Formal statement of the special principle of relativity

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Abstract

While there is a longstanding discussion about the interpretation of the extended, general principle of relativity, there seems to be a consensus that the *special* principle of relativity is absolutely clear and unproblematic. However, a closer look at the literature on relativistic physics reveals a more confusing picture. There is a huge variety of, sometimes mataphoric, formulations of the relativity principle, and there are different, sometimes controversial, views on its actual content. The aim of this paper is to develop a precise language in order to provide a precise formulation of the principle. In view of the fact that the special relativity principle is considered as a universal meta-law, which must be valid for all physical laws in all situations, we try to keep the formalism as general as possible. The benefit of the formal reconstruction is that it makes explicit all the necessary conceptual components of the principle; it brings out many subtle details and the related conceptual problems.

1 Introduction

While there is a longstanding discussion about the interpretation of the extended, general principle of relativity, there seems to be a consensus that the *special* principle of relativity (RP) is absolutely clear and unproblematic. However, a closer look at the literature on relativistic physics reveals a far more complex picture. There is a huge variety of, sometimes metaphoric, formulations of the relativity principle, and there are different, sometimes controversial, views on its actual content. Let us illustrate this with only a few quotations: "the laws of physical phenomena should be the same, whether for an observer fixed, or for an observer carried along in a uniform movement of translation" (Poincaré 1956, p. 167); "If a system of coordinates K is chosen so

that, in relation to it, physical laws hold good in their simplest form, the same laws hold good in relation to any other system of coordinates K' moving in uniform translation relatively to K." (Einstein 1923. p. 111); "it is impossible to measure or detect the unaccelerated translatory motion of a system through free space or through any ether-like medium" (Tolman 1949, p. 12); "all physical phenomena should have the same course of development in all system of inertia, and observers installed in different systems of inertia should thus as a result of their experiments arrive at the establishment of the same laws of nature" (Møller 1955, p. 4); "the laws of Physics take the same mathematical form in all inertial frames" (Sardesai 2004, p. 1); "The same laws of nature are true for all inertial observers." (Madarász 2002, p. 84)[1] "The uniform translatory motion of any system can not be detected by an observer traveling with the system and making observations on it alone." (Comstock 1909, p. 767); "The laws of nature and the results of all experiments performed in a given frame of reference are independent of the translational motion of the system as a whole. More precisely, there exists a [...] set of equivalent Euclidean reference frames [...] in which all physical phenomena occur in an identical manner. (Jackson 1999, p. 517); "If we express some law of physics using the quantities of one inertial frame of reference, the resulting statement of the law will be exactly the same in any other inertial frame of reference. [...] we write down exactly the same sentence to express the law in each inertial frame." (Norton 2013); "all inertial frames are equivalent for the performance of all physical experiments" (Rindler 2006, p. 12); "the laws of physics are invariant under a change of inertial coordinate system" (Ibid., p. 40); "The outcome of any physical experiment is the same when performed with identical initial conditions relative to any inertial coordinate system." (Ibid.); "experience teaches us that [...] all laws of physical nature which have been formulated with reference to a definite coordinate system are valid, in precisely the same form, when referred to another co-ordinate system which is in uniform rectilinear motion with respect to the first. [...] All physical events take place in any system in just the same way, whether the system is at rest or whether it is moving uniformly and rectilinearly." (Schlick 1920, p. 10); "laws must be Lorentz covariant. Lorentz covariance became synonymous with satisfaction of the principle of relativity" (Norton 1993, p. 796); "The laws of physics don't change, even for objects moving in inertial (constant speed) frames of reference." (Zimmerman Jones and Robbins 2009, p. 84); "the basic physical laws are the invariant relationships, the same for all observers" (Bohm 1996, p. viii); "laws of physics must satisfy the requirement of being relationships of the same form, in every frame of reference" (Ibid. p. 54).

We are hopeful that our analysis in this paper helps to clarify many of the aspects reflected in the above quotations. Our aim is to develop a precise language in order to provide a precise formulation of the principle. In view of the fact that the special relativity principle is considered as a universal meta-law, which must be valid for all physical laws in all situations, we try to keep the formalism as general as possible. The benefit of the formal reconstruction is that it makes explicit all the necessary conceptual components of the principle; moreover, it brings out many subtle details and the related conceptual problems.

[1] This verbal statement is formalized on page 85.

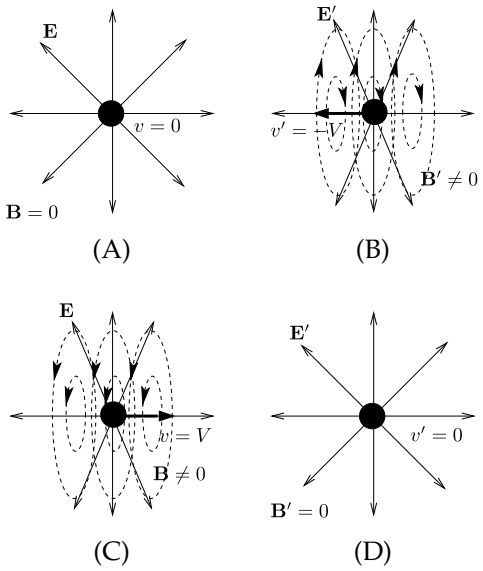

Figure 1: The descriptions of the same phenomenon in different inertial frames are different: (A) and (B). In contrast, different phenomena, (A) and (C), have descriptions of the same form in the two different (co-moving) inertial frames: (A) and (D)

2 Preliminary considerations

In trying to understand the precise meaning of the principle one encounters several obvious questions. First of all, it must be clear that the laws of physics "*in*" or "in relation to" a reference frame K are meant to be the laws of physics as they are ascertained by an observer living in reference frame K; less anthropomorphically, as they appear in the results of the measurements, such that the measuring equipments—and in some sense the objects to be measured, too—are *at rest relative to* K. At this point we encounter the first, and, as will be discussed below, highly non-trivial conceptual problem: when can we say that a physical object, in general, is at rest relative to an inertial frame of reference?

Of course, it is the *same* laws of physics which must take the same form in all inertial frames. It would be absurd to require that, say, the second law of thermodynamics in K must have the same form as Newton's force law in K'. But, what are the same laws of physics in different inertial frames? It is quite natural to say that the laws of physics can be identified by means of the physical phenomena they describe. If so, then one can think that the descriptions of the *same physical phenomenon* must have the same form in all frames of reference; the same physical phenomenon must "have the same course of development in all system of inertia". This is however obviously not the case. For example, consider the electromagnetic field of a charged particle at rest in K. This phenomenon is described in K as it is depicted in Fig. 1A. As is well known, the description of the *same* phenomenon in K' is completely different, Fig. 1B—just take the Lorentz transformation of the situation in Fig. 1A. (For more details, see Remark 6.)

