A πάρεργα und παραλειπŏμενα of J-M. Lévy-Leblond *On the Conceptual Nature of the Physical Constants*

Giuseppe Iurato
Department of Physics, University of Palermo, IT

E-mail: giuseppe.iurato@unipa.it

Abstract. Starting, at first, from some little known but notable formal remarks made by the mathematician Gabriele Darbo about the qualitative dimensional analysis of the physical quantities, for then, above all, take into account some latest science education (*d'après* diSessa) and philosophy (*d'après* Gärdenfors) research results, it shall be possible, paraphrasing a celebrated 1851 work of Arthur Schopenhauer, work out a kind of *Parerga und Paralipòmena* of the basic J-M. Lévy-Leblond paper *On the Conceptual Nature of the Physical Constants,* published in *Rivista del Nuovo Cimento* in 1977. The epistemological reconnaissance thereof, might turn out to be of some usefulness in the physics education context as regard, for instance, the meaning of the physical constants, their values and possible further epistemological and pedagogical implications.

Keywords and phrases: dimensional analysis, conceptual change, coordination class, conceptual space, physical constant, epistemic relationships between mathematics and physics.

1. Introduction

But, first of all, what do πάρεργα and παραλειπòμενα mean? The former is the plural form of the Greek male noun πάρεργον which means ''accessory'' and has scholarly Late Latin transliteration in *párergon,* from *pará* (παρά) which means ''in addition, near'' and *érgon* which means ''work'', hence with composed meaning ''addition to a work'', while the latter is a Greek term which means ''things omitted or left'', whose scholarly Late Latin transliteration is *paraleipŏmĕna* as neuter plural of the passive present participle of the verb *paraleípein* (from the Greek παραλείπειν), in turn from *paraleípô* (from the Greek παραλείπô), composed by *pará,* which also means ''over, upper'' and *leípò* (λείπω, perhaps having Indo-European origins) which means ''leave'', hence with composed meaning of ''leave out'' or ''omit''.

The first term, that is to say *parergon,* is simply an accessory addendum, a digression respect a central theme or an appendix to a given work (mostly literary but also a scientific or erudite one), often in deepening of it[1] and almost always used in its plural form (*parerga*), while the second one[2] seems dating back to the (Orthodox) Revealed Sacredness because the Greek version of the *Bible of the Seventy* define *paralipómena* the two texts of the *Old Testament,* said to be the *Books of the Chronicles,* which are an integration and/or confirmation of what narrated in the *Books of Kings* and in the *Books of Samuel* related to the same period of Israelite monarchy. Hereinafter, this term was used to denote new works which continue or complete

[1] It should be also useful to take in account the Jacques Derrida (1978) notion of *parergon* which initially was introduced in textual criticism, then extended to philosophy of the art (above all in critical comparison with Kant's parergon acception – see Barbagallo (2012)) and, at last, laids out into the philosophical framework as endeavour to have a general postmetaphysical method of destructuration (*d'après* Heidegger) of philosophical systems; moreover, one of the last Derridean attempts was just turned to extend his method to the general culture and history. Here, we are basically interested to one of the main aspects of such a Derridian conception, namely the metatextual account of it, that is to roughly say, his consideration of parergon mainly from the metatextual *deconstruction* standpoint. From this last Derridean viewpoint (even if we here does not refer to its full and quite complex philosophical meaning, but only to one aspect of it), it is possible as well to conceive as one of the main aim of this paper (precisely, through the first part of the header title) be just that of deconstructing, in the Derridian sense, the 1977 Lévy-Leblond paper, putting it into historic-critical comparison with the recent literature on science education and philosophy, to the purpose of clarifying it and explicitly highlighting its main ideas. But, for our purposes, it is enough to follow the simpler Schopenhauer model instead the Derridian one. For what has been said in this footnote, see Abbagnano (1991, Capitolo 11).

[2] For the various notes quoted in this first section, we refer to Mestica (1885, Volume II, Parte I, § XIII, p. 20).

previous others, mainly to integration of these last, like the Post-Homeric poem of Quinto Smirnèo[3] as a supplement to the Homeric *Iliad*, a work of Germano Valente Guellio[4] as a supplement to the Virgil *Aeneid*, the 1831 *Paralipòmeni della Batracomiomachia* of Giacomo Leopardi[5], the 1851 *Parerga und Paralipòmena: kleine philosophische Schriften* (whose English translation is[6] *Accessories and Postscripts: A Collection of Philosophical Essays*) of Arthur Schopenhauer, and so on. In short, with such terms, it is intended works which are, or wish to be, a simple integration or addition, or else a continuation or a supplement, to a given work or to parts of it. Finally, we have deliberately used, in the header title of the present paper, the originary Greek forms of *parerga* and *paralipòmena* just because they express the very pregnant meaning of such terms.

Well, this paper simply would like to be intended, paraphrasing, in the philological sense, a famous 1851 work of Arthur Schopenhauer, as a simple *Parerga und Paraleipòmena* of the basic J-M. Lévy-Leblond (1977) work *On the Conceptual Nature of the Physical Constants*, published in 1977 in the *Rivista del Nuovo Cimento*, taking into account both some first simple but notable (albeit yet little known) remarks made by the Italian mathematician Gabriele Darbo (1921-2003) on the formal structure underlying the main aspects of Qualitative Dimensional Analysis of physical quantities and, above all, some of the latest fundamental physics education and epistemological research results achieved, respectively, (amongst others) by Andrea A. diSessa and co-workers (on *coordination class theory*, in turn framed into the wider one of scientific conceptual changes) and by Peter Gärdenfors together with Frank Zenker (on *conceptual space theory*, also this framed into the wider scientific conceptual change theory, but distinct from the coordination class one), through which it will be thus possible to establish a kind of 'parerga und paralipòmena' to this primary 1977 Lévy-Leblond paper.

2. On Dimensional Analysis, Physics Education and all that

From an educational-pedagogical viewpoint, the dimensional analysis has been quite underestimated and considered mostly as a sort of simple heuristic and diagnostic method which assesses only a simple dimensional coherence analysis (or dimensional control) amongst terms of certain physical equations or relations. Nevertheless, the tools and methods of dimensional analysis are much more powerful of what that would may seem: in this paper, inter alia, we wish just to show an instance thereof, showing what basic role dimensional analysis may play in epistemology and what possible educational implications it may have.

For a more deeper treatment of this subject, we refer to Kasprzak et al. (1990), Straneo (1975) and Cercignani (1976), whereas for a clear, very useful and effective application of qualitative methods (amongst to which prevail the dimensional analysis techniques together a symmetry arguments) in solving physical problems, see Pescetti (1985); finally, for some examples of deduction of certain physical laws from dimensional analysis, see, for instance, the classical basic work of Bridgman (1922) or Fazio (1976). In what follows, we outline first the main points of an elementary formal algebraic theory of dimensional analysis due to the Italian mathematician Gabriele Darbo, then some of the main new results achieved by science education (mainly by Andrea A. diSessa and co-workers) and philosophy and epistemology of science (following Peter Gärdenfors and co-workers): all that will constitute the basis to build up the above mentioned 'parerga und paralipòmena' to the 1977 Lévy-Leblond paper.

2.1 On Gabriele Darbo's theory of qualitative dimensional analysis of physical quantities

The main reference for what said in this subsection is Darbo (1969), a brief summary of which is also given in Agazzi (1969, Capitolo VI, § 21, pp. 180-182) with further valuable epistemological remarks.

Let G be the class of all possible absolute physical quantities[7]. As known, it is possible to consider the addition only of homogeneous quantities, whereas the multiplication is always possible and gives rise to

[3] *Quintus Smyrnæus*, Greek epic poet, born likely in Izmir or Smyrna (Asia Minor) between the end of the II Century and the beginnings of the III Century A.C. He was also called *Quinto Calabro* because an archetype of his work was discovered in the Ancient Calabria, near Otranto.

[4] As regards this author, he is called either Germano Valente Guercio, as in Pianigiani (1907, Volume II), or Germano Valente Guellio, as in Menochio (1689, p. 169) and Vanzon (1838, Tomo V, p. 153).

[5] See Mestica (1885, Volume II, Parte I).

[6] And it would be the most adapted to make meaning to this paper.

[7] If you want, it is also possible limiting ourselves to consider a class G of certain physical quantities, like the mechanical or electromagnetic ones, etc. In such a case, therefore, the class of all the physical quantities will be the

another quantity, so that $(G, \cdot)$ is a commutative multiplicative group; furthermore, in G it is also included the multiplicative group $(\mathbb{R}^+, \cdot)$ which include the adimensional quantities (sometimes also called *pure* numbers), so that the latter is a subgroup of the former. In $(G, \cdot)$, it is possible to define the ratio of two quantities x, y as follows: we set $y/x \doteq y \cdot x^{-1}$, so that two quantities x, y are said to be *homogeneous* when y/x is a real number, that is to say, when $y/x \in \mathbb{R}^+$; in such a case, we say that y/x is the *measure* of y respect to x chosen as *measurement unit*. If we afterwards set $x \sim y$ if and only if $y/x \in \mathbb{R}^+$, then $\sim$ is an equivalent relation in G (said to be the *homogeneity* relation); on the other hand, it is immediate to verify that $(\mathbb{R}^+, \cdot)$ is a normal subgroup of $(G, \cdot)$ and that $y/x \in \mathbb{R}^+$ is equivalent to $y \cdot x^{-1} \in \mathbb{R}^+$, so that it is also immediate to verify that $G/\sim = G/\mathbb{R}^+$: this quotient group is called the group of the *species* of physical quantities (lengths, times, etc). Therefore, the canonical quotient map $\pi: G \to G/\mathbb{R}^+$ associates to each quantity $x \in G$ its species $[x] \in G/\mathbb{R}^+$, so that two quantities x, y are homogeneous among them if and only if $[x] = [y]$; the neutral element of $G/\mathbb{R}^+$ is $\mathbb{R}^+$, that is to say, multiplying a quantity for a real positive number, we obtain a quantity of the same species.

If $A \in G/\mathbb{R}^+$ is a class of homogeneous quantities, for $x, y \in A$, we define $x < y$ if and only if $y/x > 1$, this being possible because $y/x \in \mathbb{R}^+$; it is immediate then to verify that $<$ is a total well-posed order relation in A. If an arbitrary unit of measurement $u \in A$ is fixed, the correspondence $\psi_u: A \to \mathbb{R}^+$ given by $x \stackrel{\psi_u}{\to} x/u$, which associates to each quantity $x \in A$ its corresponding measure respect to u, that is to say x/u, is an isotonic bijective correspondence which allows us to define the sum of two quantities as follows: if $x, y \in A$, we set $x + y \doteq u(x/u + y/u)$ and it is immediate to prove as such a definition is well-posed, that is to say, $x + y \in A$ and it does not depend by the measurement unit u chosen. Therefore, it is easy to verify that $(A, +, <)$ is a complete ordered commutative semigroup, which is isomorphic to $(\mathbb{R}^+, +, <)$ via ψ_u, for each $u \in A$ arbitrarily fixed.

