'. . . But I Still Can't Get Rid of a Sense of Artificiality': The Reichenbach-Einstein Debate on the Geometrization of the Electromagnetic Field

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Abstract

This paper analyzes correspondence between Reichenbach and Einstein from the spring of 1926, concerning what it means to 'geometrize' a physical field. The content of a typewritten note that Reichenbach sent to Einstein on that occasion is reconstructed, showing that it was an early version of §49 of the untranslated Appendix to his *Philosophie der Raum-Zeit-Lehre*, on which Reichenbach was working at the time. This paper claims that the toy-geometrization of the electromagnetic field that Reichenbach presented in his note should not be regarded as merely a virtuoso mathematical exercise, but as an additional argument supporting the core philosophical message of his 1928 monograph. This paper concludes by suggesting that Reichenbach's infamous 'relativization of geometry' was only a stepping stone on the way to his main concern—the question of the 'geometrization of gravitation'.

> Aber ich kann auch da das Gefühl des Künstlichen nicht los werden
>
> Reichenbach to Einstein, March 16, 1926

Introduction

In the late 1950s, Hans Reichenbach's second wife Maria Reichenbach edited an English translation (Reichenbach, 1958) of his *Philosophie der Raum-Zeit-Lehre* (Reichenbach, 1928). This edition was missing a long Appendix entitled 'Die Weylsche Erweiterung des Riemannschen Raumbegriffs und die geometrische Deutung der Elektrizität' ('Weyl's Extension of Riemann's Concept of Space and the Geometrical Interpretation of Electromagnetism'). A translation of the Appendix was prepared in a nearly publishable form (including a transcription of the quite heavy mathematical apparatus), and the typescript is preserved in the Reichenbach Archives in Pittsburgh (HR, 041-2101). However, the publication must have been withdrawn eventually.

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Except for a 'dead link' to a no-longer-existing §46 on page 17, even today many readers of *The Philosophy of Space and Time* might be unaware that such an Appendix ever existed.

The decision not to publish the Appendix is understandable. The text is quite demanding for readers unaccustomed to the formalism, and struggling through it may not have been worth the effort. After Einstein's death in 1955, the very project of a unified theory of gravitation and electromagnetism, which Reichenbach discusses with a plethora of technical details, was nearly unanimously regarded as a relic of the past (but see Tonnelat, 1955)—not least of which by Hermann Weyl, one of the project's initiators (Weyl, 1956). In the same spirit, in an English translation of a selection of Reichenbach's writings from the late 1970s (Reichenbach, 1978), the pages dealing with "Weyl's generalization of Riemannian space" were omitted because, as the editors write, they had "no historical significance" (Reichenbach, 1978, 2:3).

Only a year later, however, a pathbreaking paper by Alberto Coffa (1979) proved that this judgment was hasty. Rediscovering Weyl and Reichenbach's 'elective affinities', Coffa began (with a nice pun) a fertile line of research which, much later, would bear fruit in the work of Thomas Ryckman (1995, 1996, 2005), Robert Rynasiewicz (2005) and others (see also Giovanelli, 2013b). It was in this context that Coffa provided perhaps the first and only detailed analysis of the untranslated Appendix to the *Philosophie der Raum-Zeit-Lehre*. In fact, Coffa read the Appendix as single-mindedly trying to "exhibit the vacuity of Weyl's enterprise" (Coffa, 1979, 295). Here, however, Coffa's major achievement becomes a hindrance. Despite the Appendix's somewhat misleading title, by interpreting it exclusively in terms of the Weyl-Reichenbach debate, we do not fully grasp its meaning.

Letters between Reichenbach and Einstein, preserved in the Einstein Archives in Jerusalem (AEA),[1] suggest that the Appendix should be read more broadly. In the spring of 1926, Reichenbach, after making some remarks on Einstein's newly published metric-affine theory (Einstein, 1925b), sent him a note offering what looks like his own attempt at a unified field theory. Reichenbach's note turns out to have been an early draft of §49 of the Appendix, on which he was working at the time. Einstein's objections and Reichenbach's replies reveal that criticism of Weyl's theory was only part of the story. Reichenbach was mainly interested in the very idea of the 'geometrization' of a physical field. At the time, many believed that if general relativity geometrized the gravitational field, then it was also plausible to geometrize the other known field—the electromagnetic field. To challenge this view, Reichenbach conducted what might be called an 'epistemological experiment'.

Reichenbach constructed a toy-theory establishing a connection between electricity and geometry which, he argued, was *just as good* as the one general relativity established between gravitation and geometry. Reichenbach's theory, however, was clearly not as successful as general relativity. Thus, Reichenbach could provide 'experimental' evidence that the geometrization of a physical field cannot be regarded in itself as a physical achievement. As soon as Einstein understood the 'ironical' nature of Reichenbach's enterprise, he immediately agreed with him. As Dennis Lehmkuhl (2014) has recently shown, it was in his correspondence with Reichenbach that Einstein pointed out, for the first time, that general relativity had not geometrized the gravitational field.

As we shall see, Einstein and Reichenbach's opinions about the geometrization issue were only superficially similar. However, reading Reichenbach's 1928 monograph against the background of this issue reveals a quite different view of his interpretation of general relativity. If general relativity dressed the gravitational field in a geometrical 'cloak', as Reichenbach put it, "one should not confuse the cloak [Gewand] with the body which it covers" (Reichenbach, 1928, 354; tr. HR, 041-2101, 493). The Appendix of his book was meant to show that one can, with some

[1] The correspondence will appear in the forthcoming 15th volume of CPAE.

sartorial skill, always dress a physical field in a geometrical disguise. To understand why general relativity is a successful physical theory we have to look beyond the geometrical clothing to the body it hides. In general relativity the connection turned out to be *heuristically powerful*; it led to new testable predictions. In contrast, the link between electricity and geometry established by Reichenbach's theory simply recast what was already known in geometrical terms.

This paper suggests that the geometrization issue was not just a spin-off of Reichenbach's 1928 monograph, but possibly the core message of the book. To support this claim the paper proceeds as follows. Section 1 describes the context in which Reichenbach decided to send Einstein a note on the geometrization of the electromagnetic field. Section 2 offers a reconstruction of Reichenbach's note. Section 3 describes Einstein's initially skeptical, then approving, reaction to the note. Section 4 shows what Reichenbach's *Philosophie der Raum-Zeit-Lehre* looks like if read from the perspective of the Appendix. Finally, analyzing Reichenbach's attitude towards Einstein's distant parallelism field theory, section 5 emphasizes the differences that existed behind Reichenbach and Einstein's apparent agreement on the issue of geometrization. This paper concludes by suggesting that Reichenbach's well-known 'relativization of geometry' was only a stepping stone on the way to his main concern—the question of the 'geometrization of gravitation'.

1. Reichenbach's Γ-Critique and His Note on the Unified Field Theories

On June 5, 1925 Einstein, who had just returned from a long trip to South America (see his travel diary, CPAE, Vol. 14, Doc. 455, Mar. 5–May 11, 1925), wrote to Michele Besso about the state of his research on a unified theory of the gravitational and electromagnetic fields (Goenner, 2004; Sauer, 2014; Vizgin, 1994). He revealed to Besso that he had become disillusioned with the whole "Weyl-Eddington-Schouten line of thinking," the framework in which he had been working in the previous years,[2] and that he was already "on another track, that is physically more grounded" (Einstein to Besso, Jun. 5, 1925; Speziali, 1972, 240). The paper Einstein was referring to—the first in which the term 'unified field theory' appears in the title—was presented at the Prussian Academy during its July 9, 1925 session (Einstein, 1925b).

Einstein described "the egg" he "recently laid" to Besso some weeks later (Einstein to Besso, Jul. 28, 1925; Speziali, 1972, 209–210). The theory introduced an affine connection ($\Gamma^{\tau}_{\mu\nu}$) (from which the Riemann and Ricci tensor $R_{\mu\nu}$ are derived), and independently the metric tensor $g_{\mu\nu}$ (and its correspondent contravariant tensor $g^{\mu\nu}$ and tensor density $\mathfrak{g}^{\mu\nu}$). Einstein then built the scalar density $\mathfrak{H} = \mathfrak{g}^{\mu\nu}R_{\mu\nu}$ and postulated the independent variation $\delta \int \mathfrak{H}dt = 0$, with respect to the $\mathfrak{g}^{\mu\nu}$ and $\Gamma^{\tau}_{\mu\nu}$ (Ferraris, Francaviglia, and Reina, 1982). After some manipulation he obtained, at first approximation, the already-known laws of gravitation and electromagnetism. The symmetric part of the $\mathfrak{g}^{\mu\nu}$ represents the 'gravitational potentials', and the antisymmetric part the 'electromagnetic field strength' (Einstein to Besso, Jul. 28, 1925; Speziali, 1972, 209–210).

[2]Very roughly this line of thought can be described as follows. Around 1918, in order to eliminate the last 'distant geometrical' remnants of Riemannian geometry, Weyl introduced what he called the 'length connection' φ_i. It determines the change of the length of a vector on parallel transport just as the 'affine connection' Γ^{i}_{kl} in Riemannian geometry determines the change of its direction (Weyl, 1918a,c, 1919a). φ_i could be identified with the electromagnetic four-potential. Eddington (1921) radicalized Weyl's approach, using only the affine connection as the fundamental quantity. A generally non-symmetric Ricci tensor ($R_{ik} \neq R_{ki}$) can be derived from it and split into an antisymmetric part $F_{\mu\nu}$ identified with the electromagnetic tensor and a symmetric part $R_{\mu\nu}$ corresponding to the gravitational potentials (by introducing the 'natural gauge' $R_{\mu\nu} = \lambda g_{\mu\nu}$ where λ is the cosmological constant). Einstein tried to specify the equations that govern the affine connection in Eddington's approach in three brief notes (Einstein, 1923d,e,f). In the latter theory, the electromagnetic field cannot exist in a place with vanishing current density. Schouten (1924) showed that this problem disappears if one assumes that the displacement is not symmetrical. For more details see the classical literature on the history of the unified field theory (Goenner, 2004; Vizgin, 1994). An excellent non-technical presentation is provided by Sauer, 2014.

Einstein's enthusiasm for this approach was again a flash in the pan. The paper was published in September, but by Christmas of 1925 Einstein confessed his skepticism to Besso, revealing that he had returned to a set of field equations he had presented in 1919 (Einstein, 1919), with the electromagnetic stress-energy tensor serving as the source (Einstein to Besso, Dec. 25, 1925; Speziali, 1972, 215–217; see also Einstein, 1927a[3]; cf. Vizgin, 1994, 225f. Goenner, 2004, 61f.). During those same months, Reichenbach, despite the support of Max Planck, was struggling to obtain his *Umhabilitation*[4] from Stuttgart to Berlin in order to be appointed to a chair of natural philosophy that had been created there (Hecht and Hoffmann, 1982). On March 16, 1926, Reichenbach sent a letter to Einstein in which, after discussing his academic misadventures, he remarked on the new 'metric-affine' theory (Einstein, 1925b):

> I have read your last work on the extended Rel. Th[5] more closely, but I still can't get rid of a sense of artificiality which characterizes all these attempts since Weyl. The idea, in itself very deep, to ground the affine connection independently of the metric on the Γ^i_{kl} alone, serves only as a calculation crutch here in order to obtain differential equations for the g_{ik} and the φ_{ik} and the modifications of the Maxwell equations which allow the electron as a solution. If it worked, it would of course be a great success; have you achieved something along these lines with Grommer? However, the whole thing does not have the beautiful convincing power [Ueberzeugungskraft] of the connection between gravitation and the metric based on the equivalence principle of the previous theory (Reichenbach to Einstein, Mar. 16, 1926; AEA, 20-83).

Reichenbach expressed skepticism early on towards Weyl's theory (Reichenbach, 1920, 73). Even if he partly retracted some of his concerns (Reichenbach, 1922, 367–368), he still felt that the theory did not have the same 'convincing power' (*Überzeugungskraft*) of general relativity (Reichenbach, 1922, 367), in which the identification of the g_{ik} with the gravitational potentials was solidly anchored in the principle of equivalence.[6]

[3]Einstein, 1927a was finished in January 1926. Einstein insists there that he had returned to the trace-free field equations as "a consequence of numerous failures" to pursue the "the approach proposed by Weyl and Eddington or something analogous" (Einstein, 1927a, 100). Einstein also derived the trace-free field equations using an approach suggested by Yuri Rainich (1925). In Rainich's view, in the case where the whole non-gravitational energy is electromagnetic, Einstein's field equations are already unified with Maxwell equations in empty space, because of certain algebraic property of the Riemann tensor (cf. Einstein to Rainich, Dec. 8, 1925; AEA, 20-003 and Einstein to Rainich, Mar. 8, 1926; AEA, 79-686). This 'already unified field theory' will play an important role in the history of 'geometrization' program of physics when it was rediscovered by Charles Misner and Archibald Wheeler in the late fifties (Misner and Wheeler, 1957); see also below on fn. 43.

[4]The process of obtaining the *venia legendi* at another university.

[5]Einstein, 1925b.

