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The Applicability of Mathematics and the Indispensability Arguments

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Sommaire

In the present paper I will focus on the relation between the main versions of the indispensability argument and the applicability of mathematics. I will show not only that the mathematical applicability problems are as urgent for realist philosophers as they are for anti-realist philosophers; but also that the supporters of the indispensability arguments cannot avoid to acknowledge the different roles that mathematics plays in science: I will show in this paper that ignoring these different roles might get realists into trouble.

Key words: anti-realism, applicability of mathematics, indispensability arguments, indispensability of mathematics, metaphysics, ontology, philosophy of mathematics, realism.

1 – Outline

In his famous (1960) article, physicist Eugene Wigner addressed a philosophical problem concerning the applicability of mathematics to science. Briefly speaking, the problem can be paraphrased as follows: mathematical concepts are (often) not intentionally developed for physical application and are part of a discipline which is epistemologically and ontologically different from physics;[1] so, how is it possible that these concepts are so amazingly effective in physics? We can call this problem the "Wigner's puzzle" or "problem of the mathematical applicability".

Despite its philosophical relevance, philosophers have ignored this problem for very long. Not only did they ignore Wigner's puzzle, but they also ignored the applicability of mathematics itself as a philosophically interesting topic. Only in 1998 was the topic brought back to the attention of the philosophical community, namely by Mark Steiner. Steiner showed that the problem of the applicability of mathematics is not a single problem, but that four related problems should be distinguished: a semantic problem, a metaphysical problem, a descriptive problem, and a heuristic problem. According to Steiner, some of them can be easily solved but others are quite puzzling and still without solution. An important lesson from Steiner's work is that we should refrain from conflating the different problems and address them separately.[2]

More recently, Christopher Pincock (2012) underlined how mathematics plays a lot of different roles in scientific representations.[3] Some of these roles are philosophically more problematic than others, but the important point is that, if we

1 - Just to offer few examples, mathematics studies abstract structures, whereas physics is about the real world; mathematics is mainly non-empirical, whereas physics is highly empirical.

2 - The first problem concerns the fact that mathematical terms (typically, numerical terms) seem to play different semantical roles in mathematical and mixed statements (mixed statements are statements in which both mathematical and physical terms appear). This problem, according to Steiner, has been solved by Frege, who offered a uniform account for both mathematical and mixed statements. The second problem stems from the metaphysical gap between the abstract entities described by mathematics and the concrete objects described by physics. Once again, according to Steiner, this problem has been solved by Frege, by means of his definition of mathematical objects as extensions of concepts directly related to the application of the mathematical theory at issue. The third problem concerns the descriptive applicability of single mathematical concepts. Each mathematical concept raises a specific descriptive problem; for some mathematical concepts we can account for their descriptive applicability (for example: the theory of fiber bundle), but for others (for example: Hilbert spaces, or complex analysis) we are seemingly left without a solution. The final problem concerns the effectiveness of using mathematical analogies (i.e., non-physical analogies) in predicting new laws, objects or phenomena in physics. Steiner does not offer a solution for this problem, but according to him it raises a serious problem for naturalism in contemporary physics. For more about Steiner's analysis and contemporary debates about the applicability of mathematics see (Pincock 2012).

3 - First of all, Pincock distinguishes between intrinsic *and* extrinsic *mathematics employed in representations: the first is the mathematics that appears in the* content *of a representation; the latter is the* additional *mathematics that, even though it does not appear in the representation, still contributes to the efficacy of the representation. Beyond this, Pincock specifies four dimensions along which mathematics can contribute to a scientific representation. These dimensions run along four basic dichotomies: (1) causal/acausal content, (2) concrete fixed/abstract varying content, (3) small-large representation (issues of scale), and (4) constitutive/derivative content. Thus, for a given mathematical representation, we can ask whether the mathematics involved is intrinsic or extrinsic, and then we can ask whether and how the mathematics contributes to (1), (2), (3) or (4) (it is also possible that the contribution combines different features of two or more of these dimensions).*

want to better understand the applicability of mathematics in science, we should focus very carefully on these roles, recognize them and analyze them as thoroughly as possible.

Following Steiner and Pincock, the idea that the role played by mathematics in science is much more complex than the label "applicability of mathematics" might lead one to think will be an important assumption in this paper. I will show that underestimating this point may entail unwanted consequences.

