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INTRODUCTION

The aim of this paper is to propose a metaphor which can model the course of Western science's conception of mathematics from the time of the Greeks until the present day. The image chosen is that of a horseshoe (⊃). Like any trend line that is drawn on the basis of a given set of points, this representation is of course incomplete. It cannot be imagined that all individual philosophers and their philosophies can be neatly and simplistically located within the model employed. Nonetheless, it is the contention of this paper that, as a general pattern, the 'horseshoe' image does seem to apply. Moreover, it appears that the model can be <u>used</u> to suggest a new set of interesting questions about the philosophy of science and mathematics.[1]

In what sense, then, might the history of philosophy of science be compared with a 'horseshoe'? The answer is (at least) two-fold: On the one hand, it is the horseshoe's circularity which is of particular interest. If science can indeed be said to have traced some shape through time similar to the horseshoe, then this suggests that its progress has not been linear; but, to the contrary, it has tended to curve back towards its own point of origin. That is, there must be some key respects in which modern trends have tended back towards viewpoints held near the time of Western science's own emergence. Yet, in keeping with the horseshoe model, one must suppose as well that the initial and the current portions of the 'curve' remain separated by a significant gap.

This same image of a horseshoe is significant from a second point of view. In symbolic logic, the 'horseshoe' is the symbol for material implication, the 'if...then' relationship. This logical operator plays a central role in the logical rule 'modus ponens'--a rule which many deem essential for the possibility of deductive reasoning. But it is just this rational, deductive approach--the supplanting, as it were, of myth by reason, and magic and ritual by abstractions and methodology--which characterizes the appearance of Western science among the Greeks.[2] Given this, the 'horseshoe' of logic seems a fitting symbol for the scientific era which was then begun.

In the modern era, we find that the power of the rational orientation, symbolized again by the horseshoe of logic, is still strongly felt in science. Yet its sphere of usefulness has already reached its limit. Aware that models, and the conclusions drawn from them, have only limited application (compare, for instance, how the model in which light is a 'particle' simply cannot be applied in all contexts), thinkers such as van Fraassen and his contemporaries are already holding back from claims that

that which may be named or described, provisionally, within their current theories is, by any means, a deductive certainty.

But if science is, in this sense, retreating from certainty, it is suggested in this paper that its new direction is towards a return of some kind to a pre-rationalistic mode of thought. One is reminded, for instance, of Heraclitus. Realizing the "static nature of concepts and language,"[3] and thus their inability to convey the true nature of the world, he turned instead to a mode of paradoxical expression; (as when he says, for instance: "In the same river we both step and do not step, we are and are not").[4] When science, today, speaks paradoxically of waves which are at the same time not-waves, i.e., particles, it has in some sense rediscovered this old way of revealing and describing truth. And, just as a horseshoe, if its tips were connected, would no longer be a horseshoe, but a circle; so too, if science were fully to return to that ancient perspective of myth and paradox, it would no longer be based on logic (the 'horseshoe'), but on some new principle.

We begin this discussion, therefore by exploring the birth of mathematical science, represented here by the work of Pythagoras. The radical transitions in these ideas during the so-called Copernican Revolution will next be discussed; followed by a treatment of modern Western science and its philosophies, as represented especially by van Fraassen. On the model of the horseshoe: Pythagoras appears on the one tip, van Fraassen on the other; and the scientific revolution occurs at that section of the curve which is midway between the two.

In choosing Pythagoras and van Fraassen as the virtual representatives of their respective ages, I have not meant to imply, by any means, that they and their contemporaries were in full agreement. Rather, it is thought that these particular thinkers come closer to that conceptual path in the history of thought which, according to this paper, represents the overriding trend. With respect, it is felt, to this general theme, most all of the disputes and differences within each given era can be treated as embellishments and minor variations.