The following problem arises: is it possible to coherently choice, for each quantity species $A \in G/\mathbb{R}^+$, a quantity $u_A \in A$ assumed as a unique measurement unit for each A chosen, in such a way that (*coherence* condition) $u_A \cdot u_B = u_{A \cdot B}$ whatever $A, B \in G/\mathbb{R}^+$? This problem may be more formally re-expressed as follows. If U denotes the set of all the measurement units chosen, then the conditions of such a problem are equivalent to require that such a set U, when it exists, be a system of canonical representatives of $G/\mathbb{R}^+$ (corresponding to the requirement $card\ (U \cap A) = 1$ for each $A \in G/\mathbb{R}^+$) and that it be a subgroup of G; in turn, these last conditions are equivalent to require that $G = \mathbb{R}^+ \oplus U$, that is to say, G is the direct inner product of its subgroups $\mathbb{R}^+$ and U. However, the answer to the above problem about the existence of such a set U, is affirmative thanks to a fundamental property of the multiplicative group $\mathbb{R}^+$ called *divisibility*, and that corresponds to require the existence of at least one solution x to the equation $x^n = a$ for each a and n chosen: we know that such a solution, in $\mathbb{R}^+$, there exists and it is unique. Nevertheless, the same equation in $G = \mathbb{R}^+ \oplus U$ do not have, in general, solutions if one does not admit the divisibility as hypothesis; analogously, even if there subsist existence conditions for the solutions to $x^n = a$, the related uniqueness conditions may yet fall if one does not admit a further hypothesis besides the divisibility, namely the hypothesis that G has no *torsion elements*[8], that is to say, the hypothesis that G is *torsionless*. In short, divisibility and torsionless warrant existence and uniqueness for the solutions to $x^n = a$, with a consequent affirmative answer to the above coherence problem.

Therefore, under all these hypotheses, we have the following algebraic structure $(A, +, \cdot, <)$ in which it is also possible to consider the *n-th* root of any quantity through the resolution of the equation $x^n = a$, hence having powers with rational exponent. If the group of all the physical quantities, say G, is a commutative divisible and torsionless group, then it is possible to prove that the group of the species of physical quantities $G/\mathbb{R}^+$ is also a divisible and torsionless group, so that, in it we may consider powers with rational exponent, setting $[a]^{m/n} \doteq [a^{m/n}]$ for every $m, n \in \mathbb{N}$, and having defined $a^{m/n}$ in G as above. On the other hand, it is possible to prove that any commutative divisible and torsionless group has also a natural structure of linear space on $\mathbb{Q}$, and this therefore hold for G, $\mathbb{R}^+$ and $G/\mathbb{R}^+$ in which the various linear combinations of their

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direct sum of them. *En passant*, we besides note as the first part of Darbo's paper (of which we herein have only delineated a brief summary) considers an abstract group, while at the end of it he apply what said in the first part to the case of a Lie group, till reach to the *E. Buckingham π-theorem*. In any way, the whole Darbo's considerations will deserve a major attention and further developments, even if part of them is close to much arguments treated in Kasprzak et al. (1990), Carinena et al. (1981; 1985) and Misic et al. (2010), where group theory tools are widely and profitably used both in dimensional analysis theory and its related applications.

[8] That is to say, an element $y \in G \setminus \{1\}$ such that $y^n = 1$ for some $n \geq 2$.

elements is written in multiplicative form instead of the usual linear one, that is to say, we have expressions of the type $a^\alpha \cdot b^\beta \cdot c^\gamma$ (said to be a *monomial form*[9]) instead of $\alpha a + \beta b + \gamma c$ with scalar coefficients $\alpha, \beta, \gamma \in \mathbb{Q}$ and vectors a, b, c; but, despite of this multiplicative notation, we still have a rational linear space structure. To this point, we may consider the linear dimension of the rational linear space $G/\mathbb{R}^+$, which is a crucial question far from being trivial and uniquely determined: for instance, such a dimension may depend on the physical context in which it is considered, so that for geometrical problems it is enough to choice the physical quantity length L as a unique generator, for kinematical problems it is enough to choice the physical quantities length L and time T as generators, while for dynamical ones it is enough to choice length L, time T and mass M as generators. Therefore, taking into account this last dynamical example, we can say that $\{L, T, M\}$ is a $\mathbb{Q}$-base for $G/\mathbb{R}^+$, that is to say, $dim_{\mathbb{Q}}\, G/\mathbb{R}^+ = 3$; thereafter, a coherent set of measure units is, for example $\{m, sec, kg\}$ (SI), so that any quantity $A \in G/\mathbb{R}^+$ may be written as $A = L^\alpha \cdot T^\beta \cdot M^\gamma$ with measure unit $u_A = m^\alpha \cdot sec^\beta \cdot kg^\gamma$, which besides satisfies the (above) coherence condition $u_A \cdot u_B = u_{A \cdot B}$.

In any case, and this is a crucial point (as we will see later), there is a certain indeterminateness in the linear dimension of the $\mathbb{Q}$-space $G/\mathbb{R}^+$ which can be, in part, identified by the physical context from time to time involved: it therefore has a preeminent dialectic-historical nature with also, as we will see, non-negligible epistemological features. For instance, in certain relativistic dynamics considerations, it might be suitable to identify the velocity species[10] LT^{-1} with $\mathbb{R}^+$, that is to say, make them adimensional, and hence making possible, for example, to put $c = 1$; in such a case, we'll have $dim_{\mathbb{Q}}\, G/\mathbb{R}^+ = 2$, and many other physical quantities would be identified amongst them as, for example, the energy and the mass. Nevertheless, the adimensional reduction of certain physical constants (as, for instance, due to this *unitarization* process of physical constants) might imply non-trivial physical drawbacks: for instance, the above mentioned qualitative identification $L = T$, in the relativistic context, imply non-negligible disadvantages from the experimental viewpoint because, for instance, the latest measurement accuracy of lengths (with interferometric methods – see Schödel et al. (2012)) is about 10^{-9} and those of the times is about 10^{-12}, whereas the velocity of the light in the vacuum is known with an accuracy order thereabout of 10^{-8}, so that the above length-time identification would imply a major inaccuracy if one used the meter as measurement unit of the time or the second to measure the lengths. Analogous qualitative considerations might doing to achieve a further dimensional reduction of $dim_{\mathbb{Q}}\, G/\mathbb{R}^+$ from 2 to 1 by means of identification of the masses with the inverse of the times (angular frequencies or else, less than a numerical factor equal to 2π, frequencies) through the Planck-Einstein relation $E = \hbar\omega$ in such a way, for instance, to reduce to 1 the (reduced) Planck constant $\hbar$, till to get the so-called *Planck natural units* in which $\hbar = c = 1$. Also in this case, if one compares the measurement accuracy of masses, approximately equal to 10^{-9}, with the poorer measurement accuracy of Planck constant, it is again evident the experimental and practical disadvantages in using such natural units (adopted above all in theoretical quantum physics) notwithstanding their possible theoretical advisability[11].

Thus, the choice of the various conventions characterizing the $\mathbb{Q}$-linear space $G/\mathbb{R}^+$ are also strongly influenced by the *state of the art* of the measurement accuracies and instrumentation arrangements of a given historical period[12], hence strictly depending by an *operationalistic* standpoint, this besides being closely coherent with what said in Lévy-Leblond (1977, Section 3.1) about the historical experimental determinations of the value of certain universal physical constants, like c, $\hbar$ and J. But of all this, we will discuss in-depth later.

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[9] For instance, if a, b and c are respectively the physical quantities length L, time T and mass M, then $a^\alpha \cdot b^\beta \cdot c^\gamma$ is the physical quantity $L^\alpha \cdot T^\beta \cdot M^\gamma$.

[10] In this regards, see also what said in Lévy-Leblond (1977, Section 3.1).

[11] On the other hand, not always it is convenient, also from a theoretical viewpoint, to set equal to 1 certain physical constants: for instance, the not unitary value of the gravitational constant *g* is extremely useful in Astronomy and Geophysics in determining the internal structure of planets (including the Earth), thing that will result to be inefficacious if one put *g* = 1 (see Pizzella (1993, Capitolo 4)) so that it is important to know the value of this constant with the greater possible measurement accuracy. However, for the first attempts toward a possible theory on eventual variations of the basic physical constants, see Dirac (1937), in which, amongst other, was proposed a cosmological time dependence of certain physical constants (above all, of the gravitational one), however nowadays abandoned. Similar speculative attempts were also pursued by A.S. Eddington (1946). On another basic role played by fundamental physical constants, see the paper of Brandon Carter (2011), which is a republication of his first work on the so-called *anthropic principle*; see also Uzan (2003) for a recent general review.

[12] In this regards, see, for instance, Petley (1992) and Uzan (2003).

Moreover, it results to be clear as it is not possible to define physical quantities only through the qualitative algebraic formalization given by the above mentioned group G without a physical operative description of the related measurement processes of such quantities, so that, the operationalistic point of view is unavoidable in physics. This last remark is also pointed out by Cohen & DuMond (1965) according to which, seen the ever increasing trend to specialization with a consequent plethora of *local* unit systems (whence an increasing too of $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$) to afford the highest degree of precision, it goes on towards a loss of the fundamental unifying character of Physics which instead must be guaranteed by a small number of primitive units through which to determine all the other derived ones by means of suitable conversions factors which, in turn, must be *experimentally determined*; to achieve this, Lévy-Leblond (1977) adds that these conversion factors should have been previously theoretically defined as universal constants. For a deeper discussion, see Lévy-Leblond (1977, Section 2.3), from which clearly emerges as not always it is allowed such a unitarization of physical constants (see also later).

2.2 Further epistemological remarks and some science education features

From what has been said, it therefore follows that the axiomatic structure given above for the dimensional analysis setting, is enough to completely determine its global formal structure but with an exception as regards the dimension of the $\mathbb{Q}$-linear space $G/\mathbb{R}^{+}$due to the arbitrariness in choosing its generators, which corresponds, in turn, to the arbitrariness in choosing those independent physical quantities as fundamental ones.

On the other hand, in Agazzi (1969, Capitolo VI, § 21), it is also discussed the various relationships between *derived* and *fundamental* (or *primitive*) physical quantities as well as the possible dangers to whom one may incur for an excessive reduction of the former to the latter or for the reduction to a scalar or adimensional quantity of a given one[13], that in turn are linked to the determination of the value of fundamental physical constants[14] (like in the Planck natural unit system[15] where, amongst others, it is put $c = \hbar = 1$): all this might prejudice a correct general physical sense and interpretation, like the unrealistic reduction of electromagnetism to mechanics which nevertheless is surely possible, from a purely dimensional formal viewpoint, with a suitable choice of the primitive or fundamental quantities, but that, on the other hand, leads to a non-negligible epistemological problems related of that *vexata quæstio* regarding the relationships between Physics and Mathematics[16], or else between the formal structure and the experimental bases of a physical theory. Indeed, besides what has been already said above as concern the these fundamental dialectic relationships between formal aspects and operationalistic framework of physical quantities, following Recami & Spitaleri (1975), we may say that the various mathematical relations amongst physical quantities are above all relations among (physical) quantities and not only among pure numbers, so

[13] In this regards, it plays a fundamental role the so-called *E. Buckingham π-theorem* (see Kasprzak et al. (1990)) which, amongst other things, puts in evidence what basic role may play the fundamental constants (or adimensional quantities) in determining the physical laws; inter alia, it also determines the minimal number of independent adimensional quantities to pursue this. Thereupon, see also the last part of Darbo (1969).

[14] As regard the questions inherent the variability or not of the fundamental physical constants, see Dirac (1937), Sartori (1979, Capitolo 3, § 3.5) and Uzan (2003).

[15] For the Planck natural unit system, which is mainly adopted in microscopic physics, see, for instance, Fazio (1976, Appendice A). Nevertheless, as observed in Agazzi (1969, Capitolo VI, § 21, pp. 183-184), the formal simplification obtainable putting $c = \hbar = 1$ do not always give useful results in terms of clearness for the physical meaning of the various physical relationships. Therefore, in general, an extreme reduction, albeit formally correct, of the number of primitive physical quantities provides an increasing indeterminacy both in the physical meaning and in the experimental accuracy degree of the involved physical equations and related quantities, as already said. As concerns the Planck's natural unit system, see also the interesting article of Treder (1979).

