements of which A is said) ;

(h") If (A is said of necessity of none of the elements of which B is said of necessity), then (B is said of necessity of none of the elements of which A is said of necessity).

While the latter appears to be sound[15], it is harder to make sense of the former, because there is no evident reason for the change in position of the necessity operator.

[15] In order to show the soundness of the inference of (h") from (g), let us recall what Alexander has in mind, when he speaks of the possibility of translating a proposition in its prosleptic counterpart. On the basis of this possibility of translations (that is granted, according to him, by the heterodox interpretation of the *dicta*), M. Malink expands the four propositions of the Aristotelian square in this way (cf. his paper *A Non-Extensional Notion of Conversion in the Organon*, pp. 117-118) :

1) AaB iff ∀X(BaX → AaX)
2) AeB iff ∀X(BaX → ¬AaX)
3) AiB iff ∃X(BaX & AaX)
4) AoB iff ∃X(BaX & ¬AaX).

These explanations of categorical propositions are the basis upon which it is possible to establish the soundness of categorical syllogistic, according to both M. Malink and Alexander (cf. *in Pr. An.*, p. 54, lin. 6-12 : ὑπομιμνήσκει δὲ ἡμᾶς, πῶς καὶ τὸ κατὰ παντὸς ἀπέδωκεν (ὅταν γὰρ μηδὲν ᾖ λαβεῖν τοῦ ὑποκειμένου, καθ οὗ τὸ κατηγορούμενον οὐ ῥηθήσεται), ὑπὲρ τοῦ ἐνδείξασθαι, ὅτι οὐδενὸς ἔξοθεν ἐπὶ τῆς τοιαύτης συναγωγῆς χρεία πρὸς τὸ φανερὸν γενέσθαι τὸ ἀναγκαῖον, ἀλλ᾽ ἱκανὰ τὰ κείμενα· τὸ γὰρ κατὰ παντὸς, ὅ ἐστι κείμενον καὶ εἰλημμένον διὰ τῶν προτάσεων, ἱκανὸν πρὸς τὴν δεῖξιν τῆς συναγωγῆς. διὰ τοῦτο καὶ τέλειοι οἱ οὕτως ἔχοντες συλλογισμοὶ καὶ κυρίως ἀναπόδεικτοι ; Alexander states that the validity of complete syllogisms rests on the definition of the *dicta* ; and we know that it is possible to state the soundness of the whole categorical syllogistic, by assuming the validity of complete syllogisms and of the rules of conversions — which rest on the definitions of the *dicta* too, according to Malink). Alexander thinks that the validity of necessity syllogisms (i.e., syllogisms with both necessity premisses, which infer necessity conclusions) rests too on an extension of the *dicta*, in which the propositions, which figure in the *dicta*, have the necessity operator (cf. *in Pr. An.*, p. 120, lin. 9-15 : ἡ γὰρ ὁμοία τῶν προτάσεων συμπλοκὴ καθ᾽ ἕκαστον σχῆμα μετὰ τῆς τοῦ ἀναγκαίου προσθήκης καὶ τοὺς ἀναγκαίους ποιήσει συλλογισμούς […] αἴτιον δὲ τούτου, ὅτι τό τε κατὰ παντὸς καὶ τὸ κατὰ μηδενὸς ὁμοίως καὶ ἐπὶ τοῦ ἀναγκαίου λαμβάνεται, ὡς καὶ ἐπὶ τοῦ ὑπάρχοντος, δι᾽ οὗ οἱ ἐν τῷ πρώτῳ σχήματι δείκνυται συλλογισμοί). We have thus to state the validity of a *Barbara* LLL, on the basis of a *dictum de omni* for necessity propositions. This syllogisms may be written as follows :

a) A is said of all B of necessity
b) B is said of all C of necessity
Therefore
c) A is said of all C of necessity.

As a consequence, it is plain that a universal affirmative necessity proposition ('A is said of all B of necessity') has to be translated, according to the *dictum de omni*, in this way :

(i) A is said of necessity, of all of the elements of which B is said of necessity.

Different translations would be unable to ensure the validity of a *Barbara* LLL. The same applies to universal negative propositions, which have analogous prosleptic counterparts. In Malink's notation, we may say

We could easily choose the latter translation in order to make sense of modal conversions, but this will bring us to an inconsistent way of translating standard modal premisses into prosleptic modal premisses : we will choose a double modal operator in the translation of a rule of conversion and a single modal operator in the case of the translation of premisses of syllogisms like *Barbara* LXL. Plainly such premisses could not be converted, but this seems to be against Aristotle's and Alexander's intuitions. To some extent it arises again the main exegetical thesis proposed by Becker : in Aristotle's *Prior Analytics* we have two conflicting interpretations of modalities. The only difference between Becker's modern attempt and Alexander's commentary is that these conflicting interpretations are expounded in different ways.

Since the point has not been explicitly discussed by Alexander, it seems difficult to solve this puzzle.

In my opinion there are three possible answers to this problem :

(i) it is possible to suppose that Alexander tries to avoid the problem by means of the ambiguity of the translation allowed by the modal *dictum de omni et de nullo*. If the *dictum* makes possible two translations, we will use the more suitable in each circumstance. Even though such a suggestion lacks theoretical appeal, it is plausible.

(ii) it may also be thought that Alexander simply had not in mind a possible solution to this puzzle, either because he was not aware of it, or because he thought that it was impossible to find a solution. In this case Alexander's commentary would mirror the difficulties which can be found in the *Prior Analytics*.

(iii) it is also possible to think that a reasonable attempt to develop a solution to problems like the present one comes from Alexander's semantics for modal propositions. Certainly there is no explicit and conscious claim in Alexander's commentary about such a possible solution of many puzzles which emerge from the pages Aristotle devoted to modal logic. Instead we find a quite detailed semantic theory, which is able to give a coherent interpretation of Aristotle's modal propositions and modal syllogisms. I think therefore that it is worthwhile to explore such a possible solution.

III. Alexander's semantics : the introduction of time

As is well known, some recent reconstructions of Aristotle's modal logic try to justify his claims by means of a semantic theory[16]. These reconstructions mainly

1*) $\Box AaB$ iff $\forall X(\Box BaX \rightarrow \Box AaX)$

2*) $\Box AeB$ iff $\forall X(\Box BaX \rightarrow \Box \neg AaX)$.

To summarize, it is possible to infer (h") from (g) within Alexander's logical system.

[16] The first article which advanced such a proposal is to my knowledge *Models for modal syllogisms*, published by Fred Johnson in « Notre Dame Journal of Formal Logic », 30, 1989, pp. 271-284. Johnson provided a model theoretic semantics for the presentation of Aristotle's modal syllogistic given by Storrs McCall in his book *Aristotle's modal syllogisms*, North Holland, Amsterdam 1963. Johnson did not try to catch

refer to Aristotle's theory of predication, as it is presented basically in the *Topics*. Alexander's semantics at first glance appears quite different with respect to these modern reconstructions : he thinks indeed that modal propositions mean the time in which the relation between subject and predicate, as stated by the proposition, holds or does not hold. This is, of course, only an alternative way of providing semantics for Aristotle's modal logic. There are at least two questions which can be asked from this quite brief introduction of the topic : does such semantics work in order to provide a consistent account of Aristotle's modal claims ? Has this semantics a link with Aristotle's theory of predication ? I maintain that both questions can be answered in the affirmative and I think that it is possible to argue for these claims with a close reading of the text in which Alexander presents his definition of contingency. The present and the following paragraphs are devoted to this analysis.

In *Prior Analytics*, A, 13 Aristotle presents his definition of contingency and makes some remarks on certain rules of conversion for contingency propositions. The definition is as follows : λέγω δ' ἐνδέχεσθαι καὶ τὸ ἐνδεχόμενον, οὗ μὴ ὄντος ἀναγκαίου, τεθέντος δ' ὑπάρχειν, οὐδὲν ἔσται διὰ τοῦτ' ἀδύνατον (*Pr. An.*, A, 13, 32a18-20). Modern scholars usually maintain that in this passage Aristotle is introducing the so-called 'two-sided possibility', namely the possible which is neither impossible, nor necessary. Modern interpretations of Aristotle's modal theory commonly share the idea that the Stagirite gave two meanings to the word 'contingency', namely two-sided and one-sided possibility[17]. In other words, according to contemporary scholars, if we have a proposition α, then (i) 'it is contingent that α', or, in Aristotle's language, 'ἐνδέχεται (α)', may have two meanings :

(i*) ◊α (one-sided possibility) ;

(i**) ◊α & ◊¬α (two sided possibility).