Due to its original formulation, Wigner's puzzle has often been interpreted as a puzzle for *anti-realist philosophies only*, which can be seen as one of the reasons for the philosophical neglect of the applicability of mathematics. As suggested by Colyvan (2001b), realist philosophies are often considered to be immune to Wigner's puzzle and some philosophers conceived realism as the *only* metaphysical framework in which the applicability of mathematics is not completely *unintelligible* (see for example Davies 1992, pp. 140-60; and Penrose 1990, pp. 556-7). Among the most important and compelling arguments for mathematical realism, however, are the so called "indispensability arguments". These arguments are explicitly grounded on the assumption that mathematics is not only applicable, but even *indispensable* to science. Yet, supporters of the indispensability arguments generally assume the applicability of mathematics as a brute fact without any further clarification: they usually do not acknowledge the different roles that mathematics plays in science, and in their discussions they do not pay attention to the philosophical problems connected to these roles. However, as Colyvan (2001b) showed, the supporters of the indispensability arguments are *not* exempt from Wigner's puzzle: it is a problem for them as well as for anti-realist philosophers.

In the present paper I will focus on the relation between the main versions of the indispensability argument (Quine's and Colyvan's, Putnam's, and the explanatory indispensability argument) and the applicability of mathematics. I will show not only that, as Colyvan says, the mathematical applicability problems are as urgent for realist philosophers as they are for anti-realist philosophers (in this sense, I will extend Colyvan's results by showing that also the supporters of Putnam's indispensability argument and of the so-called explanatory indispensability argument are not exempt from the philosophical problems raised by the applicability of mathematics); but also that the supporters of the indispensability arguments cannot avoid acknowledging the different roles that mathematics plays in science: I will show in this paper that ignoring these different roles might get realists into trouble.

In section 2, I will introduce the original main versions of the indispensability argument: Quine's argument for metaphysical realism and Putnam's argument for semantic realism. The indispensability arguments for metaphysical realism will be discussed in section 3. I will first discuss Quine's and Colyvan's arguments in detail (section 3.1), and I will make some remarks on the notion of "indispensability" there employed (section 3.2). Then, I will consider the employment of indispensability arguments in science and I will discuss the analogies and disanalogies between these arguments and the indispensability arguments for metaphysical realism (section 3.3). Finally, I will present and discuss the so-called "explanatory indispensability argument" (section 3.4). The indispensability arguments for semantic realism will be discussed in section 4, where I will take into consideration Putnam's argument (section 4.1) and the semantic version of the explanatory indispensability argument (section 4.2).

2 – Metaphysical and semantic realism

The first formulation of an indispensability argument is jointly credited to Quine (1961) and Putnam (1979a), and for this reason the argument is also usually called the "Quine-Putnam argument". However, many commentators have pointed out that Quine and Putnam formulate the argument in two distinct ways and that they aim to prove different claims (for a discussion about the differences between Quine's argument and Putnam's argument, see Liggins 2008). Quine's argument can be summarized in the following way:

(1_Q) we ought to have ontological commitment to all those entities that are indispensable$_Q$ to our best scientific theories;
(2_Q) mathematical entities are indispensable$_Q$ to our best scientific theories;
(3_Q) hence, we ought to have ontological commitment to (some) mathematical entities.

Putnam's argument, instead, can be summarized as follows:

(1_P) we ought to believe in the truth of any claim that plays an indispensable$_P$ role in our best scientific theories;
(2_P) mathematical claims play and indispensable$_P$ role in our best scientific theories;
(3_P) hence, we ought to believe in the truth of (some) mathematical claims.

The main difference between the two arguments is that Quine argues in favour of a *metaphysical* realism, while Putnam rather argues in favour of a *semantic* realism. As we will see, another important difference is that they bestow different meanings to the word "indispensable"—what is graphically expressed by the subscripts appended to the word "indispensable" in the different arguments.

Colyvan (2001a) probably offers the most pervasive and complete analysis of this kind of argument and proposes his version of the argument. Colyvan thinks, in agreement with Quine, that the indispensability argument is actually an argument for Platonism, and hence the metaphysical realism. Colyvan's argument is therefore very close to Quine's argument. However, as Pincock (2012, chapt. 9) points out, Colyvan's notion of indispensability is different both from Quine's and from Putnam's. So, Colyvan's argument must be kept distinct from the other two.

In the following sections I will consider each of these arguments, in order to explore the connections between them and the roles played by mathematics in science.