[6]Reichenbach already expressed reservations about Weyl's approach in his 1920 monograph on relativity, in which he accused Weyl of attempting to deduce physics from geometry (Reichenbach, 1920, 73). In early 1921, Weyl explained to Reichenbach in private correspondence that his theory was less ambitious: "I have claimed only that the concepts in geometry and field physics have come to coincide" (Weyl to Reichenbach, Feb. 2, 1921; HR, 015-68-04; tr. Rynasiewicz, 2005, 153, note 17). Briefly thereafter Weyl replied publicly to Reichenbach (Weyl, 1921, 475), who retracted his criticisms (Reichenbach, 1922, 367–368). However, Reichenbach still expressed concerns about the formalistic nature of the 'second version' of the theory. In order to circumvent Einstein's objection (1918a) that the theory was contradicted by the actual behavior of atomic clocks, Weyl (1920) abandoned the interpretation of the 'ideal' process of length-transfer in terms of the 'real' behavior of rods and clocks. According to Reichenbach, in this way the theory loses its "convincing character" and becomes "dangerously close to a mathematical formalism" (Reichenbach, 1922, 367); Reichenbach's wording is very similar to that of Wolfgang Pauli (1921, 763). Weyl's identification of the four-vector φ_ν with the electromagnetic four-potential appears to only be motivated by the goal of constructing a suitable 'action' from which he hoped to recover "the natural forms of the most general physical equations" via the 'action principle' ('Wirkungsprinzip') (Reichenbach, 1922, 367), that is, by requiring that the action-quantity assumes a stationary value. Moreover, Reichenbach criticizes Weyl's appeal to the epistemological superiority of his 'purely infinitesimal geometry', by referring to the work of

Perhaps it is not a coincidence that Reichenbach uses the very same turn of phrase in this letter. Einstein's theory introduces the affine connection independently of the metric. However, it does not attribute any physical meaning to the former; the separate variation of the metric and connection was nothing more than a 'calculation device' to find the desired field equations. Reichenbach, however, was ready to revise his negative judgment if Einstein's theory delivered the 'electron'. At the end of the paper (Einstein, 1925b), Einstein had in fact claimed that he was working with his assistant Jakob Grommer on the problem of establishing whether the theory allows for "the existence of singularity-free, centrally symmetric electric masses" (Einstein, 1925b, 419). For Einstein this was a fundamental criterion for the viability of a unified field theory (cf. e.g., Einstein, 1923a).

On March 20, 1926 Einstein replied that he warm-heartedly agreed with Reichenbach's 'Γ-Kritik': "I have absolutely lost hope of going any further using these formal ways"; "without some real new thought," he continued, "it simply does not work" (Einstein to Reichenbach, Mar. 20, 1926; AEA, 20-115). Einstein's reaction reflects his disillusion with the attempts to achieve the sought-for unification of gravitational and electromagnetic field via some generalization of Riemannian geometry. He would have probably been less ready to embrace Reichenbach's critique if he had knew what the latter exactly had in mind (see next section). However, Reichenbach was of course pleased by Einstein's endorsement. On March 31, 1926 he revealed that his remarks were not extemporary, but were the fruit of a more thorough consideration of the topic that he had jotted down at the time:

> I'm of course very glad that you agree with my Γ-critique. I have now made a few reflections on the topic, which seem to me to prove that the Weylean thought, although good mathematically, does not bring about anything new physically. The geometrical interpretation of electricity is only a visualization, which in itself still does not say anything, and can also be realized in the original relativity theory. I have attached the note and would be grateful if you could give it a look (Reichenbach to Einstein, Mar. 24, 26; AEA, 20-085).

Reichenbach attached to this letter a typewritten note. As we shall see, far more was at stake in it than a critique of Weyl's theory (which was generally considered a dead horse at the time). Reichenbach intended to call into question the very idea that, since general relativity has 'geometrized' the gravitational field, the obvious next move should be to try to 'geometrize' the electromagnetic field.

2. Intermezzo: Reichenbach's Note

In the following I will present the content Reichenbach's note, operating under the assumption that it corresponds to a part of a ten-page typescript bearing the title "Zur einheitlichen Feldtheorie von Gravitation und Elektrizität" ('On the unified field theory of gravitation and electricity'). The document is preserved at the Reichenbach Archives in Pittsburgh (HR, 025-05-10).[7] On the basis of the correspondence with Einstein that ensued (see next section) and of Reichenbach's handwritten corrections to the text, I conjecture that only pp. 1–7, that is, parts I and II of the typescript, were sent to Einstein; the bottom of p. 7 was probably added later and then included in a new part III,[8] which extended through pp. 8–10. Even though, for the sake of brevity and clarity, I will not painstakingly follow the order of Reichenbach's presentation, I

Eddington (1921) and Schouten (1922b): Weyl's geometry is only a special case of Eddington's, which in turn is only a special case of Schouten's general linear connection (Reichenbach, 1922, 367).

[7] HR, 025-05-11 is a copy of the last four pages of HR, 025-05-10.

[8] See also below in section 4.2 for more details on this hypothesis.

will adhere to his terminology and notation, which in turn seems to closely follow the German translation (Eddington, 1925a) of Eddington's textbook on relativity (Eddington, 1923).

2.1. The Physical Realization of the Operation of Displacement

Reichenbach attributes to Weyl the merit of having defined the operation of displacement (or parallel transport of vectors) given by the $\Gamma^{\tau}_{\mu\nu}$ independently of the metric $g_{\mu\nu}$. Reichenbach considered this result a "milestone" (HR, 025-05-10, 2) in the history of the mathematical problem of space. However, in Reichenbach's view, Weyl's "physical interpretation was not so fortunate" (HR, 025-05-10, 2). To explain the limits of Weyl's approach, Reichenbach refers to his distinction between *Einstellung* (adjustment)—the behavior of the physical systems that realize the comparison of length defined by $g_{\mu\nu}$—and *Beharrung* (preservation)— the behavior of physical systems that correspond to the operation of the comparison of direction defined by the $\Gamma^{\tau}_{\mu\nu}$.[9] Whereas the systems of the first type really exist in nature (i.e., rods and clocks), in Weyl's theory there are no such systems corresponding to the operation of displacement. Thus, according to Reichenbach, the separation of the operation of displacement from the metric is nothing but a "calculation aid" (HR, 025-05-10, 3) for finding the correct field equations. Reichenbach immediately extends the objection to Eddington's theory (Eddington, 1921) and to Einstein's version of the latter (Einstein, 1923e,f, 1925a), which radicalized Weyl's fundamental separation of metric and displacement.

As we have just seen, this is precisely the Γ-critique, which Einstein—who had already abandoned this operationalist point of view (Giovanelli, 2014)—had too hastily endorsed in the correspondence with Reichenbach. In fact Reichenbach intended to show that in the "original general relativity" (HR, 025-05-10, 3)—that is, in the Maxwell-Einstein theory describing gravity together with electromagnetism—it is already possible to find a realization for both fundamental operations, the metric and displacement, and thus to present the gravitational and electromagnetic fields in a common geometrical setting. In this way, the Maxwell-Einstein theory can be reformulated in a geometrical form "without any change to its physical content" (HR, 025-05-10, 1). This also means that the opposite is true: "In itself, this geometrical framework does not bring any physical innovation" (HR, 025-05-10, 1).

The $g_{\mu\nu}$ are already identified with the gravitational potentials, and measured with rods and clocks. Thus, in order to 'geometrize' the electromagnetic field, Reichenbach has to rely on the operation of displacement $\Gamma^{\tau}_{\mu\nu}$, and select suitable indicators for it. Physical theories provide no help in this respect, and thus Reichenbach tentatively searched among the physical phenomena that react to both the gravitational and electromagnetic fields, and at the same time happen to behave in the sense of the displacement $\Gamma^{\tau}_{\mu\nu}$. Gravitational and electromagnetic forces together determine the motion of particles. Thus a natural choice, though still arbitrary one, was to choose the motion of electrically charged mass points as indicators of the displacement-field.

In general relativity, the motion of such particles is subject to the already-known equations of the motion of charged particles of mass 1, which is therefore the starting point of Reichenbach's

[9] Weyl (1920) introduced the distinction between *Einstellung* and *Beharrung* to explain away the discrepancy between the non-Riemannian behavior of the 'ideal' time-like vectors implied by his theory and the Riemannian behavior of the 'real' clocks that are actually observed (Einstein's measuring rods objection, Einstein, 1918b). He suggested that atomic clocks might not *preserve* their Bohr radius if transported, but *adjust* it every time to some constant field quantity. See Ryckman, 2005, secc. 4.2.4; 6.4.2.2 for more details. Reichenbach (1922) complained that this sounds more like a restatement of the problem than a solution to it. Moreover, the adaptation has nothing to do with the parallel transport of vectors, so the latter remains physically empty (Reichenbach, 1922, 368, n. 1). However, Reichenbach made 'metaphorical' use of Weyl's 'Einstellung' in Reichenbach, 1924, 71, and again in Reichenbach, 1925, 47. See also below footnote 37.

investigation:

$$\frac{du^\tau}{ds} = \Gamma^\tau_{\mu\nu} u^\mu u^\nu - f^\tau_\nu i^\nu \text{ where } \Gamma^\tau_{\mu\nu} = -\begin{Bmatrix} \mu\nu \\ \tau \end{Bmatrix} \quad \text{(i)}$$

On the left-hand side of the equation is the acceleration of a particle, the rate of change of its velocity four-vector $u^\nu = dx^\nu/s$ (where x^ν is the four-position of a particle) with respect to a parameter s, identified with the particle's proper time. On the right-hand side the geodesic equation is supplemented by the term $-f^\tau_\nu i^\nu$ responsible for the effect of the electromagnetic field on charged particles. Notice that mass m does not appear in the force term, and thus the equation is valid only for unit masses. As is well known, this equation states that uncharged bodies free falling in a gravitational field follow geodesic paths, that is, lines of extremal length in a generally curved Riemannian space-time (particle four-acceleration vanishes identically and free-falling motion is indistinguishable from inertial motion). Charged bodies deviate from geodesic paths under the influence of an electromagnetic field, according to the Lorentz force law $K^\tau = -f^\tau_\nu i^\nu$. The $g^{\mu\nu} f^\tau_\nu = f_{\mu\nu}$ are the electromagnetic field strengths and $i^\nu = \rho u^\nu$ is the four-current, where ρ is the charge density and u^ν is four-velocity.[10]

Reichenbach aimed to rewrite these equations of motion so that charged particles under the influence of an electromagnetic field follow their 'natural path' defined by the displacement. To this end he stipulates that the physical behavior of mass points provides the 'physical realization' (or 'coordinative definition') of the geometrical operation of parallel displacement of their velocity four-vector u^τ. In general relativity, when an uncharged particle moves freely, its velocity-vector is carried along by parallel displacement along a geodesic line in Riemannian space. One can imagine a more comprehensive geometrical framework in which the displacement of the velocity four-vector of a charged body along its own direction also defines a 'privileged' path. For this reason one has to find a suitable geometrical setting.

The square of the length l of the velocity four-vector is per definition equal to c^2, and can be calculated from its components u^ν according to the formula:

$$l^2 = g_{\mu\nu} u^\mu u^\nu = 1 \text{ (by a suitable choice of units)} \quad \text{(ii)}$$

Because the length of the velocity four-vector is fixed (up to a constant), this imposes constraints on the geometrical setting one can use (HR, 025-05-10, 9)[11]:

- The length of this velocity l vector must remain unchanged under parallel transport, that is, $d(l^2) = 0$. This imposes a restriction on the 'displacement space' $\Gamma^\tau_{\mu\nu}$. It can be shown that such a condition is satisfied if a tensor of third order $K_{\mu\nu,\sigma} = 0$, where $K_{\mu\nu,\sigma}$[12] is defined as follows (cf. Eddington, 1921, 109):

$$2K_{\mu\nu,\sigma} = \frac{g_{\nu\sigma}}{x_\mu} - \Gamma_{\mu\nu,\sigma} - \Gamma_{\mu\sigma,\nu}$$

  One thus obtains a 'metrical space', that is, in Reichenbach's parlance, a space where the comparison of lengths at distance is path-independent. Weyl space differs from such a metrical space because $K_{\mu\nu,\sigma} = g_{\mu\nu}\kappa_\sigma$ (Weyl famously identified κ_σ with the electromagnetic

[10] The mixed-variant form f^τ_ν of the electromagnetic tensor $f_{\mu\nu}$ appears because of the contra-variant four-velocity u^ν in the definition of the four-current i^ν (cf. Eddington, 1925a, 273).

[11] See the cross-out paragraph on p. 5, which was later moved to pp. 8–9 in the longer version of the Note.

[12] The so-called non-metricity tensor.

four-potential).[13] In a metrical space, it is possible to define the shortest lines between two points; but in general, they are not identical with the straightest lines, defined by auto-parallel displacing of vectors.

