Mill on logic

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Godden, D. (2017). Mill on logic. In C. Macleod and D.E. Miller (Eds.), *A Companion to Mill* (ch. 12, pp. 175-191). Hoboken, NJ: John Wiley & Sons.

ABSTRACT: Working within the broad lines of general consensus that mark out the core features of John Stuart Mill's (1806–1873) logic, as set forth in his *A System of Logic* (1843–1872), this chapter provides an introduction to Mill's logical theory by reviewing his position on the relationship between induction and deduction, and the role of general premises and principles in reasoning. Locating induction, understood as a kind of analogical reasoning from particulars to particulars, as the basic form of inference that is both free-standing and the sole load-bearing structure in Mill's logic, the foundations of Mill's logical system are briefly inspected. Several naturalistic features are identified, including its subject matter, human reasoning, its empiricism, which requires that only particular, experiential claims can function as basic reasons, and its ultimate foundations in 'spontaneous' inference. The chapter concludes by comparing Mill's naturalized logic to Russell's (1907) *regressive method* for identifying the premises of mathematics.

1 The Nature of Logic

1.1 Reasoning: the Subject Matter of Logic

Logic, for Mill, is the science and art of reasoning "meaning by the former term, the analysis of the mental process which takes place whenever we reason, and by the latter, the rules, grounded on that analysis, for conducting the process correctly" (*Logic*, VII: 4). Mill's naturalization of logic begins by naturalizing its subject matter: logic studies mental, i.e., natural, processes.

Yet while logic studies reasoning as a natural process, it does so normatively, not descriptively: it "takes cognizance of our intellectual operations, only as they conduce to our own knowledge" (*Logic*, VII: 6; cf. 12).

Logic is not the science of Belief, but the science of Proof, or Evidence. In so far as belief professes to be founded on proof, the office of logic is to supply a test for ascertaining whether or not the belief is well grounded. (*Logic*, VII: 9)

Logic, as an art, is prescriptive, and hence logic is not fully naturalized. That logic studies natural processes does not commit Mill to a naturalistic account of logical norms. This, as we will see, comes later.

Nevertheless, even here Mill's position is a site of controversy. To appreciate this, consider an alternative, Fregean view. For Frege laws of logic "[are] not psychological laws of takings-to-be-true, but laws of truth" (1964: 13), and "[a] derivation from these [psychological] laws [of taking-to-be-true], an explanation of a mental process that ends in taking something to be true, can never take the place of proving what is taken to be true" (1977: 2), since "being true is different from being taken to be true, whether by one or many or everybody, and in no case is to be reduced to it. There is no contradiction in something's being true which everybody takes to be false" (1964: 13; cf. 1979: 146, 1980: vi). Reasons like these led Frege to de-naturalize the subject matter of logic, postulating a "third realm" (1977: 17) of sempiternal, insensible *thoughts* – truth-bearers or the senses of declarative sentences – for which all psychological properties, such as their being grasped by a mind, are accidental. For Frege, by mistakenly taking logic to be about thinking rather than *thought*, Mill's logic mistakenly supplies laws of takings-to-be-true rather than laws of truth (see Godden 2005, 2014).

1.2 Logic as Philosophy of Evidence

Yet for Mill, it is vital that logic be about thinking and supply laws of takings-to-be-true, since "[t]he sole object of Logic is the guidance of one's own thoughts" (*Logic*, VII: 6). Accordingly, logic supplies guidance norms and this requires its naturalization.

On Mill's view, reasoning and consciousness comprise the two sources of all our knowledge (*Logic*, VII: 6–7). Truths of which we are directly conscious provide "the original data, or ultimate premises of our knowledge," but are "known antecedently to all reasoning;" hence "[t]here is no logic for this portion of our knowledge" (*Logic*, VII: 7). Mill thus restricted "[t]he province of logic ... to that portion of our knowledge which consists of inferences from truths previously known" (*Logic*, VII: 9).

The study of reasoning, though, is not limited to demonstrative, or deductive, reasoning (which Mill called "ratiocination"). Rather logic studies all inferential means to knowledge. "Logic, as I conceive of it, is the entire theory of the ascertainment of reasoned or inferred truth" (*Logic*, VII: 206). For Mill, logic is a "Philosophy of Evidence" (*Examination*, IX: 371): a logic not of consistency, the property preserved through deductive validity, but of truth (*Logic*, VII: 208).

A logic of truth, for Mill, involves *real* rather than *verbal propositions* (*Logic*, VII: 109ff.), and *real* rather than *verbal inferences* (*Logic*, VII: 158ff.) Verbal propositions are distinguished by Skorupski's (1989: 79) criterion of *connotative inclusion* whereby "the attributes connoted by the predicate are a subset of the attributes connoted by the subject." Real propositions, by contrast, are synthetic and thereby capable of conveying new information about their subjects (*Logic*, VII: 116). Similarly, real inferences are ampliative – their conclusions assert more information than what is contained in their premises. Thus for Mill, real inferences, "in which we set out from known truths, to arrive at others really distinct from them," are alone capable of advancing knowledge (*Logic*, VII: 158ff., 162). Hence, if logic is to have an epistemic function, it must move via real inferences from premises to conclusions each expressing real propositions. As we will see below, on Mill's view, only induction is capable of producing such results.