3 – Arguments for metaphysical realism

3.1 Quine's and Colyvan's argument

That Quine's argument does not absolve the mathematical realist from the applicability problems has been already noted by many. Colyvan (2001b) himself points out that Quine's argument, whether it is right or not, does not say anything about *why* mathematics is indispensable. Rather, it assumes indispensability as a brute fact—and to answer why mathematics is indispensable we necessarily have to get deeper into the applicability of mathematics and into the many roles played by mathematics in science.[4] In this section I will go further than this argument of Colyvan and I will argue not only that the Quinean realist has still to clarify why mathematics is indispensable, but also that the argument *itself* forces her to get deeper into the effectiveness of mathematics within science.

There are two ways to defend Quine's indispensability argument. The first consists in taking the bull by the horns and trying to found the argument *in itself*, autonomously. The second consists in considering the indispensability argument as a particular case of a wider class of arguments, which share the same logical form. In this second way we try, in some sense, to 'transfer' epistemic confirmation from one argument to another, by means of analogy. These two defense strategies do not exclude each other; on the contrary, they *support* each other. The first strategy assumes that the logical form of the argument is valid and tries to show that the premises are true; the second strategy tries to show that we have good reasons to apply this argument to mathematical entities because this case is analogous to the application of (apparently) the same argument to other scientific entities—an application which we normally consider non-problematic.

3.2 Remarks on the notion of indispensability

Let us start with the first strategy. It usually implies that one focus on the two premises and how to support them. Premise (1_Q) is usually considered to be founded on two important Quinean theses: Confirmational Holism and Naturalism. The idea is that Naturalism tells us that we must commit ourselves to the existence of the entities required by our best scientific theories, and Confirmational Holism prevents us from interpreting part of these theories non-realistically. Both doctrines have been variously called into question, and there is currently a wide debate about the effectiveness of these two thesis in supporting (1_Q) (see for example Panza & Sereni 2015). However, it is not my intention to focus on these debates. I will rather focus on the meaning of "indispensable$_Q$", because Quine's argument is evidently centered around this notion, but it is not clear what exactly he meant by it.

First, it should be noted that "indispensability" should not be interpreted as generic "non-eliminability". If "indispensable" simply means "non-eliminable", it follows from Craig's theorem that *any* mathematical entity is actually *eliminable*, which makes the second premise of the indispensability argument untenable. Hence, indispensability must be intended in a different sense (see Colyvan 2001a, p. 77; for a more detailed analysis of Craig's theorem, see Putnam 1965 and Field 1980, p. 8). According to Quine, an entity ξ appearing in a theory T can be said to be indispensable only when its elimination from T produces a new theory T' that can even be equivalent to T but is notwithstanding less preferable than T. Thus, the notion of indispensable$_Q$ involves the notion of "preference for one theory over another". In order to understand how Quine sees this notion of theory preference (and, consequently, the notion of indispensability$_Q$), we must take into consideration what he called the process of "regimentation".

When we want to systematically present our beliefs, we aim to do it in a way that is as coherent and simple as possible. According to Quine, this goal is reached when we formulate our beliefs in the language of first-order logic. This process of regimentation is particularly important because Quine holds that we can clarify our ontological commitment only *after* carrying out this process of regimentation. If Quine is right in claiming that the proper language in which we should regiment our scientific beliefs is the language of first-order logic, then it is quite easy to see which entities we are ontologically committed to: we have simply to check which sentences of the form $\exists xFx$ are implied by our beliefs. It is at this level that we can prefer one theory over another. To sum up, we should understand "indispensable$_Q$" in the following way: an entity (or a class of entities) ξ is in Quine's view indispensable if, *once the regimentation has been done*, it is not possible to eliminate sentences which quantify over this entity (or over this class of entities). Quine seems to hold that mathematical entities are 'indispensable' precisely in this sense: "certain things we want to say in science may compel us to admit into the range of values of the variables of quantification not only physical objects but also classes and relations of them; also numbers, functions, and other objects of pure mathe-

4- An analogous objection has been made by Kitcher (1984, pp. 104-105).

matics" (Quine 1957, p. 16). But regimentation, as Quine puts the matter in different places, is not very different from the general process of belief choice and belief adjustment, and hence is not very different from the general process of doing science:

> Our acceptance of an ontology is, I think, similar in principle to our acceptance of a scientific theory, say a system of physics: we adopt, at least insofar as we are reasonable, the simplest conceptual scheme into which the disoriented fragments of raw experience can be fitted and arranged. [. . .]. To whatever extent the adoption of any system of scientific theory may be said to be a matter of language, the same—but no more—may be said of the adoption of an ontology. (Quine 1961, p. 16)

And again:

> We can draw explicit ontological lines when desired. We can regiment our notation [. . .]. Various turns of phrase in ordinary language that seem to invoke novel sorts of objects may disappear under such regimentation. At other points new ontic commitments may emerge. There is room for choice, and one chooses with a view to simplicity in one's overall system of the world. (Quine 1981, p. 9-10)

Regimentation seems thus to be guided by a general criterion of simplicity and transparency. However, in some cases it is just the mathematical apparatus that makes one theory preferable to another, often just in virtue of a greater simplicity granted to the theory by the mathematics itself. Mathematical concepts often make a theory easier and therefore more effective in making predictions and new discoveries. In this sense, one might say that a given mathematized theory is simpler than its non-mathematized counterpart. If this is so, the situation turns out to be quite odd: mathematics is indispensable because a scientific theory *mathematically* formulated is to be preferred over a scientific theory *non-mathematically* formulated; it is to be preferred because it is simpler; and it is simpler *because* it is mathematically formulated. This is not necessarily a problem, provided that we are able to explain *why*, *how*, and *in which sense* mathematics contributes to this preference. But this means that the Quinean realist should say something more about this 'simplifying' role of mathematics in physics if she wants to hold onto the validity of her argument. Otherwise, one might object that the notion of indispensability is, in some sense, viciously circular, since it should grant an independent and autonomous foundation for the claim that mathematics is indispensable, but mathematics turns out to play a role in shaping the notion of indispensability itself. More specifically, the question that the Quinean realist should answer is: In which sense does mathematics play a role in shaping 'simpler' physical theories? And how is it possible that mathematics can do that, provided that mathematical concepts have not been devised to accomplish such a role?

An analogous remark can be made against Colyvan's indispensability argument. Colyvan's argument in intended to support metaphysical realism as well as Quine's argument. However, his notion of "indispensability" is a bit different from Quine's. Colyvan (1999) defines "indispensability$_c$" in the following way: an entity is *dispensable* to a theory *T* if there exists a second theory *T'* with the following properties: (i) *T'* has exactly the same observational consequences as *T*, but in *T'* the entity in question is neither mentioned nor predicted; and (ii) *T'* is preferable to *T*. If an entity is not dispensable in this sense, then it is indispensable. The main difference between "indispensable$_Q$" and "indispensable$_C$" is that Colyvan's notion is not centered on the Quinean process of regimentation. Instead of arguing, like Quine does, that ontological commitment is implied by the simplest and most transparent regimented theory, Colyvan gives criteria that allow us to determine whether or not a theory *T'* is preferable over another theory *T*. As he says,

> whether an entity is indispensable or not is really a question of theory choice and so is guided by the usual canons of theory choice. These may include: simplicity, unificatory power, boldness, formal elegance and so on. It seems, then, that an entity can be indispensable even though empirically equivalent theories exist that do not quantify over the entity in question. (Colyvan 2001b, p. 270)

This point made by Colyvan rules out any attempt, standardly nominalistic or à la Field, to offer a non-mathematical reconstruction of contemporary physics: even if such a reconstruction is *theoretically* possible, it is unlikely that the result will be preferable to our standard formulation according to the criteria that Colyvan proposes, and hence the indispensability of mathematics is saved. However, in this way Colyvan exposes himself to the same remark we pointed out for Quine: in some cases it is just the mathematical apparatus that makes a theory preferable to another. In this sense, mathematics can be one of the criteria according to which we judge whether a theory is preferable over another or not. So Colyvan, as well as Quine, should offer an account of mathematics 'simplifying' role in order to avoid circularity in his definition of indispensability.

3.3 Transferring epistemic confirmation

The second way to defend the indispensability argument for metaphysical realism in mathematics is by considering it as a particular case belonging to a wider class of arguments of the same form. Those who hold that a realistic stance on mathematical entities would absolve the realist from dealing with the applicability problems usually appeal to this strategy in order to support their claim. In this section we will assess this strategy.