[16] On the other hand, about this crucial epistemic relation, the same Lévy-Leblond (1977) states as follows: «[…] *in physics, mathematics does not simply apply; it plays a far deeper, constitutive role. The identification, or synthesis of two concepts in physics thus requires first their mathematical nature to be identical (scalars or vectors, for instance) and then implies the existence of a proportionality factor. Let me stress that numerical measurements of a given quantity, as may exist in other sciences (even social ones), are not sufficient to endow it with mathematical constitutivity; it is necessary that there exist nontrivial mathematical relationships between several such quantities, expressing the ''scientific laws'' of the field*». Well, the above discussion about the epistemological meaning of $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$ with its variations, might be taken as a paradigmatic case which partially clarify the historically crucial relationships mathematics-physics. See later discussions thereon.

that the physical dimensions are not arbitrary convections but express intrinsic (or ontological[17]) characteristics of the given physical quantities, although there be a certain degree of freedom in choosing the primitive quantities. Hence, with very caution should be considered $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$ and on this, as already said, we will return later in a deeper manner, seen its significance.

Thus, the dimensional analysis should be intended as a formal and diagnostic analysis of the possible mathematical relationships among measurement units under certain prescribed conditions, and not only as a conceptual and physical-interpretative analysis: the latter therefore remain independent one from the other, so marking a notable epistemological remark as concern the relationships between mathematics and physics. In particular, the discussion made so far clearly suggests as a certain independence between these contexts should be guaranteed: a physical operative content cannot never replaced by formal considerations, and vice versa. Furthermore, if, for instance, it were more relevant to consider a non-Euclidean geometric model instead of an Euclidean one, then it would be again possible to use the international meter as unit of measurement of lengths. Hence, the *operative content* of the theory of physical quantities has a certain independence from the various degree of rationalization with which the same quantities are considered into the theoretical models from time to time adopted: for instance, again textually following Darbo (1969), the known fact whereby the product of a mass by an acceleration is a force (i.e. $F = ma$) should be intended as a consequence of a convention and not as a consequence of the Newton law which, besides, does not subsist in relativistic mechanics; such a convention may be presented as a definition of mass of a body even if such a mass is not kept to be constant during the movement (that is to say, even if one does not admit valid the Newton principle).

On the other hand, the latter remarks may also be in a certain sense correlated with some new constructs of Physics Education (see diSessa & Sherin (1998) and Levrini & diSessa (2008)) as that of *coordination class*[18] (in the *Humble Theory* sense) which, roughly, is a model of a particular kind of concept[19] that allows us to explain why the comparison with different perspectives support a deeper understanding. According to this model, framed into the broader *complex knowledge system* perspective, the concepts are large and intricately organized systems, which effectively coordinate the activation and use of many specific elements according to the context. Learning a concept is thus seen as a process of recruiting and coordinating a large number of elements in many ways according to the general constructivist perspective. The theory specifies the organization of knowledge into a well-developed coordination class and specifies several hurdles that must be overcome to achieve a ‘well-developed’ status; as such, the theory also ‘partially’ specifies the processes of acquisition of a coordination class from a *global* view towards a *local* one, and indeed a *humble theory* is just a deliberate attempt to step back from grand theories to more local ones that are, at once, more specific thus to be more easily applied to various cases. Furthermore, again following Levrini & diSessa (2008), for a concept to be a coordination class, several theoretical categories must be identified in empirical (in educational sense) data, with particular relationships amongst them, and a set of consequences is entailed. Coordination class theory has the following properties: *1)* it is specifically oriented toward *entering* the structure of each concept (rather than viewing it only in relation to other concepts) and analyzing it as a coordination of the various pieces of knowledge involved[20]; *2)* by considering the internal structure of concepts and their gradual construction, coordination class theory provides operational tools for interpreting learning difficulties and what, in general, happens during the processes of concept learning. Coordination class theory has a notion of ‘’the family of relations in which a concept participate’’ (see discussion of the ‘’inferential or semantic net’’ below), which roughly corresponds to the *semantic field* (or *semantic net*) that is given priority in other views of concepts; the emphasis in coordination class theory is on the processes by which such inferential relations are assembled and used in specific situations for doing the conceptual work characteristic of the concept under examination. The central function that ties together all the various pieces of a coordination class is to allow people reading one particular class of information out of the huge variety of situations in which the concept is useful in the real world. It is, in fact, that diversity of contexts that ensures concepts must include many context-specific elements. The relevant information that defines a

[17] It was not by chance that it speaks too of an *ontology* of measurements: see, for instance Rijgersberg et al. (2011).

[18] This notion has been introduced in by diSessa & Sherin (1998) and it is still at the centre of current physics education researches and educational learning in general, together other contexts like the historical one (see Limon (2002)). Nevertheless, its epistemological role in physics is yet underestimated.

[19] Not all the possible concepts are coordination classes; nevertheless, amongst them there are the physical quantities with which this paper has mainly to do.

[20] Concept maps and semantic networks, for example, show the *external* relationships of concepts with other ones, but coordination classes aim mainly to show the *inner* relations of parts of a concept that make it well formed and powerful.

coordination class might be, for example, *a)* the point of application, magnitude, and direction of a force or *b)* the number associated with a pair of points in space-time which satisfy the special relation called 'timelike', that we call *proper time*. Succinctly, having a concept according to coordination class theory, is in essence being able to 'see' the information that defines the concept in an appropriate range of relevant situations. Just in respect to these last points, as we will say too in the next sections, the occurrence of the universal constant c in the description of space-time, leads inter alia to the splitting of the categories of simultaneous pairs of events from that of invariant interval (now the 'lightlike' ones), questions, these, which may be related to explain some aspects of the notion of proper time (see Lévy-Leblond (1977)) that, in turn, is strictly centred upon one of the fundamental postulates of special relativity, that according to which c is a universal constant.

As already said above, the learning of a concept is seen as a process of recruiting and coordinating a large number of elements in many ways. The architecture of a coordination class summarily include *readout strategies* and *causal* (or *inferential*) *net*: the first ones concern the ways in which people focus their attention and read out any related information from the real world, whereas the second ones concern the total set of inferences which one can use to turn related information readouts into the particular information at issue; see just the case of $F = ma$. In such an educational context, it is therefore manifest what basic role may also play the possible qualitative dimensional analyses in the above sense: for instance, as we will see later, the occurrence of the universal constant c will play both a *synthetical* and a *splitting* conceptual process which are both closely related to the main pivotal points of a coordination class model in the humble theory sense, namely both that according to which it is a deliberate attempts to step back from grand theories to more local ones and that related to the functions explained by its causal net.

Later, we will see as these last considerations may be also related to the so-called (*historical*) *conceptual changes* occurring in physics education[21], of which the coordination class theory is but a model: indeed, as already said, the determination of $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$ depends both on the *state of the art* of measurement theory and instrumental setups of the time, and, as seen, all this also imply subtle epistemological questions. Furthermore, as we'll see in-depth later, on these last points it will graft too the basic Gärdenfors work on conceptual spaces, in such a way that the historical pattern $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$ will be placeable within a suitable intersection of all these perspectives.

Finally, coordination class theory is also an attempt to replace 'monolithic' treatments of displacement, or replacement of concepts, with a theory of the fine-grained elements and processes that go into such macrochanges. Despite of this observation, when many concepts are under examination within a conceptual system, it might be useful to synoptically speak of the construction of some of them, which are accomplished prior to others as 'events' rather than as 'extended processes'. However, in the final sections of this paper, as said, all these considerations will be further in-depth retaken and coherently developed.

3. Some further remarks and educational-epistemological considerations

According to Paolo Guidoni[22], in carrying out the basic educational (inseparable) pair learning-teaching of Physics, it takes place the activation[23] of a sort of complex 'tetrahedral' structure whose vertices represent *1)* the language[24] (L), *2)* the phenomenological and experimental aspects (P), *3)* the formal structure (M) and *4)* the laws of nature (N). For instance, the discussion of the Ørsted's experience seems almost always centred around the vertex *2)* with an unavoidable minimal linguistic component *1)* but also with some attention both to *3)* if one frames such an experience into a mathematical model, and to *4)* when one recognizes its fundamental unifying character between electricity and magnetism; instead, when one discusses the Euler's elastic rod stability under compressive stress (which is a very simple example of symmetry breaking phenomenon – see Radicati (1983)) as well as when one tries to determine the magnetic field at the centre of

[21] The first notion of conceptual change was given by G.J. Posner and coworkers in Posner et al. (1982), starting from some analogy parallels with the well-known epistemological theories of T.S. Kuhn and I. Lakatos which, inter alia, are also recalled in Lévy-Leblond (1977, Section 3.2, ii)) as regards the vanishing constant case (e.g. as $c \to \infty$ – see later). In any case, almost every basic paper on conceptual changes and knowledge theory in science education inevitably starts from epistemological considerations.

[22] See, for instance, Pescetti (1985, Introduzione).

[23] Even if in a different manner one from the other.

[24] Mainly, from a semantical viewpoint.

an electric current crossed squared coil, seems to prevail the vertex *3)*, whereas, finally, a science philosopher seems almost always fixed around the vertex *1)*.

The above Darbo's qualitative dimensional analysis argumentations for instance seem at the same time to involve the vertices *2)*, *3)* and *4)*, above all in determining the possible linear dimension of the $\mathbb{Q}$-linear space $G/\mathbb{R}^+$, whose variations (as already said) develop along a historical dialectic relation between theoretical and formal aspects and their experimental/operationalistic counterparts, clarifying the relationships amongst them[25]; and just in this regards, Lévy-Leblond (1977) said too that

> *«The last remark* [about the magnitude of $\hbar$ and c, computed respect to a given system of measurement units]*, trivial as it may seem when applied to our modern familiar and revered universal constants* $\hbar$ *and* c*, may be of some help in understanding the historica1 reasons for the emergence, and latter subsidence, of most universal constants, including the classical and archaic ones*[26]*. Indeed, for* c *to appear as a universal constant, it was necessary for experimental investigation to come to grip with some phenomenon where at least one combination of physical quantities with dimension* LT^{-1} *was comparable to* c*. This required a stage in the development of experimental techniques which was not reached until the 17th-Century with the first, measurements of the velocity of light*[27]*. Spatio-temporal ratios were for quite a time, the only magnitudes with the required dimensionality to be measured with the necessary precision, so that* c *could not appear but as a type-A) constant*[28]*: the velocity of light and nothing more».*

Hence, amongst other things, it is evident what notable role may play the qualitative dimensional analysis given in Darbo's formulation as concern the foundations of Physics: in particular, the occurrence of a universal constant reduces $dim_{\mathbb{Q}}\, G/\mathbb{R}^+$, which imply, in turn, an *epistemological unification/reduction* of different concepts or notions for what said in the above section 2, and this confirms that *synthetical transcending* role played by these constant as pointed out by Lévy-Leblond remarks recalled in section 3.
As seen in the previous section, the universal constant c plays a very fundamental educational and historical role in Physics because it has provided the first relativistic invariant quadratic form in the signature convention $(+,+,+,-)$ given by $x_1^2+x_2^2+x_3^2-c^2t^2$, as well as it may also play a fundamental educational role following the line of thought of Davidon (1975) which starts from the notion of *inertia* (identified with *energy* in Einsteinian relativity but with *mass* in Galileian relativity). As already said, the occurrence of the universal constant c therefore splits inertia from mass as it fuses it with energy: as already said, in the description of space-time, the splitting of the categories of simultaneous pairs of events from that of invariant intervals (now the 'lightlike' ones) may be interpreted in quite the same way (see Lévy-Leblond (1977)). However, it is well-known as the assumption of c as an invariant universal constant (besides to be an upper bound for velocities) is just one of the two initial postulates stated by Einstein himself (see Bernardini (1991)), so that it played a very fundamental role (above all historical) in the geometrical unification of the concepts of space and time: but, one of the possible aim of the present paper would also be

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[25] In this regards, it is moreover useful the reading of the beautiful and meaningful *Prologue: What is Geometry?* by Eddington (1920), about the crucial relationships between Physics and Geometry, arranged like a debate in which are involved an experimental physicists, a supporter of the new relativistic conceptions on space and time and a mathematician, whose literary style seems paraphrase the well-known historical discussions between Simplicio, Sagredo and Salviati of the landmark Galileo Galilei *Discussione sopra i due massimi sistemi del mondo* (∼1632) which marked, amongst other things, the starting point of the new scientific method.