2 Deduction

2.1 Syllogisms and Real Inferences

To understand Mill's reasons here, it is instructive to consider his treatment of an illustrative, example syllogism.

All men are mortal, The Duke of Wellington is a man, Therefore, the Duke of Wellington is mortal. (*Logic*, VII: 185)

As an aside, Mill held that all valid deductions could be represented syllogistically and that all valid syllogisms could, by means of merely verbal transformations of their constituent claims, be represented in one of the four moods of the first figure: affirmative syllogisms as either *Barbara* or *Darii*, and negative syllogisms as either *Celarent* or *Ferio* (*Logic*, VII: 168). He was wrong on both counts.¹

¹ On the second point, as Skorupski (1989: 103) observes, some transformations require a *reductio* proof, and hence appeal to the principle of non-contradiction which, according to Mill, is a real proposition.

Perhaps more seriously, it was known at least from the time of Leibniz (1646–1716) that valid inferences can have non-syllogistic forms. In his *New Essays on Human Understanding* (1704), Leibniz wrote:

It should also be realized that there are *valid non-syllogistic inferences* which cannot be rigorously demonstrated in any syllogism unless the terms are changed a little, and this altering of the terms is the non-syllogistic inference. There are several of these, including arguments from the direct to the oblique – e.g. 'If Jesus Christ is God, then the mother of Jesus Christ is the mother of God'. And again, the argument-form which some good logicians have called relation-conversion, as illustrated by the inference: 'If David is the father of Solomon, then certainly Solomon is the son of David'. (Leibniz 1996: 479f.; as quoted in Hodges 2009: 596)

In Mill's own time, several such developments were under discussion, some of which Mill was aware of (*Logic*, VII: 171 ff.; *Examination*, IX: ch.22). For example, Hamilton had proposed quantifying both the predicate and subject of syllogistic claims, while DeMorgan had discovered a 'statistical' syllogism which Mill described as follows:

DeMorgan observes, [Formal Logic, 1847: 139,] very justly, that from the premises Most Bs are Cs, most Bs are As, it may be concluded with certainty that some As are Cs, since two portions of the class B, each of them comprising more than half, must necessarily in part consist of the same individuals. (Logic, VII: 171)

DeMorgan also cited an example paralleling Leibniz's: "man is an animal, therefore the head of a man is the head of an animal" (1847: 114).

Mill's response to these developments is both instructive and curious. He rejected their relevance to his project on two basic grounds. First, he claimed that these 'expansions' to syllogistic logic were not representative of our actual reasoning processes.

Considered however as a contribution to the *Science* of Logic, that is, to the analysis of the mental processes concerned in reasoning, the new discipline appears to me, I confess, not merely superfluous, but erroneous; since the form in which it clothes propositions does not, like the ordinary form, express what is in the mind of the speaker when he enunciates the proposition. (*Logic*, VII: 173 fn)

This response is curious since, as we will see, Mill denied that the syllogism has a representative function, insisting instead that its function is purely evaluative.

Given a syllogism such as this, the question for Mill is: in what manner is the inference demonstrative? How, and to what extent, do the premises support the conclusion? The problem, Mill recognized, is the following:

the proposition [the Duke of Wellington is mortal] is presupposed by the more general assumption, All men are mortal: that we cannot be assured of the mortality of all men, unless we are already certain of the mortality of every individual man: that if it be still doubtful whether [the Duke of Wellington], or any other individual we choose to name, be mortal or not, the same degree of uncertainty must hang over the assertion, All men are mortal... (*Logic*, VII: 184; text adapted to fit example)

Because of this, Mill concluded: "It must be granted that in every syllogism, considered as an argument to prove the conclusion, there is a petitio principii" (Logic, VII: 184; emphasis added). On Mill's view, a syllogism cannot provide a reason for accepting its conclusion, since acceptance of the conclusion is already presupposed in the acceptability of the premises. As such, the general premises of syllogisms are not epistemically prior to their conclusions, and hence cannot provide a rational basis for the conclusion's acceptability.²

Accordingly, ratiocination is not a form of real inference: "no reasoning from generals to particulars can ... prove anything: since from a general principle we cannot infer any particulars, but those which the principle itself assumes as known" (*Logic*, VII: 184). This position leads Mill to several remarkable, indeed revolutionary, logical views: first on the inductive form and basis of all reasoning, second on the role of general claims in reasoning, and finally on the proper function of ratiocination.

Second, Mill claimed that the new forms being proposed did not aid in the evaluation of reasoning.

The sole purpose of any syllogistic forms is to afford an available test for the process of drawing inferences in the common language of life from premises in the same common language; and the ordinary forms of Syllogism effect this purpose completely. The new forms do not, in any appreciable degree, facilitate the process. ... The new forms have thus no practical advantage which can countervail the objection of their entire psychological irrelevancy; and the invention and acquisition of them have little value. (*Examination*, IX: 403)

This second reason is clearly false, since the expanded syllogistic systems formalize inferential structures that cannot properly be represented, and hence tested, in a classical syllogistic system. Indeed the kind of example given by Leibniz and DeMorgan requires a fully quantified predicate logic with identity in order that its validity be demonstrated.