According to Colyvan (2001a), the general form of these arguments can be presented in the following way (G):

> If apparent reference to some entity (or class of entities) ξ is indispensable to our best scientific theories, then we ought to believe in the existence of ξ. (p. 7)

As an example of an application of this kind of argument in a different context from mathematical ontology, Colyvan mentions the scientists' belief in the existence of dark matter:

> Most astronomers are convinced of the existence of so called "dark matter" to explain (among other things) certain facts about the rotation curves of spiral galaxies. [. . .] this is an indispensability argument. Anyone unconvinced of the existence of dark matter is not unconvinced of the cogency of the general form of the argument being used; it's just that they are inclined to think that there are better explanations of the facts in question. (p. 8)

As Colyvan points out, the existence of dark matter is explanatorily indispensable to our best scientific theories, and we accept it as a consequence of an inference to the best explanation. In this argument however, there seems to be talk of another kind of indispensability than in the arguments we considered before. Considering Quine's version of the indispensability argument, we see that mathematical entities play no explanatory role. Rather, mathematical entities are *referentially* indispensable; namely, there seems to be no way to formulate our best scientific theories without quantifying over mathematical entities (according to the meaning of "indispensable$_Q$" we have just seen). But this is the general meaning of "indispensable" as it is assumed by the formulation of the argument.

Thus, even if Quine's argument and the argument about dark matter both seem to be an application of (G), the two applications should not be considered to be entirely the same: in the latter case we have a (more or less) clear application of the notion of indispensability (*viz.* indispensability in terms of explanation); in the first case we don't. The furrow we have just cut between the two cases prevents us from transferring epistemic value from one case to another. What makes the latter argument plausible cannot be simply transferred on the former case, since part of this plausibility hinges on the fact that the adjective "indispensable" is employed, in the dark matter case, in a specified sense.

However, in response to this objection one might claim that there is anyway an analogy between the argument concerning dark matter and Quine's argument, and that this analogy is based on the so-called "no miracle argument". The "no miracle argument" is usually employed by scientific realists in order to justify their position: we must accept the existence of the entities postulated by our best scientific theories, because otherwise we must admit that the fact that the nature works as it works is just a miracle. For example, if we don't believe in electrons (that is, if we accept the electron theory without believing in the existence of electrons), then how can we explain the behaviour of a galvanometer? There seems to be no way other than to accept that it is a miracle. The analogy between Quine's argument and the argument in favour of the existence of electrons is underlined and criticized also by Colyvan (2001b):

> It's no miracle, claim scientific realists, that electron theory is remarkably effective in describing all sorts of physical phenomena such as lightning, electromagnetism, the generation of X-rays in Roentgen tubes and so on. Why is it no miracle? Because electrons exist and are at least partially causally responsible for the phenomena in question. Furthermore, it's no surprise that electron theory is able to play an active role in novel discoveries such as superconductors. Again this is explained by the existence of electrons and their causal powers. (pp. 270-71)

Thus, if we reject the realist claim that electrons do exist, we should admit that the extraordinary effectiveness of this theory in making predictions and describing a large class of phenomena is just a miracle. For this reason, there is indeed a pressure on anti-realists in science, since they seem unable to explain the efficacy of electron theory in describing reality (for an example of such an argument for scientific realism, see Smart 1963). But can we raise the same problem for mathematical entities? According to Colyvan (2001b), the argument can hardly be exported to the field of mathematical realism, since

> [t]here is an important disanalogy [. . .] between the case of electrons and the case of sets. Electrons have causal powers—they can bring about changes in the world. Mathematical entities such as sets are usually taken to be causally idle—they are platonic in the sense that they do not exist in space-time nor do they have causal powers. So how is that the positing of such platonic entities reduces mystery? (Colyvan 2001b, p. 271)

The fact that the existence of electrons is able to remove the aura of mystery around the theory of electrons does not come from the bare existence of the electron, but from the fact that they exist in a *certain way*, namely they are causally effective. The same seems not to hold for mathematical entities, since a Platonist usually conceives of them as abstract entities with no causal power.

These considerations suggest that it is not possible to transfer epistemic confirmation from the indispensability arguments as they are employed in empirical sciences to Quine's argument, for the very reason that the notion of indispensability refers, in the first case, to a well-specified role played by the entities at issue; in the latter, to no specific role. Still, one might insist that the two arguments are substantially the same (and that hence we can transfer epistemic confirmation from one argument to the other) by arguing that mathematical entities *do* really *have* a specified and non-generic indispensable role in science, either explanatory or causal.