- To assure that the straightest lines coincide with the shortest lines, one has to impose a further condition, that the connection $\Gamma^{\tau}_{\mu\nu}$ is symmetric in the lower indices μ and ν, that is:

$$\Gamma^{\tau}_{\mu\nu} = -\begin{Bmatrix} \mu\nu \\ \tau \end{Bmatrix} \tag{iii}$$

  where the three-index symbol on the right-hand side is the negative of the Christoffel symbols of the second kind. This specialization leads to the so-called 'Riemannian space', in which there are therefore *geodesics*; i.e., lines that are straightest and shortest at the same time.[14]

If one does not require the connection to be symmetric (by simply permuting the lower indices, one obtains a different connection; Schouten, 1922a,b),[15] then one can work in a metrical space that is not identical to the Riemannian space. In this space the straightest lines are not identical to the shortest ones.[16] Reichenbach intends to exploit this additional 'degree of freedom' to define an operation of displacement that expresses the effect of both the gravitational and electromagnetic fields. Charged mass points of unit mass move (or their velocity four-vector is parallel-transported) along the straightest lines, and uncharged particles move on the straightest lines that are at the same time the shortest ones (or rather, the line of extremal length). Let's see how Reichenbach proceeded in more detail.

2.2. An Outline of Reichenbach's Theory

Reichenbach's presentation (see HR, 025-05-10, 3–4) can be summarized in three stages:

1. Mimicking Eddington's 1921 theory (cf. section 1), Reichenbach introduced the fundamental tensor $G_{\mu\nu}$ which combines the electrical and gravitational fields:

$$G_{\mu\nu} = g_{\mu\nu} + f_{\mu\nu} \tag{iv}$$

   According to the usual procedure, this tensor can be decomposed into a symmetric part $g_{\mu\nu}$ and an anti-symmetric part $f_{\mu\nu}$ which, as one might expect, can be identified with gravitational/metrical field and the electromagnetic field:

$$g_{\mu\nu} = 1/2(G_{\mu\nu} + G_{\nu\mu}) \qquad f_{\mu\nu} = 1/2(G_{\mu\nu} - G_{\nu\mu}) \tag{v}$$

   The metric can be defined $ds^2 = G_{\mu\nu}dx_\mu dx_\nu = g_{\mu\nu}dx_\mu dx_\nu$, which is measured by using rods and clocks in absence of the electromagnetic field ($f_{\mu\nu} = 0$), that is, the more nearly the $G_{\mu\nu}$ approximate to $g_{\mu\nu}$. Thus, rods and clocks are not indicators of the $f_{\mu\nu}$ and a suitable indicator will be introduced later.

[13] cf. footnote 2.

[14] Reichenbach restricts the use of the term 'geodesic' to Riemannian geodesics.

[15] The asymmetry tensor $S_{\mu\nu,\tau} = \frac{1}{2}\Gamma^{\tau}_{\mu\nu} - \Gamma^{\tau}_{\nu\mu} \neq 0$. The term 'torsion' had been introduced already by Élie Cartan (1922) but it was still not in usage.

[16] Cf. Misner, Thorne, and Wheeler, 1973, 248–251 for an intuitive explanation.

- The $g_{\mu\nu}$ are governed by Einstein's field equations $r_{\mu\nu} - \frac{1}{2} g_{\mu\nu} r = -\kappa T_{\mu\nu}$ where r is written in lowercase to indicate that it depends on the first and second derivatives of the $g_{\mu\nu}$ alone (not on the whole $G_{\mu\nu}$).
- The $f_{\mu\nu}$ are governed by the Maxwell equations, which in four-tensor notation can be written: $\frac{\partial f_{\mu\nu}}{\partial x_\rho} + \frac{\partial f_{\nu\rho}}{\partial x_\mu} + \frac{\partial f_{\rho\mu}}{\partial x_\nu} = 0$ and $\frac{\partial f^{\sigma\rho}}{\partial x_\rho} = i^\sigma$.

2. A displacement $\Gamma^\tau_{\mu\nu}$ is introduced and decomposed into two parts (cf. Schouten, 1924, 851):

$$\Gamma^\tau_{\mu\nu} = \gamma^\tau_{\mu\nu} + \varphi^\tau_{\mu\nu} \tag{vi}$$

$$\gamma^\tau_{\mu\nu} = -\left\{ \begin{matrix} \mu\nu \\ \tau \end{matrix} \right\} \qquad \varphi^\tau_{\mu\nu} = -g_{\mu\sigma} f^\tau_\nu \frac{\partial f^{\sigma\rho}}{\partial x_\rho} \tag{vii}$$

   - The first $\gamma^\tau_{\mu\nu}$ are defined as the negative of the Christoffel symbols of the second kind, which are functions of the $g_{\mu\nu}$ and their first-order partial derivatives.
   - The definition of the $\varphi^\tau_{\mu\nu}$ (which Reichenbach does not explain further) seems to be obtained from the right side of four-dimensional Lorentz force law $K^\tau = -f^\tau_\nu i^\nu$, in which Maxwell equations with sources ($\frac{\partial f^{\sigma\rho}}{\partial x_\rho} = i^\sigma$) substitute the four-current. The term $g_{\mu\sigma}$ is added to lower the indices $g_{\mu\sigma} i^\sigma = i_\mu$. The direct product of two tensors (multiplying components from the two tensors together, pair by pair) increases the rank of the tensor by the sum of the ranks of each tensor,[17] keeping the character of the indices. Thus Reichenbach obtains $\varphi^\tau_{\mu\nu} = -f^\tau_\nu i_\mu$, that is, a skew-symmetric three-rank tensor with which to lower the indices.

   Without pointing it out explicitly, Reichenbach exploits the fact that the difference of two displacements transforms like a mixed tensor of third rank (Eddington, 1921, 109; eq. 4.6). In particular a non-symmetric displacement is always the sum of a symmetric displacement and a skew symmetric tensor. Thus, Reichenbach seems to have obtained the hoped-for result in a formally correct way: $\Gamma^\tau_{\mu\nu}$ is the sum of the usual Christoffel symbols, which in turn depend on the gravitational field $g_{\mu\nu}$, and a tensorial part, which depends on the electromagnetic field $f_{\mu\nu}$.

3. The reason for the definitions (vii) immediately becomes apparent (HR, 025-05-10, 6–7). Using (vii), Reichenbach can rewrite (i) so that the force term is, so to speak, absorbed into a suitably defined $\Gamma^\tau_{\mu\nu}$:

$$\frac{du^\tau}{ds} = \Gamma^\tau_{\mu\nu} u^\mu u^\nu \tag{viii}$$

   According to (vi), this equation is equivalent to the following:

$$\frac{du^\tau}{ds} = \gamma^\tau_{\mu\nu} u^\mu u^\nu + \varphi^\tau_{\mu\nu} u^\mu u^\nu \tag{ix}$$

   Because of (vii), the three-index symbol $\gamma^\tau_{\mu\nu}$ is defined as the Christoffel symbols of the second kind; thus the first summand of (ix) is simply the right-hand side of the general relativistic

[17] A vector is of course a tensor of first rank.

geodesic equation. To see the trick behind Reichenbach's less than obvious definition of the tensor $\varphi^{\tau}_{\mu\nu}$, a little more effort is needed. Plugging this definition into eq. (ix), the second summand becomes $-g_{\mu\sigma}f^{\tau}_{\nu}\frac{\partial f^{\sigma\rho}}{\partial x_{\rho}}u^{\mu}u^{\nu}$. Keeping in mind Maxwell's equations with sources (where $i^{\sigma} = \rho u^{\sigma}$), one obtains $-g_{\mu\sigma}f^{\tau}_{\nu}\rho u^{\sigma}u^{\mu}u^{\nu}$. According to (ii), $g_{\mu\sigma}u^{\mu}u^{\sigma} = 1$, thus the expression reduces to $-f^{\tau}_{\nu}i^{\nu}$. The final result is the following:

$$\frac{du^{\tau}}{ds} = -\left\{\begin{matrix}\mu\nu\\ \tau\end{matrix}\right\}u^{\mu}u^{\nu} - f^{\tau}_{\nu}i^{\nu} \qquad \text{(x)}$$

This is of course nothing but eq. (i), from which we started. By defining the displacement space $\Gamma^{\tau}_{\mu\nu}$ in a suitable way (via (vi) and (vii)), Reichenbach was able to dress this well-known equation's physical content in the more appealing geometrical garb of eq. (viii).

Just like eq. (i) in general relativity, eq. (viii) in Reichenbach's theory describes the motion of test particles under the influence of the combined gravitational and electromagnetic fields. However, now the difference in the behavior of charged and uncharged particles can be expressed in terms of geometrical differences within Reichenbach's non-Riemannian space-time. The velocity-vectors of charged particles of unit mass are parallel transported along the straightest lines defined by $\Gamma^{\tau}_{\mu\nu}$. When the charge of these particles is zero (i.e., when the tensorial component of $\varphi^{\tau}_{\mu\nu}$ vanishes) the straightest lines coincide with the shortest one.

In this way Reichenbach believed himself to have achieved the sought-for geometrization of the Einstein-Maxwell theory. Gravitation $g_{\mu\nu}$ and the electromagnetic field $f_{\mu\nu}$ are components of the geometrical field $G_{\mu\nu}$. Physically, the metric $g_{\mu\nu}$ is defined via the behavior of rods and clocks in which the gravitational field manifests itself. The displacement space $\Gamma^{\tau}_{\mu\nu}$, which has been defined as a function of $g_{\mu\nu}$ and $f_{\mu\nu}$, finds its physical counterparts in the velocity vectors of particles of unit mass and arbitrary charge, which are indicators of the electromagnetic field.

3. Einstein's Comments on Reichenbach's Note

Einstein must have immediately read or at least glanced at Reichenbach's attempt at providing a unified field theory, and he replied a few days later on March 31, 26. His initial reaction to the theory was not very encouraging:

> You've run over to the theoretical physicists and moreover at a bad spot. Of course I immediately found some flies in the ointment. First of all, your approach $\varphi^{\tau}_{\mu\nu} = -g_{\mu\sigma}f^{\tau}_{\nu}\frac{\partial f^{\sigma\rho}}{\partial x_{\rho}}$ is really arbitrary. Second, no metric should belong to your $\Gamma^{\tau}_{\mu\nu}$. It is unnatural to ascribe a metric to the summand γ of Γ. Your equations of motion do not have any physical meaning, since they describe the behavior of matter only for a value of the relationships between electrical and ponderable density. Finally, your theory does not connect electricity and gravitation, since there are no mathematically unified field equations that provide the field law for gravitation and electromagnetism simultaneously; it does not even provide a connection between electricity and gravitation in the sense that one could infer from the theory which electromagnetic quantities produce the gravitational field. I would not publish it; otherwise, what happened to me will happen to you: you'll have to disown your children (Einstein to Reichenbach, Mar. 31, 1926; AEA, 20-116).

Einstein deconstructs Reichenbach's theory piece by piece. (1) The definition (vii) of the $\varphi^{\tau}_{\mu\nu}$ does not seem to have any physical motivation. (2) A Riemannian metric determines an affine connection; however, in general this is not true the other way around (cf. e.g., Einstein, 1923c, 9). If Reichenbach started from a general displacement space $\Gamma^{\tau}_{\mu\nu}$ and defined it via parallel transport

independently from the metric $g_{\mu\nu}$, then he should not have reintroduced the $g_{\mu\nu}$ surreptitiously by defining the $\gamma^{\tau}_{\mu\nu}$ as the negative of the Christoffel symbols of the second kind (which are expressed in terms of the $g_{\mu\nu}$) (see eq. iii).[18] (3) Reichenbach's equations of motion can be valid only for a certain charge-density-to-mass-density ratio ρ/μ (or, in the case of particles, a certain charge-to-mass ratio e/m). In a given displacement, there is only one straightest line passing through a point in a given direction, but different test particles with different charge-to-mass ratios accelerate differently in the same electric field. Thus they cannot all travel on the same straightest line (see below in section 4.2). After all, this is the precise difference between gravitational and non-gravitational forces. Finally, (4) in the note, the $g_{\mu\nu}$ and $f_{\mu\nu}$ are governed respectively by the well-known Einstein and Maxwell equations; thus the theory not only fails to yield a single set of field equations governing both the gravitational and electromagnetic fields, but it does not even bother to supplement the gravitational field equations with electromagnetic terms so that they contain the gravitational effect of the electromagnetic field. That is, the theory does not even yield a geometrization of the Einstein-Maxwell theory.

Reichenbach replied by return post on April 4, 1926. There are two aspects to his response, which we will deal with separately for the sake of clarity. First, Reichenbach replied to Einstein's technical objections:

> 1) The approach for the $\varphi^{\tau}_{\mu\nu}$ is not only arbitrary (*willkürlich*), but even artificial (*künstlich*); but why is one not allowed [to do something like this]? Here, from a purely logical point of view, one can define what one wants; one can define the $[\varphi^{\tau}_{\mu\nu}]$ in such a way that [the definition] agrees with already known law of motion of charged particles 2) you say that no metric should pertain to my $\Gamma^{\tau}_{\mu\nu}$; however, it is exactly the opposite. Eddington assumes the field $\Gamma^{\tau}_{\mu\nu}$ as primary and deduces from it the field $G_{\mu\nu}$, which he splits into a symmetrical and antisymmetrical part. One can also assume a field $G_{\mu\nu}$ as primary and derive from it a field $\Gamma^{\tau}_{\mu\nu}$; this is logically equivalent 3) My law of motion is not valid only for a certain ratio of charge and mass, but for arbitrary charge and mass = 1. However from the point of view of the geometrical visualization this disadvantage is no worse than the fact that not every measuring rod defines the ds, but only the one of length 1 (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086).