² As Scarre (1989: 52–3) observes, Mill is mistaken on this key point, since there are clear cases where even an empiricist must admit that general claims are not known inductively. Scarre gives the following example: "All U.K. citizens over the age of 18 can vote in parliamentary elections. Jim is a U.K. citizen over the age of 18, therefore Jim can vote in parliamentary elections," where the general premise is established by statute and known my reading the statute. While Mill is correct in saying that any doubt concerning the conclusion extends equally to the premise (perhaps we might wonder whether Jim can actually vote if his is not also a British resident, or if he is a member of the House of Lords, or if he is presently incarcerated), Mill is wrong to say that general claims cannot be epistemically prior to their conclusions and hence cannot act as reasons in argument. Insofar as there are legitimate means of knowing general claims other than by enumerative induction, e.g., by mathematical induction, ratiocinative inference can be real inference.

If not the premises of the syllogism, what, according to Mill, provides the real reason on the basis of which we accept its conclusion?

The true reason why we believe that the Duke of Wellington will die, is that his fathers, and our fathers, and all other persons who were cotemporary with them, have died. Those facts are the real premises of the reasoning. (*Logic*, VII: 195)

3 Empiricism in Logic

3.1 Our Knowledge of General Truths

Here we discover the second key component of Mill's naturalization of logic: his empiricism. Mill found the rationalist view of intuitionist philosophers that substantive truths can be known a priori to be "the great intellectual support of false doctrines and bad institutions" (*Autobiography*, I: 233). Against this, Mill sided with the "School of Experience" which he described as follows:

Of nature, or anything whatever external to ourselves, we know, according to this theory, nothing, except the facts which present themselves to our senses, and such other facts as may, by analogy, be inferred from these. There is no knowledge *a priori*; no truths cognizable by the mind's inward light, and grounded on intuitive evidence. Sensation, and the mind's consciousness of its own acts, are not only the exclusive sources, but the sole materials of our knowledge. (*Coleridge*, X: 125)

Thus, considering the general proposition 'all men are mortal,' Mill asked,

whence do we derive our knowledge of that general truth? Of course from observation. Now, all which man can observe are individual cases. From these all general truths must be drawn, and into these they may be again resolved. (*Logic*, VII: 186)

Because of its sources in experience, our knowledge occurs, and is acquired, first in individual cases. Only subsequently by means of inductive inference is this knowledge of particular instances collected and organized into generalizations.

3.2 Induction as Real Inference

The result is that induction is the sole form of real inference, and indeed the basis of all other inference. Since "all experience begins with individual cases, and proceeds from them to generals," the fundamental, primary, and basic operation of inference cannot be deduction, or ratiocination, which Mill defined as "inferring a proposition from propositions *equally* or *more* general" (*Logic*, VII: 163, 162). Rather induction, "inferring a proposition from propositions *less general* than itself" (*Logic*, VII: 162) must be primary. Indeed induction, being the sole form of ampliative inference, is the sole form of real inference

In every induction we proceed from truths which we knew, to truths which we did not know; from facts certified by observation, to facts which we have not observed, and even to facts not capable of being now observed; future facts, for example; but which we do not hesitate to believe on the sole evidence of the induction itself. (*Logic*, VII: 163)

4 Deduction Revisited

4.1 The Function of General Claims and Principles in Inference

Since they must be supported inductively, general claims do not have the inferential function they are typically taken to have. Generalizations, whether occurring as premises or as inferential principles, are not load-bearing structures in reasoning. Rather, they are *inferentially inert*.

Consider again our example syllogism, and recall that for Mill our sole evidence supporting our belief in the conclusion is our prior experience of individual cases of human mortality.

The mortality of John, Thomas, and others is, after all, the whole evidence we have for the mortality of the Duke of Wellington. Not one iota is added to the proof by interpolating a general proposition. Since the individual cases are all the evidence we can possess, evidence which no logical form into which we choose to throw it can make it greater than it is; and since that evidence is either sufficient in itself, or, if insufficient for the one purpose [i.e., of providing sufficient reason for the particular claim of the conclusion], [it] cannot be sufficient for the other [i.e., of providing sufficient reason for the general claim of the premise]; I am unable to see why we should be forbidden to take the shortest cut from these sufficient premises to the conclusions and constrained to travel the 'high priori road' by the arbitrary fiat of logicians. (Logic, VII: 187)

For Mill, articulating our reasoning syllogistically, such that it passes through a general claim from which the conclusion logically follows, is not a means of adding to the evidence we have gleaned from experience. Instead, it is merely a means of referring to that evidence. When we conclude that

the Duke of Wellington is mortal like the rest; we may, indeed, pass through the generalization, All men are mortal, as an intermediate stage; but it is not in the latter half of the process, the descent from all men to the Duke of Willington, that the *inference* resides. The inference is finished when we have asserted that all men are mortal. What remains to be performed afterwards is merely deciphering our own notes. (*Logic* VII, p. 187)

General claims, on Mill's account, do not, properly speaking, function as premises. Rather, Mill described their function variously as "memoranda" and as "registers of ...

inferences already made" (*Logic*, VII: 194–5; 193), and explicitly denied them an evidentiary function: "when we conclude that the Duke of Wellington is mortal, we do not infer this from the memorandum, but from the former experience" (*Logic*, VII: 195). The role of general claims in inference is not evidential but notational: "a general truth is but an aggregate of particular truths; a comprehensive expression, by which an indefinite number of individual facts are affirmed or denied at once" (*Logic*, VII: 186). As such, while general claims are cognitively useful – Skorupski (1989: 115) describes them as functioning like currency in an economy: they are a store and measure of value, and a means of exchanging real goods – they are, nevertheless, inferentially inert.