This argument has been developed in the literature in two different ways. Bigelow (1988) and Maddy (1990), for example, have argued for a causal effectiveness of mathematical entities. However, in her subsequent works Maddy seems to have abandoned such a view (see Maddy 1997, 2007). This option is quite marginal in the contemporary debate, and actually the fact that the Quinean realist is compelled to say that mathematical entities are causally active is rather seen as a *difficulty* for Platonism (see for example Cheyne & Pigden 1996). On the other side, a great part of recent debates around the indispensability argument tries to show that mathematical entities actually play an indispensable *explanatory* role in empirical sciences (see for example Lyon & Colyvan 2008; Colyvan 2002, 2010; Baker 2005, 2009). In both cases, the supporters of the indispensability argument try to boost the indispensability argument by showing that the indispensable role mathematical entities play in science can be specified in one sense or in another.

3.4 Explanatory indispensability argument

Of the above two strategies, the first seems to be the most difficult. Unfortunately, at least for now, there is no convincing argument that can satisfactorily clarify in which sense mathematical entities are causally active. One might say that we can employ the mathematical entity (or class of entities) ξ in describing the physical phenomenon *P because* ξ is *part of the causal chain* that determines the nature of *P*. However, this seems hardly promising for our purposes; this claim cannot be considered as a satisfying account of the applicability of mathematics until we offer a satisfying account of what it means for a mathematical entity to be part of the causal chain of a certain phenomenon.

The second strategy seems more promising, and has been widely discussed in recent times. We can formulate an explanatory version of the indispensability argument as follows:

(1_{EXP}) we ought to have ontological commitment to any entity that plays an indispensable explanatory role in our best scientific theories;
(2_{EXP}) mathematical entities play an indispensable explanatory role in our best scientific theories;
(3_{EXP}) hence, we ought to have ontological commitment to mathematical entities.

This argument needs some clarification. First of all, one should specify what is meant by "explanatorily indispensable" and "mathematical explanation" in science. Further, we need to establish that there are entities that play an indispensable explanatory role, hence we need an example of indispensable mathematical explanation in science. Usually (see for example Pincock 2012), a scientific explanation is said to be "mathematical" if it makes use of a mathematical claim. But in order to understand that the explanation is really mathematical (i.e. the mathematics in it is *non-eliminable*) we should put it on probation by means of a replacement test: if we eliminate the mathematical claim from the explanation and the explanatory value disappears, then the explanation can be said to be "mathematical". However, this 'replacement test' clarifies what we mean by mathematical explanation in science, but it does not exhaust the meaning of "explanatorily indispensable", since we can have different, competing explanations that do not make use of mathematical claims. Hence, we must submit the explanation to another test, let us call it the "comparison test": consider all the possible explanations of a certain phenomenon; if the mathematical one is the best one (i.e. if it has the greatest explanatory power), then the mathematics employed is *explanatorily indispensable*.

Are there indispensable mathematical explanations in science? The answer to this question is widely debated among philosophers. On one side in this debate, there are philosophers like Colyvan, Baker, Batterman, and Pincock who argue that there really are examples of such explanations (see for example Baker 2005; Batterman 2002, 2010; Pincock 2011a, 2011b, 2012). Three examples have become paradigmatic: the explanation of the periodic life cycle of some species of cicada, the explanation of the hexagonal form of the bee's honeycomb and the explanation of why it is not possible to cross all the bridges of Königsberg exactly once in a circuit that returns to the starting point. On the other side, philosophers like Melia, Daly and Langford have tried to show that these explanations are not real examples of mathematical explanations (see for example Melia 2000, 2002; Daly & Langford 2009). Without going in too much detail concerning this debate, let me make some remarks about how this relates to the applicability of mathematics.

First of all, it must be noted that, even if we admit that there are indispensable mathematical explanations in science, this is of no help for the analysis of the applicability of mathematics. For two reasons: first, the explanatory role of mathematics is not the only role for mathematics in science, and hence we have still to clarify the effectiveness of mathematics in all these other roles;[5] second, the indispensability argument does not offer an account of why mathematics is helpful in explaining physical phenomena. The eliminability test and the comparison test permit us to say whether a mathematical claim plays an indispensable role in explaining a certain phenomenon or not, but they are not aimed at accounting for the conditions that a mathematical claim has to satisfy in order to have an explanatory power. In this sense, the explanatory indispensability argument seems also not to be of any help in clarifying the applicability of mathematics, and supporters of this argument are in no sense exempted from the applicability problems.