[26] In Lévy-Leblond (1977, Section 2.2, p. 198), there is a classification of universal constants into three subclasses according to their historical evolutive status, namely in: *i)* the *modern* ones, such as $\hbar$ and c, the conceptual role of which is still dominant; *ii)* the *classical* ones, such as *k* or *J*, which today appear essentially as unit conversion factors, their conceptual role having become almost implicit; *iii)* the *archaic* ones, which have been so well assimilated and digested as to become utterly invisible. These classifications of physical constants into classes of type *A)*, *B)* and *C)* (see next footnote [27]) as well as *i)*, *ii)* and *iii)*, play a fundamental epistemological role far from having a historical importance only.

[27] See Sanders (1965).

[28] That is to say, regarding physical properties of particular objects: precisely, Lévy-Leblond classify the physical constant by order of increasing generality, distinguishing in the following types: *A)* constants expressing physical properties of particular objects (e.g. masses of fundamental particles, their magnetic moments, etc); *B)* constants characterizing whole classes of physical phenomena (e.g. coupling constants of fundamental interactions, Fermi constant of weak interactions, the gravitational one, etc); *C)* universal constants (like c, $\hbar$, etc).

oriented toward possible attempts of using such a historical evolution of the physical constants to improve the acquisition of some main aspects of the theory in which they emerge or are involved. Thereby, the paper of Davidon (1975) was also written to this aim, focussing the discussion on the role played by the inertial equivalence of energy $1/c^2$.

3.1 On conceptual change modelling: brief recalls

In diSessa & Sherin (1998), before to present their new *coordination class* model, the authors review first the previous standard models of conceptual change, then that of a concept, hence reaching to the conclusion that it is necessary to replace the latter with a variety of more carefully defined theoretical constructs, one of these being just that of a *coordination class* which, inter alia, explain systematicities and connections between ideas rather than focusing predominantly on the quasi-independence of the various elements; there is therefore a convergence towards the center of these perspectives in order to account for coherence, systematicity and transition (see Özdemir & Clark (2007)).

The standard model of a conceptual change as due to S. Carey[29], at first focus on the changes related to the relations among the concepts that appear in a given belief, then she turns to the history of science with a special emphasis to the work of Thomas S. Kuhn from which borrows the basic notion of *knowledge restructuring* distinguishing between a *weak* and a *strong* type: in the former, new relations among concepts are represented and new schemata come into being which allows the solution of new problems and which change the solutions to old problems; the latter, instead, involve changes at the level of individual concepts at the core of the successive systems. Just this last type of stronger variety of restructuring will be called a *conceptual change* (in the Carey's sense), which constitutes what is said to be the *standard model of conceptual change*. Subsequently, D. Gentner and co-workers distinguish three grades of change: a) *belief revision*, as a simple change in facts, b) *theory change* as a change in the global knowledge structure, and c) *conceptual change*, the most drastic, as a change in the fundamental concepts that compose the belief structure; conceptual change therefore requires at least locally non-alignable or incommensurable beliefs, this last conception of conceptual change being very similar to the Carey's one. In any case, it is worth emphasizing that the standard model originated in literature from the history of science: for instance, in the seminal paper Posner et al. (1982), the authors develop their version of standard model of a conceptual change precisely by making analogies with the history of science, taking into account both the Kuhnian notions of 'normal science' and 'scientific revolution' and the 'research programs' of Imre Lakatos, hence starting just from the philosophy of science[30] as main source and epistemological basis for their attempt to explain the notion of conceptual change also by means of a revisiting of the originary Piagetian assimilation and accommodation; the Kuhnian scientific revolutions or Lakatosian research programs are considered as analogous patterns of what a conceptual change in learning and educational science might be[31]. Hence, not only many ideas are borrowed from history of science, but much of the conceptual change literature uses historical examples for basic illustration and as starting points.

Thereafter this historical recognition, necessarily it reaches to the notion of what a concept is and how they change if one wants carrying out a reasonable conceptual change model. After a further brief discussion of the various cognitive psychology analyses of the notions of category and concept (mainly from a Piagetian stance) with a comparative and critical appraisal of them, diSessa & Sherin (1998) itemize, but not in an exhaustive manner, the main models of what a concept might be, namely: j) the *relational theory of concept*, in which concepts gain their meaning by participating into a net of relations with other ones, the changes corresponding to the adding and/or deletion of net nodes as well as variations in the existing relations between these nodes; jj) the *neural net activation pattern*, where there may be no simple localized mental representation that correspond to patterns of activation that emerge during the activity of neural net, giving rise to a concept; and jjj) the *actional/situated perspective* in which, rejecting not only a simple localization

[29] For the various authors and related references herein quoted, see diSessa & Sherin (1998), while for a recent survey paper on conceptual change theories, see Özdemir & Clark (2007).

[30] The authors suggest a certain number of skills and abilities to develop in students, amongst to which just that stimulating the awareness of the epistemological and historical foundations of modern science.

[31] Hence, there have been also glimpsed certain analogies between theory changes in the history of science and student's learning of science, recalling into question a kind of *Haeckel-Müller biogenetic fundamental law* according to which the ontoepigeny (read the individual learning) recapitulates the phylogeny (read the historical evolution); in this regards, see above all Thagard, (1992, Chapter 10), which is a very good reference for the general theory of conceptual changes and their historical development.

of a concept in a particular mental representation but also any sort of localization of concepts in the individual mind, it considers the concepts as arising from abstractions over people acting in setting. Subsequently, recalling the Kantian philosophy and the Piaget's work, diSessa and Sherin introduce their notion of coordination class as a model concerning a class of systematically connected (i.e. coordinated) ways to get information by ''seeing world's things in the various situations or differently presented in their context[32]'', laying out into a class of knowledge and reasoning strategies (*readout strategies*) that inferentially determine when and how such observations are related to the information under examination[33] (*causal net*).

The coordination process underlying such a class of readout strategies has a double meaning. First, it refers to the fact that, within a given situation, multiple observations or aspects may need to be coordinated in determining the necessary information, or else what we wish to 'see'; this version of coordination might be described as *integration*. Second, it refers to the fact that, across instance and situations, the knowledge that accomplishes readout of information must reliably determine the same information, or else how observations in different circumstances can manage to obtain it; this last sense of coordination might be called *invariance*[34]. Integration and invariance are the two primary performance specifications of coordination classes, beyond 'getting information' by means of readout strategies, while the third primary structural component of coordination classes is the causal net, where most difficulties with conceptual change in physics are present and from which arise. The relationships between readout strategies and casual net are very tight: one looks for things that are related (via casual net) in order to determine some quantity, and they co-evolve as learning occurs. To explain the relevance of the causal net, diSessa and Sherin argue on the well-known Newton's second law, bringing this example as a special case study illustrating what potential pivotal role the physical equations may play in the causal net for quantities, stating that non-quantitative (that is to say, qualitative[35]) connections among quantities, and other aspects of the physical situation, are critical in coordinating. In the case of physical quantities, the related equations in which they are involved are not properly coordination classes but one can use a physical equation in coordinating and relating some observations to particular needed information: for instance, in this case the dimensional analysis play a basic role to this purpose, so that it is a powerful and useful coordinating tool. The author also stress that specific quantitative use of equations is less important than qualitative uses of the connections evident in equations[36].

3.2 Some further brief outlines on coordination class model

In continuation to what already said at the end of the previous section 2 and in synthesis of what said in section 3.1, we briefly sketch, following almost textually Levrini & diSessa (2008), the *coordination class model* in the following categories: *(a)* the architecture organizational structure of the internal elements of a coordination class, *(b)* the processes that build up a coordination class, and *(c)* the characteristic difficulties that the students encounter in building up a coordination class.

(a) As regards the *architecture*, we may say that, in general, people do not directly and transparently see the relevant information of a coordination class. Instead, typically they *read out* some related information and then *infer* the coordination-characteristic information. For example, in order to see the magnitude of a force, one may 'observe' mass and acceleration and then 'infer' force by multiplying mass times acceleration. These two steps motivate the splitting of the architecture of a well-formed coordination class into two parts, namely into: *1) readout strategies*, that is to say, the ways in which people focus their attention and read out any related information from the real world (where 'related' can be exemplified by the fact that mass and acceleration information are related to force); *2) causal net* (sometimes called the *inferential net*), that is to say, the total set of inferences which one can use to turn related information readouts into the particular information at issue.

(b) While coordination class theory accepts that entirely new elements may be created in constructing a new concept, an overriding care is understanding how *prior knowledge* contributes to or detracts from

[32] Which may be, for instance, the characteristic attributes of a concept modeled in different situations: for example, for a physical quantity, this corresponds mainly to determine its values in particular situations.
[33] As, for instance, those related to the determination of the values of a given quantity under examination.
[34] Thereupon, the below notion of a *concept projection* will play a prime role.
[35] Just like the dimensional analysis.
[36] Like those given by dimensional analysis (see previous footnote). On this we will return later, above all when we talk about Gärdenfors' conceptual space theory in which qualitative dimensional analysis play a very primary role in the wider conceptual change theory.

the construction of a coordination class. This emphasis represents the claim that coordination class theory provides a model for *conceptual change*, not just ‘’blank slate’’ learning. With respect to prior knowledge, the process of building up a coordination class involve the following two generic processes: *1) incorporation*, that is to say, recruiting elements of prior conceptualization into partial encoding of the new concept, typically ‘partial encoding’ meaning that those elements will be used in some circumstances but not in others; *2) displacement*, that is to say, ‘dismissing’ elements of prior conceptualization that may initially and inappropriately ‘take over’ the function of the coordination class in certain circumstances: for example, novices often determine the existence and magnitude of a force by using the inference ‘if there is motion, there must be a force’ (which is a mistake).

Coordination class theory hypothesizes two particular and characteristic difficulties – namely, the *1)* and *2)* below – that students have in creating new coordination classes and that they must overcome for reaching to a right knowledge. These have to do with the characteristic ability to the coordination class properly ‘work out’ across a wide range of situations in which it is useful; the theory presumes that ‘working out the concept’ may use different knowledge in different situations, and this case is very far from being rare. The particular knowledge used in specific applications of the concept is called a *concept projection*. Said this, the above mentioned difficulties are: *1)* the problem of *span*, that is to say, to have adequate conceptual resources to operate the concept across a wide range of contexts in which it is applicable or may be laid out; *2)* the problem of *alignment*, that is to say, being able to determine the *same* concept-characteristic information across diverse possible circumstances; this is a well-formedness or coherence principle that ‘what you see’ does not vary when you use different methods of seeing it[37]. Said in terms of concept projections, alignment means that each concept projection provides the same results: indeed, the ability to the coordination class properly ‘’work out’’ across a wide range of situations in which it is useful, coordination class theory presuming too that during ‘working the concept’ it may be possible to use different knowledge in different situations. As students work from one context to another, they should improve both span and alignment; span will be improved, prototypically, by adding knowledge that allows their concept to work in contexts where it could not work before. These last considerations will turn out to be useful later.