4.2 The Basic Axioms of Syllogistic Reasoning

Mill's position on the primacy of induction and the inferential inertness of general claims in reasoning extends also to the basic axioms or principles of deduction itself.

Mill held that the fundamental principle of all ratiocination is the transitivity of coexistence, having two formulations corresponding to affirmative and negative syllogisms respectively (*Logic*, VII: 178). Importantly for Mill, these principles are not merely verbal (e.g., conceptual or definitional) but are real, universal laws of nature.

These axioms manifestly relate to facts, and not to conventions; and one or the other of them is the ground of the legitimacy of every argument in which facts and not conventions are the matter treated of. (*Logic*, VII: 178)

Yet while they mark the legitimacy of syllogistic inference, they do not have an evidentiary or warranting function. Instead they are, like any other general truth, supported by induction. As real propositions, they are known first in their particular instances and only subsequently is this evidence drawn together inductively to conclude the general axiom.

Similarly, concerning the principles of non-contradiction and excluded middle as axioms of rationality or deduction, Mill wrote:

I consider it [the law of non-contradiction] to be, like other axioms, one of our first and most familiar generalizations from experience. The original foundation of it I take to be, that belief and disbelief are two different mental states, excluding one another. (*Logic*, VII: 277)

Because of this, the basic principles of ratiocination lack any argumentative role in deduction, even as second-order or meta-theoretic principles. Stating the principle "things which coexist with the same thing, coexist with one another" (*Logic*, VII: 178) together with the Duke of Wellington syllogism provides the conclusion with no additional evidence or warrant beyond the evidence of any particular cases supplied by experience.

Thus the supposed axioms of ratiocination have the same function as any other general claim. First, as already mentioned, rather than function evidentially, they function as memoranda of the evidence collected in induction.

All inference is from particulars to particulars: General propositions are merely registers of such inferences already made, and short formulae for making more: The major premise of a syllogism, consequently, is a formula of this description: and the conclusion is not an inference drawn *from* the formula, but an inference drawn *according to* the formula: the real logical antecedent, or premise, being the particular facts from which the general proposition was collected by induction. (*Logic*, VII, p. 193)

Second, as will be discussed further below, rather than functioning as parts of arguments, providing reasons, or principles for reasons, from which conclusions may then be justifiably inferred, general claims and principles provide rules according to which inferences may be made, and against which the correctness of inferences may be checked. Importantly though, these rules do not act as inference licenses, authorizing the step from premises to conclusion, since the rule itself is inferentially inert. Rather than in the rule itself, the authorization for the inference is to be found in the evidence collected under the rule.

4.3 The Proper Function of Ratiocination

Given his picture of the role of general claims in reasoning, Mill must offer an alternative account of the proper function of the syllogism. On Mill's account, syllogisms, and deduction generally, cannot function argumentatively: they cannot supply reasons on the basis of which the acceptability of their conclusions rest. Nor, according to Mill, is the syllogism representative of our actual reasoning processes.

[T]hough there is always a process of reasoning or inference where a syllogism is used, the syllogism is not a correct analysis of that process of reasoning or inference; which is, on the contrary, (when not a mere inference from testimony) an inference from particulars to particulars. (*Logic*, VII: 196)

Rather than having an argumentative or representative function, Mill claimed that the proper function of the syllogism is evaluative.

[T]he syllogism is not the form in which we necessarily reason, but a test of reasoning: a form into which we may translate any reasoning, with the effect of exposing all the points at which any unwarranted inference can have got in. ... [T]he syllogistic theory is only concerned with providing forms suitable to test the validity of inferences. (*Examination*, IX: 390; cf. *Logic*, VII: 198, 205)

Together with the formulation of general premises or principles, syllogistic logic provides a mechanism to test the validity of our ordinary reasoning. If we can formulate our reasoning in one of the valid syllogistic forms, and if, having done so, we are prepared to admit the general premise(s) of the syllogism, then we may be assured of the acceptability of our conclusion – at least to the extent that we are justified in our willingness to admit the premises. Yet to reiterate, this operation is only a test, since the real argumentative work (both evidentiary and warranting) has been done inductively, in

amassing the evidence required to establish the general premises or principles in the first place.

5 Induction

5.1 Analogical Reasoning: The Basic Structure of Induction

Induction, then, is the foundation of all inferential knowledge. Further, given that it provides the basis for all other forms of inference and inferential principles, Mill's system requires that induction itself be self-supporting. What then is the structure of inductive reasoning, and how is it free-standing while being the sole load-bearing structure in Mill's logic?

Induction, for Mill, "consists in inferring from some individual instances in which a phenomenon is observed to occur, that it occurs in all instances ... which *resemble* the former, in ... the material circumstances" (*Logic*, VII: 306). Though induction includes both inductive generalization and induction to a particular, for Mill both the alpha and omega of induction, and hence of all reasoning, is reasoning from particulars to particulars (*Logic*, VII: 193). Indeed it is the first and most natural way in which we ordinarily reason.