How is it possible to state that Aristotle distinguishes these two senses of 'possible' ? This assumption is indeed commonly shared by contemporary interpretations

Aristotle's semantic intuitions, and his paper cannot properly be considered an interpretation of Aristotle's own doctrine. The scholar who mainly developed this perspective is Richard Patterson, who wrote many articles on Aristotle's modal syllogistic and is the author of the book *Aristotle's modal logic. Essence and entailment in the Organon*, Cambridge University Press, Cambridge 1995. His intuitions influenced the works of Paul Thom (*The logic of Essentialism. An interpretation of Aristotle's modal syllogistic*, Kluwer Publishers, Dordrecht-Boston-London 1996) and of Marko Malink (*A reconstruction of Aristotle's modal syllogistic*). These studies try to present a semantic theory consistent with Aristotle's text and which can exactly capture Aristotle's claims on the validity and the invalidity of all possible connections of premisses.

[17] Cf. T. Ebert, U. Nortmann, *Aristoteles. Analytica Priora. Buch I*, Akademie Verlag, Berlin 2007, pp. 470-471.

of Aristotle's syllogistic, and thus it is worth looking at which passages scholars do refer, in order to maintain a view in some sense opposite to the one of Alexander. As we have cursorily seen[18], Alexander thinks that Aristotle expands three senses of the terms 'ἐνδεχόμενον'/'ἐνδέχεσθαι' in *Pr. An.,* A, 3, 25a37-39 (πολλαχῶς λέγεται τὸ ἐνδέχεσθαι (καὶ γὰρ τὸ ἀναγκαῖον καὶ τὸ μὴ ἀναγκαῖον καὶ τὸ δύνατον ἐνδέχεσθαι λέγομεν)) : a) 'ἐνδεχόμενον'/'ἐνδέχεσθαι' as 'necessary' ; b) 'ἐνδεχόμενον'/'ἐνδέχεσθαι' as merely assertive — namely, a sense which may be added to a proposition which says that something is the case, without changing the proposition's truth value ; c) 'ἐνδεχόμενον'/'ἐνδέχεσθαι' as possible. It is perhaps surprising to note that contemporary scholars, while expounding this very passage, tend to suggest something which is not sharply different from Alexander's explanation. David W. Ross, for example, says that « at first sight it looks as if he [*scilicet* : Aristotle] distinguished three senses, τὸ ἀναγκαῖον, τὸ μὴ ἀναγκαῖον, τὸ δύνατον » ; certainly Ross adds — departing from Alexander's view — that « these are plainly not three *senses* of ἐνδεχόμενον, which could not be said ever to *mean* either 'necessary' or 'not necessary'. He can only mean that there are three kinds of case to which ἐνδεχόμενον can be applied. When he [*scilicet* : Aristotle] says τὸ ἀναγκαῖον ἐνδέχεσθαι λέγομεν, he clearly means that that which is necessary may *a fortiori* be said to be possible »[19]. Analogously T. Ebert and U. Nortmann think that « Aristoteles drei Fälle vor Augen hat, in denen Möglichkeitsaussagen wahr sein können, und daß er an dieser Stelle nicht verschiedene Sinne von ‚können' (ἐνδέχεσθαι) bzw. ‚möglich(erweise)' unterscheiden will. Denn er kann nicht behaupten wollen, daß das Wort ‚notwendig' einen der *Sinne* von ‚möglich' abdecke. Plausibel ist es dagegen zu sagen, daß in jedem Fall, in dem etwas sogar notwendigerweise der Fall ist (oder eine Eigenschaft irgendeinem Ding notwendigerweise zukommt), das Betreffende auch möglicherweise ist (oder zukommt), die entsprechende Möglichkeitsasussage also wahr ist »[20]. Both the commentaries by Ross and by Ebert and Nortmann follow thus a similar line of thought : they suggest that, *prima facie*, Aristotle seems not to endorse different *senses* of the terms 'ἐνδεχόμενον'/'ἐνδέχεσθαι' in *Pr. An.*, A, 3, 25a37-39, but that he merely wants to state that these terms may be applied to necessity proposition, or to non necessary states of affairs, or to genuinely possible states of affairs. Hence, it seems that the only difference with Alexander's interpretation lays in the fact that, according to the commentator of Aphrodisias, these three possible applications of the term 'ἐνδέχεσθαι' to a proposition are in fact different *senses* of the term itself. I think indeed that the textual evidence could hardly support a different interpretation — namely, an interpretation according to which also for Alexander there is only one sense of 'ἐνδέχεσθαι', and this sense is applied to different kinds of proposition. The

[18] A more detailed demonstration of this claim may be found in Gili, *La sillogistica di Alessandro di Afrodisia*, pp. 145-150 ; cf. also pp. 239-244.

[19] W. D. Ross ed., *Aristotle's Prior and Posterior Analytics*, Clarendon Press, Oxford 1949, p. 295.

[20] Ebert, Nortmann, *Aristoteles. Analytica Priora. Buch I*, p. 267.

commentator says that the term 'ἐνδεχόμενον' is homonymous[21], and, according to Aristotle, homonyms have various *meanings*[22] ; furthermore, Alexander specifies that, inasmuch as necessity and categorical propositions are considered, if they are said to be contingent, one must remember that this way of saying is only a label — and thus, has to recall what these 'contingency' propositions really *mean*, namely necessity or categorical propositions[23] : it is thus plain that, according to Alexander, Aristotle is not saying that if a proposition is a necessity, or a categorical proposition, then it is *a fortiori* a contingency proposition (as D. W. Ross would have suggested) ; the commentator rather thinks that if a proposition, which is said to be a contingency one, turns to be identical with a necessity, or with a categorical proposition, then in this case 'contingency' must be understood as a label, which refers to something else. In other words, the proper *meaning* of the label 'it is contingent that (x)' (ἐνδέχεσθαι [x]) appears to be either 'it is necessary that (x)', or '(x) is the case' (where 'x' is a proposition without any modal operator).

However, modern commentators add that *de facto* Aristotle has two senses of contingency in mind, when he writes *Pr. An.*, A, 3, namely one-sided possibility (which D. W. Ross labels as 'the possible'), and two-sided possibility (or 'the contingent' in D. W. Ross's words)[24]. They make this claim basically because it is necessary to use rules of conversion of both sorts of contingency propositions in order to prove the validity of modal syllogisms. It is plain, for example, that every proof by *reductio ad absurdum* within apodeictic syllogistic (namely, syllogisms with both necessity premisses and necessity conclusion) rests on auxiliary syllogisms for proving the absurd with at least a one-sided possibility premiss. It is true that Aristotle avoids this solution — because he does not want to justify the validity of apodeictic syllogistic on the basis of what has not already expounded by him, namely syllogisms with contingency premisses. However, since these auxiliary syllogisms could theoretically be introduced, we are entitled to think that Aristotle certainly uses a notion of one-sided possibility. And,

[21] Cf. *in Pr. An.*, p. 37, lin. 28.

[22] Cf. Aristotle, *Categories*, 1, 1a1-4.

[23] Cf. *in Pr. An.*, p. 38, lin. 30-35 : ἀλλ᾽ ὅσα μὲν ἀναγκαίως μὴ ὑπάρχοντα ὑπὸ τοῦ ἐνδέχεσθαι σημαίνεται, ἢ ὅσα μὴ ἐξ ἀνάγκης ὑπάρχοντα […], τὰ μὲν τούτων σημαντικὰ ἐνδεχόμενα ὁμοίως ἀντιστρέφει εἰκότως· οὐδὲν γὰρ ἄλλο ἢ ὀνόματος ὑπαλλαγὴ γέγονεν ὑπ᾽ αὐτῶν, τὰ δὲ σημαινόμενα ταὐτά ἐστι τοῖς φθάνουσι δεδεῖχθαι.