Concerning 1), the awkward definition of one of the summands of the $\varphi^{\tau}_{\mu\nu}$ in eq. (vii), Reichenbach did not hide that his theory was an operation of 'reverse engineering'; and for this reason anything goes, even a cheap trick like the one he used in the note. Objection 2), for Reichenbach, was the consequence of Einstein's hasty reading. Reichenbach did not start from the displacement and then define the tensor $G_{\mu\nu}$ in terms of the latter, as Eddington did, but the other way around. Thus the metric was not obtained as a by-product, but was introduced from the beginning. To make his point, Reichenbach explains to Einstein the geometrical structure he resorted to, a metrical space, in which the symmetry of the lower indices of the $\Gamma^{\tau}_{\mu\nu}$ is dropped. In Riemannian space the operation of displacement delivers the same comparison of length as the metric. From this the "Riemannian values $\left\{ {\mu\nu \atop \tau} \right\}$ of the $\Gamma^{\tau}_{\mu\nu}$ " follow, if one assumes that the latter are symmetric in the lower indexes μ and ν. If one drops this assumption, then "one has at one's disposal a somewhat more general $\Gamma^{\tau}_{\mu\nu}$ " (Reichenbach to Einstein, Apr. 4, 1926; AEA,

[18] Einstein's insistence on this point probably should be understood against the background of his own recent attempts at a unified field theory. In his reformulation of Eddington's theory, Einstein, in contrast to Eddington, constructed a Lagrangian density $\mathcal{H}$ depending only on a connection and its first derivatives, and considered the variation $\delta\{\int \mathcal{H} = 0\}$ only with respect to the connection (Einstein, 1923e, 34). As we have seen, in his last theory he assumed that the variation of the affine connection and the metric are independent of one another (Einstein, 1925b).

20-086). One can then define the operation of displacement so that charged mass-points move on auto-parallel lines, which in general do not coincide with lines of extremal length: "in this way one obtains a full geometrical visualization of the law of motion" (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086).[19] Thus Reichenbach also makes it clear that he was not concerned with finding the field equations, as Einstein seems to have implied; he was only trying to reinterpret the equations of motion geometrically. Replying to objection 3) Reichenbach insists that these equations are valid for the motion of a body with unit mass *and arbitrary charge*. If one rewrites the tensorial part of the displacement as $\varphi^{\tau}_{\mu\nu} = -\rho f^{\tau}_{\nu} u_{\mu}$, one sees that the charge density ρ (but not the mass density μ) appears as a parameter. Setting $\mu = 1$, Reichenbach argues, is no worse than setting $ds = 1$.[20] So Reichenbach seems to have interpreted Einstein's objection as a misunderstanding. However, the validity of Reichenbach's equations of motion for arbitrary charge might have been precisely the severe flaw that Einstein envisaged in his approach, as Reichenbach himself possibly realized later (see below section 4.1 and section 4.2).[21]

The second aspect of Reichenbach's defense is even more important for properly understanding his philosophical goals. Einstein misunderstood the spirit of the typescript. Reichenbach makes clear that the physicists should in no way think that he had some "secret physical intention" (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086). Thus, Reichenbach recounted to Einstein why he decided to write the note. He was working on a philosophical presentation of the problem of space (see below in section 4.2), and of course he felt compelled to add a chapter about 'Weyl space', or more generally about attempts to 'geometrize' the electromagnetic field by using some generalization of Riemannian geometry: "Thereby I wondered what the geometrical presentation of electricity actually means" (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086).

Reichenbach concluded that such alleged geometrizations were actually only 'graphical representations' (*graphische Darstellungen*)—an expression he clearly borrowed from Eddington (1925a, 294ff.).[22] They were comparable to the account of the "Lorentz transformations as rotations in Minkowski space" (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086), which is only a formal analogy. To prove his point, Reichenbach decided to construct an *Abbildung* or mapping of the Einstein-Maxwell theory onto a non-Riemannian space, "*without any change of its physical content*" (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086; my emphasis).

Reichenbach was even more ambitious. He aimed to present a geometrical transcription (*Umschreibung*) that was in some respects better than the one provided by Weyl and his successors, including Einstein. Reichenbach's geometrical interpretation, he insisted, had "the advantage over other geometrical representations in that *the operation of displacement possesses a physical realization [Realisierung]*" (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086; my emphasis), namely, the velocity-vector of charged mass particles of unit mass. In Eddington's parlance[23] it is a 'natural geometry'.

This point is essential to Reichenbach's argument. It was precisely because his toy-geometrization

[19] As Dennis Lehmkuhl pointed out to me, it is curious that neither Reichenbach nor Einstein ever mention Theodor Kaluza's theory, in which (under certain conditions) such a geometric visualization seems to have been already achieved: both charged and uncharged particles move on geodesics of R^5 (Kaluza, 1921, 970). Einstein (possibly influenced by Klein, 1926) showed renewed interest in the theory in the following months (Einstein, 1927b,c). Dennis is working on a paper on this topic.

[20] See below footnote 35.

[21] I thank Dennis Lehmkuhl for a discussion on this point.

[22] For Reichenbach's use of this term see below footnote section 4.2. For Eddington, 'graphical representations' are geometrical visualizations of physical quantities, e.g., pressure-volume diagrams of an ideal gas, which, however, do not make any hypothesis as to the ultimate nature of the quantities represented. In Eddington's view, Weyl's non-Riemannian geometry is not the real geometry of space-time as Weyl claimed, but merely a 'graphical representation'. The 'natural geometry' is the geometry of rods and clocks, which is exactly Riemannian (Eddington, 1925a, 296).

[23] See previous footnote.

was not envied by its more titled competitors that Reichenbach believed himself to be in an excellent position to "attack the view that with a geometrical presentation of electricity, one would already gain something" (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086). For such a geometrical interpretation of electromagnetism to become a physical theory as successful as general relativity, more was required than a mere geometrization. It called for something *new*:

> If one succeeds in establishing unified field equations that admit the electron as a solution, this would be *something new*. To this end one should do something more than establish a simple formal pooling [Zusammenfassung] of the Maxwell eq. and the gravitational equations; these eq. *should be changed* in their content. This is the problem on which you are working and of course also what Weyl and Eddington meant. *But the geometrical representation of electricity in itself does not lead to this goal*. It can at most be an aid [Hilfsmittel] to guessing the right equations; maybe what looks most simple from the standpoint of Weyl geometry, also happens to be correct. But this would be only a coincidence. [...] Inasmuch, however, as the present theories do not provide the electron as a solution, they also provide nothing more than a simple transcription [Umschreibung] of the old Th. of Rel (Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086; my emphasis).

Thus, Reichenbach argued that the geometrical interpretation of a physical field can only be successful if it leads to a 'change' in the equations and does not simply rewrite in geometrical terms the equations that are already known. One could object that Weyl, Eddington and Einstein's theories also changed the equations and did not simply rewrite them. However, Reichenbach seems to consider the derivability of solutions that correspond to the electron as a litmus test for a real change in the field equations. Maxwell's field equations are valid in free space and cannot explain why the separate, equally charged parts do not fly apart without introducing a non-electromagnetic cohesion force (the so-called Poincaré stress). On the other hand, Einstein's field equations, in their original form, do not entail any effect of gravitation on charge and cannot provide the cohesion force. It is only by changing the currently available field equations that it would become possible to establish a connection between $g_{\mu\nu}$ and $f_{\mu\nu}$, thereby assuring the equilibrium of the electron.

To fully understand Reichenbach's stance on this issue, one must keep in mind that in a paper published in April (Reichenbach, 1926b) he expressed strong skepticism about the possibility of solving the problem of the 'grainy' structure of matter and and most all of the "proper quantum-riddle" (Reichenbach, 1926b, 424) in a field-theoretical/geometrical context. In Reichenbach's view, the "casuality [Zufälligkeit]" (Reichenbach, 1926b, 424) of the 'quantum jumps' (the transition between orbital energy levels in Bohr's atom) suggests that the problem should be tackled from a different angle, by considering whether the very notion of causality in physics should be replaced by that of probability (Reichenbach, 1926b, 424). After all, one should appreciate Reichenbach's clairvoyance, considering that Max Born's paper (Born, 1926) on the statistical interpretation of the wave function appeared only in June.[24]

Reichenbach offered to send Einstein the corresponding epistemological sections of the text on which he was working (possibly §50 of the Appendix). In a letter from April 8, 1926 Einstein did not comment on this offer, but his reaction to Reichenbach took a different tone. Even if Einstein did not reply to Reichenbach's more technical remarks, Reichenbach's philosophical point clearly resonated with him:

> You are completely right. It is incorrect to believe that 'geometrization' means something essential. It is instead a mnemonic device [Eselsbrücke] to find numerical laws. If one combines

[24]For Reichenbach's take on the emerging quantum mechanics at this time, see the posthumously published manuscript, Reichenbach, 1926a.

> geometrical representations [Vorstellungen] with a theory, it is an inessential, private issue. What is essential in Weyl is that he subjected the formulas, beyond the invariance with respect to [coordinate] transformation, to a new condition ('gauge invariance').[25] However, this advantage is neutralized again, since one has to go to equations of the 4. order,[26]which means a significant increase of arbitrariness (Reichenbach to Einstein, Apr. 8, 1926; AEA, 20-117).

Recently the importance of this letter has been emphasized in Einstein scholarship (Lehmkuhl, 2014). It is the first instance where Einstein explicitly claims that general relativity did not geometrize gravitation, thus suggesting a very different interpretation of the achievement of the theory than what we are used to. The geometrization was only a means to the end of finding the field equations, which are 'numerical laws'. It is worth noticing that Einstein goes further in claiming that even Weyl's theory should not be seen as an attempt to 'geometrize' the electromagnetic field. The core of Weyl's theory consists in the formal requirement of 'gauge invariance', which, however, led to equations where the choice of the Lagrangian becomes non-unique. For our purposes it is interesting that Einstein not only endorsed Reichenbach's claim that a 'geometrization' is not an essential achievement of general relativity, but also questioned the meaning of the notion of 'geometrization', and for that matter the very notion of 'geometry' (Lehmkuhl, 2014). This latter step was not taken by Reichenbach, who preferred to speak of general relativity as a 'geometrical interpretation of the gravitational field', albeit not a 'geometrization' of the latter.

4. From the Unpublished Note to the Appendix of *Philosophie der Raum-Zeit-Lehre*

4.1. The Stuttgart Talk

Reassured by Einstein's endorsement, in the ensuing weeks Reichenbach must have continued to work on the note. Corrections made by hand suggest that he probably added some remarks at the bottom of p. 7 which later were included in an entirely new part III that extended through pp. 8–10. Reichenbach apparently believed himself to have refuted most of Einstein's objections, since no corrections were made to the first two parts of the note. However, he may have realized that the claim in his theory that unit mass particles of *arbitrary* charge travel on the privileged paths of $\Gamma^{\tau}_{\mu\nu}$ was less straightforward than he initially thought. Reichenbach now acknowledged that this point "requires a further clarification" (HR, 025-05-10, 7).[27]

As we have seen, Reichenbach had defined the $\Gamma^{\tau}_{\mu\nu}$ as the sum of the Christoffel symbols $\gamma^{\tau}_{\mu\nu}$ and the tensorial correction $\varphi^{\tau}_{\mu\nu}$, which depends on the divergence of the electromagnetic field strengths $\frac{\partial f_{\rho\sigma}}{\partial x_{\rho}}$ (see def. vii). Reichenbach explains that in a region of space-time free of charge, where $\frac{\partial f_{\rho\sigma}}{\partial x_{\rho}}$ vanishes, the tensorial component of the connection vanishes as well $\varphi^{\tau}_{\mu\nu} = 0$. The displacement becomes identical to the Riemannian displacement $\Gamma^{\tau}_{\mu\nu} = \gamma^{\tau}_{\mu\nu}$; the parallel transport of the velocity-vector of uncharged mass points describes a geodesic, a line that is the shortest and straightest at the same time. If the divergence $\frac{\partial f_{\rho\sigma}}{\partial x_{\rho}}$ for the entire electrical field $\neq 0$ (that is $= i^{\nu}$), then the tensorial contribution $\varphi^{\tau}_{\mu\nu}$ to the connection appears; and with it

[25]That is, invariance by the substitution of g_{ik} with λg_{ik} where λ is an arbitrary smooth function of position (cf. Weyl, 1918b, 468). Weyl introduced the expression 'gauge invariance' (*Eichinvarianz*) in Weyl, 1919a, 114.

[26]Cf. Weyl, 1918b, 477. Einstein regarded this as one of the major shortcomings of Weyl's theory; see Einstein to Besso, Aug. 20, 1918; CPAE, Vol. 8b, Doc. 604, Einstein to Hilbert, Jun. 9, 1919; CPAE, Vol. 9, Doc. 58.