Thus, the opposite must be true: the RP is about *different* physical phenomena; different phenomena must have descriptions of the same form in the different inertial frames of reference. In our example, 'the static electromagnetic field of the rest charge' is *one* phenomenon (Fig. 1A) and 'the time-dependent stationary electromagnetic field of the same charge in motion with velocity $v = V'$ is the *other* (Fig. 1C). What the RP asserts is this: the description in the co-moving inertial frame K' of the phenomenon depicted in Fig. 1C takes exactly the same form as the description of the phenomenon in Fig. 1A in inertial frame K (see Fig. 1D). But, in what general sense these two phenomena are the counterparts of each other?

The next problem is how the phrase "same form" should be understood. For, formulas (equations, relations, functions, etc.) which are—for example logically—*equivalent* may have completely different forms/shapes in some algebraic/typographic sense. So, "same form" must be understood as "same form up to some equivalent transformations". Generally, two formulas must be regarded as equivalent if they express the same physical content, in the sense that they determine the same relations between the same physical quantities.

This immediately raises the next question: How do we identify the physical quantities defined by the different observers in different inertial frames? As Grøn and Vøyenli (1999, p. 1731) points out:

> A law fulfilling the restricted covariance principle, has the same mathematical form in every coordinate system, and it expresses a physical law that may be formulated by the same words (without any change of meaning) in every reference frame [...]

In our understanding, the "meanings of the words" by which a physical law is formulated are determined by the empirical/operational definitions of the quantities appearing in the law. The obvious solution is, therefore, that we identify the physical quantities which have identical empirical definitions. It is however far from obvious how these identical empirical definitions are actually understood. For the empirical/operational definitions require *etalon* measuring equipments. But how do the observers in different reference frames share these *etalon* measuring equipments? Do they all base their definitions on the same *etalon* measuring equipments? Is the principle of relativity really understood in this way? Is it true that the laws of physics in K and K', which ought to take the same form, are expressed in terms of physical quantities defined/measured with the same standard measuring equipments? Not exactly. Consider how Einstein describes a simple application of the relativity principle in his 1905 paper:

> Let there be given a *stationary* rigid rod; and let its length be l as *measured by a measuring-rod which is also stationary*. We now imagine the axis of the rod lying along the axis of x of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity v along the axis of x in the direction of increasing x is then imparted to the rod. We now inquire as to the length of the *moving* rod, and imagine its length to be ascertained by the following *two* operations:
>
> (a) The observer *moves together with the given measuring-rod and the rod to be measured*, and *measures the length* of the rod directly

by superposing the measuring-rod, *in just the same way as if all three were at rest*.

(b) By means of *stationary* clocks set up in the *stationary* system and synchronizing in accordance with [the light-signal synchronization], the observer ascertains at what points of the *stationary* system the two ends of the rod to be measured are located at a definite time. The distance between these two points, measured by the measuring-rod already employed, which in this case is *at rest*, is also a length which may be designated "the length of the rod."

In accordance with the principle of relativity the length *to be discovered by the operation (a)*—we will call it "the length of the rod in the moving system"—must be equal to the length l of *the stationary* rod.

The length to be discovered by the operation (b) we will call "the length of the (*moving*) rod in the *stationary* system." This we shall determine on the basis of our two *principles*, and we shall find that it *differs* from l. [all italics added] (Einstein 1905, pp. 41–42)

That is to say, if the standard measuring equipment by means of which the observer in K defines a physical quantity ξ is at rest in K and, therefore, moving in K', then the observer in K' does not define the corresponding ξ' as the physical quantity obtainable by means of the original standard equipment—being at rest in K and moving in K'—but rather as the physical quantity obtainable by means of the standard equipment *in another state of motion*; namely, at rest relative to K' and in motion relative to K. Consequently the measurement operations and the measurement outcomes in K' are not the same physical phenomena as their counterparts in K.

Taking into account the above considerations, as a first step toward the precise formulation, we give the following preliminary formulation of the principle (Szabó 2004):

The description of a phenomenon exhibited by a physical system co-moving as a whole with an inertial frame K, expressed in terms of the results of measurements obtainable by means of measuring equipments co-moving with K, takes the same form as the description of the same phenomenon exhibited by the same physical system, except that the system is co-moving with another inertial frame K′, expressed in terms of the measurements with the same equipments when they are co-moving with K′.

3 Conceptual components of the RP

Let K and K' be two *arbitrary* inertial frames of reference; and let $\mathbf{V}$ be the velocity of K' relative to K.

Measurement outcomes in K Denote by Σ the set of all possible measurement operations with certain measuring devices being *at rest* in inertial frame K. Let σ_s denote the set of the possible outcomes of measurement $s \in \Sigma$. We assume that $\sigma_{s_1} \cap \sigma_{s_2} = \emptyset$ for all $s_1 \neq s_2$. Let E denote the union of all possible outcomes of all possible measurement: $E \overset{def}{=} \bigcup_{s\in\Sigma} \sigma_s$.

Measurement outcomes in K' Similarly, denote by Σ' the set of all possible measurement operations with certain measuring devices being *at rest* in inertial frame K'. Let $\sigma'_{s'}$ denote the set of the possible outcomes of measurement $s' \in \Sigma'$. We assume that $\sigma'_{s'_1} \cap \sigma'_{s'_2} = \varnothing$ for all $s'_1 \neq s'_2$. Finally, $E' \stackrel{def}{=} \bigcup_{s' \in \Sigma'} \sigma'_{s'}$.

The counterparts The RP requires identification between the physical quantities in different inertial frames; we need to be able to say which measurement operation and measurement outcome in K correspond to which measurement operation and measurement outcome in K'. In other words, we need to have a pair of one-to-one maps

$$P_1 : E \to E'$$
$$P_2 : \Sigma \to \Sigma'$$

such that for all $s \in \Sigma$,

$$P_1(\sigma_s) = \sigma'_{P_2(s)} \tag{1}$$

Remark 1. P_1 and P_2 are not simply arbitrary one-to-one relations between the measurement operations and measurement outcomes somehow assigned to inertial frames K and K', satisfying (1), but they must express the following physically meaningful correspondence: $P_2(s)$ must be the same measurement operation with the same measuring equipment as measurement s, except that everything is in a collective motion with velocity $\mathbf{V}$ relative to K. Similarly, the measurement outcome $P_1(\omega_s)$ must be the same physical phenomenon as the measurement outcome ω_s, except that everything is in a collective motion with velocity $\mathbf{V}$ relative to K. For example, if measurement outcome ω_s consists in that the pointer of a measuring device at rest relative to K is in a certain position, then the measurement outcome $P_1(\omega_s)$ must be the phenomenon consisting in that the pointer of the same measuring device is in the same position, except that everything—the device, the pointer, the scale—is in a collective motion with velocity $\mathbf{V}$ relative to K. ⌟