[24] See also Ebert, Nortmann, *Aristoteles. Analytica Priora. Buch I*, p. 269 : « zu Beginn haben wir die Annahme geäußert, Aristoteles wolle (noch) nicht verschiedene Sinne von ἐνδέχεσθαι unterscheiden. Der weitere Verlauf des Gedankenganges des Kapitel zeigt dann aber, daß Aristoteles schließlich einen Sinn von Möglichkeitsausdrücken, nach dem diese sich auf das einseitig Mögliche beziehen (also auf Fälle der ersten beiden eingangs unterschiedenen Arten), abheben möchte gegenüber einem anderen Sinn, dem Kontingenzsinn (womit die dritte Art von Fällen abgedeckt ist) ». Ross makes similar claims. The idea of these commentators is that Aristotle outlines the rules of conversions for contingency propositions, while he has in mind one-sided possibility — and, however, the Stagirite wants to apply the same rules to two-sided possibility propositions.

alongside with this notion, he introduces two-sided possibility in *Pr. An.*, A, 13, as all scholars are open to admit. Since Aristotle speaks of rules of conversion for all the types of propositions he is going to deal with in the exposition of the system (*Pr. An.*, A, 4-22), it is reasonable to expect a description of the rules of conversion for both sorts of contingency propositions. Now, there is no passage but *Pr. An.*, A, 3 in which these rules could be found, and thus it must be urged that the Stagirite has in mind both senses of contingency when he speaks of conversions of contingency propositions — and this is the interpretative framework of distinguished scholars like D. W. Ross, T. Ebert and U. Nortmann. What is problematic in this picture is that Aristotle does not explicitly introduce these different senses of contingency. Even if the first sort (one-sided possibility) seems to be referred to by speaking of what is 'not necessary' (cf. *Pr. An.*, A, 3, 25a37-38), two-sided possibility has still to be defined, though, as we have seen, it seems reasonable to assume that in *Pr. An.*, A, 3 we find the rules of conversion for two-sided possibility propositions as well. As we stated above, a good and straightforward definition says that this sort of possible is what is neither necessary, nor impossible, and similar definition is that of *Pr. An.*, A, 13, 32a18-20, according to modern scholars[25] : here Aristotle says that it is possible what is not necessary, and what, if it is the case, has no impossible consequences. Now, if X entails something impossible, X must also be impossible. In other words, in *Pr. An.*, A, 13, 32a18-20 Aristotle suggests that the possible is what is neither necessary, nor impossible — and this is indeed two-sided possibility. In this way modern scholars holds both that (a) the Stagirite speaks of the two sorts of contingency, and (b) that he gives rules of conversion for both groups of propositions, in order to employ them in his modal system.

Which is Alexander's position ? I have tried to make clear the similarities with modern interpretations, but I think also that we could point at a major discrepancy between his picture and our reading of Aristotle's text : Alexander seems to be not aware of the distinction among one-sided and two-sided possibility. Or, better perhaps, he has a distinction as clear and explicit as the one we can find in Aristotle's text ; there is indeed no passage in which the distinction is explicitly stated, though, however, it could be argued that the distinction is at work in Alexander's commentary as well ; and, above all, the commentator shares the idea that contingency has a univocal meaning (which pretends to embrace both one-sided and two-sided possibility, as I will expound in what follows).

[25] Cf. Ross, *Aristotle's Prior and Posterior Analytics*, p. 327 ; Ebert, Nortmann, *Aristoteles. Analytica Priora. Buch I*, pp. 470-471 ; A. Rini, *Aristotle's modal proofs.* Prior Analytics *A, 8-22 in predicate logic*, Springer, Dordrecht - Heidelberg - London - New York 2011, p. 119. For an evaluation of this last attempt at reconstructing Aristotle's syllogistic from the standpoint of first order logic see my review of Rini's book, in « Ancient Philosophy », 32/1, 2012, pp. 206-211.

First, it is worth noting that Alexander has clear in mind that if a proposition X entails something impossible, then X is impossible too : ᾧ γὰρ ἀδύνατόν τι ὑποτεθέντι ἕπεται, ἀδύνατον τοῦτο (*in Pr. An.*, p. 157, lin. 7). It is thus evident that Alexander has all the elements for claiming that, in *Pr. An.*, A, 13, 32a18-20, we have a definition of two-sided possibility. To what extent does he develop this thought ? If two-sided possibility is generically expanded as the possibility for a given subject of having and of not having a certain property, then Alexander without doubt holds that this account is a proper explanation of what Aristotle has in mind. However, if we trace a distinction among two-sided and one-sided possibility, by means of the recourse to a transcription of these ideas in lower predicate logic[26], then it is necessary to make a sharp distinction among the two notions — and Alexander will probably not follow us in this attempt. But if it is illegitimate to translate his notion of contingency, by saying that if (i) 'it is contingent that α', then (i*) ◊α & ◊¬α, he certainly thinks that the possible of *An. Pr.*, A, 13, 32a18-20 describes the possibility of having and not having a property. By doing so, however, Alexander has to face still this alternative : either (a) he claims that there is only one sense of contingency — and this is the sense of two-sided possibility, which we have outlined — and this only sense is the third genuine sense of contingency, of which Aristotle speaks in *An. Pr.*, A, 3 (the first two senses being the necessary and the existent), or (b) he admits (at least implicitly) two

[26] This attempt cannot be considered in any way 'old fashioned', though it certainly owes its widespread popularity to scholars like A. Becker and J. Łukasiewicz, whose main works on Aristotle's syllogistic appeared in 1933 (Becker's *Die aristotelische Theorie der Möglichkeitsschlüsse*) and in 1957 (first edition of Łukasiewicz's *Aristotle's syllogistic from the standpoint of modern formal logic*). Even though in the last two decades we had various attempts at providing logical reconstructions of Aristotle's syllogistic, without the employment of first order logic, it must be noticed that some scholars still use this 'classical' tool : among them we may recall the works by U. Nortmann (*Modale Syllogismen, mögliche Welten, Essentialismus : eine Analyse der aristotelischen Modallogik*, de Gruyter, Berlin-New York 1996), by K. J. Schmidt (*Die modale Syllogistik des Aristoteles. Eine modal-prädikatenlogische Interpretation*, Mentis Verlag, Paderborn 2000), and by A. Rini (*Aristotle's modal proofs*). With respect to the problem I am dealing with, the advantage of this tool is that it shows in a crystal-clear way the impossibility of considering a contingency proposition alternatively either a two-sided possibility, or a one-sided possibility proposition in every occurrence. For example, it is evident that a necessity proposition α entails ◊α, but it cannot entail (◊α & ◊¬α). I think that we should avoid this transcription into the lower predicate calculus of Alexander's modal ideas for two reason : (a) if I am right in claiming that Alexander thinks that there is only a genuine sense of contingency, which embraces both two-sided and one-sided possibility, then the assumption of first-order modal logic as a key for understanding the commentator's ideas will lead us to an inconsistency ; I am not claiming that Alexander's theory is perfectly sound, but I would rather explore the way in which he could have defended his claim about the uniqueness of the sense of contingency ; and (b), more importantly, Alexander never explicitly states a distinction between the two senses, and thus it seems reasonable to avoid a tool which would lead us immediately to such a distinction. I am aware that it is rather difficult to avoid such a distinction among the two senses, even if we do not adopt any lower predicate logic transcription : what I am suggesting is rather that Alexander may underestimate the need for a distinction, since he does not work with formalization as clear as that of modern logic textbooks.

senses of contingency : one-sided possibility (which was outlined in *An. Pr.*, A, 3), and two-sided possibility. Both alternatives are problematic for him.