[27]This reconstruction is merely conjectural. This 'further clarification' is strangely never mentioned in the correspondence with Einstein. Thus I surmise that Reichenbach added it when he realized that his claim that charge particles of *arbitrary* charge all travel on privileged paths was problematic as Einstein had pointed out.

Reichenbach's non-Riemannian $\Gamma^{\tau}_{\mu\nu}$; the velocity-vector of charged unit mass points is parallel transported along the straightest lines, whereas uncharged mass points move along the shortest lines (HR, 025-05-10, 7).

Reichenbach must have found it somewhat unsatisfying that the $\Gamma^{\tau}_{\mu\nu}$-field depends not only on the electromagnetic field, but also on the charge of the test particles. At the bottom of p. 7 he squeezed in a footnote reassuring his readers that this "is not something like a blemish but an essential trait" (HR, 025-05-10, 7; footnote) of every theory attempting to 'geometrize' the effect of the electromagnetic field on its probes.[28]: "[a] charged mass point produces [...] its own transfer geometry [Verpflanzungsgeometrie] depending on the strength of its charge" (HR, 025-05-10, 7–8).

It was probably at a later stage that Reichenbach decided to transform these additional remarks into a new part III of the note. A loose leaf with the handwritten indication *Einlage zu S. 7* (insertion to p. 7) contains a few introductory lines that were supposed to be inserted before the passages just mentioned. The reassuring footnote at the bottom of p. 7 was crossed-out and its content moved into the main text. In the subsequent lines, Reichenbach added some further remarks to persuade those who still may have been perplexed. There is ultimately nothing wrong in assuming that, in addition to the 'space-field' (*Raumfeld*) $f_{\mu\nu}$ (the geometrized electromagnetic field as the second components of $G_{\mu\nu}$), the presence of a charged test particle produces 'an extra-field' $\Gamma^{\tau}_{\mu\nu}$ (*Zusatzfeld*) which, via the tensorial term $\varphi^{\tau}_{\mu\nu} = -f^{\tau}_{\mu} i_{\nu}$, also depends on ρ.[29] However, beneath his self-assured facade, Reichenbach might have sensed that the status of this extra displacement field was rather unclear. Thus, he was keen to let us know that, for test particles, the contribution of ρ to the tensorial part of the displacement is after all negligible with respect to the contribution of f^{τ}_{ν} (HR, 025-05-10, 8).

Despite having to engage in considerable hand waving, Reichenbach did not seem to lose confidence in his theory. After making some remarks about the geometrical meaning of non-symmetric displacements[30]—which took the place of a shorter crossed-out paragraph on p. 5—he concluded the new part III by proudly proclaiming that his theory had achieved a prototypical geometrization of the Einstein-Maxwell theory. Other geometrizations, he claimed, can differ "in their physical content," but not in their "logical structure" (HR, 025-05-10, 10). To avoid the misunderstandings that had emerged in his correspondence with Einstein, Reichenbach makes clear that even by deriving the field equations of the combined gravitational-electromagnetic field from an action principle, this would only be "progress in the mathematical formulation," and not something new physically.

To achieve the latter goal a further step is needed: "The unification of gravitation and electricity," Reichenbach writes, "can only have a formal character inasmuch as the concept of matter in the theory of gravitation is conceived only phenomenologically" (HR, 025-05-10, 10). In general relativity matter is 'black-boxed' in the matter tensor and treated as a continuum; matter, however, is known to be built of electrically charged particles. According to Reichenbach, unification of the gravitational and electromagnetic fields can have a real physical meaning only if it delivers "an electrical theory of matter" (HR, 025-05-10, 10), which would account not only for the existence of elementary particles with a certain mass and charge (electrons and hydrogen nuclei),

[28] As we shall see, this is indeed an essential trait of all attempts to impose a geodesic equation on electromagnetism in a four-dimensional setting (see below footnote 44), but it can hardly be said that it is not a blemish.

[29] Recall that $i^{\nu} = \rho u^{\tau}$.

[30] A symmetric displacement is characterized by the existence of infinitesimal parallelograms HR, 025-05-10, 9-10: If four neighboring infinitesimal vectors are parallel in pairs and equally long in the sense of the displacement, they will form a quadrilateral (cf. note # added to p. 9). This assumption is in general not true for a non-symmetric displacement.

but also for their quantum behavior (the electrons' privileged orbits around the nucleus and the discontinuous transitions from one state to another).[31]

On May 26, 1926 Reichenbach might have possibly have presented this improved version of the note in Stuttgart at the Gauvereinstagung of the Deutsche Physikalische Gesellschaft (the regional meeting of the German Physical Society). The abstract of this presentation was published under the title "Die Weylsche Erweiterung des Riemannschen Raumes und die geometrische Deutung der Elektrizität" (Reichenbach, 1926c). It is worth quoting at length, since it constitutes a good summary of what Reichenbach's theory looked like after his correspondence with Einstein:

> The meaning of Weyl's extension of the type of space is formulated such that Weyl recognized the independence of the operation of displacement and of the metric. The application of the extended type of space to physics is however characterized by a certain arbitrariness because it remains open to finding certain objects that behave like the operation of displacement. It is shown that these objects are the velocity vectors of electrically charged mass points. With the aid of this coordination, it is possible to interpret gravitational and electrical phenomena as expressions of the geometry of a Weylean space, so that electricity finds a geometrical interpretation *in the same sense as gravitation*. The remarkable thing here, however, is that this presentation does not change the content of Einstein's theory of gravitation at all; the geometrical interpretation is only a different parlance, which does not entail anything new physically. Of course this geometrical interpretation of electricity cannot solve the problem of the electron, because it cannot achieve anything more than Einstein's theory. The goal of this investigation was only to show the limit of a geometrical interpretation as such. A detailed publication will appear elsewhere (Reichenbach, 1926c, 25; my emphasis).

However, this abstract registers an aspect not mentioned in either the correspondence with Einstein or in the note. Reichenbach revealed that what he wanted to achieve was a geometrical interpretation of a physical field 'in *the same sense as gravitation*' in Einstein's theory, i.e., one that was *just as good* as that attained by general relativity. The geometrical operation of displacement has a physical interpretation in Reichenbach's toy-theory, just like the *ds* does in general relativity Thus, Reichenbach claims to have provided not just a successful 'geometrical interpretation' of the electromagnetic field, but an interpretation that was of the same 'quality' as the one general relativity provided for the gravitational field. However, this was Reichenbach's point: the theory was not a successful physical theory like general relativity. Thus, he concluded, providing a geometrical interpretation of a physical field is not in itself a physical achievement.

4.2. The Appendix and its §49

At the end of the abstract, Reichenbach mentioned that a more detailed version of his presentation was in preparation. He was clearly referring to a larger project he was involved with at that time. In December 1926 Reichenbach wrote to Schlick that he was working on a two-volume book bearing the title 'Philosophie der exakten Naturerkenntnis'. "The first volume that deals with space and time," he wrote, "is finished" (Reichenbach to Schlick, Dec. 6, 1926; SN). Reichenbach wanted to publish it in the forthcoming Springer series, 'Schriften zur wissenschaftlichen Weltauffassung', directed by Schlick and Frank. The next July Reichenbach wrote to Schlick that he had a publication agreement with De Gruyter (Reichenbach to Schlick, Jul. 2, 1927; SN). The *Vorwort* of *Philosophie der Raum-Zeit-Lehre* is dated October 1927.

The Appendix of the book is long 45 pages. It was constructed around the note that Reichenbach sent to Einstein in March 1926 (cf. section 1). In particular, §49 is a redrafted seven-page long version of the first two parts of the note. As pointed out on p. 358, n. 1, the content of §49

[31] Cf. Einstein, 1923b for a description of this field-theoretical program.

was presented as a talk in Stuttgart, together with an abstract summarizing the whole Appendix. Reichenbach added §§46–48 (27 pages) to describe in detail the geometrical setting of the theory that he had previously only sketched. §50 (10 pages) draws the epistemological consequences. I do not want to give a detailed presentation of the Appendix here, which would require a separate paper (see Coffa, 1979, for more details).[32] I will concentrate mainly on the differences between the note and §49.

At first glance, nothing much seems to have been changed also in this final version. Reichenbach remained confident that Einstein's objections had been answered. The definitions of the two summands of the connection $\varphi^{\tau}_{\mu\nu}$ and $\gamma^{\tau}_{\mu\nu}$ are not modified, despite Einstein's criticisms; no attempt is made either to derive the field equations governing both fields from a variational principle, or, more simply, to indicate how the electromagnetic field contributes to the gravitational field. There is, however, a part of §49 that has been heavily modified with respect to the note. Interestingly, it again concerns the interpretation of the equations of motion.

Reichenbach possibly came to realize that Einstein had seized on the weak spot of his geometrization here: since the charge-to-mass ratio e/m varies from particle to particle, the trajectories of charged particles in an electromagnetic field cannot be construed as moving along the privileged paths of any *single* connection (cf. Friedman, 1983, 197). Thus, Reichenbach was forced to paper over the cracks.

(a) As he did in the note, Reichenbach concedes that if his equations of motion are supposed to be valid for unit mass particles of *arbitrary charge*, then unit mass particles of the same charge "will engender [their] own displacement geometry" (Reichenbach, 1928, 362; tr. HR, 041-2101, 506) (depending on the strength of their charge), and will run along their 'own' straightest lines defined by it (cf. Reichenbach, 1928, 363; tr. HR, 041-2101, 508). However, in §49, the tone is quite different. Reichenbach now recognizes that this solution is "questionable" (Reichenbach, 1928, 363; tr. HR, 041-2101, 508), since the existence of a field should not depend on the properties of its probes. Reichenbach is forced to admit explicitly that, in contrast to the field $G_{\mu\nu}$ (and its two components $g_{\mu\nu}$ and $f_{\mu\nu}$), the displacement space $\Gamma^{\tau}_{\mu\nu}$ does not exist in itself as a property of space-time, but also depends on the properties of the test particles, that is, on their charge ρ. The tensorial part of the connection can be rewritten as $\varphi^{\tau}_{\mu\nu} = -\rho f^{\tau}_{\nu} u_{\mu}$ to make this more transparent.[33] Since the $\Gamma^{\tau}_{\mu\nu}$-field does not have independent existence, Reichenbach admits that one can doubt that a geometrization has been achieved at all. Clutching at straws, Reichenbach tries to suggest that the ambiguous status of the $\Gamma^{\tau}_{\mu\nu}$-field should be seen as an argument in favor of "Weyl's conception or perseverance," which would acquire a "deeper significance" (Reichenbach, 1928, 362; tr. HR, 041-2101, 406): the paths of charged particles are a 'line of preservation'

[32] I will limit myself to a terminological clarification. As we have seen, Reichenbach attributes to Weyl the merit of having discovered that the 'displacement space' (the affine connection) can be defined (via the operation of the parallel transport of vectors) independently from the metric. He refers us to "Gravitation und Elektrizität" (Weyl, 1918a) and to § 34 of the third edition of *Raum, Zeit, Materie* (Weyl, 1919b). Reichenbach, however, adopted the more general view introduced by Eddington (1921), in which the displacement does not even allow for the comparison of length at the same place; Eddington restricted his approach to a symmetric displacement in order to avoid what he called an 'infinitely crinkled' world (Eddington, 1921, 107). Reichenbach, however, abandoned this restriction following Schouten (1922a,b). Thus, what Reichenbach calls 'Weyl's extension of Riemann's Concept of Space' should not be confused with what we usually call 'Weyl geometry'. The latter is only a particular case of a symmetric displacement where $K_{\mu\nu,\sigma} = \kappa_{\sigma} g_{\mu\nu}$. Reichenbach's odd nomenclature, which is reflected in the title of the Appendix, is probably one of the reasons the latter was read unilaterally in relationship to Weyl's unified field theory.

[33] Note that the charge density ρ (for an incoherent charge fluid or e for particles), but not the mass density μ (or the mass m) appears as a parameter; hence Reichenbach's insistence that his equations of motion are valid only for unit masses.

Beharrungslinien) and only uncharged particles 'adapt' to the $G_{\mu\nu}$-field.

(b) Reichenbach must have sensed that not everyone would buy into such an argument. Thus, in order "to avoid this peculiarity of our formulation" (Reichenbach, 1928, 367; tr. HR, 041-2101, 506), he also suggested an alternative version of the theory which was not present in the note. The tensorial part of the displacement is defined $\varphi^{\tau}_{\mu\nu} = -f^{\tau}_{\nu}u_{\mu}$ and now depends only on the electromagnetic field (since ρ is set $= 1$). Reichenbach now sees an additional difficulty; the displacement depends on the particle four-velocity u_{μ}. Reichenbach, however, does not realize that this is also true for the previous definition (the only difference is that $\rho \neq 1$). Consequently, he treated the problem as a "mathematical complication" (Reichenbach, 1928, 363; tr. HR, 041-2101, 507) of the new definition, which, he claims, fortunately disappears when the latter is plugged into the equation of motion (viii).[34] However, the most worrying issue was this: now Reichenbach had to swallow Einstein's objection. The equations of motion now apply only to unit mass particles of a certain *unit charge* (Reichenbach, 1928, 363f.; tr. HR, 041-2101, 508ff.). Under the influence of the electromagnetic field, a class of charged particles with an arbitrarily chosen charge-to-mass ratio move on the straightest lines and uncharged particles always move on the shortest lines. Since there are two 'norms' that one would naturally choose, the e/m of the positive and that of the negative electron, there would only be two 'natural' geometries. Clutching at straws once again, Reichenbach attempts to convince his readers that, for this reason, this version of the theory provides an analogon of the equivalence principle. After all, in general relativity the ratio of the gravitational-charge-to-mass ratio is also arbitrarily set $= 1$ (cf. Reichenbach, 1928, 366; tr. HR, 041-2101, 513).