For the sake of simplicity, in what follows we restrict the discussion for a finite number of measurements $s_1, s_2, \ldots s_n \in \Sigma$. Let Ω denote the set of the possible outcome combinations:

$$\Omega \stackrel{def}{=} \times_{i=1}^{n} \sigma_{s_i}$$

The counterparts are $P_2(s_1), P_2(s_2), \ldots P_2(s_n) \in \Sigma'$ and

$$\Omega' \stackrel{def}{=} \times_{i=1}^{n} \sigma'_{P_2(s_i)}$$

Putting primes Maps P_1 and P_2 determine the following bijection

$$\begin{aligned} P : \quad & \Omega \to \Omega' \\ & (\omega_1, \omega_2, \ldots \omega_n) \mapsto (P_1(\omega_1), P_1(\omega_2), \ldots P_1(\omega_n)) \end{aligned} \tag{2}$$

Admissible values In the above sense, the points of Ω and the points of Ω' range over all possible measurement outcome combinations $(\omega_1, \omega_2, \ldots \omega_n)$ and $(\omega_1', \omega_2', \ldots \omega_n')$. It might be the case however that some combinations are impossible, in the sense that they never come to existence in the physical world. Let us denote by $R \subseteq \Omega$ and $R' \subseteq \Omega'$ the physically admissible parts of Ω and Ω'. Note that $P(R)$ is not necessarily identical with R'.[2]

Transformation law For the formulation of the RP we need to introduce the concept of what we usually call the "transformation" of physical quantities; another bijection between Ω and Ω', more precisely, between R and R'. It is conceived as a bijection

$$\Lambda : \Omega \supseteq R \to R' \subseteq \Omega' \tag{3}$$

determined by the observable fact that whenever the measurements $s_1, s_2, \ldots s_n$ have outcomes $(\omega_1, \omega_2, \ldots \omega_n) \in R$ then the outcomes of the measurements $P_2(s_1), P_2(s_2), \ldots P_2(s_n)$ are $\Lambda(\omega_1, \omega_2, \ldots \omega_n) \in R'$, and vice versa. Since $s_1, s_2, \ldots s_n$ can be various physical measurements in the various contexts, nothing guarantees that such a bijection exists. We assume however the existence of Λ.

Numeric values To bring our formalism closer to the ordinary language of physics, without serious loss of generality, we assume that the measurement outcomes can be labeled by real numbers. That is, there exist two coordinate maps $\phi : \Omega \to \mathbb{R}^n$ and $\phi' : \Omega' \to \mathbb{R}^n$, such that for all $\omega \in \Omega$,

$$\phi(\omega) = \phi'(P(\omega)) \tag{4}$$

Let us denote the coordinates by $(\xi_1, \xi_2, \ldots \xi_n) = \phi(\omega)$ and $(\xi_1', \xi_2', \ldots \xi_n') = \phi'(\omega')$. We will refer to them as real valued physical quantities measured by the measurements $s_1, s_2, \ldots s_n$ and $P_2(s_1), P_2(s_2), \ldots P_2(s_n)$, respectively.

Remark 2. In spite of (4), it must be emphasized that $\xi_1, \xi_2, \ldots \xi_n$ and $\xi_1', \xi_2', \ldots \xi_n'$ are *different* real valued physical quantities, due to the fact that the operations by which the quantities are defined are performed under *different* physical conditions. Thus, the same numeric values, say, $(5, 12, \ldots 61) \in \mathbb{R}^n$ correspond to different states of affairs when $\xi_1 = 5, \xi_2 = 12, \ldots \xi_n = 61$ versus $\xi_1' = 5, \xi_2' = 12, \ldots \xi_n' = 61$.

Remark 3. It is worthwhile to consider several examples.

(a) Let (ξ_1, ξ_2) be (p, T), the pressure and the temperature of a given (equilibrium) gas; and let (ξ_1', ξ_2') be (p', T'), the pressure and the temperature of the same gas, measured by the moving observer in K'. In this case, there exists a one-to-one Λ:

$$p' = p \tag{5}$$
$$T' = T\gamma^{-1} \tag{6}$$

[2]One can show however that $P(R) = R'$ if the RP, that is (9), holds.

where $\gamma = \left(1 - \frac{V^2}{c^2}\right)^{-\frac{1}{2}}$ (Tolman 1949, pp. 158–159).[3] A point $\omega \in \Omega$ of coordinates, say, $p = 101325$ and $T = 300$ (in units Pa and $^\circ K$) represents the class of physical constellations—the class of possible worlds—in which the gas in question has pressure of 101325 Pa and temperature of 300 $^\circ K$. Due to (6), this class of physical constellations is different from the one represented by $P(\omega) \in \Omega'$ of coordinates $p' = 101325$ and $T' = 300$; but it is identical to the class of constellations represented by $\Lambda(\omega) \in \Omega'$ of coordinates $p' = 101325$ and $T' = 300\gamma^{-1}$.

(b) Let $(\xi_1, \xi_2, \ldots \xi_{10})$ be $(t, x, y, z, E_x, E_y, E_z, r_x, r_y, r_z)$, the time, the space coordinates where the electric field strength is taken, the three components of the field strength, and the space coordinates of a particle. And let $(\xi'_1, \xi'_2, \ldots \xi'_{10})$ be $\left(t', x', y', z', E'_x, E'_y, E'_z, r'_x, r'_y, r'_z\right)$, the similar quantities obtainable by means of measuring equipments co-moving with K'. In this case, there is no suitable one-to-one Λ, as the electric field strength in K does not determine the electric field strength in K', and vice versa.

(c) Let $(\xi_1, \xi_2, \ldots \xi_{13})$ be $(t, x, y, z, E_x, E_y, E_z, B_x, B_y, B_z, r_x, r_y, r_z)$ and let $(\xi'_1, \xi'_2, \ldots \xi'_{13})$ be $\left(t', x', y', z', E'_x, E'_y, E'_z, B'_x, B'_y, B'_z, r'_x, r'_y, r'_z\right)$, where B_x, B_y, B_z and B'_x, B'_y, B'_z are the magnetic field strengths in K and K'. In this case, in contrast with (b), the well known Lorentz transformations of the spatio-temporal coordinates and the electric and magnetic field strengths constitute a proper one-to-one Λ. ⌟

Description of a phenomenon Next we turn to the general formulation of the concept of *description of a particular phenomenon* exhibited by a physical system, in terms of physical quantities $\xi_1, \xi_2, \ldots \xi_n$ in K. We are probably not far from the truth if we stipulate that such a description is, in its most abstract sense, a *relation* between physical quantities $\xi_1, \xi_2, \ldots \xi_n$; in other words, it can be given as a subset $F \subset R$.

Remark 4. Consider the above example (a) in Remark 3. An isochoric process of the gas can be described by the subset F that is, in ϕ-coordinates, determined by the following single equation:

$$F \quad \{p = \kappa T \tag{7}$$

with a certain constant κ.

To give another example, consider the case (b). The relation F given by equations

$$F \quad \begin{cases} E_x(t, x, y, z) = E_0 \\ E_y(t, x, y, z) = 0 \\ E_z(t, x, y, z) = 0 \\ \qquad r_x(t) = r_0 + v_0 t \\ \qquad r_y(t) = 0 \\ \qquad r_z(t) = 0 \end{cases} \tag{8}$$

[3] There is a debate over the proper transformation rules (Georgieu 1969; Sewell 2008).