Secondly, as Pincock (2012, pp. 206-207) points out, the notion of "indispensable mathematical explanation" has been analysed by means of two tests that require to eliminate or

5 - See p. 2 and relative footnote.

substitute the mathematical *claims* appearing in the scientific explanation. However, premise (2_{EXP}) refers to mathematical *entities*. Hence, we need an argument that shows the link between these two tests and the relevance of mathematical *entities* in explanation. In absence of this link, it would be better to reformulate the whole argument as an argument for *semantic*, rather than *metaphysical*, realism. In other words, the argument seems to fall short of its proponents' expectections.

Since this argument fails to argue in favour of metaphysical realism and seems to be better an argument for *semantic* realism, I will not discuss here whether a clarification of the applicability of mathematics is in any sense required in order to support this explanatory indispensability argument. I will rather discuss this question in section 4.2, where I will take into consideration the semantic version of this argument.

4 – Arguments for semantic realism

4.1 Putnam's argument for semantic realism

As we already saw, Putnam argues in favour of semantic realism. His conclusion is that confirmational holism commits us to believe that mathematical claims employed in science are true, as are other true scientific claims that appear in our best scientific theories. To do this, he relies on a notion of "indispensability" which differs both from Quine's and Colyvan's. Whereas Quine's notion of "indispensable$_Q$" is centered around the practice of regimentation, Putnam's indispensability aims rather to grasp the real meaning of the scientists' claims. According to him, "one of our important purposes in doing physics is to try to state 'true or nearly true' (the phrase is Newton's) laws, and not merely to build bridges or predict experiences" (Putnam 1979a, p. 338). Therefore, one of the main tasks of philosophy consists in acknowledging this purpose and respecting this character of physical inquiry. Thus, Putnam's argument assumes that scientific claims have a more or less clear meaning that philosophical reflection should respect and should not demand to reformulate (as in Quine's regimentation). It follows that the notion of "indispensable$_P$" concerns the scientific claims in their original formulation: something is indispensable$_P$ if it is required by the scientists in their formulations of a theory.

This feature of Putnam's indispensability seems to fit better with an argument for metaphysical realism: if there is no need for regimentations or reformulations (that, depending on their fulfillment, could eliminate the reference to abstract entities), and since scientific claims on this account directly refer to mathematical entities, the argument could easily argue for the existence of mathematical entities rather than for semantic realism. However, Putnam is cautious and prefers to opt for a weaker conclusion. The reason for this is that Putnam is worried about what he calls "equivalent constructions" in mathematics, something that in his view is intrinsic to the mathematical formulation: "the chief characteristic of mathematical propositions is the very wide variety of equivalent formulations that they possess" (Putnam 1979b, p. 47). Among these possible formulations, some of them do not need abstract objects (for example, modal formulations *à la* Hellman 1989), and hence it may not be necessary to include abstract entities in our ontology.

That said, let us go back to our main question in this section: how can semantic realism be of any help in solving the applicability problems? A first possible answer is that if mathematical propositions are true, then we can apply them just because they are true. The applicability of mathematical claims is therefore justified by their being true. However, this answer cannot be considered satisfying: after all, the proposition "My right incisor is chipped" is true, but I can hardly believe that such a proposition can be of any employment in science. In other words, there are billions of true propositions that have no application in science. Why are mathematical propositions different?

A slightly more articulated answer is that mathematical claims can be successfully employed in science because they are true and pertain to the nature of the world. However, this answer too can hardly be considered satisfying, since it does not avoid the difficulties of the previous answer: in which sense, exactly, do mathematical claims 'pertain to' the natural world? In the absence of an answer to this question, semantic realism cannot be of any help in accounting for the effectiveness of mathematics.

4.2 Explanatory indispensability argument for semantic realism

As we saw at the end of section 3.4, it is also possible to formulate an *explanatory* indispensability argument for semantic realism:

(1_{exp}) We ought to believe in the truth of any claim that plays an indispensable explanatory role in our best scientific theories;
(2_{exp}) mathematical claims play an indispensable explanatory role in our best scientific theories;
(3_{exp}) hence, we ought to believe in the truth of mathematical claims.