The limits of Reichenbach's approach coincides with the limits of this latter analogy. The electric-charge-to-mass ratio is not the same for all particles, as the gravitational-charge-to-mass ratio is. Despite Reichenbach's insistence, the theory precisely misses a good analogon of the equivalence principle. Let's drop Reichenbach's curious restriction to unit masses to see this point more clearly.[35] In order to "construct a space which is independent of the indicator" (Reichenbach, 1928, 363; tr. HR, 041-2101, 508), that is, of mass and charge of the test particles, one is forced to admit that only *one* class of particles with a certain charge-to-mass ratio (say, electrons) follows the privileged paths defined by the *same* displacement. To allow for *all* charged particles with whatever mass and charge to move on privileged paths, then a parameter k depending on the charge-to-mass ratio should appear in the tensorial part of the displacement $\varphi^{\tau}_{\mu\nu} = -kf^{\tau}_{\nu}u_{\mu}$ (as in Droz-Vincent, 1967). In this way, however, the displacement "does not exist independently"

[34] Given that $du^{\tau} = \frac{dx_{\tau}}{ds}$, one can write (viii) as $du^{\tau} = \Gamma^{\tau}_{\mu\nu}u_{\mu}dx_{\nu}$. Then, since $u_{\mu}u^{\mu} = 1$, one gets: $du^{\tau} = \left\{ {\mu\nu \atop \tau} \right\} dx_{\nu} - f^{\tau}_{\nu}dx_{\nu}$. The additional term (beyond the one entailing the Christoffel symbols) depends now only on position and not on the velocity-vector (Cf. Reichenbach, 1928, 362-363; tr. HR, 041-2101, 507-508 for more details). Actually, one can proceed in the very same way with the previous definition of the tensor $\varphi^{\tau}_{\mu\nu}$, with the only difference being that the charge density $\rho \neq 1$. However, in both cases if one switches back to eq. (viii) with s as a scalar parameter of motion, the velocity vector reappears. Thus, I am not sure why Reichenbach believed himself to have gotten rid of the velocity-dependent connection with this trick. On velocity-dependent connections, see Vargas, 1991.

[35] The reason for this restriction is not completely clear to me. Reichenbach possibly wanted to avoid making the displacement also depend on the mass of particles (particles with the same charge, but different mass, would produce their own connection). He insists on several occasions (cf. Reichenbach to Einstein, Apr. 4, 1926; AEA, 20-086, cited above in section 3) that setting $m = 1$ amounts to nothing but the choice of a norm, which does not differ with the choice of unit rods and clocks to norm the $ds = 1$. The analogy seems to me only partially successful. The choice of the norm is of course arbitrary in both cases; however, in Reichenbach's theory only particles with $m = 1$ travel on geodesics, but of course not only intervals $ds = 1$ have length.

(Reichenbach, 1928, 362; tr. HR, 041-2101, 506) from the internal degrees of freedom of the test particles (cf. Cohn, 1972; Quale, 1972). Test particles with different charge-to-mass ratios would follow privileged paths defined by *different* displacements, one for each value of the parameter k. The situation becomes even more desperate if one keeps in mind that the displacement also depends on the four-velocity of particles u_μ.

Reichenbach's theory clearly shows that, in a four-dimensional setting, using a geodesic equation to describe a non-universal force is possible, but the price one has to pay for it is extremely high. Quite surprisingly,[36] Reichenbach decided that the price was worth the message he intended to convey, which evidently was close to his heart. In the last chapter of his *Philosophie der Raum-Zeit-Lehre* (the one proceeding the Appendix), Reichenbach concedes that, since the gravitational field is measured by the same measuring instruments as those used for geometry (light rays, rods and clocks) general relativity has established a peculiar and previously unknown connection between geometry and gravitation. However, Reichenbach says loud and clear how this should be interpreted: "*it is not the theory of gravitation that becomes geometry, but it is geometry which becomes an expression of the gravitational field*" (Reichenbach, 1928, 294; Reichenbach's emphasis; tr. 1958, 256).[37] The long Appendix that follows was meant to add a powerful argument to support this claim. The geometrical interpretation of gravitation is merely "the visual cloak" (Reichenbach, 1928, 353f.; tr. HR, 041-2101, 493) in which the gravitational field can be dressed. However, it would be a mistake to confuse "the cloak with the body it covers" (Reichenbach, 1928, 354; tr. HR, 041-2101, 493). The electromagnetic field can be dressed in an equally nice geometrical cloak without reaching any significant result from a physical point of view.

The fact that the Appendix was not included in the widely-read English translation of *Philosophie der Raum-Zeit-Lehre* is probably the main reason so little attention has been paid to this issue. The Appendix is much more 'technical' than the rest of the book, which has a somewhat semi-popular tone. However, Reichenbach reassures readers unable to manage the formal language that the section "presents nothing new, philosophically speaking, but merely constitutes an

[36]The surprise derives from the fact that this attitude seems to be in open conflict with Reichenbach's well-known definition of gravitation as a 'universal force'. As is pointed out in the first part of Reichenbach's book, one 'geometrizes' the gravitational field but not, say, the temperature field, because in the latter case we would have different geometries for materials with different coefficients of heat expansion (see §6). Notice that when Reichenbach claims that, in the second version of his theory, there are only two 'natural geometries', he explicitly remarks that the situation is nevertheless better than the case of a temperature field (Reichenbach, 1928, 364; tr. 1958, 513). However, he should have then concluded that, in the first version of his theory, there is a different 'natural geometry' for every charge-to-mass ratio. As we have seen, Reichenbach attempts to avoid this conclusion by claiming that the displacement $\Gamma^\tau_{\mu\nu}$, since it depends on the properties of test particles, does not really exist as an independent geometrical field, and the real field is represented by the $G_{\mu\nu}$. As Reichenbach sensed, however, one can hardly speak of a 'geometrization'. Reichenbach, as we have seen, could avoid this conclusion only through some philosophical hand-waving.

[37]To fully understand this point one must keep in mind that according to Reichenbach, in addition to the problem of the measurement of physical geometry, which was introduced by Riemann, Helmholtz and Poincaré, Einstein introduced the problem of a *scientific explanation* of physical geometry. Since the gravitational field affects all measuring instruments in the same way, one may regard them as 'free from deforming forces'; nevertheless one may still consider the gravitational field as the *cause* of the fact that all measuring instruments happen to agree on the same geometry. Reichenbach borrowed from Weyl the concept of 'adjustment' (*Einstellung*) (Reichenbach, 1928, 294; tr. 1958, 257), to emphasize that 'causation' here does not mean deviation from an alleged correct behavior (e.g., the Euclidean one), but the surprising convergence toward a non-trivial one (which in the general case is non-Euclidean). What is important in this context is the fact that in Reichenbach's view, since the gravitational field is the *cause of* the behavior of our geometrical measuring instruments, it exists independently from its peculiar geometrical manifestation and cannot be reduced to it (cf. Reichenbach, 1928, 357; tr. HR, 041-2101, 491). However one might judge Reichenbach's notion of causation in this context, it must be emphasized that it does not represent a minor aspect of his philosophy. Reichenbach uses the same Weylian term 'adjustment' to also refer to special relativity (Reichenbach, 1924, 70–71). The whole issue would merit a separate paper.

application of the epistemological principles developed earlier in this book" (Reichenbach, 1928, 331; tr. HR, 041-2101, 461). If one takes Reichenbach's claim at face value, then the reading of *Philosophie der Raum-Zeit-Lehre* with the inclusion of the Appendix is a valuable tool for understanding one of the main messages of the book, if not the main one. Reichenbach's monograph was not a plea for the 'relativization of geometry' induced by the gravitational field, but rather a *j'accuse* against the rhetoric of the 'geometrizaton' of gravitation, which, in the 1920s, had spread through popular and technical readings of the theory of general relativity (Reichenbach, 1928, 294; tr. 1958, 256). If there is something we can learn from general relativity, Reichenbach argues, it is that abstract geometry has been lowered to physics, and certainly not that physics has been absorbed into geometry.

Weyl, Eddington and Einstein had provided 'graphical representations'[38] of the electromagnetic field in which they had intentionally forgone providing "'tangible' realizations" (Reichenbach, 1928, 371; tr. HR, 041-2101, 519) of the operation of displacement; the tacit assumption is that, once the field has been cast in geometrical form, what looks "*simple* and *natural*" (Reichenbach, 1928, 370; tr. HR, 041-2101, 518) would lead to the correct field equations. This, however, would be nothing more than a fortunate coincidence. In Reichenbach's view the Appendix shows that, with some effort, one can do better; one can construct a 'proper geometrical interpretation' of the combined gravitational/electromagnetic field, providing a concrete interpretation of the operation of displacement which allows a comparison of the theory with experience. Going even further, by suggesting an analogon of the equivalence principle, Reichenbach believed himself to have achieved a 'geometrization' of the electromagnetic field that was "no 'worse'" (Reichenbach, 1928, 366; tr. HR, 041-2101, 512) than the one provided by general relativity. Nevertheless, in contrast to general relativity, Reichenbach's theory "tells us nothing about reality that we did not know before" (Reichenbach, 1928, 368; tr. HR, 041-2101, 516). Thus 'geometrizing' a physical field does not give us any privileged access to the physical world. Reichenbach does not hide his hopes that this result would contribute to freeing physicists from the "Sirens' song [Sirenenzauber] of a unified field theory" (Reichenbach, 1928, 373; tr. HR, 041-2101, 521). The "many ruins along this road," Reichenbach argued, "urgently suggest that solutions should be sought in an entirely different direction" (Reichenbach, 1928, 373; tr. HR, 041-2101, 521).

[38] As we have mentioned, Reichenbach borrowed the expression from Eddington (1925b, 312). Cf. Reichenbach, 1928, 365; tr. HR, 041-2101, 510. However, in § 15 of *Philosophie der Raum-Zeit-Lehre*, he attempts a more general and formal definition: 'graphical representations' are structural analogies between different physical systems (e.g., compressed gases, electrical phenomena, mechanical forces, rigid bodies and light rays etc.) which are realizations of the same conceptual system (e.g., the axioms of Euclidean geometry). In this context this means roughly the following. In Weyl's theory the vector field φ_i determines the change of length of vectors; the curl of φ_i is the length-curvature tensor f_{ik} (Weyl tensor). The f_{ik} satisfy the identity $\frac{\partial f_{kl}}{\partial x_i} + \frac{\partial f_{li}}{\partial x_k} + \frac{\partial f_{ik}}{\partial x_l} = 0$. This formula *looks a lot like* Maxwell-Minkowski equations in empty space. Thus it was very suggestive to interpret φ_i as the electromagnetic four-potential and its curl f_{ik} as the electromagnetic tensor. This conclusion is based only on a structural analogy, that is, it is only a 'graphical representation'. When Weyl attempted to give a concrete physical meaning to φ_i in terms of the behavior of clocks which measure the length of a time-like vector ds, the theory turned out to be empirically inadequate: atomic clocks do not change their periods as a function of their space-time path (cf. Einstein, 1918a). On the contrary, in Reichenbach's view, general relativity is not simply a graphical representation, but a natural geometry, or 'proper geometrical interpretation' in Reichenbach's parlance; it contains assertions about the system of rods and clocks themselves—measuring the line element ds (and thus the g_{ik})—and not about its structural equivalence with other physical systems (Reichenbach, 1928, 365f.; tr. HR, 041-2101, 511). At that time there was confidence that the prediction of the theory concerning atomic clocks would turn out to be correct.

5. Reichenbach and Einstein's Distant Parallelism Field Theory

It is of course highly significant that the critique of the geometrization program—which has been neglected by most recent readers of Reichenbach's monograph—is precisely the one that attracted Einstein's attention. On December 1, 1927 Reichenbach wrote to Einstein that he knew from Paul Hinneberg, the editor of the *Deutsche Literaturzeitung*, that Einstein intended to write a review of his forthcoming book. Reichenbach sent him the drafts of the book and also added that he would send the Appendix some days later, since it was still being typeset (Einstein to Reichenbach, Dec. 1, 1927; AEA, 20-090). Einstein's review appeared in the first 1928 issue of Hinneberg's weekly magazine (Einstein, 1928c). It is interesting to note that the only point where Einstein expressed agreement with Reichenbach's approach concerned the Appendix: "In the Appendix the foundation of the Weyl-Eddington theory is treated in a clear way and in particular the delicate question of the *coordination* of these theories to reality" (Einstein, 1928c, 20; my emphasis). He then went further: "In this chapter *just like in the preceding*—in my opinion quite rightly—it is argued that the claim that general relativity is an attempt to *reduce physics to geometry* is unfounded" (Einstein, 1928c, 20; my emphasis). Einstein mentions Reichenbach's treatment of the 'delicate question' of the 'coordination' (*Zurodnung*) of the theory to reality; however he only explicitly agrees with Reichenbach's stance on the problem of 'geometrization'.