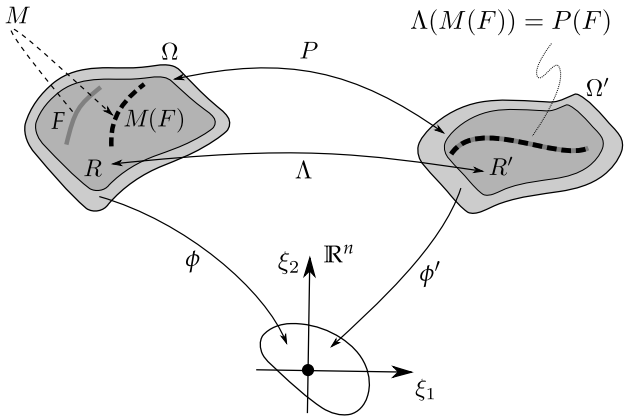

Figure 2: The relativity principle

with some specific values of E_0, r_0, v_0 describes a neutral particle moving with constant velocity in a static homogeneous electric field. ⌟

Physical equations Of course, one may not assume that an arbitrary relation $F \subset R$ has physical meaning. Let $\mathcal{E} \subset 2^R$ be the set of those $F \subset R$ which describe a particular behavior of the system. We shall call $\mathcal{E}$ the *set of equations* describing the physical system in question. The term is entirely justified. In practical calculations, two systems of equations are regarded to be equivalent if and only if they have the same solutions. Therefore, a system of equations can be identified with the set of its solutions. In general, the equations can be algebraic equations, ordinary and partial integro-differential equations, linear and nonlinear, whatever. So, in its most abstract sense, a system of equations is a set of subsets of R.

Now, consider the following subsets of Ω', determined by an $F \in \mathcal{E}$:

Primed solution $P(F) \subset \Omega'$: the "primed F", that is a relation "of exactly the same form as F, but in the primed variables $\xi'_1, \xi'_2, \ldots \xi'_n$". The quotation marks are important. Since one and the same $F \subset \Omega$ can be given in many different "forms", by means of different numbers of different equations, functions, relations, of different types. That is why we formalized the concept of a description of a phenomenon as an abstract relation between quantities $\xi_1, \xi_2, \ldots \xi_n$, given in the form of a subset of Ω. Similarly, subset $P(F)$ is an abstract relation between $\xi'_1, \xi'_2, \ldots \xi'_n$, which can be thought of in many different equivalent "forms". So, whether F and $P(F)$ are "of the same form" may or may not be manifestly apparent (cf. Friedman 1983, p. 150). Also note that relation $P(F)$ does not necessarily describe a true physical situation, since it can be not realized in nature.

Same solution expressed in primed variables $\Lambda(F) \subseteq R'$: which is the same description of the same physical situation as F, but *expressed* in the primed variables.

The same but in different state of motion We need one more concept. The RP is about the connection between two situations: one is in which the system, as a whole, is at rest relative to inertial frame K, the other is in which the system shows the similar behavior, but being in a collective motion relative to K, co-moving with K'. In other words, we assume the existence of a map $M : \mathcal{E} \to \mathcal{E}$, assigning to each $F \in \mathcal{E}$, stipulated to describe a phenomenon exhibited by a system co-moving with inertial frame K, another relation $M(F) \in \mathcal{E}$, that describes the same physical system exhibiting the same phenomenon as the one described by F, except that the system is *in motion* with velocity $\mathbf{V}$ relative to K, that is, co-moving with inertial frame K'.

4 The formal statement of the RP

Now, applying all these concepts (Fig. 2), what the RP states is the following:

$$\Lambda\left(M(F)\right) = P(F) \quad \text{for all } F \in \mathcal{E} \tag{9}$$

or equivalently,

$$P(F) \subset R' \text{ and } M(F) = \Lambda^{-1}\left(P(F)\right) \quad \text{for all } F \in \mathcal{E} \tag{10}$$

Remark 5. Notice that, for a given fixed F, everything on the right hand side of the equation in (10), P and Λ, are determined *only by* the physical behaviors of *the measuring equipments* when they are in various states of motion. In contrast, the meaning of the left hand side, $M(F)$, depends on the physical behavior of *the object physical system* described by F and $M(F)$, when it is in various states of motion. That is to say, the two sides of the equation reflect the behaviors of *different parts* of the physical reality; and the RP expresses a law-like regularity between the behaviors of these different parts. ⌟

Remark 6. Let us illustrate these concepts with a well-known textbook example of a static versus uniformly moving charged particle. The static field of a charge q being at *rest* at point (x_0, y_0, z_0) in K is the following (Fig. 1 (A)):

$$F \begin{cases} E_x(t,x,y,z) = \dfrac{q\,(x-x_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_y(t,x,y,z) = \dfrac{q\,(y-y_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_z(t,x,y,z) = \dfrac{q\,(z-z_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ B_x(t,x,y,z) = 0 \\ B_y(t,x,y,z) = 0 \\ B_z(t,x,y,z) = 0 \end{cases} \tag{11}$$

The stationary field of a charge q *moving* at constant velocity $\mathbf{V} = (V, 0, 0)$ relative to K can be obtained (Jackson 1999, pp. 661–665) by solving the equations of electrodynamics (in K) with the time-depending source (Fig. 1 (C)):

$$M(F)\begin{cases} E_x(t,x,y,z) = \dfrac{qX_0}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_y(t,x,y,z) = \dfrac{\gamma q\,(y-y_0)}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_z(t,x,y,z) = \dfrac{\gamma q\,(z-z_0)}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ B_x(t,x,y,z) = 0 \\ B_y(t,x,y,z) = -c^{-2}VE_z(t,x,y,z) \\ B_z(t,x,y,z) = c^{-2}VE_y(t,x,y,z) \end{cases} \tag{12}$$

where where (x_0, y_0, z_0) is the initial position of the particle at $t = 0$, $X_0 = \gamma\,(x - (x_0 + Vt))$.

Now, we form the same expressions as (11) but in the *primed* variables of the co-moving reference frame K' (Fig. 1 (D)):

$$P\,(F)\begin{cases} E'_{x'}(t',x',y',z') = \dfrac{q'\,(x'-x'_0)}{\left((x'-x'_0)^2+(y'-y'_0)^2+(z'-z'_0)^2\right)^{3/2}} \\ E'_{y'}(t',x',y',z') = \dfrac{q'\,(y'-y'_0)}{\left((x'-x'_0)^2+(y'-y'_0)^2+(z'-z'_0)^2\right)^{3/2}} \\ E'_{z'}(t',x',y',z') = \dfrac{q'\,(z'-z'_0)}{\left((x'-x'_0)^2+(y'-y'_0)^2+(z'-z'_0)^2\right)^{3/2}} \\ B'_{x'}(t',x',y',z') = 0 \\ B'_{y'}(t',x',y',z') = 0 \\ B'_{z'}(t',x',y',z') = 0 \end{cases} \tag{13}$$

By means of the Lorentz transformation rules of the space-time coordinates, the field strengths and the electric charge (e.g. Tolman 1949), one can express (13) in terms of the original variables of K (Fig. 1 (C)):

$$\Lambda^{-1}\,(P(F))\begin{cases} E_x(t,x,y,z) = \dfrac{qX_0}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_y(t,x,y,z) = \dfrac{\gamma q\,(y-y_0)}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_z(t,x,y,z) = \dfrac{\gamma q\,(z-z_0)}{\left(X_0^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ B_x(t,x,y,z) = 0 \\ B_y(t,x,y,z) = -c^{-2}VE_z(t,x,y,z) \\ B_z(t,x,y,z) = c^{-2}VE_y(t,x,y,z) \end{cases} \tag{14}$$

We find that the result is indeed the same as (12) describing the field of the moving charge: $M(F) = \Lambda^{-1}(P(F))$. That is to say, the RP seems to be true in this particular case.