Not only *may* we reason from particulars to particulars without passing through generals, but we perpetually do so reason. All our earliest inferences are of this nature. ... We all, where we have no definite maxims to steer by, guide ourselves in the same way. (*Logic*, VII: 188)

The structure of reasoning from particulars from particulars is analogical (*Logic*, VII: 202ff; cf. 554ff.): we pass from premises that note particular properties in observed cases to a conclusion that projects at least one of those properties onto unobserved cases resembling the former in respect of the other observed properties. Mill gave the following general formula for analogical reasoning:

Form of Analogical Reasoning
Two things resemble each other in one or more respects;
a certain proposition is true of the one;
therefore it is true of the other. (Logic, VII: 555)

This formulation, he claimed, "will serve for all reasoning from experience ... the strictest induction, equally with the faintest analogy" (*Logic*, VII: 555). Yet, clearly such inferences are not always successful: they do not always, or even generally, conduce to true conclusions even when their premises are true. The question for Mill, then, was what distinguishes the successful, cogent applications of analogical reasoning, which Mill identified with induction, from those that unreliably project similarities from premises to conclusions?

5.2 Induction and Causal Laws of Nature

For Mill, the difference between successful induction and failed analogy is that in cases of induction our projection of properties tracks some causal law of nature: "every well-grounded inductive generalization is either a law of nature, or a result of laws of nature, capable, if those laws are known, of being predicted from them" (*Logic*, VII: 318). When a property is reliably projected in induction, this reliability is explained by the fact that the principle at work in the inference corresponds to some actual regularity in the world – a law of nature.

Thus, in order to know whether our inductions are warranted we must discover the laws of nature, or the actual regularities according to which the universe operates. (Importantly, for Mill "the expression, Laws of Nature, *means* nothing but the uniformities which exist among natural phenomena" which he claimed to be synonymous with "the results of induction" (*Logic*, VII: 318).) In this respect, the aims of inductive logic and natural science are the same.

To achieve this end, Mill proposed a series of Baconian methods of experimental inquiry for "singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an invariable law" (*Logic*, VII: 388ff.). These canons of induction prescribe sequences of controlled observations designed to isolate particular aspects of correlated phenomena and subsequently (i) to exclude them as not part of the cause, because they can be absent yet the same result obtain (method of agreement), or (ii) identify those aspects acting causally, because when they alone are absent the result fails to obtain (method of difference). (The remaining methods either combine or are built on the results of these.)

Because successful, warranted inductions track laws of nature, whenever an indication to a particular is warranted, so too is an inductive generalization that states the relevant nomological regularity.

If, from observation and experiment, we can conclude to one new case, so may we to an indefinite number. If that which has held true in our past experience will therefore hold in time to come, it will hold not merely in some individual case, but in all cases of some given description. Every induction, therefore, which suffices to prove one fact, proves an indefinite multitude of facts: the experience which justifies a single prediction must be as such to bear out a general theorem. (*Logic*, VII: 196)

Mill's thesis that successful inductions track natural laws is at the core of his dispute with Whewell (1794–1866) concerning the nature of induction. The question was whether induction involved the mind adding anything to what was given in experience. According to Whewell, induction involves a *colligation* of facts: a bringing together of particular facts under some general, uniting conception. This uniting conception, Whewell claimed, is not found among the facts, but is supplied by the mind.

The facts are known, but they are insulated and unconnected, till the discoverer supplies from his own store a principle of connexion. The pearls are there, but they will not hang together till someone provides the string. (Whewell 1858: 73)

Against this Mill argued that, while the mind must conceive of this generalization for itself, when induction is rightly conducted it is because the generalization conceived in the mind corresponds to a fact in the world – specifically to a law of nature. "If the facts are rightly classed under the conception, it is because there is in the facts themselves something of which the conception is itself a copy" (*Logic*, VII: 296).

5.3 The Ground of Induction

As with ratiocination, Mill granted that there was a "fundamental principle, or general axiom, of induction," a real proposition or "universal fact, which is our warrant for all inferences from experience," namely the *uniformity principle* [UP]: "that the course of nature is uniform; that the universe is governed by general laws" (*Logic*, VII: 306–7).

Mill claimed that our acceptance of UP is warranted by experience. "The truth that every fact which has a beginning has a cause, is coextensive with human experience" (*Logic*, VII: 325; cf. 306). Yet he also claimed that our experience of the uniformity of nature was not, itself, uniform. "The course of nature, in truth, is not only uniform, it is also infinitely various" (*Logic*, VII: 311). Hence, "[t]he general regularity [UP] results from the coexistence of partial regularities [the laws of nature]" (*Logic*, VII: 315).

[T]he uniformity of the course of nature ... is itself a complex fact, compounded of all the separate uniformities which exist in respect to single phenomena. These various uniformities, when ascertained by what is regarded as sufficient induction, we call in common parlance, Laws of Nature. (*Logic*, VII: 315)

Consequently, our entitlement to UP is itself grounded on induction.³

³ Mill's commentators frequently note that he failed to address or even take notice of the sceptical problem of induction (e.g., Skorupski 1994: 100; Scarre 1998: 116). Some have sought to excuse this by pointing out that the problems of induction were not well known in Mill's time. Scarre (1998: 117) for example, claims that "there was nowhere a lively interest in this sceptical problem of induction before the Green and Grose edition of Hume's work in 1874 – and by that date Mill was dead." Ducheyne and McCaskey (2014) claim that Hume's association with induction was not made until the 1920's with the publication of Keynes's *Treatise on Probability* (1921).