3.3 A coordination class theory case study: the notion of proper time of special relativity

In Levrini & diSessa (2008), some relevant educational problems concerning the notion of proper time are considered within the theoretical framework given by the coordination class theory which, as seen, is an evolving model of concepts and conceptual changes in which different contexts are a critical focus.

According to Levrini & diSessa (2008), most textbooks teach special relativity following paths similar to Einstein’s original 1905 presentation: they introduce the historical context by presenting the inconsistencies between classical mechanics and electromagnetism, they focus on Lorentz transformations, and they use algebraic representations predominantly. However, unlike Einstein’s papers, they pay little attention to what it means to measure space and time, taking into account the new constraints of the theory, namely the unsurpassable and constant speed of light (universal constant c). In other words, despite following Einstein’s reasoning, they do not attach relevance to his original *operational* perspective, which we view as consistent with the idea that space and time are special names we give to ways of relating events by measurement. We have seen that the *operationalization* of ‘’observing’’ plays a special role in coordination class theory, so this ‘slight’ of Einstein’s orientation is substantial, and it is more general than just ‘a way to teach relativity’. In particular, the arguments and empirical results of Levrini & diSessa (2008) case study, affirm a contention of other research: the crucial necessity of introducing students to special relativity by *operatively* constructing, for a «single frame of reference», the time between events (colloquially, the duration of an event) and the procedures for defining the time of a distant event. In this manner, one arrives to the analysis of the Einstein’s thought experiments, concerning the consequences of the speed of light invariance with respect to different frames of reference, with a structured and operational concept of frame of reference as a lattice of rules and synchronized clocks. In particular, the teachers thought that the comparison could be a way to support different ways of creating relations between events: *measuring*, according to the Einstein’s operational perspective, and *calculating*, according to the Minkowski’s geometrical perspective based on the space-time interval invariance; we interpret these as ways to support reflection on different projections (or classes of projections). In passing, we note that framing the consideration of multiple projections as historical

[37] And, in this sense, we reconnect to the above mentioned *invariance*.

perspectives, by itself, seems a useful strategy. Historical perspectives are potentially interesting and important in their own right, they may have independent good pedagogical properties, and they constitute an easier 'handle' on diverse projections for students who, of course, have no reference in coordination class theory.

In any case, the related educational analysis made by the research group of Olivia Levrini of the Department of Physics and Astronomy of the University of Bologna, has ascertained and reported in Levrini & diSessa (2008) as the richness and diversity of the thinking about the historical context was the ''secret ingredient'' in achieving greater conceptual competence; the historical context, or better, the possible different historical perspectives (here viewed as proxies for different classes of projections), *per se* have had valuable influences in the effectiveness of the instruction. Nevertheless, it would have also been of a some usefulness a further critical-comparative discussion of this case study related to proper time, made in the Levrini & diSessa (2008), in respect to the meaning or conceptual change due to the limiting process[38] $c \to \infty$ which, within the general framework of the vanishing constant processes, plays a fundamental epistemological role in physics, as rightly and remarkable emphasized by Lévy-Leblond (1977, Section 3), but whose, besides possible, educational implications seem having been quite underestimated.

3.4 On Peter Gärdenfors conceptual space theory and other

The Peter Gärdenfors's *conceptual space theory*[39], explaining the main aspects of conceptual changes, is of fundamental importance for our purposes, laying out itself into an its own proper position respect to the wide frame of general conceptual change theories. The basic papers of Gärdenfors and Zenker (2011; 2012), which we shall closely follow herein in an almost textually manner, offer a new way of reconstructing the conceptual change in empirical theory, starting (as usual) from a brief recall of the main Kuhnian and Lakatosian epistemological ideas compared with the structuralist program, these three positions continuing to provide the anchoring points for the contemporary debate on scientific changes and their educational implications. One of the main aims of the present paper is also that to present a quite new way of analyzing changes of the conceptual content of empirical theories through the Gärdenfors' theory of *conceptual spaces*, formerly exposed in Gärdenfors (2000), which have many interesting points in common with the above exposed Darbo's $\mathbb{Q}$-linear space $G/\mathbb{R}^+$ theory (of section 2.1).

The primary Gärdenfors and Zenker thesis is that, starting from the assumption according to which the meaning of a scientific theory do not reside in the symbolically expressed laws (physical equations) that are formulated to express connections between the dimensions terms[40], but instead it is exhausted by the structure of conceptual frameworks and their associated measurement procedures, it follows as many types of scientific changes can best be understood as systematic change operations applied to a previous framework, also arguing as the relations between different historical stages of a scientific conceptual framework may become clearer at the dimensional level[41]. In particular, the *conceptual framework* of a scientific theory become comparable via the mathematical properties of the dimensions, in agreement with the above Darbo's claims. Conceptual spaces provide just a meta-framework through which theory frameworks can be reconstructed and analyzed, the basic components of a conceptual space being, as said, its dimensions and their connections to the measurement procedures, the notion of a dimension should being understood literally in the Bridgman's operationalistic sense. They assume too that each dimension is endowed with certain *geometrical* structures, and, in this regard, again may turn out to be useful just the above mentioned seminal Darbo's arguments, which might be besides considered as a formalization of the related conceptual space. The relations between different historical stages of a scientific conceptual framework may become clearer at the dimensional level, the conceptual frameworks of scientific theories may becoming comparable among them via the mathematical properties of the related dimensions. Conceptual changes in empirical theories occur in terms of the (formal) structure of the dimensions – that is to say, the conceptual spaces – underlying the conceptual framework within which a given empirical theory is formulated. When the conceptual framework of an empirical theory is modelled as a conceptual space,

[38] Moreover, another possible interesting comparative analysis between the notions of physical time (involved into this case study on proper time) and psychological time, it could turn out also to be of a some educational usefulness.
[39] See Gärdenfors (2000).
[40] In agreement with what said at the end of section 3.1.
[41] Coherently with the already mentioned crucial dialectic epistemic relationships between formal structure and experimental bases of Physics, inter alia epitomized as well by the historical variations of $dim_{\mathbb{Q}}\, G/\mathbb{R}^+$.

changes of a theory framework divide naturally into some particular epistemic modes, this providing a finer grain than distinguishing normal from revolutionary change; namely, there exist five main types of changes, which ordered by increasing severity are identified in the following : (1) addition/deletion of special laws, (2) change in scale or metric, (3) change in the importance of dimensions, (4) change in the separability of dimensions, and (5) addition/deletion of dimensions. The authors state that only the most extreme type, that is to say the replacement of dimensions[42], comes close to a scientific revolution.

First of all, it is necessary to clarify in what sense we formally understand the notion of dimension. Dimensions can be framed into *domains*[43] which are defined through a distinction between *integral* and *separable* dimensions: for instance, the three usual dimensions of the ordinary Euclidean spatial coordinates (x, y, z) are separable from the time coordinate t under a Galilean transformation (as well as in Newtonian mechanics), but not under a Lorentz transformation (Einsteinian special relativity); similarly, mass is separable from everything else in Newtonian theory, but not from energy in special relativity: it is part of the meaning of ''integral dimensions'' that the dimensions share a metric[44]. A theory's *domain* can now be defined as the set of integral dimensions that are separable from all other dimensions. More precisely: domain C is *separable* from D in a theory if and only if the invariance transformations of the dimensions in C do not involve any dimensions from D; similarly, the dimensions of a domain C are *integral* if and only if their invariance class does not involve any other dimension. This criterion for identifying domains is tightly connected to measurement procedures for the domains (*operationalism*).

Subsequently, the study approach of these authors for reconstructing the conceptual framework of empirical theories via their underlying conceptual dimensions, is fruitfully compared with dimensional analysis: namely, provided that the meaning of a scientific concept is determined by the dimensions that constitute it and by their respective measurement procedures, a scientific law can now be defined as the expression of a constraint upon the related dimensions underlying the associated theory[45]. The conceptual spaces approach treat all dimensions principally on the same level, so that a theory domain need not consist of fundamental dimensions only (in the sense of fundamental measurement magnitudes). The approach thus leaves room for, but does not require assigning epistemological privileges to *fundamental* over *derived* quantities; it makes no attempt either to answer at ontological questions that arise when assessing which dimensions to take as primitive nor does it require drawing Sneed's (1979) distinction between T-theoretical terms (e.g. *mass* and *force* in Newtonian mechanics) and T-nontheoretical ones (e.g. *space* and *time*). However, in relying on measurement procedures to define the separability of domains, the authors follow the Sneed's pragmatic standpoint: a T-theoretical dimension is one for which the value of an object or entity cannot be determined without applying the theory T itself. As already said, what is commonly referred to as *scientific revolution*, by the authors is categorized primarily under the last of the above five change operations: the replacement of dimensions, in turn strictly related to the variations of $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$. For our purposes, we are mainly interested to the types (3), (4) and (5), for the first two referring to Gärdenfors & Zenker (2012, Sections 3.1 and 3.2). As regards change operation type (3), a considerable amount of scientific debate concerns the *ontology* of the dimensions involved in a theory[46]: for example, in Newtonian mechanics it was discussed whether forces really exist, or whether they are just defined entities.

However, changing the importance of a dimension does not change the empirical content of a theory[47], that is to say its testability, so this cannot be the crucial rationale (epistemological) reason for the emergence of a scientific change. The ontological debates are important for the rationality of a change, as they concern for the most part which dimensions are fundamental and which are derived, and what is deemed fundamental or

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[42] Hence, those including the types (3), (4) and, above all, (5).

[43] Originally, such a notion comes from psychology and this is explicitly said in Gärdenfors & Zenker (2012, Section 2.2), where is quoted the related literature thereof.

[44] On the other hand, as we will see later (when we shall talk about Lévy-Leblond paper), these last considerations turn out to be explainable through the emergence of certain universal constants, namely the velocity of light in this case (and the Planck's constant in the quantic case).

[45] And in this, once again, it is no possible does not descry tight analogies with the Darbo's formal arguments of section 2.1.

[46] Recently, there has been a revival of scientific interest for the epistemology of measurements (see, for instance, Mari, (2003) and references therein) and its ontological status (see Rijgersberg et al. (2012)).

[47] In agreement with what has been above said about the epistemic implications (on the relationships between mathematics and physics) of an extreme theoretical reduction of $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$, from which it has emerged the inevitable appreciate of an operationalistic component in building up the theoretical framework of a physical theory (at least, in the macroscopic context).

derived changes (whence variations of $dim_{\mathbb{Q}}\ G/\mathbb{R}^{+}$) as science develops; as it will be said later, changes of the importance of dimensions can also pave the way for the unification of two theories. As regards thereafter change operation type (4), the paradigmatic 20*th*-Century example of change in separability is the transition (ascribed to Einstein but early prepared, amongst others, by Poincaré) from Newton's *space* and *time* to Hermann Minkowski's *space-time*. In Newtonian physics, there is no interaction in the measurements of space and time. Under special relativity, spatial and temporal coordinates (*x, y, z, t*) become instead integrated and this, as we will see later when we shall argue on the synthesizer role played, amongst other, by the universal constants, in this case c. Changing the separability of dimensions (hence, a variations of $dim_{\mathbb{Q}}\ G/\mathbb{R}^{+}$) is a comparatively radical move, since it involves a change in the *measurement methods* that may be applied. In most of historical examples, such a change is connected with a unification of two theories, so that the change can be rationalized as a step in the unification process.