Reversely, *assuming* that the particle + electromagnetic field system satisfies the RP, that is, (10) holds for the equations of electrodynamics, one can *derive* the stationary field of a uniformly moving point charge (12) from the static field (11). ⌟

5 Covariance

Now we have a strict mathematical formulation of the RP for a physical system described by a system of equations $\mathcal{E}$. Remarkably, however, we still have not encountered the concept of "covariance" of equations $\mathcal{E}$. The reason is that the RP and the covariance of equations $\mathcal{E}$ are not equivalent—in contrast to what is so often claimed in the literature. As Norton (1993, p. 796) writes:

> The lesson of Einsteins's 1905 paper was simple and clear. To construct a physical theory that satisfied the principle of relativity of inertial motion, it was sufficient to ensure that it had a particular formal property: its laws must be Lorentz covariant. Lorentz covariance became synonymous with satisfaction of the principle of relativity of inertial motion and the whole theory itself, as Einstein (1940, p. 329) later declared:
>
> > The content of the restricted relativity theory can accordingly be summarized in one sentence: all natural laws must be so conditioned that they are covariant with respect to Lorentz transformations.

In fact, the precise relationship between the two conditions is much more complex. To see this relationship in more detail, we previously need to clarify a few things.

Consider the following two sets: $P(\mathcal{E}) = \{P(F)|F \in \mathcal{E}\}$ and $\Lambda(\mathcal{E}) = \{\Lambda(F)|F \in \mathcal{E}\}$. Since a system of equations can be identified with its set of solutions, $P(\mathcal{E}) \subset 2^{\Omega'}$ and $\Lambda(\mathcal{E}) \subset 2^{R'}$ can be regarded as two systems of equations for functional relations between $\xi_1', \xi_2', \ldots \xi_n'$. In the primed variables, $P(\mathcal{E})$ has "the same form" as $\mathcal{E}$. Nevertheless, it can be the case that $P(\mathcal{E})$ does not express a true physical law, in the sense that its solutions do not necessarily describe true physical situations. In contrast, $\Lambda(\mathcal{E})$ is nothing but $\mathcal{E}$ expressed in variables $\xi_1', \xi_2', \ldots \xi_n'$.

Now, covariance intuitively means that equations $\mathcal{E}$ "preserve their forms against the transformation Λ". That is, in terms of the formalism we developed:

$$\Lambda(\mathcal{E}) = P(\mathcal{E}) \tag{15}$$

or, equivalently,

$$P(\mathcal{E}) \subset 2^{R'} \text{ and } \mathcal{E} = \Lambda^{-1}(P(\mathcal{E})) \tag{16}$$

The first thing we have to make clear is that—even if we know or presume that it holds—covariance (16) is obviously *not sufficient* for the RP (10). For, (16) only guarantees the invariance of the set of solutions, $\mathcal{E}$, against $\Lambda^{-1} \circ P$, but

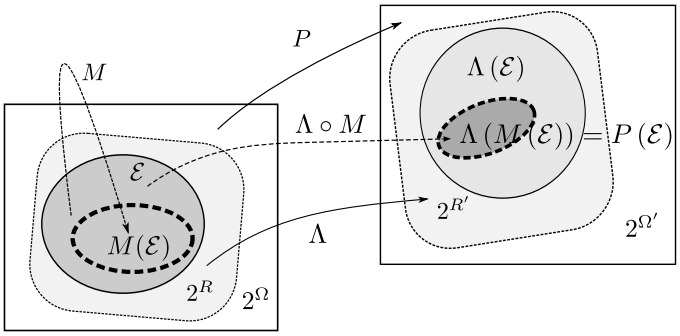

Figure 3: The RP only implies that $\Lambda(\mathcal{E}) \supseteq \Lambda\left(M\left(\mathcal{E}\right)\right) = P\left(\mathcal{E}\right)$. Covariance of $\mathcal{E}$ would require that $\Lambda(\mathcal{E}) = P(\mathcal{E})$, which is generally not the case

it says nothing about which solution of $\mathcal{E}$ corresponds to which solution. The concept of covariance does not even refer to the concept of a particular solution describing a particular behavior of an object. In contrast, it is the very essence of the statement of RP that $\Lambda^{-1}\left(P(F)\right)$ is *the* solution that describes the same physical system exhibiting the same phenomenon as the one described by F, except that the system is in motion with velocity $\mathbf{V}$ relative to K. For example, the mere covariance of the physical laws only implies that the Lorentz contracted configuration of a solid rod is *one of the possible* configurations admitted by the laws of physics governing the rod's behavior. But it does not imply that this configuration is the one that constitutes the rod in motion with velocity $\mathbf{V}$ relative to K.

It must be clear that the fact that covariance (16) does not imply the RP (10) is simply a *logical* fact. This fact is prior to the physical problem of whether the RP holds for a given physical situation or not; whether we have physically meaningful maps P_1,P_2; whether we have an unambiguous meaning of $M(F)$ describing "the system set anyhow in motion". The confusion between covariance and the RP differs from the more sophisticated problem discussed by J. S. Bell in his famous two-spaceship paper. The essence of Bell's observation is this:

> Lorentz invariance alone shows that for any state of a system at rest there is a corresponding 'primed' state of that system in motion. But it does not tell us that if the system is set anyhow in motion, it will actually go into the 'primed' of the original state, rather than into the 'prime' of some *other* state of the original system. (Bell 1987, p. 75)

That is to say, in some situations, in spite of the fact that the physical laws in question are covariant, the Lorentz boosted solution $\Lambda^{-1}\left(P(F)\right)$ is not identical with the one describing the system set in motion (also see Jánossy 1971, pp. 207–210; Szabó 2004). The mere logical possibility of such a situation means that covariance does not imply the RP.