Whatever the case, it seems clear that Mill found the combined tools of empiricism and naturalism sufficient to solve the problem of induction. As we will see, Mill's naturalism took for granted the primitive cogency of spontaneous induction, from which he used empiricism to build the more rigorous scientific induction. In view of reasons like this, Macleod (2014) argues that, rather than ignoring the problem of induction, Mill's naturalism provides a Kantian solution to it by taking for granted the starting point of our common reasoning faculties.

Additionally, Mill had little patience for the purely sceptical aspects of the problem of induction. For example, when considering the question of what evidence we have for UP, Mill engaged with the familiar sceptical argument that our experience that the course of nature was uniform is not good evidence that it will continue to be uniform, which is precisely what is required to establish the universal generalization UP, that the course of nature is, always, uniform. To this point Mill replied:

Dr. Ward's ... strongest argument, is the familiar one of Reid, Stewart, and their followers – that whatever knowledge experience gives us of the past and present, it gives us none of the future. ... I confess that I see no force whatever in this argument. Wherein does a future fact differ from a present or past fact, except in their merely momentary relation to the human beings at present in existence? The answer made by Priestley, in his *Examination of Reid* [1774], seems to me sufficient, viz. that

[T]his great generalization [UP] is itself founded on prior generalizations. ... We should never have thought of affirming that all phenomena take place according to general laws, if we had not first arrived, in the case of a great multitude of phenomena, at some knowledge of the laws themselves; which could be done no otherwise than by induction. (*Logic*, VII: 307)

As such, UP has the same epistemic grounding in experience and induction, and hence the same inert argumentative status, as any other real general claim. By using UP, "every induction may be thrown into the form of a syllogism" such that "the uniformity of the course of nature, will appear as the ultimate major premise of all inductions" (*Logic*, VII: 308). Yet, like any other major premise its function is merely notational, rather than evidentiary or warranting. To all inductions, UP stands to their conclusions as any major premise in a syllogism: "not contributing at all to prove it, but being a necessary condition of its being proved" (*Logic*, VII: 308).

5.4 'Spontaneous' Inference: Mill's Naturalization of Induction

What then is the ultimate ground for induction in Mill's logic? Here we find a third, and most trenchant, aspect of Mill's logical naturalism. As already noted, Mill held that reasoning from particulars to particulars is the first and most natural way we ordinarily reason. Additionally, Mill held that the kinds of spontaneous induction human beings naturally engage in is primitively cogent.

Assuredly, if induction by simple enumeration were an invalid process, no process grounded on it could be valid; just as no reliance could be placed on telescopes, if we could not trust our eyes. But though a valid process, it is a fallible one. (*Logic*, VII 7: 567–8)

Thus, at the very core of Mill's logic one finds a naïve naturalism about the reliability of induction and the epistemic responsibility of its use as a means to knowledge. Though inferential, induction is a basic source of justification, and rather than provide it with some further ground, the task of the logician is to supply guidance norms for its proper use.

Induction, reflectively practiced and rigorously articulated, e.g., by representing and testing our inferences syllogistically, and by using Mill's methods to identify correct generalizations, constitutes a refinement of, indeed an improvement upon, our spontaneous and unreflective inferential proclivities. For example, Mill claimed, "Though not necessary for reasoning, general propositions are necessary to any considerable progress in reasoning" (*Logic*, VII: 199). Such refinements can improve our success in using induction, and hence the reliability of induction in particular applications. Yet, mere improvements they remain. "[T]he most scientific proceeding can be no more than an improved form of that which was primitively pursued by the human understanding, while undirected by science" (*Logic*, VII: 318).

though we have had no experience of what is future, we have had abundant experience of what was future. (Logic, VII: 577)

Indeed the very processes of refinement, which Mill called "rigorous" or "scientific" induction, presupposes rather than establishes or bolsters the reliability of the initial, spontaneous practice.

As, however, all rigorous processes of induction presuppose the general uniformity, our knowledge of the particular uniformities from which it was first inferred was not, of course, derived from rigorous induction, but from the loose and uncertain mode of induction *per enumerationem simplicem*; and the law of universal causation, being collected from results so obtained, cannot itself rest on any better foundation. It would seem, therefore, that induction *per enumerationem simplicem* not only is not necessarily an illicit logical process, but is in reality the only kind of induction possible; since the more elaborate process depends for its validity on a law, itself obtained in that inartificial mode. (*Logic*, VII: 567)

Thus, rather than provide any further justification for induction, Mill was content to provide a natural history of it, seemingly because that was all the justification it needed, or at least because that was all the justification there was to be found.