Indeed, one can find this attitude mirrored in the review of Émile Meyerson's book on relativity (Meyerson, 1925), which Einstein published in the same year (Einstein, 1928a). Meyerson regarded relativity as a stage in a long process of the progressive geometrization of physics, which had started with Descartes. Einstein of course disagreed; he regarded the 'unification' of inertia and gravity as the major achievements of general relativity; a unified field theory should further unify gravitational and electromagnetic fields rather than 'geometrize' the latter (Lehmkuhl, 2014). According to Einstein, "the term 'geometrical' used in this context is entirely *devoid of meaning*" (Einstein, 1928a, 165; my emphasis): we do not regard the Hertz-Heaviside field equations as a 'geometrization' of the electromagnetic field because of the geometrical concept of vector that occurs in these equations. On the contrary, Einstein fully endorsed Meyerson's rationalist epistemology, that is, Meyerson's attitude about how the theory is "coordinated with [zugeordnet] the objects of experience" (Einstein, 1928a, 162). Einstein found in Meyerson's work an emphasis on 'the deductive-constructive character' of relativity theory, which fit his pursuit for a unified field theory.

Thus the apparent agreement between Reichenbach and Einstein on the geometrization issue actually hides a somewhat complicated dialectic. For Einstein, the very idea of a geometrical interpretation of a physical field was meaningless, and what he wanted to achieve was a unification of two different fields. On the contrary, Reichenbach regarded the geometrical interpretation of a physical field as a meaningful enterprise, which, however, offered no guarantee of physical unification. Moreover according to Reichenbach a good geometrical interpretation implies a 'Zurordnung' between the fundamental geometrical structures of the theory and the behavior of suitably chosen probes; on the contrary Einstein had come to realize that this operationalist approach was not only unnecessary, but a detriment to very project of a unified field theory. This dialectic emerges more clearly in Reichenbach's discussion of Einstein's new attempt to develop a unified field theory.

In spring 1928, during a period of rest after a circulatory collapse, Einstein, as he wrote to Besso, "laid a wonderful egg in the area of general relativity" (AEA, 40-69). On June 7, 1928 he presented a note to the Prussian Academy on a 'Riemannian Geometry, Maintaining the Concept of Distant Parallelism' (Einstein, 1928d), a flat space-time that is nonetheless non-Euclidean since the connection is non-symmetrical. On June 14, 1928 he submitted a second paper in which the field equations are derived from a variational principle (Einstein, 1928b). Reichenbach wrote to

Einstein with some comments on the theory on October 17, 1928:

> Dear Herr Einstein,
> I did some serious thinking on your work on the field theory and I found that the geometrical construction can be presented better in a different form. I send you the ms. enclosed. Concerning the physical application of your work, frankly speaking, it did not convince me much. *If geometrical interpretation must be, then I found my approach simply more beautiful, in which the straightest line at least means something.* Or do you have further expectations for your new work? (Reichenbach to Einstein, Oct. 17, 1928; AEA, 20-92; my emphasis).

There are two aspects of this passage that should be considered separately.

The first part refers to the mathematical-geometrical aspect of Einstein's papers. The manuscript to which Reichenbach refers seems to have been lost. However, from Einstein's reply on October 19, 1928 one can easily infer that Reichenbach must have sent him the classification of geometries which would appear in an article Reichenbach submitted in February 1929 (Reichenbach, 1929c, see below in this section). Einstein agreed that in principle it was possible to proceed as Reichenbach suggested, "starting with displacement law, and to specialize it on the one hand with the introduction of a metric on the other side with the introduction of integrability properties" (Einstein to Reichenbach, Oct. 19, 28; AEA, 20-094). Reichenbach in fact defines a metrical space by imposing the condition $d(l^2) = 0$ to the displacement space $\Gamma^{\tau}_{\mu\nu}$, which in general is non-symmetrical; he then obtains Einstein space by requiring that the Riemann tensor $R^{\tau}_{\mu\nu\sigma}(\Gamma)$ vanishes.[39] Einstein, in contrast, preferred the classification he had given in his paper: Weyl's geometry allows for the comparison over finite distances neither of lengths nor of directions; Riemannian geometry allows the comparison of lengths, but not directions; and Einstein's geometry directions but not lengths (Sauer, 2006).

This, however, was only a minor point. Reichenbach's further remark concerning the physical application of Einstein's geometrical setting is, from a philosophical standpoint, more interesting, even if Einstein did not comment on it. Reichenbach claims that, if one really wants to provide a geometrical interpretation of gravitation and electricity, then his own approach was better after all. Reichenbach uses his own toy-theory as a benchmark for a good 'geometrical interpretation' (but of course not for a good physical theory). Reichenbach's theory provides a physical meaning to the displacement operation and thus a physical definition of a straightest line. On the contrary, Einstein's theory did not attempt to provide a physical interpretation of the notion of displacement, nor even the field quantities; if the theory has nothing more to offer, Reichenbach claims, (i.e., if the theory does not solve the problem of the electron) it is merely a 'graphical representation' (cf. also Eddington, 1929 for a similar judgment).

In a note added by hand at the bottom of the typewritten letter, Einstein invited Reichenbach and his first wife Elisabeth for a cup of tea on November 5, 1928, mentioning that Erwin Schrödinger[40] would also be present (Reichenbach to Einstein, Oct. 17, 1928; AEA, 20-92). It was probably on that occasion that Einstein told Reichenbach about the physical consequences of the theory he was working on. In the meantime, on November 4, 1928, an article by Paul Miller appeared in *The New York Times* with the sensational title "Einstein on Verge of Great Discovery; Resents Intrusion". The paper triggered the curiosity of the press. In the late 1920s Reichenbach was a regular contributor to the *Vossische Zeitung*, at that time Germany's most prestigious newspaper; not surprisingly he was asked for a comment on Einstein's theory. With

[39] The Γ alludes to the fact that this condition can be defined without reference to the $g_{\mu\nu}$.

[40] Schrödinger succeeded Max Planck at the Friedrich Wilhelm University in Berlin in 1927. He held his inaugural lecture on July 4, 1929 (Schrödinger, 1929).

the advantage of having personally discussed the topic with Einstein, Reichenbach published a brief didactic paper on Einstein's theory on January 25, 1929 (Reichenbach, 1929b).

Reichenbach conceded that Einstein's theory provided a unification of gravitation and electricity which had more than just formal significance, since it made "new assertions concerning the relation between gravitation and electricity in relatively complicated fields" (Reichenbach, 1929b). However, he maintained his skepticism by claiming that the theory was "only a first draft, lacking the persuasive powers of the original relativity theory because of the *very formal method by which it is* established" (Reichenbach, 1929b, ; my emphasis). Reichenbach was clearly not the only one to write about Einstein's new theory in the press. On January 12, 1929—one day after Einstein submitted a third paper on distant parallelism (Einstein, 1929b) to the Academy—*The New York Times* published an article entitled 'Einstein Extends Relativity Theory'.

It was amid this atmosphere that, at the end of January, Einstein wrote an angry letter to the *Vossische Zeitung* lamenting Reichenbach's "tactless behavior" in violating the academic code (Einstein to the Vossische Zeitung, Jan. 25, 1925; AEA, 73-229). On January 26, 1929, the curator of the literary section, Monty Jakobs (cf. Badenhausen, 1974), defended the behavior of the newspaper and forwarded Einstein's letter to Reichenbach (Jakobs to Einstein, Jan. 26, 1925; AEA, 73-230). Reichenbach wrote to Einstein the next day with feelings ranging from surprise to anger; he complained that Einstein did not write directly to him after all he had done to defend relativity theory (Hentschel, 1982), and he denied any wrongdoing (Reichenbach to Einstein, Jan. 27, 1925; AEA, 20-096). On January 30, 1928 Einstein replied that he was somewhat pleased by Reichenbach's annoyance, which was the "fair equivalent" of the annoyance he had caused by feeding the press private information (Einstein to Reichenbach, Jan. 30, 1920; AEA, 20-099). However, Einstein quickly settled the dispute to Reichenbach's relief (Reichenbach to Einstein, Jan. 31, 1920; AEA, 20-101).

On January 30, 1929 Einstein's paper was finally published in the proceedings of the Academy with the vague title 'On the Unified Field Theory' (Einstein, 1929b). On February 2, 1929 another semi-popular paper by Reichenbach was published in the *Zeitschrift für Angewandte Chemie* (Reichenbach, 1929a) without any reaction from Einstein. Einstein's anger at Reichenbach (which might at first seem rather exaggerated) is understandable if one keeps in mind the attention that the theory was attracting among the public; Einstein might have been upset that a colleague and friend would also contribute to the craze. At the beginning of February the *New York Herald Tribune* (February 1) printed a translation of the entire paper. Several days later *The New York Times* (February 3) and the *London Times* (February 4) published Einstein's own popular account. The 'irrational exuberance' towards the theory is well attested to by a letter Eddington sent to Einstein a few days later, recounting that Selfridges—a British chain of high-end department stores—had pasted all six pages of Einstein's papers in its window (Eddington to Einstein, Feb. 11, 1929; AEA, 9-292).

In the meantime, on January 22, 1929, Reichenbach had already submitted a second and more technical paper, which only appeared in the *Zeitschrift für Physik* in September (Reichenbach, 1929c). The paper offers a readable presentation of Einstein's new theory; Reichenbach again presented his own take on the relationship between displacement and metrical space, and located Einstein space as an alternative to Riemannian space, rather than a generalization of it (Reichenbach, 1929c, 684–687). He then showed how in Einstein's theory the $\Gamma^{\tau}_{\mu\nu}$ and the $g_{\mu\nu}$ are considered as functions of a parameter h^{ν}_{α} (the ν projections on the α orthogonal unit vectors forming the so-called n-bein)[41]:

[41] In contrast to Reichenbach, Einstein assigned Greek letters to the *Koordinaten-Indizes* and Latin ones to the *Bein-Indizes* or tetrads indices: h^{a}_{ν}.

$$g_{\mu\nu} = h_{\mu\alpha}h_{\nu\alpha} \qquad \Gamma^{\tau}_{\mu\nu} = -h^{\tau}_{\alpha}\frac{\partial h_{\mu\alpha}}{\partial x_{\nu}}$$

The $n^2 = 16$ quantities h^{ν}_{α}—enough to include both the gravitational and electromagnetic field—play the role of the field potentials defined at every point (Reichenbach, 1929c, 687). The goal is to construct a suitable Lagrangian $\mathcal{H}$ from which the 16 field equations for the field variables h^{ν}_{α} could be obtained, as usual, from a variational principle $\delta \int \mathcal{H}$, with variation respect to the h^{ν}_{α} (cf. Sauer, 2006, for more details).

After this semi-popular presentation of Einstein's geometry and its physical application, Reichenbach added some remarks that are interesting from a philosophical point of view. He pointed out that there are two ways to unify two different physical theories. The first is a *formal unification*, comparable to the relationship between Lagrangian and Hamiltonian formalism in classical mechanics (the first can be Legendre transformed into the other without adding any new physical knowledge); the second is an *inductive unification*, exemplified by the relationship between Kepler and Newton's laws (something new is of course added by moving from Kepler's laws to Newton's theory of gravitation).

The first approach was the one used by Reichenbach himself in his own 'unified field theory':

> The author [Reichenbach] has shown that the first way can be realized in the sense of a combination of gravitation and electricity to one field, which determines the geometry of an extended Riemannian space; it is remarkable that thereby *the operation of displacement receives an immediate geometrical interpretation, via the law of motion of electrically charged mass-points*. The straightest line is identified with the path of electrically charged mass-points, whereas the shortest line remains that of uncharged mass points. In this way one achieves *a certain parallelism to Einstein's equivalence principle*. By the way [the theory introduces] a space which is cognate to the one used by Einstein, i.e., a metrical space with non-symmetrical $\Gamma^{\tau}_{\mu\nu}$. The aim was to show that the geometrical interpretation of electricity does not mean a physical value of knowledge per se (Reichenbach, 1929c, 688; my emphasis).

Notice that, according to Reichenbach, the advantage of his own approach consists in the fact that it provides a physical realization of the displacement operation, and also (Reichenbach insists) an analogon to the equivalence principle (at least for particles of certain charge-to-mass-ratio). The disadvantage is that it is only a *unification of the representations* of two physical fields in a common geometrical setting. The second approach is the one used by Einstein, and it presented the opposite characteristics:

> On the contrary Einstein's approach of course uses the second way, since it is a matter of increasing physical knowledge; it is the goal of Einstein's new theory to find such a concatenation of gravitation and electricity, that only in first approximation it is split in the different equations of the present theory, while is in higher approximation reveals a reciprocal influence of both fields, which could possibly lead to the understanding of unsolved questions, like the quantum puzzle. However, it seems that this goal can be achieved only *if one dispences with an immediate interpretation of the displacement, and even of the field quantities themselves*. From a geometrical point of view this approach looks very unsatisfying. Its justification lies only on the fact that the above mentioned concatenation implies more physical facts that those that were needed to establish it (Reichenbach, 1929c, 688; my emphasis).