Finally, let us note that, in a precise sense, covariance is not only not sufficient for the RP, but it is *not even necessary* (Fig. 3). The RP only implies that

$$\Lambda(\mathcal{E}) \supseteq \Lambda\left(M\left(\mathcal{E}\right)\right) = P\left(\mathcal{E}\right) \tag{17}$$

(9) implies (15) only if we have the following *extra* condition:

$$M(\mathcal{E}) = \mathcal{E} \tag{18}$$

6 Initial and boundary conditions

Let us finally consider the situation when the solutions of a system of equations $\mathcal{E}$ are specified by some extra conditions—initial and/or boundary value conditions, for example. In our general formalism, an extra condition for $\mathcal{E}$ is a system of equations $\psi \subset 2^{\Omega}$ such that there exists exactly one solution $[\psi]_{\mathcal{E}}$ satisfying both $\mathcal{E}$ and ψ. That is, $\mathcal{E} \cap \psi = \{[\psi]_{\mathcal{E}}\}$, where $\{[\psi]_{\mathcal{E}}\}$ is a singleton set. Since $\mathcal{E} \subset 2^{R}$, without loss of generality we may assume that $\psi \subset 2^{R}$.

Since P and Λ are injective, $P(\psi)$ and $\Lambda(\psi)$ are extra conditions for equations $P(\mathcal{E})$ and $\Lambda(\mathcal{E})$ respectively, and we have

$$P([\psi]_{\mathcal{E}}) = [P(\psi)]_{P(\mathcal{E})} \tag{19}$$

$$\Lambda([\psi]_{\mathcal{E}}) = [\Lambda(\psi)]_{\Lambda(\mathcal{E})} \tag{20}$$

for all extra conditions ψ for $\mathcal{E}$. Similarly, if $P(\mathcal{E}), P(\psi) \subset 2^{R'}$ then $\Lambda^{-1}(P(\psi))$ is an extra condition for $\Lambda^{-1}(P(\mathcal{E}))$, and

$$\left[\Lambda^{-1}(P(\psi))\right]_{\Lambda^{-1}(P(\mathcal{E}))} = \Lambda^{-1}\left([P(\psi)]_{P(\mathcal{E})}\right) \tag{21}$$

If equations $\mathcal{E}$ satisfy the covariance condition (16), we have

$$\left[\Lambda^{-1}(P(\psi))\right]_{\mathcal{E}} = \Lambda^{-1}\left([P(\psi)]_{P(\mathcal{E})}\right) \tag{22}$$

That is to say, solving the primed equation with the primed extra conditions is equivalent to first expressing the primed extra conditions in the original quantities and then solving the original equations (cf. Houtappel, Van Dam, and Wigner 1963). Notice however that it by no means follows *from the covariance* of equations $\mathcal{E}$ that the primed extra conditions determine the solution describing the moving object; that is, it can be the case that $\left[\Lambda^{-1}(P(\psi))\right]_{\mathcal{E}} \neq M([\psi]_{\mathcal{E}})$—this is the difference between the RP and the covariance requirement.

Now consider a set of extra conditions $\mathcal{C} \subset 2^{2^{R}}$ and assume that $\mathcal{C}$ is a *parametrizing set of extra conditions* for $\mathcal{E}$; by which we mean that for all $F \in \mathcal{E}$ there exists exactly one $\psi \in \mathcal{C}$ such that $F = [\psi]_{\mathcal{E}}$; in other words,

$$\mathcal{C} \ni \psi \mapsto [\psi]_{\mathcal{E}} \in \mathcal{E} \tag{23}$$

is a bijection.

$M : \mathcal{E} \to \mathcal{E}$ was introduced as a map between solutions of $\mathcal{E}$. Now, as there is a one-to-one correspondence between the elements of $\mathcal{C}$ and $\mathcal{E}$, it generates a map $M : \mathcal{C} \to \mathcal{C}$, such that

$$[M(\psi)]_{\mathcal{E}} = M([\psi]_{\mathcal{E}}) \tag{24}$$

Thus, from (19) and (24), the RP, that is (9), has the following form:

$$\Lambda([M(\psi)]_{\mathcal{E}}) = [P(\psi)]_{P(\mathcal{E})} \quad \text{for all } \psi \in \mathcal{C} \tag{25}$$

or, equivalently, (10) reads

$$[P(\psi)]_{P(\mathcal{E})} \subset R' \text{ and } [M(\psi)]_{\mathcal{E}} = \Lambda^{-1}\left([P(\psi)]_{P(\mathcal{E})}\right) \tag{26}$$

One might make use of the following theorem:

Theorem 1. *Assume that the system of equations $\mathcal{E} \subset 2^R$ is covariant, that is, (15) is satisfied. Then,*

(i) for all $\psi \in \mathcal{C}$, $\Lambda(M(\psi))$ is an extra condition for the system of equations $P(\mathcal{E})$, and, (25) is equivalent to the following condition:

$$[\Lambda(M(\psi))]_{P(\mathcal{E})} = [P(\psi)]_{P(\mathcal{E})} \tag{27}$$

(ii) for all $\psi \in \mathcal{C}$, $P(\psi) \subset 2^{R'}$, $\Lambda^{-1}(P(\psi))$ is an extra condition for the system of equations $\mathcal{E}$ and (26) is equivalent to the following condition:

$$[M(\psi)]_{\mathcal{E}} = \left[\Lambda^{-1}(P(\psi))\right]_{\mathcal{E}} \tag{28}$$

Proof. (i) Obviously, $\Lambda(\mathcal{E}) \cap \Lambda(M(\psi))$ exists and is a singleton; and, due to (15), it is equal to $P(\mathcal{E}) \cap \Lambda(M(\psi))$; therefore this latter is a singleton, too. Applying (20) and (15), we have

$$\Lambda([M(\psi)]_{\mathcal{E}}) = [\Lambda(M(\psi))]_{\Lambda(\mathcal{E})} = [\Lambda(M(\psi))]_{P(\mathcal{E})} \tag{29}$$

therefore, (27) implies (26).

(ii) Similarly, due to $P(\psi) \subset 2^{R'}$ and (16), $\mathcal{E} \cap \Lambda^{-1}(P(\psi))$ exists and is a singleton. Applying (21) and (16), we have

$$\Lambda^{-1}\left([P(\psi)]_{P(\mathcal{E})}\right) = \left[\Lambda^{-1}(P(\psi))\right]_{\Lambda^{-1}(P(\mathcal{E}))} = \left[\Lambda^{-1}(P(\psi))\right]_{\mathcal{E}} \tag{30}$$

that is, (28) implies (26). □

Remark 7. Let us note a few important—but often overlooked—facts which can easily be seen in the formalism we developed:

(a) The covariance of a set of equations $\mathcal{E}$ does *not* imply the covariance of a subset of equations separately. It is because a smaller set of equations corresponds to an $\mathcal{E}^* \subset 2^R$ such that $\mathcal{E} \subset \mathcal{E}^*$; and it does not follow from (15) that $\Lambda(\mathcal{E}^*) = P(\mathcal{E}^*)$.

(b) Similarly, the covariance of a set of equations $\mathcal{E}$ does *not* guarantee the covariance of an arbitrary set of equations which is only satisfactory to $\mathcal{E}$; for example, when the solutions of $\mathcal{E}$ are restricted by some extra conditions. Because from (15) it does not follow that $\Lambda(\mathcal{E}^*) = P(\mathcal{E}^*)$ for an arbitrary $\mathcal{E}^* \subset \mathcal{E}$.