The argument is similar to the analogous argument for metaphysical realism, but premise (2_{exp}) refers to mathematical claims instead of mathematical entities. In this sense, the two tests we previously discussed about the explanatory value of a claim can directly support premise (2_{exp}), while for premise (2_{EXP}) we needed another argument to show that if a mathematical claim plays an indispensable explanatory role in our best scientific theory then the entities entailed by that claim are indispensable as well.

Apparently, this argument seems to be able to remedy the difficulty of the previous argument. There I noted that semantic realism cannot be of any help in explaining the effectiveness of mathematics in science because it does not explain in which sense mathematical (true) claims would pertain to the natural world. This argument seems to suggest that mathematical claims pertain to the scientific description of the natural world precisely in the sense that they are explanatorily relevant to this description. So, one might say, on the basis of this argument, that mathematics is effective in science *because* mathematical claims are true *and* they *explanatorily* pertain to scientific subjects. However, it must be noted that the addition of the adverb "explanatorily" does not add very much to the sense of "to pertain to". This is just a way to beg the question: if we are interested in understanding why and how mathematics is so effective in science, then we are interested in understanding in which sense mathematical claims can be, among other things, explanatorily relevant for the scientific discourse. Thus, once again, it seems that a stance on mathematical realism, be it semantic or metaphysical, is of no help in clarifying the applicability problems.

But there is something more. Let us consider the following examples (for more about these examples, see Maddy 1992). It is common, in the analysis of water-wave dispersion, to assume that water depth tends to infinity. Analogously, in fluid dynamics, scientists usually assume that matter is continuous. Both assumptions are clearly false,[6] nevertheless, scientists make such assumptions in order to explain the behaviour of water waves and matter, respectively. Thus, it turns out that the assumptions employed here have an indispensable explanatory role, but they remain *false* assumptions and nobody will hold them as true. These assumptions are not mathematical, strictly speaking. However it seems that now we have two different classes of claims: (1) true claims having an indispensable explanatory role in our best scientific theories, and (2) *false* claims (idealizations, for the most part) that notwithstanding play an indispensable explanatory role in our best scientific theories. The problem is: how can we say that mathematical claims fall within the first of the two classes? It seems that we need a specific argument to show that, but how can such an argument be? To prove that a mathematical claim *p* falls into class (1) we should prove that it is true, but if we have an argument to prove that p is true we don't need any indispensability argument to prove what we have already proved in a different way!

However, this does more than point out the risk of a vicious circularity in the argument; it also shows a conflict between naturalism and confirmational holism: scientists do not hesitate to assume patently false claims in order to get the job done, but confirmational holism does not permit us to account for this peculiarity of scientific practice, thus pushing us away from naturalism.[7]

5 – Conclusion

Indispensability arguments are our best and most compelling arguments for mathematical realism. However, we have seen that these arguments do not clear their supporters from the applicability problems raised by Wigner (1960) and Steiner (1998). This holds not only for supporters of Quine's indispensability argument—as Colyvan (2001b) already pointed out—but, as I have shown, also for supporters of other versions of indispensability arguments, like Putnam's version or the explanatory indispensability arguments. As I have already noted, these arguments do not directly deal with the mathematical applicability problems, and in this sense it is not a surprise that they do not solve them. What really is a surprise is the fact that, besides these arguments explicitly based on the applicability of mathematics, supporters of these arguments ignored for very long time the philosophical problems arising from the applicability of mathematics.

However, in the present paper I showed that the problem at issue is even more compelling, since, apparently, if the supporters of the indispensability arguments do not take into account that mathematics has many different applications in science, they incur difficulties that may undermine their arguments. The applicability of mathematics cannot be simply assumed as a brute fact, and be left unanalyzed. Indeed, a better comprehension of mathematical applicability seems to be necessary in order to avoid serious difficulties. In other words, the indispensability of mathematics cannot be uncritically assumed to justify ontological realism, but it is important (and— let me say—*indispensable*) to deal with the concrete problems posed by the application of mathematics.

6 - The first assumption concerning depth of water is patently false; the second one is false if we assume that our best scientific theories about the ultimate composition of matter are true.

7 - It is not clear to which extent confirmational holism is needed to support premise (1_{exp}) and this point is currently a matter of discussion (see for example Panza & Sereni 2015). If we can do without it, then these last criticisms do not really undermine the indispensability argument.

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