Einstein's theory was claimed to be a *unification of the dynamics* of two physical fields, i.e., a unification of the fundamental interactions. However, Reichenbach argues that Einstein could achieve this result only at the cost of dispensing with a physical interpretation of the fundamental quantities.

Thus, according to Reichenbach, his own theory had the ambition of being a '*proper geometrical interpretation*' (or, one might say, to provide a 'natural geometry'), but it was physically sterile; Einstein's theory sought to be physically fruitful, but it was merely a '*graphical representation*' (see also Eddington, 1929). Clearly, for Reichenbach, only general relativity was able to combine both virtues: it was a proper geometrical interpretation (the ds, and thus the $g_{\mu\nu}$ are measured using rods and clocks) that leads to new physical results. Reichenbach did not seem to realize (or at least does not explicitly point out) that this epistemological standard had become hard to comply with in precisely the context of the field-theoretical explanation of the electron that he was calling for.

In the 1930s the physics community was divided. In July 1929 Cornelius Lanczos, who had started to work with Einstein in Berlin at the end of 1928, also published a little semi-popular paper on the *Fernparallelismus* theory (Lanczos, 1929). In a more extended but also non-technical account, Lanczos (1931) distinguished two different ways of conceiving of the coordination between theory and experience: (1) a *positivistic-operationalist entrenchment* of relativity theory, which requires a direct definition of the fundamental variables $g_{\mu\nu}$ or $f_{\mu\nu}$, in terms of the behavior of some physical systems used as probes. However, this approach fails in the domain of the elementary particles, since there are no probes smaller than the electrons. Thus the pursuit of a field-theoretical interpretation of matter seems to require (2) a *metaphysical-realistic perspective*, based on the conviction that the deep structure of nature is understandable only by means of speculative mathematical constructions. There are few doubts that, around 1930, Einstein was leaning towards the second way (Dongen, 2010; Norton, 2000). In his *Fernparallelismus* approach, e.g., no attempt is made to give a direct physical meaning to the fundamental field variables $h_{\nu a}$ (or ${}^{a}h_{\nu}$ in the notation suggested by Roland Weitzenböck; Einstein, 1929a,b) considered in isolation. The justification of the field equations relies on the fact that they are the most simple and natural laws that the h-field can satisfy (Einstein, 1930, 6; tr. 10).

Conclusion

Einstein soon abandoned the distant parallelism approach, later taking up a five-dimensional approach again, but this time using the tetrad formalism (Einstein and Mayer, 1931, 1932). While Einstein was visiting Caltech in 1933, the Nazis came to power. He never returned to Germany and instead landed at Princeton, where he remained until the end of this life. Reichenbach moved to Istanbul, attracted, like many other German academics, by Atatürk's secular Turkey. The enthusiasm was short-lived. In April 1936 Reichenbach, fearing Weyl's opposition, wrote to Einstein asking for his support in obtaining a position at Princeton (Reichenbach to Einstein, Apr. 12, 1936; AEA, 10-107). Einstein answered that he had heard from Rudolf Carnap that Princeton did not want to hire more Jews: "also up here not all that glitters is gold," he remarked bitterly (Einstein to Reichenbach, May. 2, 1936; AEA, 20-118). Reichenbach obtained a position at UCLA in 1938, from which he would exert an enormous influence on American philosophy of science.

The confrontation between Einstein and Reichenbach about the philosophy of space and time was resurrected only a decade later by Reichenbach's contribution (Reichenbach, 1949) to the Schilpp-Volume in Einstein's honor (Schilpp, 1949). It concerned the very same issue we discussed at the end of the preceding section. In an unpublished and overall positive commentary on Reichenbach's paper, Einstein disagreed in particular with Reichenbach's claim that 'the meaning of a statement is reducible to its verifiability': "it seems to me doubtful whether one can maintain

this conception of *meaning* for the individual *statement*" (AEA, 2-057).[42]

As is well known, Einstein raised this precise objection against Reichenbach in the so-called "Reply to Criticisms" of the Schilpp-Volume (Einstein, 1949b). At the end of a fictional dialogue between Reichenbach and Poincaré (Einstein, 1949b, 677–678), Einstein entrusted his epistemological views to the persona of an 'anonymous non-positivist' (or, as he put it elsewhere, a 'tamed metaphysician'; Einstein, 1950, 3): a theory has a 'meaning', a physical content, only as a whole, even if its parts, in isolation, do not find a direct physical interpretation (Einstein, 1949b, 678). Einstein's remark, as he confessed to Besso a year later, must be understood against the background of the "Don Quixotian situation" in which one finds oneself in the search for a unified, non-dualistic, field theory. The material structures (e.g., rods and clocks), which are used as probes and give physical content to the field quantities governed by the field equations, are supposed to be solutions of the field equations themselves. Thus, no real definition of such quantities seems to be possible: "To really understand my point of view you must read my answer in the [Schilpp]-volume [Sammelband]" (Einstein to Besso, Apr. 15, 1950; Speziali, 1972, 438–439).

Interestingly, in the Schilpp-Volume, Einstein points out another consequence of this epistemological stance. In a context in which the geometrical measuring instruments, rods and clocks, would be treated as physical systems just like any other (Giovanelli, 2014), the strict opposition between the 'interval', the geometrical variable they measure, and all other non-geometrical variables, also seems to lose its *raison d'être* (Einstein, 1949a, 61). Thus the very program of "reducing physics to geometry" (Einstein, 1949b, 61) becomes meaningless. Although Einstein made this remark only in passing, as we have seen, his reflections on this topic date back over twenty years and were occasioned precisely by Reichenbach's toy-geometrization. Concluding our reconstruction of this forgotten Reichenbach-Einstein debate, I think that there are two lessons we can draw from it.

From (a) a *historical standpoint*, it turns out that the Einstein-Reichenbach correspondence inaugurated a philosophical reflection about the role played by geometric considerations in physical theories. This issue, which is rarely addressed today, was not only relevant for Einstein, as Lehmkuhl (2014) has recently shown, but played an important role in Reichenbach's philosophy as well. In particular, Reichenbach's 1928 monograph should be read as an attempt to present general relativity as the crowning achievement of a process of 'the physicalization of geometry', against the prevailing opinion that it marked the beginning of the epoch of 'the geometrization of physics'. The decision not to include the Appendix in the 1958 translation of *Philosophie der Raum-Zeit-Lehre* (Reichenbach, 1928) is probably why this issue has never attracted the attention of Reichenbach's interpreters, despite the fact that it is precisely this aspect that Einstein himself emphasized in his review of the book. Reichenbach's philosophy of space and time looks quite different if this issue is taken into account, and, given his enormous influence, may have contributed in quite different ways to the debate on the foundation of space-time theories.[43] The

[42]The italicized words are in English in the original German text.

[43]The question of whether the fundamental role of geometrical concepts is only of "historical and traditional" or rather "logical nature" (Dantzig, 1956, 48) was in fact robustly discussed in the physics community in the years following the English translation of *Philosophie der Raum-Zeit-Lehre*. As is well known, beginning at the end of the 1950s, John Archibald Wheeler had tried to pursue what he called 'geometrodynamics'; starting from the successful geometrization of the gravitational field provided by general relativity, Wheeler investigated the possibility of treating "fields and particles" not as foreign entities immersed in geometry, but as "nothing but geometry" (Misner and Wheeler, 1957, 526). See above on fn. 3 for the so called 'already unified field theory'. However, the hope that "physics could be brought into a geometric formulation," as Steven Weinberg pointed out in his celebrated textbook a decade later, "has met with disappointment" (Weinberg, 1972, 147). Thus Weinberg could defend what even today is a quite heterodox position, that the "geometric interpretation of the theory of gravitation has dwindled to a mere analogy" (Weinberg, 1972, 147). What is relevant is the ability to make predictions about images on photographic plates, frequencies of spectral lines, and so on, and it "simply doesn't

infamous 'relativization of geometry' (cf. Giovanelli, 2013a,c), with which Reichenbach's reading of general relativity is usually identified, suddenly recedes into the background and what may have been his main concern begins to emerge—the 'geometrization of gravitation'.

Reichenbach's toy theory might, however, also be interesting from (b) a *systematical point of view*, though for reasons Reichenbach would not have appreciated. His theory shows that, in a four-dimensional setting, "the price for imposing a geodesic equation of motion to describe a non-universal interaction" (Aldrovandi and Pereira, 2013, 120) is *not* worth paying. The simple reason is that a good analogon of the (weak) equivalence principle that Reichenbach repeatedly brags about is missing. Ironically, Reichenbach's Appendix offers possibly the best argument of his 1928 book to show that gravitation is a universal force, though electromagnetism is not. The gravitational-to-inertial-mass ratio m_g/m_i is a constant, but the charge-to-mass ratio e/m is not; thus, *pace* Reichenbach, it is extremely cumbersome to impose a geodesic equation upon electromagnetism, and in this sense, to geometrize the latter. One can easily construe an affine connection in which *one* type of particle with a certain e/m travels on geodesics. However, if one wants to have *all* charged particles moving on geodesics under the influence of the electromagnetic field, there is no other way than to introduce a separate connection for each value of e/m. One can in principle proceed in this way (Droz-Vincent, 1967). Given a connection (say the Levi-Civita connection), one can obtain a new connection simply by adding a suitable three-rank tensor with two lower indices, and so on. If the latter depends on e/m, then one would have as many connections as one needs.[44] However—if I am allowed to borrow my punchline—the situation became "not unlike that in alchemy where a new 'essence' is invented to explain any phenomena not covered by the previous 'essence'" (Earman and Friedman, 1973, 357).[45]

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matter whether we ascribe these predictions to the physical effect of gravitational fields to a curvature of space and time" (Weinberg, 1972, 147). The opposition between the geometrization of physics vs. the physicalization of geometry was also discussed by Peter G. Bergmann (Bergmann, 1979), Einstein's former assistant in Princeton and one of the major relativists of his time.

[44]This approach was suggested, e.g., by Philippe Droz-Vincent (1967), and is astonishingly similar to that of Reichenbach (although Droz-Vincent must have been unaware of Reichenbach's Appendix). Just like Reichenbach, Droz-Vincent resorts to a non-symmetric affine connection which is the sum of the Christoffel symbols and a (1,2) tensor, which depends on the electromagnetic field and the four-velocity, and the charge-to-mass ratio of the particle: $P^{\rho}_{\alpha\beta} = kF^{\rho}_{\alpha}u_{\beta}$ (one can easily recognize Reichenbach's definition behind the slightly different notation). The covariant derivatives of the metric tensor are supposed to vanish, just like in Reichenbach's theory. The consequence (the same one Reichenbach was forced to acknowledge) is that the "affinity is not an 'external' property of space, independent of the particle" (Burman, 1970); in particular, it depends on the charge-to-mass ratio (and also on the four-velocity of every particle). On the problem of incorporating the particle properties into the space-time geometry, see Cohn, 1972; Quale, 1972; see Vargas, 1991 on the problem of velocity-dependent connections. An alternative might be to use a particular case of the Finsler metric, the so called Randers metric (Randers, 1941) $ds = \sqrt{g_{\mu\nu}dx^{\mu}dx^{\nu}} + e/m\,\varphi_{\mu}dx^{\mu}$ (where φ_{μ} is the electromagnetic potential). Geodesics $\delta\int ds$ in a Randers space are not geodesics in a Riemannian space, so that charged particles of different types can have different Riemannian paths (Stephenson and Kilmister, 1953). In this setting there is only one connection; however, the metric depends explicitly on e/m and thus one needs (again) different metrics for each type of particle. The only way to avoid this multiplication of geometrical structures is to move to higher dimensions. In Kaluza-Klein-type theories, the motion of any particle with an arbitrary value of e/m is associated with the same geodesic in the five-dimensional space, but with different four-dimensional projections of the latter (Leibowitz and Rosen, 1973). An attempt in six dimensions has been made by Bown, 1970. An opposite thought experiment might be to try to transform the geodesic equation of general relativity into a force equation as in the case of teleparallel gravity (Aldrovandi and Pereira, 2013). This might be useful in the case where the equivalence principle turns out to be violated. In this case geometrized gravity would not make much sense precisely for the reason we have suggested: "test particles with different relations m_g/m_i would require connections with different curvatures to keep all equations of motion given by geodesics" (Aldrovandi and Pereira, 2013, 120).

[45]Earman and Friedman refer to Droz-Vincent (1967); see previous footnote.

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Abbreviations

AEA *The Albert Einstein Archives at The Hebrew University of Jerusalem* (1879–1955).
CPAE Albert Einstein (1987–). *The Collected Papers of Albert Einstein*. Ed. by John Stachel et al. 14 vols. Princeton: Princeton University Press.
HR *Archives of Scientific Philosophy: The Hans Reichenbach Papers* (1891–1953).
SN *Vienna Circle Foundation — Wiener Kreis Stichting: Schlick Nachlass* (1882–1936).

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