(a) With respect to the first, it is hard seeing how it could be interpreted as two-sided possibility the description of genuine contingency, that he gives in his general scheme of modal propositions : here he says that, among the propositions which are not necessity propositions, namely propositions in which the predicate holds not at every instant of time of the subject, we find genuine contingency propositions, which are distinguished from categorical propositions in this way :

> εἰ δὲ μὴ ὑπάρχοι ἐπὶ τοῦ παρόντος τὸ κατηγορούμενον τῷ ὑποκειμένῳ δυνάμενον αὐτῷ ὑπάρχειν καὶ οὕτως, ὡς δυνάμενον, λαμβάνοιτο, ἐνδεχομένη καταφατικὴ ἀληθὴς ἡ πρότασις (*in Pr. An.*, p. 26, lin. 10-12) ; ἔτι ἐπεὶ πᾶν τὸ ὑπάρχον τινὶ ἢ ἀχώριστον αὐτοῦ ἐστι καὶ καθ᾽ αὑτὸ ὑπάρχον, ἢ χωριστόν, εἰ μὲν ἀχώριστον ἦν, ἡ τοῦτο δηλοῦσα πρότασις ἀναγκαία, εἰ δὲ χωριστόν, ἐνδεχομένη, ἧς ἡ μὲν τὸ παρὸν ἤδη δηλοῦσα ὑπάρχουσα, ἡ δὲ τὸ κεχωρισμένον ἢ τὸ μήπω παρὸν οἷόν τε δὲ ὑπάρξαι ἐνδεχομένη ἰδίως (*ibid.*, p. 26, lin. 18-22).

Genuine contingency propositions are propositions whose predicate may hold of the subject, namely whose predicate does not yet hold of the subject, but will hold in a future instant of time.

There is no reference to the possibility for the predicate of not holding of the subject, and thus it seems reasonable to infer that Alexander is working in this passage with a one-sided notion of possibility.

(b) Should we hence posit that he knows both senses, even though he avoided any explicit distinction among them ? Even this proposal should be rejected. Alexander indeed clearly identifies the sense of ἐνδεχόμενον outlined in *in Pr. An.*, p. 26, lin. 10-12, with the sense defined by Aristotle in *Pr. An.*, A, 13, 32a18-20. The commentator indeed thinks that Aristotle in this latter passage wants to state a univocal meaning of the term ‘ἐνδεχόμενον’. There are at least two reasons that underline this : (i) first, Alexander presented Aristotle’s first exposition of the ‘contingent’ in the *Prior Analytics* (cf. *Pr. An.*, A, 3, 25 a38-39) as a rough meaning, which can be applied to necessity propositions, to non-modal assertoric propositions and to genuine contingency propositions ; the proper meaning of the term was still awaited by the reader of Alexander’s commentary ; (ii) furthermore Alexander says that Aristotle is giving a proper definition of the term — thus, the definiendum cannot be taken as homonymous. Now, for the commentator, Aristotle is saying that what is contingent is something which is neither necessary, nor actually existent, but if one posits that it is the case, nothing impossible would follow from that assumption. With this paraphrase of Aristotle’s definition, Alexander removes two of the three meanings to which the generic contingent outlined in *Pr. An.*, A, 3, 25a38-39 could apply, namely the necessary and the

existent[27]; in other words, Alexander does not think that the definition says that it is contingent what is neither necessary, nor impossibile; the first elements of the definition of 'contingent' are rather aimed to distinguish it from the necessary and the existent. And this perfectly fits with his general scheme of modal propositions, where genuine contingency is distinguished from both necessity and actual existence.

As we have seen, since a necessity proposition states that the predicate always holds of the subject, the contingency proposition will state that it not always holds. But when does it hold? Certainly not now, because the contingent is different from the actually existent, and only the latter designates that the predicate holds now of the subject. This assumption clearly restates the semantic account of propositions given in *in Pr. An.*, pp. 25, lin. 24 - 26, lin. 22. Alexander finds a confirmation of such an account in the rules of conversion proper to modal contingency proposition, which allow the conversion from affirmative into negative and *vice versa*. Aristotle discusses these rules in *Prior Analytics*, A, 13, 32a29-b3: since contingency propositions are not necessity propositions, if it is contingent that A holds of B, then it is also contingent that A does not hold of B. It is worthwhile to notice that such rules were not accepted by Theophrastus and his disciples. The concept of contingency they had in mind was probably that of one-sided possibility, which indeed does not allow such conversions. Alexander's claim is on the other hand that if Theophrastus had had a better understanding of the definition of contingency, he would certainly have agreed with his master with regard to these rules of conversion[28]. The thesis that Theophrastus' account of contingency, namely one-sided possibility, cannot explain the Aristotelian rules of conversion for contingency propositions is straightforwardly correct; but the endorsement of such a claim would imply that, according to Alexander, contingency must be understood in terms of two-sided possibility. And indeed I think that he had such an intuition, even though he expressed it in a way that may appear quite odd to our understanding. According to him a proposition like

(a) it is contingent that B belongs to A

means that there will be an instant of time *t* in the future in which B actually belongs to A. But since in the future there will also be instants in which B actually does not belong to A, it is also possible to say that *now*, namely when (a) is uttered, it is possible either that B belongs to A, or that B does not belong to A.

[27] Cf. *in Pr. An.*, p. 156, lin. 21-29: τὸ γὰρ μὴ ὂν οὐδ᾽ ἀναγκαῖον, οὐ μὴν τὸ μὴ ἀναγκαῖον ἤδη καὶ μὴ ὄν. ἢ ἀμφότερα ἀπέφησε τοῦ ἐνδεχομένου, καὶ τὸ ἀναγκαῖον καὶ τὸ ὑπάρχον, τὸ μὲν ἀναγκαῖον διὰ τοῦ εἰπεῖν οὗ μὴ ὄντος ἀναγκαίου, τὸ δ᾽ ὑπάρχον διὰ τοῦ τεθέντος δ᾽ ὑπάρχειν· τὸ γὰρ τεθέντος ἀπέφησεν αὐτοῦ καὶ τὸ ὑπάρχον. ἢ διὰ τοῦ εἰπεῖν οὗ μὴ ὄντος ἀναγκαίου ἀπέφησεν αὐτοῦ καὶ τὸ ὑπάρχον· κατηγορεῖται γὰρ κατ᾽ αὐτὸν καὶ κατὰ τοῦ ὑπάρχοντος τὸ ἀναγκαῖον· τὸ γὰρ ὑπάρχον τινὶ ἀναγκαῖον ὑπάρχειν αὐτῷ, ἔστ᾽ ἂν ὑπάρχῃ.

[28] See for example *Physics*, Λ, 7; *Metaphysics*, Λ, 2-3.

According to Alexander, this is the core reason which grants the soundness of the rules of conversion for contingency proposition. It is philosophically worthwhile to notice that Alexander believed that these propositions, in which contingency is explained in terms of two-sided possibility, are employable in order to give an account of natural changes. Change is explained by Aristotle in terms of a subject which is without a certain form, and which acquires this form at the end of the process[29]. According to Alexander this process can be accounted for with a contingency proposition of the type of (a), in which B is the form acquired at the end of the process and A is the subject of it. Contingency, which in this case too appears to be two-sided possibility, explains the fact that at certain instants A is without the form B and in other instants, such as at the end of the process of change, A has the form B[30].

These considerations may bring us to see a possible solution of the unsatisfactory alternative we had to face above. As I said, Alexander must have worked with a notion of one-sided possibility too, but he does not distinguish it from two sided possibility. Presumably, this is possible because of this reason : if we have a proposition like

(i) it is contingent that P belongs to S,

then, according to what we have stated above, it must be thought that P does not hold of S at the present time t_p, but that it will hold of it in a future instant $t_{n(n>p)}$. However, there will also be future instants of time, different from t_n (we can label them $t_{m\,(m>p\,m\neq n)}$), in which P does not belong to S. Thus, it is right to say that S, at the present instant of time, may have the property P (one-sided possibility). But if we take into consideration the instants of time t_m, then we may also say that S may have, and may not have the property P (two-sided possibility). In other words, in Alexander semantics it is reasonable to suppose that there are states of affair (we could name contingent facts or events), which may be described either by two-sided possibility propositions, or by one-sided possibility propositions. All in all, when Alexander defines contingency, he rather intends to speak of these contingent facts or events, and not of propositions that describe them. Indeed, if we take that he is actually defining propositions, we are not only faced with an ambiguous use of the terms 'ἐνδεχόμενον' and 'δυνατόν' (this happens in Aristotle's text too), but we have also the undesired consequence of having a definition, which is explicitly said to be the only and univocal description of what is intended with the terms 'ἐνδεχόμενον' and 'δυνατόν' in all their occurrences within the whole syllogistic. Hence, I think that it is more economic to suggest that Alexander has in mind contingent states of affairs, when he describes contingency in terms of both one-sided, and two-sided possibility, though he wants to give a univo-

[29] Cf. *in Pr. An.*, p. 160, lin. 26-28.