(c) The same holds, of course, for the combination of cases (a) and (b); for example, when we have a smaller set of equations $\mathcal{E}^* \supset \mathcal{E}$ together with some extra conditions $\psi \subset 2^R$. For, (15) does not imply that $\Lambda(\mathcal{E}^* \cap \psi) = P(\mathcal{E}^* \cap \psi)$.

(d) However, covariance is guaranteed if a covariant set of equations is restricted with a *covariant* set of extra conditions; because $\Lambda(\mathcal{E}) = P(\mathcal{E})$ and $\Lambda(\psi) = P(\psi)$ trivially imply that $\Lambda(\mathcal{E} \cap \psi) = P(\mathcal{E} \cap \psi)$. ⌟

7 Concluding discussions and open problems

As we have seen, the RP does not reduce to the covariance of the physical equations, and the precise formulation of the RP is a much more difficult matter. It requires several conceptual plugins, without which the RP would be simply meaningless. In section 3 we gave the explicit formulation of these concepts in our formalism. The concrete meanings of these generally formalized conceptual plugins are to be specified in the concrete physical contexts.

It must be mentioned that one of these concepts, $M : \mathcal{E} \to \mathcal{E}$, which carries an essential part of the physical content of the RP, seems to be especially problematic. For, what does it generally mean to say that a solution, $M(F)$, describes the same physical system exhibiting the same phenomenon as the one described by F, except that the system is in motion relative to K, with velocity $\mathbf{V}$, together with inertial frame K'? As it was pointed out in (Szabó 2004), there is no clear and unambiguous answer to this question, even in very simple situations.

In fact the same question can be asked with respect to the definitions of maps P_1 and P_2—and, therefore, with respect to the actual meanings of Λ and P. For, according to Remark 1, $\xi'_1, \xi'_2, \ldots \xi'_n$ are not simply arbitrary variables assigned to reference frame K', in one-to-one relations with $\xi_1, \xi_2, \ldots \xi_n$, but the physical quantities obtainable by means of the same operations with the same measuring equipments as in the operational definitions of $\xi_1, \xi_2, \ldots \xi_n$, except that everything is in a collective motion with velocity $\mathbf{V}$. Therefore, we should know what we mean by "the same measuring equipment but in collective motion". From this point of view, it does not matter whether the system in question is the object to be observed or a measuring equipment involved in the observation.

At this level of generality we only want to point out two things. First, whatever is the definition of $M : \mathcal{E} \to \mathcal{E}$ in the given context, the following is a *minimum* requirement for the RP to have the assumed physical meaning:

(M) Every relation $F \in \mathcal{E}$ must describe a phenomenon which can be meaningfully characterized as such that the physical system exhibiting this phenomenon is co-moving with some inertial frame of reference.

Recall that this minimum requirement is, tacitly, already there in Galileo's principle. As Brown points out:

> The process of putting the ship into motion corresponds [...] to what today we call an active pure boost of the laboratory. A key aspect of Galileo's principle that we wish to highlight is this. For Galileo, the boost is a clearly defined operation pertaining to a certain subsystem of the universe, namely the laboratory (the cabin and equipment contained in it). The principle compares the outcome of relevant processes inside the cabin under different states of inertial motion of the cabin relative to the shore. It is simply assumed by Galileo that the same initial conditions in the cabin can always be reproduced. What gives the relativity principle empirical content is the fact that the differing states of motion of the cabin are clearly distinguishable relative to the earth's rest frame. (Brown 2005, p. 34)

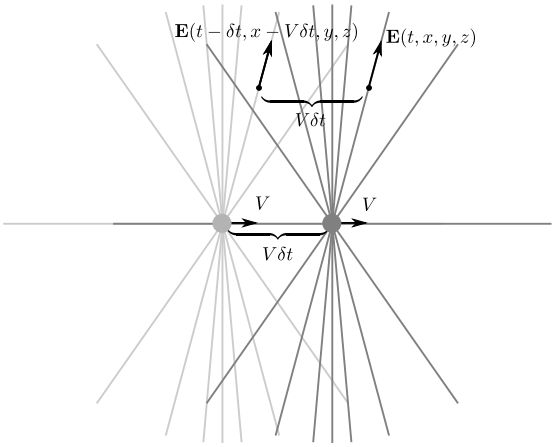

Figure 4: The stationary field of a uniformly moving point charge is in collective motion together with the point charge

A simple example for a system satisfying condition (M) is the one discussed in Remark **6**: solutions (11) and (12) both describe a system of charged particle + electromagnetic field which are in collective rest and motion respectively. The electromagnetic field is in collective motion with the point charge of velocity V (Fig. 4) in the following sense:

$$\mathbf{E}(t,x,y,z) = \mathbf{E}(t-\delta t, x-V\delta t, y, z) \tag{31}$$
$$\mathbf{B}(t,x,y,z) = \mathbf{B}(t-\delta t, x-V\delta t, y, z) \tag{32}$$

But, generally, condition (M) by no means requires that the system be in a simple stationary state and all parts move with the same collective velocity—the objects contained in Galileo's cabin may exhibit very complex time-dependent behavior; the fishes may swim with their fins, the butterflies may move their wings, the particles of the smoke may follow a very complex dynamics.

Notice that requirement (M) does not even say anything about whether and how the fact that the system in question is co-moving with a reference frame is reflected in a solution $F \in \mathcal{E}$. It does not even require that this fact can be expressed in terms of quantities $\xi_1, \xi_2, \ldots \xi_n$. It only requires that each $F \in \mathcal{E}$ belong to a physical situation in which it is meaningful to say—perhaps in terms of quantities different from $\xi_1, \xi_2, \ldots \xi_n$—that the system is at rest or in motion relative to an inertial reference frame. How a concrete physical situation can be so characterized is a separate problem, which can be discussed in the particular contexts.[4]

The second thing to be said about $M(F)$ is that it is a notion determined by the concrete physical context; but it is *not* equal to the "Lorentz boosted solution" $\Lambda^{-1}\left(P(F)\right)$ *by definition.* —as the following reflections show:

(a) In this case, (10) would read

$$\Lambda^{-1}\left(P(F)\right) = \Lambda^{-1}\left(P(F)\right) \tag{33}$$

[4]For example, even this minimum requirement can raise non-trivial questions in electrodynamics (Gömöri and Szabó 2011).

That is, the RP would become a tautology; a statement which is always true, independently of any contingent fact of nature; independently of the actual behavior of moving physical objects; and independently of the actual empirical meanings of physical quantities $\xi'_1, \xi'_2, \ldots \xi'_n$. This would contradict to the view—shared by a number of physicists and philosophers (see Brading and Castellani 2008)—that the statement of relativity/covariance principle, like many other symmetry principles, must be considered as a contingent, empirically falsifiable, statement. As Houtappel, Van Dam, and Wigner (1963) warn us:

> The discovery of Lee, Yang, and Wu, showing, among other facts, that the laws of nature are not invariant with respect to charge conjugation, reminded us of the empirical origin of the laws of invariance in a forcible manner. Before the discoveries of Lee, Yang and Wu, one could quote Fourier's principle as an earlier example of an invariance principle which had to be abandoned because of empirical evidence.