In short, the emphasis of this paper is to present, in outline form, a general schema for a history of philosophy of science (with special emphasis on its treatment of mathematics). For the sake of narrowing its field, it will focus on the outset, the turning point, and the present era of this process. Also, as space permits, the sketch will be coloured in with limited reference to contemporary debates.

Before concluding the paper, some mention will be made of the work of Hartry Field. His work is useful in that it shows, in practice, the limits to the applicability of any model--including the model presented by this paper. For though, according to the horseshoe image, Field's contribution belongs most forcefully to the middle stage (as will be shown), he is, nonetheless, of the present age chronologically. In the attempt to map

Field's ideas onto the horseshoe model, it is believed that both the strengths and inevitable limits of that model will be revealed. It is therefore hoped that the image which this paper presents of the progression of scientific thinking through time will be instructive, essentially accurate, and--most importantly--provocative of new ideas.

THE HORSESHOE

TIP OF THE HORSESHOE: PYTHAGOREAN PERIOD

To discover a time of origin for mathematical thinking would seem an impossible task. Although, as stated, this paper will begin with Pythagoras (fl 570 BC), it has been convincingly argued that "all the factual mathematical knowledge which is ascribed to the early Greek philosophers was known many centuries before," in Europe and Babylon.[5] According to Neugebauer, in The Exact Sciences in Antiquity, even the Pythagorean theorem, itself, predates Pythagoras by over one thousand years.[6]

Thus, what distinguishes Pythagoras and the other early Greek philosophers from their predecessors is not so much that the Greeks invented the mathematics with which they are credited. Rather, as expressed by Maziarz and Greenwood, in Greek Mathematical Philosophy., their contribution was especially to discover and emphasize "the abstractive and deductive process in mathematics", and to begin, thereby, "the rational tradition which characterizes Greek philosophy and science."[7] For instance, whereas the Egyptian interest in geometry "consisted of empirically obtained simple propositions about areas and volumes, Thales [the earliest recorded Greek philosopher] visualized a geometry of simple lines, an essentially abstract subject which has remained the basic part of geometry."[8]

If this new approach to mathematics was already evident in the work of Thales, it was nurtured and greatly enhanced by Pythagoras. Yet, Pythagoras' own links with tradition were still quite strong. No less than the adherents of the contemporary mystery religions, he too was directing his search towards the discovery of that Divine Soul or God which, in their common belief, underlies the nature of all things. But, although he retained the mystical base, Pythagoras was distinguished by his view that this Divine underpinning of the world could, in fact, be identified with Number and mathematics.

Like the believers of the mystery religions--for whom their secret rituals were taken as the divine re-enactments of the cosmic world drama--Pythagoras too acknowledged the importance of this first-hand "emotional experience of re-union". Thus, in the Brotherhood he founded, mathematics was not to be taken as simply an area for intellectual study; but, rather, it should be contemplated--as being the central focus for a whole way of life. That is, for him the ritual experience was expanded to encompass one's whole life; and what was required was "the passionless contemplation of rational unchanging truth, and...[the pursuit] of wisdom."[9]

We thus find that, with Pythagoras, mathematics provided a most suitable interface for the emerging transition from cult to philosophy, and from emotion to reason.[10] To be sure, numerical relations had long played a part in mystic speculations, and their practicality in "commerce and everyday social intercourse" was well known.[11] It was left to Pythagoras, however, to observe that <u>every</u> experience of life--in whatever realm--seemed touched by number. Even the harmonies of music, he discovered, can be related to the ratios of the strings which produce the tones.