[30] See his *Quaestiones*, I, 19 and II, 15.

cal meaning of genuine contingency. Contingency, indeed, is employed in physical description of natural processes.

In the fifth section of this paper I shall argue that Alexander's temporal semantics is the counterpart of his theory of description of natural processes. Hence modal propositions are employable in order to describe such processes.

IV. A puzzle about Alexander's temporal semantics : stochastic modalities

In order to analyze the relationship between Alexander's temporal semantics for modal propositions and his theory of predication, it may be useful to consider a puzzle which arises at this stage of our inquiry. So far we have stated that for Alexander a contingency proposition of the structure 'A is contingently B' means that there will be an instant of time in which it will be true that A is B. Even though this is still a rough picture of Alexander's conception, one could ascribe to the commentator the endorsement of the so-called 'principle of plenitude'. It is an object of dispute whether Aristotle held such assumption. Despite the famous claim made by J. Hintikka[31], it is not certain that Aristotle believed that every possibility would be actualized. Furthermore this thesis apparently supports determinism, and Aristotle's discussion against the Megarians seems to imply that the Stagirite was in no way a determinist philosopher[32]. The same problems arise for Alexander, whose philosophy was intended to be a faithful explanation and systematization of Aristotle's thought[33]. In particular, Alexander also faced the determinism of Stoic philosophers, and he argued in many texts that events are not predetermined : if there is something which governs the course of events, this is nature, and nature is indeed what mythology called fate[34].

I think that the comparison with Stoic and Megaric doctrines led Alexander to a redefinition of contingency, which softens his sharp division of propositions according

[31] Cf. his *Time and necessity. Studies in Aristotle's theory of modality*, Clarendon Press, Oxford 1974, in particular p. 28ff.

[32] See Aristotle, *Met.*, Θ, 3-4. Hintikka maintains that these rather complex passages support his interpretation (see for example J. Hintikka, *Aristotle on the realization of possibility in time*, in S. Knuuttila ed., *Reforging the great chain of being. Studies of the history of modal theories*, Kluwer Academic Publishers, Dordrecht 1981, p. 63).

[33] On Alexander's rejection of determinism see R. Sharples, *Determinism, responsibility and chance*, in P. Moraux ed., *Der Aristotelismus bei den Griechen. Von Andronikos bis Alexander von Aphrodisias. Dritter Band : Alexander von Aphrodisias*, de Gruyter, Berlin - New York 2001, pp. 513-592 ; B. Strobel, *Zur Konzeption von τὸ ἐφ' ἡμῖν bei Alexander von Aphrodisias*, in J. Müller, R. Hofmeister Pich eds., *Wille und Handlung in der Philosophie der Kaiserzeit und Spätantike*, de Gruyter, Berlin - New York 2010, pp. 131-174. For a different reading of Alexander's opinions on determinism see D. Frede, *The dramatization of determinism : Alexander of Aphrodisias' De fato*, « Phronesis », 27, 1982, pp. 276-298 : according to D. Frede, Alexander develops « a kind of compatibilist position of his own » (p. 287) among determinism and human free will.

[34] As is well known, this is the core thesis which Alexander defends in his *De fato*.

to their realization in different instants of time[35]. Such a redefinition can be found in a digression of the commentary on *Pr. An.*, A, 15 : after having pointed out that contingency either has the definition given by Aristotle in *Pr. An.*, A, 13, 32a18-20 or means the same as necessity, Alexander observes that other philosophers also presented different accounts of contingency[36]. Among them, Diodorus Chronos presented a sense of contingency which is not the proper one, in Aristotelian terms, but rather the contingency which has the necessity of what is the case. Diodorus indeed said that the possible is what either is or will be (δύναται λέγειν καὶ περὶ τῶν Δυνατῶν, τοῦ τε, ὃ Διοδώρειον λέγεται, ὃ ἢ ἔστιν ἢ ἔσται, *in Pr. An.*, pp. 183, lin. 34 - 184, lin. 1)[37]. This definition could be applied also to Alexander's notion of the possible, insofar as he roughly presented it. And indeed what is surprising is that the commentator does not directly criticize Diodorus' account, but he puts alongside it Philo's definition of the possible, which seems guided by an opposite conception. While for Diodorus the possible cannot fail to be realized, Philo thinks that not only may what is possible not happen, but also that there is no natural cause that makes its realization likely : the 'possible' for Philo is what can be predicated of a certain subject in virtue of the terms themselves (ὁμοίως καὶ περὶ τοῦ κατὰ Φίλωνα· ἦν δὲ τοῦτο τὸ κατὰ φιλὴν λεγόμενον τὴν ἐπιτηδειότητα τοῦ ὑποκειμένου, κἂν ὑπό τινος ἔζωθεν ἀναγκαίου ᾖ γενέσθαι κεκωλυμένον, *in Pr. An.*, p. 184, lin. 6-8). If Diodorus is determinist and in this sense develops a thesis not compatible with Alexander's philosophy, Philo too presents a theory which cannot satisfy the commentator, because contingency propositions are not suitable for expressing natural processes in Philo's view. According to Alexander, Philo is right in saying that what is possible could fail to be actualized, but such a failure must be explained with the introduction of an external obstacle. Since what is contingent is a natural process for Alexander, if there are no external causes which prevent the actualization of the contingent, and if all the natural causes of this natural process are enabled to cause it, what is contingent will be actually existent at a certain instant in the future. On this basis, Alexander concludes that Aristotle's (and his own) definition of contingency must be regarded as something halfway between the opposite and extreme

[35] Cf. *in Pr. An.*, pp. 25, lin. 24 - 26, lin. 22.

[36] Cf. *in Pr. An.*, p. 183, lin. 29-33 : Τὸ δὲ καὶ ὁσακῶς ἄλλως λέγεται τὸ δυνατόν· ἐν ἅπασι γὰρ ὁμοίως ἔξει δύναται μὲν λέγεσθαι καὶ ἐπὶ τοῦ ὡς ἐπὶ τὸ πλεῖστον καὶ τοῦ ἀορίστου καὶ τοῦ ἐπ᾽ ἔλαττον, ὃ ἦν ὑπὸ τὸ ἐν γενέσει δυνατόν, ἢ καὶ ἐπὶ τοῦ ἀναγκαίου, εἰ εἴη πρότερον τὸ δυνατὸν τὸ ὡς ἀληθὲς εἰληφώς· ἢ εἰ τότε τὸ ἀναγκαῖον ἔλαβε, νῦν ἂν τὸ ἀληθὲς λέγοι.

[37] Severinus Boethius presents the same description of Diodorus' account of the possible : « Diodorus possibile esse determinat, quod aut est aut erit ; inpossibile, quod cum falsus sit, non erit verum ; necessarium, quod cum verum sit, non erit falsum ; non necessarium quod aut iam est aut erit falsum » (*In Arist. De Interpretatione II*, ed. C. Meiser, Teubner, Leipzig 1880, p. 234, lin. 22-26). In general for an accurate presentation of Diodorus' definition of the 'possible' see S. Bobzien, *Determinism and freedom in Stoic philosophy*, Clarendon Press, Oxford 1998, pp. 97-112.

definitions given by Diodorus and Philo : ὧν ἐστι μεταξὺ τὸ ὑπ᾽ Ἀριστοτέλους λεγόμενον· δυνατὸν γὰρ καὶ τὸ οἷόν τε γενέσθαι ἀκώλυτον ὄν, κἂν μὴ γένηται (*in Pr. An.*, p. 184, lin. 10-12). The introduction of the specification 'ἀκώλυτον ὄν' redefines the contingency in a way which avoids the endorsement of the principle of plenitude. Modalities are definitely not stochastic for Alexander, although certain texts apparently support this idea, since the specification 'ἀκώλυτον ὄν' is implicit in the majority of cases.

Now it is possible to ask what can be prohibited so that it fails to be actualized. The answer at this stage is clear : only natural changes can fail to be actualized, because of the intervention of certain external causes which prohibit the process.