Many of the uniformities existing among phenomena are so constant, and so open to observation, as to force themselves upon involuntary recognition. ... The first scientific inquirers assumed these and the like as known truths, and set out from them to discover others which were unknown ... [as well as to revise] these spontaneous generalizations ... when the progress of knowledge ... showed their truth to be contingent on some circumstance not originally attended to. ... [T]here is no logical fallacy in this mode of proceeding; ... any other mode is rigorously impracticable: since it is impossible to frame any scientific method of induction, or test of the correctness of induction, unless on the hypothesis that some inductions deserving of reliance have already been made. (Logic, VII: 318–9)

6 Conclusion

6.1 Summary

In summary, Mill's logic concerns human reasoning insofar as it is an inferential path to knowledge, and seeks to provide reasoners with guidance norms for inferential knowledge. As an empiricist, Mill held that all our knowledge is acquired experientially and hence, in the first instance, is of concrete particulars. Since its original inputs are particular claims, the primary form of reasoning is from particulars to particulars. This kind of analogical reasoning becomes properly inductive when our extrapolation of properties from premise to conclusion conforms to an actual regularity in nature. When this occurs, we may infer not only to a particular but also to a generalization (i.e., to the regularity), and it is by this method that all real general propositions are properly inferred. The reliability of our inductions may be improved when we explicitly formulate the generalizations our inferences rely upon, and undertake to identify the actual regularities at work in the world, the causal laws of nature, ensuring that the former are instances of

the latter. Ratiocination, or syllogistic logic, provides a mechanism to test our reasoning. When our reasoning conforms to a valid syllogistic form and the general premises are properly nomological we may be assured that our conclusions are soundly derived. Yet, none of the general premises of ratiocination, or the principles of deduction (the transitivity of coexistence) or induction (the uniformity of nature), have any argumentative (evidentiary or warranting) function. Rather, in every case they are themselves the product of induction and their justification is reliant on induction. As such, induction is free-standing as the basic source of inferential justification. Its primitive cogency is established naturalistically through our spontaneous tendency to rely on it and our successes when doing so.

6.2 Mill's Naturalism and Russell's Regressive Method

Fumerton has charged that:

Where Mill is most original, he is often least plausible. His apparent endorsement of induction as the source of even elementary knowledge of arithmetic ... truths, for example, isolates him even from his most staunch fellow empiricists. (Fumerton 2009: 147)

On the face of it, Mill's empiricist and naturalist account of the structure and foundations of inference seems deeply at odds with the kind of account that would become prevalent in Anglo-American analytical philosophy. For example, in as much as Mill's empiricism was embraced by the logical positivists concerning synthetic knowledge, his natrualized and empirical account of the nature and foundation of putatively a priori, analytic knowledge was soundly rejected in favor of a formalist, logicist approach of the sort afforded by the new logical calculus. Take arithmetic, for example. While Frege recognized that simple arithmetical theorems (e.g., 2+2=4) and laws (e.g., the associativity of addition) are "amply established by the countless applications made of them every day" (1980: 2), he claimed that empirical observations play no part in the proof of such claims, which are properly demonstrated via derivations from first principles, as in his own *Grundgesetze* (1893–1903).

Similarly, in describing his early views on Mill's logic, Russell wrote: "In spite of [a] strong bias towards empiricism, I could not believe that 'two plus two equals four' is an inductive generalization from experience" (1959: 11). Yet, in 1907 Russell voiced his *regressive method* for discovering the premises of mathematics. The paper begins with the recognition of a striking paradox:

There is an apparent absurdity in proceeding, as one does in the logical theory of arithmetic, through many rather recondite propositions of symbolic logic, to the 'proof' of such truisms as 2+2=4: for it is plain that the conclusion is more certain than the premises, and the supposed proof therefore seems futile. (Russell 1973: 272)

The paradox lies in the fact that the logical axioms from which theorems such as the truism 2+2=4 are derived, while putatively supplying the logical bases for the theorem, are in fact accepted by us only because they produce the truism as a theorem. "[W]e

tend," Russell (1973: 273-4) claimed, "to believe the [logical] premises because we can see that their consequences are true, instead of believing the consequences because we know the premises to be true." That is to say, our acceptance of the truism is primary in the order of *our* reasons, and our acceptance of the axioms is explained by the fact that they generate as theorems the truisms we *already* and *independently* accept.

Although I do not wish to claim that Russell's views here were influenced by Mill's – indeed their projects were quite different – Russell's position bears an unmistakable resemblance to Mill's naturalized, empirical account of our knowledge of arithmetical truths. For example, Russell wrote that, while we now accept 2+2=4 as obvious and might thereby use it as a reason to demonstrate that combining two pairs of sheep would yield four sheep,

the proposition '2 sheep + 2 sheep = 4 sheep' was probably known to shepherds thousands of years before the proposition 2+2=4 was discovered; and when 2+2=4 was first discovered, it was probably inferred from the case of sheep and other concrete cases. (Russell 1973: 272)

Not only does this account embrace Mill's naturalization of the subject matter of arithmetic (the "gingerbread or pebble arithmetic" so derided by Frege (1980: viii) in the introduction to his *Grundlagen* (1884)) but it also accepts Mill's naturalized and empiricist account of the epistemic foundations of arithmetic. We accept the general axioms of a system (arithmetic, in this case) because of a prior and independent acceptance of the particular instances which are logical consequences of the axioms.