Earman points out a more general epistemological aspect:

> [V]iewing symmetry principles as meta-laws doesn't commit one to treating them *a priori* in the sense of known to be true independently of experience. For instance, that a symmetry principle functions as a valid meta-law can be known *a posteriori* by a second level induction on the character of first-order law candidates that have passed empirical muster. (Earman 2004, p. 6)

Notice that even the transformation rules must be considered as empirically falsifiable laws of nature. For, how can we verify even a single instance of the covariance principle? One might think that the verification of the covariance of a given law of physics is only a matter of mathematical verification. But this is true only if we know the transformation laws of the physical quantities—against which the physical law in question must be covariant. Consequently, we must have an independent knowledge of the transformation rules expressible in terms of the physical behavior of the measuring equipments—in various states of motion—by means of which the physical quantities are operationally defined.[5] For, as Einstein emphasizes:

> A Priori it is quite clear that we must be able to learn something about the physical behavior of measuring-rods and clocks from the equations of transformation, for the magnitudes z, y, x, t are nothing more nor less than the results of measurements obtainable by means of measuring-rods and clocks. (Einstein 1920, p. 35)

Note that a tautology is entirely different from a fundamental principle, even if the principle is used as a fundamental hypothesis or fundamental premise of a theory, from which one derives further physical statements. For, a fundamental premise, as expressing a contingent fact of nature, is potentially falsifiable by testing its consequences; a tautology is not.

[5] For a case study illustrating this, see Gömöri and Szabó 2013.

(b) Even if accepted, $M(F) \overset{def}{=} \Lambda^{-1}(P(F))$ can provide physical meaning to $M(F)$ only if we know the meanings of Λ and P, that is, if we know the empirical meanings of the quantities denoted by $\xi_1', \xi_2', \ldots \xi_n'$. But, the physical meaning of $\xi_1', \xi_2', \ldots \xi_n'$ are obtained from the physical meanings of the maps P_1 and P_2. But they are based on the concepts of the same measurement operations and the same measurement outcomes with the same equipments when they are co-moving with K' with velocity $\mathbf{V}$ relative to K. Symbolically, we need, priory, the concepts of $M(\xi_i\text{-}equipment\ at\ rest)$. And this is a conceptual circularity: in order to have the concept of what it is to be an $M(brick\ at\ rest)$ the (size)' of which we would like to ascertain, we need to have the concept of what it is to be an $M(measuring\ rod\ at\ rest)$—which is exactly the same conceptual problem.

(c) One might claim that we do not need to specify the concepts of $M(\xi_i\text{-}equipment\ at\ rest)$ in order to know the *values* of quantities $\xi_1', \xi_2', \ldots \xi_n'$ we obtain by the measurements with the moving equipments, given that we can know the transformation rule Λ independently of knowing the operational definitions of $\xi_1', \xi_2', \ldots \xi_n'$. Typically, Λ is thought to be derived from the assumption that the RP (10) holds. If however M is, by definition, equal to $\Lambda^{-1} \circ P$, then in place of (10) we have the tautology (33), which does not determine Λ.

(d) Therefore, unsurprisingly, it is not the RP from which the transformation rules are routinely deduced, but the covariance (16). As we have seen, however, covariance is, in general, neither sufficient nor necessary for the RP. Whether (10) implies (16) hinges on the physical fact whether (18) is satisfied. But, if M is taken to be $\Lambda^{-1} \circ P$ by definition, the RP becomes true—in the form of tautology (33)—but does not imply covariance $\Lambda^{-1}(P(\mathcal{E})) = \mathcal{E}$.

(e) Even if we assume that a "transformation rule" function $\phi' \circ \Lambda \circ \phi^{-1}$ were derived from some independent premises—from the independent assumption of covariance, for example—how do we know that the Λ we obtained and the quantities of values $\phi' \circ \Lambda \circ \phi^{-1}(\xi_1, \xi_2, \ldots \xi_n)$ are correct plugins for the RP? How could we verify that $\phi' \circ \Lambda \circ \phi^{-1}(\xi_1, \xi_2, \ldots \xi_n)$ are indeed the values measured by a moving observer applying the same operations with the same measuring equipments, etc.?—without having an independent concept of M, at least for the measuring equipments?

(f) One could argue that we do not need such a verification; $\phi' \circ \Lambda \circ \phi^{-1}(\xi_1, \xi_2, \ldots \xi_n)$ can be regarded *as the empirical definition* of the primed quantities:

$$(\xi_1', \xi_2', \ldots \xi_n') \overset{def}{=} \phi' \circ \Lambda \circ \phi^{-1}(\xi_1, \xi_2, \ldots \xi_n) \tag{34}$$

This is of course logically possible. The operational definition of the primed quantities would say: ask the observer at rest in K to perform the measurements $s_1, s_2, \ldots s_n$ with the equipments at rest in K, and then perform the mathematical operation (34) on the results $\xi_1, \xi_2, \ldots \xi_n$ so obtained. In this way, however, even the transformation rules would become

tautologies; they would be true, no matter how the things are in the physical world.

(g) Someone might claim that the identity of M with $\Lambda^{-1} \circ P$ is not a simple stipulation but rather an analytic truth which follows from the identity of the two *concepts*. Still, if that were the case, RP would be a statement which is true in all possible worlds; independently of any contingent fact of nature; independently of the actual behavior of moving physical objects.

(h) On the contrary, as we have already pointed out in Remark 5, $M(F)$ and $\Lambda^{-1}(P(F))$ are *different concepts*, referring to different features of different parts of the physical reality. Any connection between the two things must be a contingent fact of the world. The map $\Lambda^{-1} \circ P$ is completely determined by the physical behaviors of the *measuring* equipments. Consequently, even if F is a description of a particular phenomenon exhibited by a object physical system, nothing guarantees that $\Lambda^{-1}(P(F))$ has anything to do with the behavior of the object system; the physical behaviors of the measuring equipments do not grantee that $\Lambda^{-1}(P(\mathcal{E})) \subseteq \mathcal{E}$, nor that the elements of $\mathcal{E}$ satisfy condition (M). For example, from the information of how the static field of a charge at rest looks like—formula (11)—and how the transformation laws of electrodynamic quantities look like—regarded as independent empirical facts about the measuring equipments—one can determine the Lorentz boosted field (14), no matter how the system of equations of electrodynamics looks like, no matter whether (14) is a solution of these equations or not, and no matter whether this solution is the one describing the field of the uniformly *moving* charge.

(i) In the standard applications of the RP, M is used as an independent concept, without any prior reference to the Lorentz boost $\Lambda^{-1} \circ P$. Continuing the above example, we do not need to refer to the transformations laws of the electrodynamic quantities in order to understand the concept of 'the electromagnetic field of a uniformly moving point charge'; as we are capable to describe this phenomenon by solving the electrodynamic equations for such a situation within one single frame of reference. ⌟

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