Inspired by observations such as these, Pythagoras saw in numbers both the divine subjects for contemplation, and the basis for a rational understanding of the universe. Since numbers appeared to be the unifying principle behind the varied manifestations of being, they were considered divine; and so, to ponder them, was a sacred contemplation. Yet, beyond this mystical significance, the fact that numbers represented the fundamental principle of the universe, while having properties which can be discovered and explored, provided a basis for rational inquiry about the world. Namely, one could learn about the world by studying the mathematics of which it is comprised.[12]

However, once the 'secularization' of science had become more pronounced and sophisticated (i.e. once the departure of science from its own mythological and religious roots became more complete), then this lofty role of numbers could no longer be maintained. To be sure, the sense of awe among thinkers at the properties of number remained for centuries, and still continues. Yet, Pythagoras' assertion that "Number is the essence of all things"[13] could not withstand the criticisms which were soon to follow. Nonetheless, many of those arguments which were subsequently made against Pythagoras failed to grasp that clear sense of Pythagoras' own mystic vision, which, had it been taken into account, might otherwise have made his meaning transparent.

For instance, consider Pythagoras' aim in relating how the universe unfolded from a central Monad; or else, in affirming that "the whole Heaven is harmony and number." Throughout, his central purpose was to express a mystical truth: that one should seek "the meaning and nature of the whole in every part."[14] For him, this 'meaning of the whole' was best expressed by number; since in every occurrence or phenomenon he found evidence of number, and hence of the number One, the Monad. In speaking of numbers, he thus encapsulated the mystical ideas by which alone, in his view, one could comprehend the mystery of the world--as a "processional movement [of divinity] out of unity into plurality, out of light into darkness."[15]

However, many of the criticisms which were later directed against the Pythagoreans assume a distinction has been made between the "procession of [idealized numbers from] the Monad and [some supposed physical] procession which generates the visible world in space." Thus arise such questions as how the extended number-atoms could possibly be related to, or 'participate in' the 'pure' numbers, abstracted by thought. And, to be sure, the later Pythagoreans, themselves, had begun to speak in such terms (as about atom-like numbers) as to actively invite criticisms of this sort. Nevertheless, it should be emphasized that by the time mathematical numbers had been conceived in this way (i.e. as extended, separate 'atoms' of which things are said to be physically constructed), the core mystic doctrine of unity had already begun to be lost.[16]

Cornford describes this process, which occurred within as well as outside of Pythagoreanism, as the tendency to dualism.[17] Elsewhere in this paper, I have referred to it as a process of 'secularization' of science. What this involves, primarily, is the removing of the immanence of divinity from the world. Nature is taken to exist, in some sense, independently of the divine, and to be a subject of inquiry in its own right. In fact, in the extreme example of Aristotle's self-contemplating deity, divinity has almost ceased to interact with the world at all. To ask, therefore, how Pythagoras' numbers could serve as causes on earth, is already to assume their separateness from the world described.[18] Yet, it is just this dualistic premiss which Pythagoras himself had denied.

However, the distinction between the views of Pythagoras and his ancient critics can by no means be rigidly drawn. For the tendency towards dualism was already present among the Orphics, by whom Pythagoras was himself strongly influenced. The Orphic religion, in turn, developed from the older mystery religion of Dionysus that believed in the endless cycle of life-death-rebirth. But the rebirth, in their case, was not considered a rebirth for the individual person; the eternal soul was the group soul, not the individual soul.[19] By introducing the idea of an individual soul which persists through reincarnation, the Orphics made possible a hope of personal release and redemption; yet, in the process, they divided the unity of Being.

Of course, the Orphics too were expressing the impulse of the time. In Finley's Four Stages of Greek Thought, he describes the cultural process which led to the rational orientation attained in Greece by the time of Plato and Aristotle. As life itself became more diverse, complicated, and individuated, the "desire for reasonable decency [in contrast to the 'irrational' excitements of the Mysteries] set the tone."[20]

Of those who came after Pythagoras, perhaps Plato came closest to a Pythagorean form of expression when he spoke of the Forms as the unifying principle behind appearances. Perceived as immanent, they serve a similar mystic role to Pythagoras' numbers. Yet just as later Pythagoreans allowed their numbers to become crystallized into separate entities--thereby sacrificing the original aim for unity--even Plato himself succumbed to attempts at rational analysis of his mystical concept of participation; and thus allowed the forms to "ultimately dry up into mere 'concepts' or 'logical objects' of thought--immutable still and independent of the subject which knows them, but without life and power." The trend, in other words, was for forms to become simply "the relation of logical subject to universal predicate"[21] and for numbers to become, as for Aristotle, "a mere elaboration of the catgory of quantity."[22] In short, the dualistic tendency was, even then, very strongly in evidence.