V. Alexander's general solution of the puzzle : the theory of predicables

After having presented his general definition of contingency, Aristotle specifies that contingency may be understood in two ways : Διωρισμένων δὲ τούτων πάλιν λέγωμεν ὅτι τὸ ἐνδέχεσθαι κατὰ δύο λέγεται τρόπους, ἕνα μὲν τὸ ὡς ἐπὶ τὸ πολὺ γίνεσθαι καὶ διαλείπειν τὸ ἀναγκαῖον [...] ἄλλον δὲ τὸ ἀόριστον (*Pr. An.*, A, 13, 32b4-10). Commenting on this passage, Alexander is able to outline his theory of predication more clearly.

(i) The first meaning of 'contingency' refers to what is natural and happens in most cases : indeed, what is natural sometimes fails to happen, although it happens in most cases (τοιαῦτά ἐστι τὰ φύσει γινόμενα, ἃ οὐκ ἐξ ἀνάγκης μὲν γίνεται τῷ ἐπὶ τινῶν συμπίπτειν ποτὲ καὶ μὴ οὕτως, οὐ μὴν ἀλλ᾽ ὡς ἐπὶ τὸ πολὺ γίνεται, *in Pr. An.*, p. 162, lin. 4-6). This interpretation of Aristotle's text is based on the general account that Alexander gives of natural phenomena and of the processes of generation and corruption in the sublunar world. In this world only the species are eternal, whereas individuals which fall under them change and are governed by the 'πρόνοια'. And 'πρόνοια' acts with the mediation of the causal influence of the celestial bodies[38]. These individuals are the subjects of contingency propositions which express something which happens in most cases and by nature. According to Alexander there are two reasons why what happens by nature is not necessary and hence sometimes fails to be actualized. (a) The first reason is that what happens by nature is not eternal. Since for Alexander if something is necessary, it is always true[39], what happens by nature cannot be necessary (cf. *in Pr. An.*, p. 162, lin. 15-18 : τῷ οὖν μὴ αἰεὶ ταῦτα εἶναι, οἷς ἐνδέχεσθαί φαμεν τὰ κατὰ φύσιν ὑπάρχειν (τοῖς γὰρ καθ᾽ ἕκαστα ταῦτα ὑπάρχει), οὐκ ἐξ ἀνάγκης τὰ κατὰ φύσιν). For the commentator it is obvious that what happens by nature is not eternal. (b) The

[38] See for example *Quaestiones*, II, 19, p. 66, lin. 22-26 : ὅσον δὲ αὐτοῦ [scilicet : τοῦ κόσμου] γενητόν τε καὶ φθαρτὸν καὶ τῆς ἄλλου βοηθείας δεόμενον πρός τε τὸ εἶναι καὶ πρὸς τὴν διὰ τῆς εὐτάκτου μεταβολῆς κατ᾽ εἶδος ἀϊδιότητα, τοῦτ᾽ ἐστὶ τὸ προνοούμενον ὑπὸ τῆς τοῦ θείου μέρους τοῦ κόσμου κινήσεως εὐτάκτου καὶ ποιᾶς σχέσεως πρὸς αὐτὸ κυβερνώμενον.

[39] This view could have been developed on the basis of texts of Aristotle like *De Generatione et corruptione*, B, 11, 338a1-2 : ὥστ᾽ εἰ ἔστιν ἐξ ἀνάγκης, ἀΐδιόν ἐστι, καὶ εἰ ἀΐδιον, ἐξ ἀνάγκης.

second reason why something which happens by nature is not necessary presupposes an assumption. If we consider a proposition like :

(1) Men become grey-haired when they are 60 years old

it is possible to think that it is contingent because not every man lives until his 60th birthday. This would be an explanation of the type of (a) of the contingency of proposition (1). Now, let us suppose that every man lives until his 60th birthday. Even in this case, says Alexander, a proposition like (1) will be contingent. Plainly this cannot be explained by (a). The reason which Alexander proposes is rather elliptical :

> δεύτερον δὲ τὰ κατὰ φύσιν γινόμενα οὐκ ἐξ ἀνάγκης ἐστίν, ὅτι, κἂν ᾖ οὕτος, ᾧ ὑπάρχει τὸ κατὰ φύσιν, εἰς ἑξήκοντα ἔτη προεληλυθώς, ὡς ἐπὶ τὸ πλεῖστον μὲν πολιωθήσεται, οὐ μὴν ἐξ ἀνάγκης. ὃ ἐδήλωσεν εἰτὼν ὄντος μέντοι ἀνθρώπου ἢ ἐξ ἀνάγκης ἢ ὡς ἐπὶ τὸ πολύ ἐστιν· εἰ γὰρ ὡς ἐπὶ τὸ πολύ, δῆλον ὅτι οὐκ ἐξ ἀνάγκης (*in Pr. An.*, p. 162, lin. 26-30).

The point is that (1) remains contingent, because even if every man lives until his 60th birthday, there will be only a majority of men who became grey-haired. Since 'being-grey-haired' fails to apply to all men who are more than 60 years old, it is possible to infer that such a property does not necessarily belongs to his proper subject (namely, more-than-60-year-old men). On this basis, Alexander is able to infer that (1) is a contingency proposition even under the assumption that all men live until their 60th birthday.

(ii) The second sense of contingency applies to what happens rarely or by chance. Alexander specifies that this sense has two internal subdivisions : it refers (a) either to what has the same probability of happening or not happening, or (b) to what is opposed to what happens in most cases, for example not becoming grey-haired after your 60th birthday (cf. *in Pr. An.*, p. 167, lin. 1-7). Aristotle makes two examples of this sense of contingency : going for a walk, and an earthquake while you are walking. According to Alexander, the first example refers to the sense (ii.a), because there is the same probability that I am walking and that I am not walking ; the earthquake on the other hand is something which happens rarely and almost by chance.

On the basis of this observation Alexander is able to outline a hierarchy among contingency propositions, depending on the number of instants of time in which the connection expressed by the proposition is realized : sense (i) is the most frequent and the nearest to necessity, followed by sense (ii.a) and, at the bottom of the hierarchy, by sense (ii.b) (cf. *in Pr. An.*, p. 163, lin. 15-18).

So far, I have presented how Alexander believes that modal propositions can express natural changes. Now I would point out that this description of natural changes has a

close connection with Aristotle's and Alexander's theory of predication[40]. In order to discuss this theme, it is probably worthwhile to raise a question about our description of the second reason why what happens by nature is not necessary.

Let us consider a proposition like

(2) A belongs κατὰ φύσιν to B.

One could ask which is the term in virtue of which it is right to say that (2) happens in most (but not all) cases. From the picture I have presented one could answer that the logical subject, which refers to the persisting subject of natural change, cannot be the term in virtue of which we say that (2) is contingent. But as we have seen, Alexander considers such a proposition contingent for more than one reason. If we examine the example

(2') Being-grey-haired belongs κατὰ φύσιν to all men after their 60th birthday,

it is possible to notice that the proposition is contingent either because not all men live for more than 60 years, or because not all 60-year-old men are grey-haired (cf. *in An. Pr.*, p. 162, lin. 21-26). As always, it is possible to give a prosleptic reading of the proposition, which can help us understand the structure of the second condition :

(2") Being-grey-haired is said κατὰ φύσιν of all of which more-than-60-year-old man is said.