Finally, in view of the proceeding considerations, Russell proposed a picture of the function of derivation from logical premises that strikingly agrees with Mill's view. In deriving the truths of arithmetic from logical first principles, Russell claimed, "But of course what we are really proving is not the truth of 2+2=4, but the fact that from our premises [i.e., the logical axioms] this truth can be deduced" (Russell 1973: 272). While Russell did not claim that the function of the derivation is to provide a check of our untutored arithmetical reasoning, his position does grant that the derivation of arithmetic from logic does not function epistemically or argumentatively, providing us with reason to accept arithmetical truths. Russell concluded that

If the contentions of this paper have been sound, it follows that the usual mathematical method of laying down certain premises and proceeding to deduce their consequences, though it is the right method of exposition, does not, except in the more advanced portions, give the order of knowledge. (Russell 1973: 282)

Rather than as an attempt to discover the epistemological foundations of mathematics, the project of Russell and Whitehead's *Principia* (1910–1927) is better understood as a "rational reconstruction" of mathematics as a science – the exhibition of its logical structure through the discovery of a set of assumptions that would sufficiently support it. Rather than to provide mathematics with a foundation in logic that would make it more secure or certain than it had been previously, the expectation was that it would be better understood and less philosophically puzzling. While it is true, then, that Russell and Mill were up to very different things, it also seems true that Russell's views about what Mill

was up to (i.e., about the epistemic structure of our knowledge of mathematics) accord rather strikingly with Mill's own, at least as they are expressed in the 1907 "Regressive Method" paper.

Mill's interest in the *System of Logic* was to analyse and evaluate the order of our knowledge. His aim was to provide us with norms for the guidance of our thoughts. And, he recognized, perhaps better than any other in his time, that the content and structure of logical reasons need not, and typically do not, correspond to the content and structure of empirical reasons. Today such a view is commonplace: it is widely recognized that logical norms (e.g., consistency and deductive closure) are not fit as rational norms, and quite often they fail even to prescribe rational norms in any straightforward way. Moreover, it is the empirical reasons that constitute the actual bases for our beliefs: they explain why we hold the beliefs we do, and they are what is really at issue when it comes to changing minds. In providing a naturalized account of the subject matter and foundations of inference, Mill hoped to articulate the logic of our *real* reasons and our *actual* inferential practices. Viewed in this way, perhaps his system of logic is best approached and understood as a system of reasoning.

References

- DeMorgan, A. (1847). Formal Logic: or, The Calculus of Inference, Necessary and Probable. London: Taylor and Walton.
- Ducheyne, S. and McCaskey, J. (2014). The sources of Mill's views of ratiocination and induction. In A. Loizides (ed.), *Mill's* A System of Logic: *Critical Appraisals*. New York: Routledge, pp. 63–82.
- Frege, G. ([1893] 1964). *The Basic Laws of Arithmetic: Exposition of the System*. M. Furth (ed.) and (trans.). Berkeley: University of California Press.
- Frege, G. ([1897] 1979). Logic. In H. Hermes, F. Kambartel and F. Kaulbach (eds.), P. Long and R. White (trans.), *Posthumous Writings*. Chicago: University of Chicago Press, pp. 126–51.
- Frege, G. ([1918] 1977). Thoughts. In P. Geach (ed.), P. Geach and R. Stoothoff (trans.), *Logical Investigations*. Oxford: Basil Blackwell, pp. 1–30.
- Frege, G. ([1884] 1980). *The Foundations of Arithmetic*, 2nd edn. J. Austin (trans.). Evanston, IL: Northwestern UP.
- Fumerton, R. (2009). Mill's logic, metaphysics and epistemology. In W. Donner and R. Fumerton, *Mill*. Oxford: Wiley-Blackwell, pp. 145–95.
- Godden, D. (2005). Psychologism in the logic of John Stuart Mill: Mill on the subject matter and foundations of logic. *History and Philosophy of Logic*, 26, pp. 115–43.
- Godden, D. (2014). Mill's *System of Logic*. In W.M. Mander (ed.), *Oxford Handbook of British Philosophy in the Nineteenth Century*. Oxford: Oxford UP, pp. 44–70.
- Hodges, W. (2009). Traditional logic, modern logic and natural language. *Journal of Philosophical Logic*, 38, pp. 589–606.
- Leibniz, G. ([1704] 1996). *New Essays on Human Understanding*. P. Remnant and J. Bennett (eds.). Cambridge: Cambridge UP.
- Macleod, C. (2014). Mill on the epistemology of reasons: A comparison with Kant. In A. Loizides (ed.), *Mill's* A System of Logic: *Critical Appraisals*. New York: Routledge, pp. 151–69.
- Mill, J.S. (1963–91). *The Collected Works of John Stuart Mill*, 33 vols. J.M. Robson (ed.). Toronto: University of Toronto Press, and London: Routledge and Kegan Paul.
- Russell, B. (1959). My Philosophical Development. New York: Simon and Schuster.
- Russell, B. ([1907] 1973). The regressive method of discovering the premises of mathematics. In D. Lackey (ed.), *Essays in Analysis*. New York: George Braziller, pp. 272–83.
- Scarre, G. (1989). Logic and Reality in the Philosophy of John Stuart Mill. London: Kluwer.
- Scarre, G. (1998). Mill on induction and scientific method. In J. Skorupski (ed.), *The Cambridge Companion to Mill*. Cambridge: Cambridge UP, pp. 112–38.

- Skorupski, J. (1989). John Stuart Mill. New York: Routledge.
- Skorupski, J. (1994). J.S. Mill, logic and metaphysics. In C. Ten (ed.), *The Nineteenth Century: Routledge History of Philosophy*, vol 7. New York: Routledge, pp. 98–121.
- Whewell, W. (1858). Novum Organon Renovatum (Being the Second Part of the Philosophy of the Inductive Sciences), 3rd edn. London: John W. Parker and Son.