Finally, the most extreme form of conceptual change occurs when a dimension is added to (or deleted from) its associated conceptual framework[48], that is to say, when takes place the change operation type (5), the most radical type of change and also the most difficult to rationalize: also in this case, all that is strictly connected with a variation of $dim_{\mathbb{Q}}\ G/\mathbb{R}^{+}$. Again, the aim to *unify* theories seems to be common to many historical dimensional changes – like, for example, in Newton's unification of terrestrial and celestial mechanics by the introduction of the new dimension ''mass'' – even if it seems that the (epistemic) rationality of a dimensional change can only be evaluated *post hoc*, that is to say, historiographically. In any case, the five types of dimensional changes so far presented constitute an *epistemic toolbox* for the analysis of historical changes of scientific conceptual frameworks. Thereafter, as main case study on their theory, Gärdenfors and Zenker consider the conceptual change involved in the passage from Newtonian mechanics to special relativity.

Therefore, the authors hope to have shown as the conceptual framework of a scientific theory can be repeatedly revised without necessarily lead to a scientific revolution; moreover, their analysis can likewise taking into account the long-term guidance which conceptual frameworks provide in seeking empirical ''cooperation'' from nature. On the other hand, the same authors claim that their classification of changes of theoretical frameworks may be also seen as a new historiographical tool for history and philosophy of science, historians of science hoping find it useful when treating a field's history. More generally, historians of science may find the above exposed methodological toolbox especially helpful insofar as the addition/deletion of dimensions – whence variations of $dim_{\mathbb{Q}}\ G/\mathbb{R}^{+}$ and vice versa[49] – can be used as a main criterion for identifying a radical scientific change: for instance, as they have sought to explain in the case study treated by them, the conceptual change in transitioning from Newtonian mechanics to Einsteinian special relativity mainly consists in integrating the three-dimensional space and one-dimensional time into four-dimensional space-time, and making *energy* and *mass* convertible, at the expense of demoting *forces*. In this regards, afterwards, the authors also discuss a particular aspect, namely that related to the 4-momentum and its Newtonian limit as $c \to \infty$ (this being very close with what shall be said later as regards the 1977 Lévy-Leblond paper).

Their proposal of reconstructing scientific change within conceptual spaces seeks thus to show as the main aims of epistemological work should turn toward the analysis of the status of the underlying conceptual dimensions, how they are related, which are deemed fundamental and which are not, and what are their measurement procedures[50]. Their basic strategy has been that to identify empirical theories not via theory *cores* (in the technical sense of structuralism and constructivism), but by using conceptual dimensions in a more direct way: a reconstruction of the framework underlying a theory in conceptual spaces bring into focus

[48] Which can be subsumed into the wider class of adding/deletion of net nodes as well as of variations in the existing relations between these, already mentioned at the point j) of section 3.1, belonging to the relational theory of concept.

[49] For instance, in the case of Einsteinian special relativity, it was first axiomatically introduced as universal constant the speed of light c, from which it followed first a reduction of $dim_{\mathbb{Q}}\ G/\mathbb{R}^{+}$, hence a radical scientific change in the space and time conceptions, then a series of experimental testing. A quite similar argument holds for the case of the introduction of Planck's constant $\hbar$.

[50] And this in agreement with the main programmatic features of diSessa (1993), when he says that […] «*The framework* [of his 1993 work] *is aimed at answering, at a coarse level of detail, a set of questions central to a full theory of knowledge: What are the elements of knowledge; how do they arise; what level and kind of systematicity exists; how does the system as a whole evolve; and what can be said about the underlying cognitive mechanisms that are responsible for the normal operation of the system and its evolution?*», promising to answer to them through an its own original epistemological perspective.

those dimensions that belong to a theory at a particular point of the time and the measurement procedures associated with them[51]. They have adopted this strategy from dimensional analysis and extended it to revisions of conceptual frameworks, rejecting in part the notion of a (Kuhnian) paradigm shift as a historical episode to which cannot be given a rational reconstruction without citing extra-scientific factors belonging to the general sociology of scientific revolutions. In doing this, they have shown as a conceptual change in science be a rather regular process which can be (epistemological) rationalized and classified according to methodological severity, to this end having classified such changes into five main types of which only the addition/deletion of dimensions of type (5) is of truly radical nature.

3.5 Conceptual changes in learning: other general perspectives

Following Aalto et al. (2004), Özdemir & Clark (2007) and Treagust & Duit (2008) (together to the references therein quoted), conceptual change research has inspired the general learning research during the two last decades. However, there has been very little research with regard to students ideas on particular historical concepts (see (Limon, 2002)) because most of the research has dealt with science concepts. Conceptual change has been roughly defined as domain-specific form of learning, a gradual and complex process of restructuring domain-specific knowledge: according to Vosniadou (1994), the simplest form of conceptual change is called *enrichment*. In the process of enrichment, a learner is simply adding new information to an existing theoretical explanation, according to a general constructivist view. If the changes are targeted at the level of the learner's framework theory, then it is considered as the most difficult form of conceptual change. Moreover, the theories of conceptual change stress the relationships between the prior knowledge and the information to be learned. According to these theories, the relationship between prior knowledge and information to be learned is one of the most crucial factors in determining the quality of learning. The researchers (see, amongst others, Vosniadou (1994)) make a distinction between different qualities of learning processes targeted at conceptual change: namely, a continuous growth and discontinuous change. The easier level of learning is the above *enrichment* process, which suggest continuous growth, improving of the existing knowledge structure, and so on. A discontinuity of learning then means a situation where prior knowledge is incompatible with the new information and needs *revision*; where significant reorganization – not merely enrichment – of existing knowledge structures is needed, this kind of knowledge acquisition being typical in specific domains of science. Some recent progresses have been also made as concern the possible applications of conceptual change approach to mathematics (see Vosniadou &Verschaffel (2004)), hopefully with promising features.

4. On 1977 J-M. Lévy-Leblond paper

The Lévy-Leblond paper is one of the main (amongst the few ones of epistemological tendency) works related to the epistemological and educational analysis of the *status* of physical constant: herein, we will restrict to those points of his seminal paper which may be put into a some relationship to what said above, namely both to the questions inherent the linear dimensions of $G/\mathbb{R}^+$ and its diachronic variations (deduced from Gabriele Darbo work) and to the above variously treated science education (sections 3.1 and 3.2) and philosophy (section 3.4) arguments and questions.

Lévy-Leblond, throughout his 1977 work, tries answering to nine initial fundamental questions concerning the nature and the role of fundamental constants in Physics (as well as in other scientific contexts, as, for instance, the social one), always bearing in mind both the historic-critical and methodological-comparative point of view, with epistemological and educational implications where possible. Furthermore, the author above all specifies as be possible answering to these questions only relying on the understanding of physical science mainly as a *historical process*, and this coherently with what said in the previous sections; precisely, he states as follows

> *«I will try to show that the answer to these questions and other ones rely on the understanding of physical science as a historical process. Only by studying the conditions for the appearance, or disappearance, of physical constants can we understand their nature. Only by emphasizing the variations in status of a given constant, can we understand its role. Only by contrasting the opposite effects of theoretical and experimental practices upon the fate of such a constant, can we analyse its*

[51] Once again, in perfect agreement with the Darbo's considerations.

significance. The present investigation thus takes piece within a definite vision of physics, and science in general, as a social endeavour. Its ensuing historicity should then be put into light even at its seemingly most abstract and formal levels. The case of physical constants thus epitomizes this view, since their constant numerical values make sense only through a changing conceptual nature.

[…] *Not only does the type of a fundamental constant (or its absence thereof) depend on the history of physics, but it may also vary according to one's implicit epistemological position».*

Therefore, the historical emergence of a universal constant always imply certain unavoidable repercussions on the epistemic relationships between formal-theoretical structures and operationalistic-experimental bases of those physical theories in which such a constant is framed or, in a some way, involved; in particular, the discussions made above as regard the eventual variations of $dim_{\mathbb{Q}}\ G/\mathbb{R}^+$ and their epistemological meaning, especially according to the Gärdenfors' work (but also in agreement with the coordination class theory, as pointed out in section 3.4), have just shed light upon these basic epistemic relationships, pointing out the crucial role played by addition/deletion of dimensions in scientific conceptual changes, which are also related to the variations of $dim_{\mathbb{Q}}\ G/\mathbb{R}^+$, as seen. Furthermore, in the light of what said about conceptual changes, it is clear as both the 'changing conceptual nature' due to the occurrence of a physical constant and the same variations of $dim_{\mathbb{Q}}\ G/\mathbb{R}^+$, may be laid out into the general framework of conceptual changes in the sense said in section 3.1, as it will be clearer later; in particular, the latter are tightly related to a kind, as say, of *process of de-fundamentalization*[52] of physical quantities (from primitive to derived) which takes place just for the occurrence of a physical constant, whence a consequent variations of $dim_{\mathbb{Q}}\ G/\mathbb{R}^+$.

In Lévy-Leblond (1977, Section 2), the author focus on the so-called *conceptual synthesis* and *analysis* roles played by universal constants, and in order to understand these he considers the particular case of Planck constant[53] of the relation $E = \hbar\omega$, which is customarily interpreted as associating an energy E to the pulsation ω of a physical phenomenon: the connection thus established between a concept of particle mechanics, the energy of a discrete entity, and one of wave theory, the pulsation, leads to the so-called *wave-particle duality* in quantum physics and, further, to the philosophy of *complementarity*[54]. Nevertheless, the objects of quantum physics are not either waves or particles, as duality would want us to believe: there are neither waves nor particles, even though they do exhibit, under very particular circumstances, two types of limit behaviour as classical waves or classical particles

Again according to Lévy-Leblond

«Coming back to Planck constant, the quantic relationship $E = \hbar\omega$, according to this point of view, is not to be interpreted as linking two classical concepts, but rather as transcending them through their synthesis, to establish a new single concept with a broader scope. The quantum energy indeed is a new concept, since it associates to any physical state a whole spectrum of numerical values and is to be represented by a Hermitian operator, as opposed to the numerical function which represents energy in classical mechanics. Here again a new name should have been given to stress the emergence of this concept, as an intrinsic one. Energy and pulsation then appear as two particular facets of a more general notion, each of which being the only visible one from either one of two quite specific points of view. The role played here by Planck constant in bringing together these two facets is characteristic of universal constants. Any universal constant may be so described as a concept synthesizer expressing the unification of two previously unconnected physical concepts into a single one of extended validity. It will be shown below that classical constants such as k (Boltzmann constant) and J (mechanical equivalent of heat) play exactly the same role: precisely, they not only introduced new concepts but whole new theories: statistical mechanics for the first one, thermodynamics for the second».

[52] In turn, closely related to processes of *adimensionalization*.

[53] Another, much more intricate case thereof, is that regarding the Einstein field equations of General Relativity in which is involved the known *cosmological constant Λ* which, inter alia, has played a crucial role in the late Einstein attempts towards a unified field theory. However, the role of this constant is nowadays recognized to be very basic for General Theory of Relativity and its applications, whose nature and significance are yet far from being fully understood. In this regards, see also Dirac (1937).

[54] In the Niels Bohr's sense.

Hence, any universal constant, beyond to express a whole theory[55], is nevertheless not to be interpreted as simply linking two different concepts but rather as a *concept synthesizer* that, transcending them, establishes a new single concept with a broader scope and extended validity by the unification of the two previously unconnected physical concepts, so that it is evident the conceptual change role brought by them, in agreement both with the (semantic net aspect of) coordination class theory and the conceptual space one. As we will see later, the *unitarization* process, underwent by certain physical constants, expresses just such their synthesizer nature when the theory, in which they are involved, is expressed in its *natural* units, all this, in turn, being also correlated with the *adimensionalization* process (and vice versa).