(2") is contingent since not being-grey-haired fails to apply to all of which more-than-60-year-old man is said. It could seem that the things of which 'more-than-60-year-old man' is predicated are individual men (who are actually more than 60 years old). If this is true, that would bring Alexander to endorse the orthodox reading of the

[40] Alexander's theory of predication has its obvious roots in Aristotle's own theory, and on the latter there is, of course, a huge scholarly literature. Among the titles we could be referred to, I would like to mention in particular M. Mignucci, *Aristotle's theory of predication*, in I. Angelelli, M. Cerezo eds., *Studies in the History of Logic*, de Gruyter, Berlin - New York 1996, pp. 1-20. It is object of dispute whether Aristotle's theory of predication is of some help, in order to make sense of his modal syllogistic. Jonathan Barnes, for example, argues that such an approach is wrong-headed : « Of course, Aristotle's syllogistic is essentially tied to the concept of predication ; for the argument forms which it examines are fixed by a certain logical structure, namely the subject-predicate structure. But nothing in the syllogistic requires, or even suggests, any classification of predicates : that a predicate is substantial or qualitative, relational or a matter of *habitus* — all that is of supreme indifference to the syllogistic » (Barnes, *Truth, etc.*, p. 133). However, many scholars, like M. Malink and A. Rini, have produced challenging and innovative works with this approach — hence, I suggest that it is useful to test it in the case of Alexander's texts too : the final remarks of this paper have this aim.

dictum de omni et de nullo. This interpretation[41] is not necessarily the only possible reading of the passage, even though it appears to be straightforwardly true. The heterodox reading of the *dictum* could be defended by saying that 'more-than-60-year-old man' is predicated of other collective terms (e.g. 'more-than-60-year-old Britons' etc.) or it might be said that such example is misleading. However things may be, what is worthwhile is that (2") must be understood as a contingency proposition, because the logical predicate fails to apply to every items of the logical subject.

Following Aristotle (cf. in particular his *Top.*, A, 8, 103b6-19), Alexander thinks that every term which figures in a proposition is either convertible with the other term of the proposition, or not convertible. Let us consider a proposition of the structure :

(3) A is said of all B.

According to Alexander, B is convertible with A either because it is its definition, or because it is a *proprium* of A. If B is not convertible, either it belongs to the essence of A (and in this case it is either a genus or a *differentia*), or it does not belong to the essence of A, and hence B is an accident. The text in which such a doctrine is presented is the following :

> πᾶν τὸ κατηγορούμενόν τινος ἀνάγκη ἢ ἐπ᾽ ἴσης αὐτῷ λέγεσθαι (καὶ ἀντικατηγορεῖται ταῦτα ἀλλήλων) ἢ μὴ ἴσης, ὧν τὸ μὲν ἀντικατηγορούμενον ἢ ὅρος ἢ ἴδιον, τὸ δὲ μὴ ἀντικατηγπρούμενον ἢ ἐν τῇ οὐσία καὶ τῷ ὁρισμῷ τοῦ πράγματός ἐστιν ἢ οὔ. καὶ εἰ μὲν τῶν ἐν τῷ ὁρισμῷ, γένος ἢ διαφορὰ ἂν εἴη, εἰ δὲ μὴ ἐν τῷ ὁρισμῷ, συμβεβηκὸς ἂν εἴη· τὸ γὰρ συμβεβηκὸς ἦν ὃ μήτε ὅρος μήτε ἴδιον μήτε γένος ὂν ὑπῆρχε τῷ πράγματι. εἴη δ᾽ ἂν ὁ συλλογισμὸς κατηγορικὸς ἐν πρώτῳ σχήματι ἐκ διαιρέσεως ἔχων οὕτως πᾶν τὸ κατηγορούμενόν τινος ἢ ἐπ᾽ ἴσης αὐτοῦ κατηγορεῖται ἢ οὔ, πᾶν τὸ ἐπ᾽ ἴσης ἢ μὴ ἐπ᾽ ἴσης τινὸς κατηγορούμενον κατηγορεῖται αὐτοῦ ἢ ὡς ὅρος ἢ ὡς ἴδιον ἢ ὡς γένος ἢ ὡς συμβεβηκός, πᾶν ἄρα τὸ κατηγορούμενον ἢ ὡς ὅρος ἢ ὡς ἴδιον ἢ ὡς γένος ἢ ὡς συμβεβηκὸς κατηγορεῖται (*in Arist. Top.*, pp. 63, lin. 26 - 64, lin. 7).

What is convertible with the subject and what belongs to the essence of the subject cannot fail to apply to the items of which the subject is predicated, if the subject is not an accident. These core relations between terms govern in some sense the modal propositions which figure in a syllogism. Alexander is quite explicit on this point, and says that terms which figure in premisses and conclusions of syllogisms fall under the ten categories[42]. These terms, if taken as predicates, are predicated of their subjects either *simpliciter*,

[41] Jonathan Barnes has proposed such an interpretation for both Aristotle and Alexander, in his already cited *Truth, etc.*, pp. 386-412.

[42] See *in Pr. An.*, p. 366, lin. 22-24 : λέγει, ποσαχῶς ἀληθές ἐστιν εἰπεῖν ἄλλο ἄλλῳ ὑπάρχειν· ὁσαχῶς γὰρ αἱ κατηγορίαι καὶ τὰ τῶν ὄντων γένη διῄρηται.

or in part. Those which are predicated *simpliciter* are genus, *differentiae*, *propria* and definitions, while accidents are predicated only partially and not *simpliciter*:

> καὶ τούτων τῶν κατηγορουμένων καὶ ἀληθευομένων κατά τινος ἢ ἁπλῶς καὶ καθόλου ληπτέον τὰ κατηγορούμενα κατηγορεῖσθαι ἢ πῇ· τὰ μὲν γὰρ γένη καὶ αἱ διαφοραὶ καὶ τὰ ἴδια καὶ οἱ ὁρισμοὶ ἁπλῶς κατηγοροῦνταί τε, ὧν εἰσι, καὶ ἀληθεύονται κατ᾽ αὐτῶν, τὸ δὲ συμβεβηκὸς ποτὲ μὲν ἁπλῶς, ὡς ἐπὶ τῆς χιόνος τὸ λευκόν, ποτὲ δὲ πῇ, ὡς τοῦ ὀφθαλμοῦ τὸ λευκόν (*in Pr. An.*, pp. 366, lin. 33 - 367, lin. 3).

From passages like this, one may reasonably infer that if the predicate is a genus, or a *differentia*, or a *proprium*, or the definition of the subject, the proposition which states the connection between these two terms is a necessity one. A contingency proposition, on the other hand, states the connection between a predicate which expresses an accident of the subject. This frame does not rule out the temporal semantics we have presented, but rather it provides the metaphysical basis on which such semantics is grounded. So far, I have argued for the two exegetical claims that this paper was designed to defend, namely that Alexander develops temporal semantics for modal propositions, and that the temporal analysis is only the counterpart of a theory of predication in which every term which figures in a proposition is a *praedicabile* and in which the relations among *praedicabilia* explain the modality of a proposition.

VI. Conclusion

Our presentation of Alexander's semantic ideas is designed to solve the apparently conflicting intuitions which are at the basis of his modal syllogistic. His solution to the puzzle of the two modal *Barbaras*, by means of the modal version of the *dictum de omni*, works perfectly, but unfortunately it creates some other problem if the same translation into prosleptic propositions is used in the case of the rules of conversion of modal premisses. Our task is now to examine whether Alexander's semantics works in order to provide a better solution to these problems. This inquiry for the most part answers questions raised by contemporary Aristotelian scholarship, but I think that there is quite surprising evidence in Alexander's texts about the employment of his semantic intuitions in order to solve some puzzles of modal syllogistic.

First, it is quite easy to show how temporal semantics may explain the rules of conversion for modal premisses. Let us consider again the conversion of universal negative necessity propositions:

(1) If 'A is said necessarily of no B', then 'B is said necessarily of no A'.

The antecedent says that there is no instant of time in which A is said of one of the Bs. Therefore, for every instant of time it is true that A is said of no B; thus, by

the rule of conversion of universal negative non-modal propositions, for every instant of time it is true to say that B is said of no A. From this, we infer the consequent of (1), and hence (1) is proved.

Even if one considers which praedicabile makes true a proposition like (1), it is possible to reach the same conclusion. If A is said necessarily of no B, A must be either a proprium, or a genus, or a differentia, or a definition of a term 'not-B', namely of a term different from B in the division : this is the only way in which (a) A is said necessarily of something, and (b) A is said of no B — in other words, that something of which A is necessarily said must be different from B ; it cannot be neither a genus, nor a species of B (otherwise in both cases A would have been said necessarily of some Bs). Hence, this something must be different from B in the division. Let us call C this term 'not-B'. A is thus said necessarily of C, and from this it follows that A is either a proprium, or a genus, or the differentia, or the definition of C. At this point we have two possibilities.

a) If A is a proprium, or the differentia, or the definition of C, then C is necessarily said of all A. But B was necessarily said of no C, because our hypothesis posited that they were opposed in division ; therefore B is necessarily said of no A. And in this way we prove our proposition (1).

b) If A is a genus of C, we must add that A cannot be the genus of B too, otherwise A would have been necessarily said of all B. On the basis of the *arbor Porphyrii*, it is plain to argue that if C and B are opposed in division, then also the genus of C, which is not the genus of B, namely A, and B must be opposed in division. And thus B is said necessarily of no A, and we prove proposition (1).