Afterwards, the author considers the case of two particular constants, that of Planck and the velocity of light, whose role (above all, that of c) epitomizes what just said above, namely stating as follows

> *«In any case, the same analysis*[56] *may be applied to* c*, one use of which, for instance, is to bring together the concepts of spatial interval* Δx*, on the one hand, and time interval* Δt*, on the other hand. These are but two aspects of the more general notion* [due to Hermann Minkowski] *of a space-time interval* $(\Delta s)^2 = (\Delta t)^2 - c^{-2}(\Delta x)^2$*, which reduces to one or the other under special circumstances. Of course, any one universal constant usually brings about several such synthetical concepts. Planck constant also unifies momentum and wave number through the de Broglie relationship* $\vec{p} = \hbar\vec{k}$*, while* c *unifies mass and energy through the Einstein relationship* $\Delta E = c^2 \Delta m$*. This is easily understood, since any physical concept by essence belongs to a theoretical framework which relates it to other concepts*[57]*. The synthesis of two concepts thus is a local aspect of a more global unification of two pre-existing consistent theoretical structures. Bringing them into contact at one point usually requires fitting together of other parts as well. This is what happens when the spatio-temporal consistency of particle mechanics, on the one hand, and wave theory, on the other, requires* $\hbar$ *to play the same role with respect to momentum and wave number (space aspect) as it does with respect to energy and pulsation (time aspect). Following this point of view may lead to a better understanding of the new concepts».*

The last points of what just said, thus, also show some possible common claims between the epistemological role of physical constants and the coordination class aspects as exposed in Levrini & diSessa (2008): namely, the synthesis of two concepts, played by such constants, is a local aspect of a more global unification of two pre-existing consistent theoretical structures, this being just very near to some of the main aspects of a coordination class, that is to say, both to the processes of acquisition (by a coordination class) from a *global* view towards a *local* one (if the coordination class is thought laid out within the *humble theory* framework[58], as said in section 2) and in explaining systematicities and connections between ideas rather than focussing predominantly on the quasi-independence of the various elements, as said in section 3.1; all this is then also contemplated by Gärdenfors theory, as seen. To further confirmation thereof, the author states that

> *«Universal constants thus express synthetical transcending not of isolated pairs of concepts, but of whole conceptual arrays. In this sense, a universal constant is a ''theory synthesizer'', more than a mere ''concept synthesizer''. From this abstract point of view, the various specific syntheses expressed through a universal constant between various pairs of concepts belonging to two theoretical frameworks (for instance, the three formulae* $E = \hbar\omega$*,* $\vec{p} = \hbar\vec{k}$ *and* $L = \hbar m$*) are but equivalent consequences of the general theoretical unification of these frameworks. Nevertheless, because of historical considerations and epistemological motivations, they are not actually given an equal status, especially in educational practice. Some of them are taken as a starting point or fundamental hypothesis, such as* $E = \hbar\omega$ *or* $(\Delta s)^2 = (\Delta t)^2 - c^{-2}(\Delta x)^2$*, while other ones are considered as derived relations, or consequences, such as* $\vec{p} = \hbar\vec{k}$*, or* $\Delta E = c^2 \Delta m$*. Since one has to start*

[55] To further corroboration of this, in Lévy-Leblond (1977, Section 3.1), it is discussed a meaningful critical comparison between Galileian and Einstenian relativities, in respect to the electromagnetic field transformations, which result to be related to the aforesaid idea that a universal constant does not underlie a single concept but a whole theoretical framework.

[56] That is to say, the aforementioned synthesizer and splitting epistemological analysis played by fundamental constants.

[57] Take into account what said about coordination class model.

[58] And indeed, as already said, a humble theory is, amongst other, a deliberate attempt to step back from grand (or global) theories to more local ones that are, at once, more specific thus more easily applicable to various cases.

somewhere, it is probably true that the equivalence of all such expressions, as reflecting various aspects of one and the same synthetical process through a given universal constant, is bound to remain a rather abstract statement. Its acceptance, however, may pave the way to a modification of the traditional hierarchy. As an example, it has been recently proposed to develop Einsteinian relativity by starting directly from the mass-energy relationship $\Delta E = c^2 \Delta m$*, by building upon it the ''relativistic'' concepts of energy and momentum, and then by deriving from them the theoretical structure of space-time*[59]*. After all, this corresponds much more closely to the real needs of physics in which the Lorentz transformation formulae or invariant expressions are actually much more used for momentum-energy quantities them for space-time ones. Moreover the role of* c *in relating energy and inertia (rather than mass) is deeply rooted in the immediate prehistory of Einsteinian relativity, and 'could' have been the starting point of another historical path towards this theory. These considerations, clearly, are of some epistemological and pedagogical importance*[60]*»*.

In particular, when he says that […] *«Since one has to start somewhere, it is probably true that the equivalence of all such expressions, as reflecting various aspects of one and the same synthetical process through a given universal constant, is bound to remain a rather abstract statement. Its acceptance, however, may pave the way to a modification of the traditional hierarchy»*, it is clear the recalls above all to alignment problems of a coordination class (outlined at the end of section 3.2), according to which each possible concept projection should get the same result, but also to the epistemological basis of conceptual change; in this regards, the author further states as follows

«[…] *the role of universal constants in the synthesis and unification of previously unrelated concepts or sets thereof, if it is the prime one in historical order of appearance, has for a corollary the fact of their leading to split and separate previously fused, if not confused, concepts. Two simple examples in relativity theory may be given here. The first one deals with the impossibility in Einsteinian relativity of a concept with the following two properties of the velocity in Galilean relativity: i) being an additive quantity, that is the canonical parameter of Galilean transformations, obeying the simple composition law* $v_{12} = v_1 + v_2$ *and ii) giving the time rate of spatial change, namely* $v = dx/dt$*, for uniform motions. In Einsteinian relativity, if the second property is used as a definition of what we will keep calling '' velocity''* v*, the first one will hold true for another quantity, the so-called ''rapidity''* φ*. The two quantities are related by* $v = tgh\,\varphi$*, or, with dimensional notations, by* $v = c\,tgh\,(\varphi/c)$*, which makes apparent their fusion in the limit as* $c \to \infty$*. The introduction of the concept of rapidity is of a major help for educational purposes*[61]*. It not only yields a more compact and more significant expression for Lorentz transformations via hyperbolic functions, but it explains away the pseudoparadoxes associated to the idea of a limiting velocity or non-additivity of velocities, as simply due to a bad choice of parameter, such as would occur if rotations were labelled through the tangent of the angle instead of the angle itself. In recent years, the concept of rapidity has also been*

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[59] See Davidon (1975). In this work, the author first points out the need for a separation between kinematical and dynamical quantities, avoiding illegal and indiscriminate commingling amongst them, for then outlines the axiomatic structure of special relativity, distinguishing between its kinematics part from the dynamical one. Thereafter, he assumes the inertial equivalence of energy as main basic assumption of relativistic dynamics, focusing on the so-called *inertial equivalence of energy*, that is to say $1/c^2$, hence discussing some related physical problems, as moving clocks, Lorentz contractions and certain electromagnetic questions, always identifying and distinguishing the various kinematical and dynamical symmetry aspects of the question. In this regards, he consider the possible cases inherent the sign of $1/c^2$, relating these to symmetry properties arising from Lorentz and Poincaré groups, as well as their relationships via group contractions techniques concerning the mathematical limiting processes as $1/c^2 \to 0$ (equivalent to $c \to \infty$) and their possible physical implications. Precisely, from this, it follows that the inertial equivalent of energy allows to distinguish between physical theories having different symmetry groups, hence identifying the various involved geometrical properties, as well as, with a plausible extrapolation, to characterize, through it, each physical theory in which c play a considerable role; all that, however, will be clearer later when we'll argue on the general limiting processes. At last (but not least, as usually it is said), the author also affirms that […] *«Each approach builds upon certain historical developments. An awareness of these can contribute both to an increased appreciation for coherent human effort and accomplishment as well as to better grasp of contemporary physical concepts»*, hence emphasizing what fundamental role may play the historical approach also from an education viewpoint (in agreement with the last conclusions of Levrini & diSessa (2008)).

[60] See Lévy-Leblond (1976).

[61] See Taylor & Wheeler (1966).

fruitfully used in high-energy phenomenology. A similar clarification may be achieved in relativistic dynamics, by introducing, with the concepts of energy and mass, the one of inertia, defined as the coefficient of the velocity in the expression for the momentum[62]*. It is seen then that inertia is to be identified with energy in Einsteinian relativity, but with mass in Galilean relativity*[63]*. The occurrence of the universal constant* c *then splits inertia from mass us it fuses it with energy. In the description of space-time, the splitting of the categories of simultaneous pairs of events from that of invariant intervals (now the 'lightlike' ones) may be interpreted in quite the same way. Other examples can be found at will. To use the same material metaphor as above, it may be said that the fitting of two conceptual structures, while bringing into contact previously separated pieces, also generates stresses requiring various splits within the new body».*

Thus, just when the author states that […] *«The occurrence of the universal constant* c *then splits inertia from mass us it fuses it with energy. In the description of space-time, the splitting of the categories of simultaneous pairs of events from that of invariant intervals (now the 'lightlike' ones) may be interpreted in quite the same way»*, it is again evident the recalls to one of the characteristic properties of a coordination class in part mentioned in *1)* of section 2.2, namely that according to which it is specifically oriented toward *entering* the structure of each concept (rather than viewing it only in relation to other concepts) and analyzing it as a coordination of the various pieces of knowledge involved; concept maps and semantic networks, for example, show the *external* relations of concepts with other ones[64], but coordination classes aim to show the *inner* relations of parts of a concept that make it well formed and powerful (see Levrini & diSessa (2008)).

Subsequently, Lévy-Leblond (1977, Section 2.2) argues on the question of units and unitarization (hence, also of adimensionalization) of physical constants, in partial agreement with what already said above in discussing the variations of $dim_{\mathbb{Q}}\, G/\mathbb{R}^+$, which is also endorsed by Gärdenfors arguments. Namely, he first states as follows

[…] *«While all textbooks and articles of the twenties keep a detailed record of all* $\hbar$*'s and* c*'s in their formulae, it is the common use today to take them as unity, which only means adopting a more adapted system of units. This convention has become almost tacit in the recent years, so that, except perhaps at the educational level, it will soon be obvious that there is no difference of nature between* $\hbar$, c, *on the one hand, and* k, J, *on the other. This then is the ordinary fate of universal constants: to see their nature as concept synthesizers being progressively incorporated into the implicit common background of physical ideas, then to play a role of mere unit conversion factors and often to be finally forgotten altogether by a suitable redefinition of physical units. Once this is realized, one may well ask how many of these forgotten universal constants are lying around*[65]*».*

Hence, he discuss many examples in which are involved physical constants that may be, or not, unitarized in dependence on the physical problem context at issue: when a physical constant undergoes the process of unitarization/adimensionalization, it fades into a sort of 'Nirvâna of unity and oblivious', and, in this regards, Lévy-Leblond (1977, Section 2.3) says what follows

«It is only from the theorist's point of view that the life of a universal constant reaches the happy end of such a drift into the Nirvâna of unity and oblivion. The experimentalists working in the laboratory, when making measurements, must use concrete definitions of their units and cannot at will identify two operationally independent standards as the theorists on the paper do. It is a fact that, whatever fundamental system of units is adopted, based on the theoretical knowledge of the time, the use of units belonging to various other systems adapted to such and such domain of physics, cannot be eliminated together. There are two reasons for this state of affairs. The first one is historical social inertia, which, for instance, forces the experimental physicists on the other side of the Atlantic to plan and order the

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[62] See Davidon (1975).

[63] See Lévy-Leblond (1976).

[64] And this may be correlated with what previously said about the 'concept synthesizer' role played, inter alia, by universal constants.

[65] Thereafter, the author reports another interesting critical discussion drawn from Taylor & Wheeler (1966) against this indiscriminate tendency towards such a unitarization process.

nuts, bolts, plates, rods, etc. of their apparatus, by stating their dimensions in feet and inches rather than in metres and centimetres. The universal constant χ *entering the relationship* $l_{US} = \chi l_{EU}$ *between the length of some object in the United States and the length of the same one in Europe (so that the subscript « EU» refers to us, while «US» refers to you) thus can be taken as unity in principle – but in principle only.*

[…] *The relationship between volumetric measurements with the litre as a primary unit and linear measurements in metres (for instance) requires the experimental determination of the universal constant in the relationship* $V = \beta L^3$ *between volumes and lengths; the constant is in fact* $\beta = 1.000028 \pm 0.000004\, litre \cdot dm^{-3}$*, which, of course, can be taken as unity – by the theorists. Once more, it should be pointed out that the ''noble'' universal constant* c *is not different in principle from this* β*».*

The remainder of the section 2.3 of Lévy-Leblond (1977) is a beautiful and careful examination of why it is not always possible or advantageous an indiscriminate unitarization/adimensionalization of physical constants[66] within the natural unit system of the given theory in which they are involved, notwithstanding the theoretical synthesizer role that such a process might have. Indeed, only motivated historical reasons can carry out such a program which, as seen, has also a fundamental epistemological role in order to conceptual changes may arise

Subsequently, the final Section 3 of Lévy-Leblond (1977) deals with the case of vanishing constants and related limiting passages as, for instance, those involving the passages to the limit given as $c \to \infty$ (or as $1/c \to 0$), in the reduction of Einsteinian relativity to the Galileian one, and as $\hbar \to 0$ in the classical limit of quantum mechanics. The author stress as these passages have only a mere mathematical sense within the related mathematical frameworks since a universal physical constant cannot freely change its value in such a manner, hence arguing on what should be the real physical meaning of these formal passages and in what approximation schemata should be laid out. These limiting processes, then, are analyzed respectively in their *unicity*, *singularity* and *validity* aspects (using a Kuhnian terminology): as concerns their unicity, it is pointed out as they may lead to different limit theories like in the emblematic case given by the Galileian and the Carrollian ones as limit cases of the Einsteinian theory when is put $c = 1$, under certain approximation conditions[67]. In this regards, then, the author states that

«If one is to study the possible limits of a theory, one must start from this theory as such, expressed within its autonomous system of concepts and intrinsic units. Once more it is seen how much the universal constants, even in the very most technical formulae, bear the mark of the historical developments of physics. It has been mentioned above […] *that the Carroll group probably is of little physical interest, so that the above considerations might seen of academic significance»*,

in accordance with one of the main characteristics of a coordination class recalled in section 2.2, namely that according to which it is specifically oriented towards *entering* the structure of each concept (rather than viewing it only in relation to other ones) and analyzing it as a coordination of the various pieces of knowledge variously involved; concept maps and semantic networks, for example, show the *external* relations of concepts with other ones, but coordination classes aim to show the *inner* relations of parts of a concept that make it well-formed and powerful. Subsequently, Lévy-Leblond (1977) argues on the singularity of these limit processes remembering the Kuhnian epistemological theory of scientific revolutions and normal science and focusing in particular on *paradigm's shift* undergoing any radical change in the historical development of a given scientific doctrine field[68] and declaring as this paradigm replacements by limiting passages take place just like a singular limit resembling the mathematical one; precisely, he states that

«Kuhn has argued that the history of science proceeds through ''scientific revolutions'' in between which scientific activity would consist of ''normal science''. These revolutions would bring about the replacement of old paradigms by new ones, such that the ideas and concepts would undergo radical

[66] To this purpose, see also what has already been said at the end of section 2.1.

[67] See also Bernardini (1991, Capitolo 1).

[68] With some severe reservation as regards the mathematics which, according to Kuhn, it does not fall in this general process of scientific revolution; but, in this regards, see also what say Vosniadou & Verschaffel (2004).

changes. For instance, mechanics is supposed to be so affected by the Einsteinian scientific revolution that our ideas on space-time kinematics and dynamics have nothing in common any more with those of Newtonian physics. Such strong statements, obviously, are contrary to all our experience as working and teaching physicists. The difficulty here is that of the apparent dilemma between a continuous view of the history of science which would deny any qualitative change, and a discontinuous one which, finally, cannot interpret the process of change from one stage to the other. This is not the place to attempt a global evaluation of Kuhn's sociological history of science. One restricted aspect of his views, however, is closely related to the present investigation, namely the nature of the relationship between two successive ''paradigms'' in a given scientific domain. Taking as an example Kuhn's one of Einstein vs. Newton mechanics, let us try to put it into perspective. This historical perspective it must be emphasized first, needs a backward look. Obviously, the relationship between some physical theory and a more general successor cannot be studied until the generalization has succeeded. It is then necessarily from the point of view of the new, more encompassing, paradigm that the old one in to be judged. There is no vantage point, outer to both, from which their borderline could be seen and the transition analysed. We have to assess Newtonian mechanics starting from the Einsteinian one. In other words, the epistemological approach is necessarily opposite to the chronological one. It may be suggested then that this approach is that of a singular limit in the mathematical sense. This statement, first, is certainly true at the factual level. Indeed the restricted theories, Galileian relativity or, more generally, Newtonian mechanics, are obtained from the modern more general ''relativistic'' theory by a limit process which is necessarily singular. If it were not, the change would amount to a simple rescaling, without any conceptual modification. It is only in the limit in which c goes to infinity, and not when it is arbitrarily large but finite, that the old theory is recovered. In the case of relativity theory, a definite mathematical framework exists, the theory of contraction of groups[69], showing how a continuous family of group isomorphisms depending upon some parameter may tend towards a singular limit, whereby a new, non-isomorphic, group is obtained. But the idea of the old paradigm as a singular limit of the new one is proposed here in a wider, metaphorical sense, as well. It may help in understanding how the transition from one to the other, as expressed by the vanishing (or infinity) of some universal constants, brings about qualitative changes into the conceptual tools of the trade. Indeed, if a universal constant brings about the synthesis and the unification of two previously unconnected concepts, its vanishing must be shown to give rise to the converse disjunction, clearly a very singular phenomenon. This is the only way to understand, for instance, how the quantum energy-pulsation branches off into classical particle energy and classical wave pulsation. As another example, Einstein mechanics knows of only two conserved quantities, energy and momentum, while Newtonian mechanics imposes the further conservation of mass, but introduces another, non-conserved quantity, namely internal energy. In this case, clearly, it is the Einsteinian mass, which in the Galileian limit yields both a conserved mass m and non-conserved internal energy U; of course Einstein's role was precisely to operate the inverse synthesis through the relationship $U = mc^2$. Let it be clear, however, that the singularity may be that of a coalescence of concepts as well as of a disjunction, since we deal here with the converse processes to both the syntheses and the splittings described above. But this aspect is rather trivial here, consisting for example in the merging of the rapidity φ with the velocity $v = tgh\,\varphi$ in the Galileian limit».

It is evident as what has been just said remembers what above said about conceptual changes and their occurrence, in particular being noteworthy the fact that «[…] *the epistemological approach is necessarily opposite to the chronological one*[70]*. It may be suggested then that this approach is that of a singular limit in the mathematical sense. This statement, first, is certainly true at the factual level.* […] *If it were not, the change would amount to a simple rescaling, without any conceptual modification»*, just resembling much of what has been already said about conceptual changes according to Gärdenfors, as well as in agreement with what stated by Vosniadou & Verschaffel (2004) according to which such changes cannot be achieved through additive or constructive mechanisms but rather both by means of the general mechanisms of

[69] See Inönü & Wigner (1953); see also Saletan (1961) for general methods of Lie group contraction and what said in the previous footnote [59] as regards the possible physical meaning of such limiting processes as $c \rightarrow \infty$ which is equivalent to $1/c^2 \rightarrow 0$.

[70] Coherently with what Gärdenfors says about the fact that the rationality of a dimensional change (hence, of a conceptual change) can only evaluated *post hoc*.

incorporation and displacement, and overcoming the span and alignment problems (see subsection 3.2). Since, thereafter, it has been pointed out as not all limiting processes lead to physically meaningful theories, a related validity problem naturally arise in warranting the physical relevance of such possible limiting theories, and, in this regards, Lévy-Leblond argues upon the paradigmatic case of quantum mechanics with its classical limits. On the other hand, as seen in sections 3.3 and 3.4, just the passage to limit as $c \to \infty$ plays a fundamental role from the conceptual change stance: in any way, both in Levrini & diSessa (2008) and in Gärdenfors & Zenker (2012), it is not fully useless to consider, in their respective cases study, this limiting process applied respectively to the proper time and to the rest mass (in discussing the 4-momentum and its limit[71]), also in view of their possible educational features (which besides are often mentioned in many points of Lévy-Leblond paper). We conclude this section with the last words of the 1977 Lévy-Leblond paper, namely the following

> *«It may serve as a useful conclusion then by regarding us that understanding the role of the physical constants is but the beginning of a concrete physical analysis, and only helps in asking the right questions, which now are left to be answered».*

5. Conclusions

Simply, both the Gärdenfors and co-workers epistemological theory, in part formalized by the Darbo's considerations, and the diSessa and co-workers work on coordination classes and conceptual changes, have constituted the main opportunities to make a reasonable *parerga und paralipòmena* of the basic 1977 J-M. Lévy-Leblond paper, by means of the various possible grafts made to it in the preceding section.

To sum, on the one hand, the Darbo's theory of physical quantities, *in primis* may to be considered as a tool for the formalization of the Gärdenfors' conceptual spaces which, as seen, is fundamentally centred on dimensional analysis; *in secundis*, it has given, through the diachronic evolution of $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$, a clear demarcation line between mathematics and physics, specifying which roles they may reciprocally played and pointing out on the impossibility of avoiding an *operationalist* stance in Physics both from epistemological and educational perspective. Since, then, all these considerations are variously present, both explicitly and implicitly in the 1977 Lévy-Leblond paper, we may therefore state as the Darbo's work be, in first instance, a formal work which has been used as *parergon* to the latter, also by means of the basic Gärdenfors' work. Hence, both the Darbo's work and part of the Gärdenfors one, may be considered as *parerga* to this basic 1977 Lévy-Leblond paper. On the other hand, instead, the same Gärdenfors work together with the diSessa and co-workers one on coordination classes, may be considered as *paralipòmena* of such a Lévy-Leblond paper, which then show as the physical constant framework, with its synthetical and splitting processes, the historical variations of $dim_{\mathbb{Q}}\, G/\mathbb{R}^{+}$ and so on, might be also considered both as a coordination class construct[72] as well as a conceptual space model, hence belonging into the intersection between the two classes of coordination class models and conceptual space ones, in turn both falling within the wider class of conceptual change patterns. And all this it has been possible mainly thanks to the qualitative dimensional analysis, in accordance with what the same diSessa and Sherin (1998) have stated about a certain importance played by qualitative methods in coordination class theory (as recalled in Section 3.1) which, from what sketched in the present paper, seems to deserve a major attention in physics education research (PER).

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[71] Just the case study related to the 4-momentum $p = \gamma m_0 v = mv$, where γ is the Lorentz factor, and its limiting passage as $c \to \infty$, it has been taken by T.S. Kuhn as a paradigmatic case of radical meaning change (called *rupture*), which can be related with the analogous case study discussed by Levrini & diSessa (2008) in account of proper time.

[72] We use the term *coordination class (or system)* to describe a collection of readout strategies and causal net elements that do not necessarily meet the criteria of integration and invariance (see Thaden-Koch et al. (2003)).

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