This discussion will appear clearer with an example. Let us replace A with 'animal', B with 'stone' and C with 'man' (case b). The proposition we have to prove, with the help of the *praedicabilia*, is the following :

> (1*) If animal is necessarily said of no stone, then stone is necessarily said of no animal.

We find that among the things of which 'animal' is necessarily said of, we have 'man', which is opposed in division to 'stone' ; but so must also be the man's genus 'animal'. Thus it is true that 'stone is necessarily said of no animal'. By replacing C with 'living creature able to perceive' we have an example of our case (a).

It is not easy to see how Alexander's semantics may work in the case of the two modal *Barbaras*, but with some preliminary remarks his line of argument will appear quite clear.

The valid *Barbara* has the following structure :

(1) A is said of all B of necessity
(2) B is said of all C
Therefore (3) A is said of all C of necessity.

According to the theory of predication that has been presented, the major premiss says that A is either a genus, or a differentia, or the definition, or a proprium of B. In each of these cases B must be a substance-term. Now, since in syllogistic we do not deal with singular terms[43], C must also be a kind term. But a substance-term, like B, can be predicated of a kind-term, like C, only if C is a sub-species of B. B, as a substance-term, could be predicated also of primary substances which fall under it, but this possibility has to be excluded in this particular case. The minor premiss is indeed a universal affirmative proposition, and therefore it is impossible that C is a primary substance. The predicative relation between B and C is thus transitive : everything which is said of a genus, namely of B, is said also of its subspecies, namely of the Cs[44]. Alexander is rather elliptical when he justifies the validity of *Barbara* LXL, but in another passage it is evident that he interprets the *dictum de omni* in this case as referring to this transitive predicative relation, which is expressed by the minor premiss. It is useful to quote the entire passage, in which Alexander is explaining why the dictum does not validate *Barbara* XLL :

> ἀλλ᾽ ἔοικεν ἐπακολουθῆσαι μόνῳ τῷ κατὰ παντὸς ἐξ ἀνάγκης ὡς σημαίνοντι ‘ὅταν μηδὲν ᾖ λαβεῖν τοῦ ὑποκειμένου, καθ᾽ οὗ τὸ κατηγορούμενον οὐ ῥηθήσεται ἐξ ἀνάγκης’[45]. εἰ μὲν οὖν οὕτως ἔχοντα λαμβάνει τὰ ὑπὸ τὸ Β ὡς τοῦ Β τινὰ ὄντα, ἐξ ἀνάγκης αὐτῶν ἔλαβε τὸ Α κατηγορεῖσθαι. τοῦτο δὲ ἦν ἀληθές, εἰ πάντα τὰ ὑπὸ τὸ Β μέρη τοῦ Β ἦν καὶ οὕτως αὐτοῦ τινα ὡς ἐν τῇ οὐσία αὐτοῦ εἶναι. εἰ δὲ δύναταί τινα τῶν ὑπὸ τὸ Β καὶ χωρίζεσθαι αὐτοῦ, οὐκέτι τοῖς οὕτως οὖσιν ὑπὸ τὸ Β ἐξ ἀνάγκης τὸ Α ὑπάρξει. ἦν δὲ ἐπὶ τῆς καθόλου ὑπαρχούσης τῆς παραγωγῆς ταύτης αἰτία· ἐπεὶ γὰρ ἐν ἐκείνῃ πάντως ὑπάρχειν δεῖ τὸ Α τοῖς ὑπὸ τὸ Β, ἂν παντὶ τῷ Β ὑπάρχῃ, ἀκολουθεῖν δοκεῖ καὶ τὸ Α ἀναγκαίως αὐτοῖς ὑπάρξειν, ἐὰν ἀναγκαίως ὑπάρχῃ παντὶ τῷ Β· τοῦτο δέ, ἐπεὶ τὰ ὑπὸ τὸ Β τινὰ τοῦ Β ἐστίν (*in Pr. An.*, pp. 129, lin. 33 - 130, lin. 10).

[43] There are syllogisms in which one of the three terms is a primary substance like Choriscus or Mikkalus, but it is a matter of controversy whether those syllogisms belong to the system outlined in *Prior Analytics*, A, 1-22. However things may be, it is reasonable to exclude the possibility that in the case of a syllogism like *Barbara* one of the three terms could be replaced with a primary substance.

[44] The thesis is part of Aristotle's doctrine and it has been clearly stated in *Cat.*, 5, 3a37-b5 : τῶν δὲ δευτέρων οὐσιῶν τὸ μὲν εἶδος κατὰ τοῦ ἀτόμου κατηγορεῖται, τὸ δὲ γένος καὶ κατὰ τοῦ εἴδους καὶ κατὰ τοῦ ἀτόμου· ὡσαύτως δὲ καὶ αἱ διαφοραὶ καὶ κατὰ τῶν εἰδῶν καὶ κατὰ τῶν ἀτόμων κατεγοροῦνται. καὶ τὸν λόγον δὲ ἐπιδέχονται αἱ πρῶται οὐσίαι τὸν τῶν εἰδῶν καὶ τῶν γενῶν, καὶ τὸ εἶδος δὲ τὸν τοῦ γένους — ὅσα γὰρ κατὰ τοῦ κατηγορουμένου λέγεται, καὶ κατὰ τοῦ ὑποκειμένου ῥηθήσεται.

[45] I put the inverted commas here, departing from Wallies' edition, which puts them after ῥηθήσεται. Wallies probably thought that Alexander was quoting Aristotle's definition of the *dictum de omni*, but the context shows that the commentator was instead giving his own version of the modal extension of the *dictum de omni* ; hence ‘ἐξ ἀνάγκης’ is a constituent part of this revised *dictum*.

From this passage it is clear that the predicative relation introduced by the minor premiss of *Barbara* LXL is transitive and the Cs are said to be parts of B. Hence, if A is said necessarily of B, it will also be said necessarily of all of its subspecies, namely of all C. So far, we have proved the validity of *Barbara* LXL. The task of proving the invalidity of *Barbara* XLL is simpler: it is sufficient to recall that the major premiss ('A is said of all B') could express a relation of an accident (A) to its subject (B). Even if the relation between B and C is transitive and necessary, A cannot be necessarily predicated of any subject, if it is understood as an accident. Therefore, it is impossible to infer that 'A is said of all C of necessity' from such premisses.

I believe that these observations show that Alexander saw the main difficulties of Aristotle's modal syllogistic and that he tried to offer a reasonable solution. To present a fully detailed description of this account of modal syllogistic on the basis of his semantics goes beyond the intentions of this paper and, I think, beyond the limits of a merely historical reconstruction, because Alexander seems not to have developed all the logical details of the modal system, which he outlined in his extant works. I believe, however, that Alexander's texts too, as I have shown, contain engaging and original semantic reflections on Aristotle's modal syllogistic.

ABSTRACT

In this paper I dwell on Alexander of Aphrodisias' reconstruction of a famous puzzle of Aristotle's modal logic. Aristotle's claim that a LX-L *Barbara* is a valid syllogism, whilst a XL-L *Barbara* is an invalid formula within the syllogistic system has been contested not only in recent times, but by ancient Peripatetics too. I show that Alexander accepted Aristotle's claim, and criticized Theophrastus, who firstly contested the above thesis. My claim is that Alexander introduced a 'dictum de omni et de nullo' for modal propositions, and that, on the basis of it, he proved the validity of the LX-L *Barbara* syllogism. However, this proof is problematic, because it seems to be inconsistent with Alexander's proof for the validity of the rules of conversions for modal propositions. I suggest that Alexander had good arguments for avoiding the above difficulty, by making appeal to his semantics for modal propositions. This semantics is at the same time a temporal semantics and is rooted in Alexander's theory of predication. The second part of the paper offers a brief account of this semantics and presents its theoretical advantages.

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