THE TURNING POINT: THE SCIENTIFIC REVOLUTION

If any philosopher of the time could be said to represent the essence of the new natural philosophy and science of the 17th Century, Descartes is perhaps the most likely candidate. As aptly described by Westfall in his excellent survey, The Construction of Modern Science, Descartes, with his unshakable faith in reason, uttered the clarion "call for the abolition of wonder by understanding."[23]

In the previous section of this paper, it was shown how the tendency towards dualism was inherent in the very emergence of science and rational thinking among the Greeks. But what was latent, or only just developing, at that time reached full maturity in the writings of Descartes. Here, the mind/body distinction has been made complete; and all traces of the spiritual or psychic have been removed from matter.[24] Not only the occult qualities of Scholasticism, and the enspirited nature of Renaissance Naturalism gave way to this new vision, but even the apparently 'real' qualities of heat and colour and the rest, proposed by Aristotle, were dismissed; since even these could not conform within the reigning dichotomy of mind vs. matter.[25]

To be sure, this new philosophy did not arise at once, 'full blown'. In fact, the science of the time was characterized throughout by a dynamic tension between the so-called mechanical philosophy which was being created, and a still strong attachment to the Pythagorean tradition in mathematics. This tension, in turn, can be related to the shifting aim and focus of the scientific enterprise, itself.

According to Cornford, in his talk on the <u>Laws of Motion in Ancient Thought</u>, the science of the Greeks was simply not addressing the same problems as those confronted by modern science. To the contrary, says Cornford, the Greeks lay stress on discovering the essence of what <u>is</u>. Since their science was dissociated "from the pursuit of power and wealth," they were "not bent on influencing future facts to [their own] advantage." Thus, he continues, "Greek speculation took geometry in particular--that static science--as the pattern and ideal of all knowledge."[26]

In opposition to the Greeks, however, in Cornford's account, more modern science is not so much concerned with static existence (i.e. with that which Mill dubs the uniformities of "simultaneity among co-existent phenomena"). Instead, commencing with the work of Copernicus and Kepler, science has shifted its emphasis to the study of motion, and the laws of succession; since it is these which give power-- the power to predict correctly and to act on the basis of these predictions.[27] As Westfall expands this image: The Pythagoreans' search for order was "satisfied to discover exact mathematical description, which it understood as an expression of the ultimate structure of the universe. The mechanical philosophy, in contrast, concerned itself with the causation of individual phenomena"[28] in order both to vanquish uncertainty, and to provide the basis for prediction and control.

But this distinction between the two opposing viewpoints of science, for the Greeks and for the science of the 17th century, is not just the contrast between two ages. In practice, the allure of Pythagorean mathematics still lingered, and had its effects in the science of the new era. This explains, for instance, why Kepler, who is renowned for his discovery of the three laws of planetary motion, which we still accept today, held also to diverse speculations (e.g. relating musical harmonies to planetary motion, or regarding the geometric architecture of the universe) which now seem unfamiliar and outdated.[29] Ironically, Galileo, who helped discover the more modern concepts of mechanics, could not himself resist a return to the more traditional picture of the physical solar system (which was based on that staple of the Pythagoreans, the circle); a picture which Kepler had already discarded as unworkable.[30]

Even today, this Pythagorean strain can still be detected in science. This 'throwback', as Cornford calls it, is evident wherever the laws of science are conceived not merely as statements of causal relations, but in a metaphorical aspect as 'timeless', universal properties, inherent in the world.[31] The advantage of this approach is that if such eternal laws could indeed exist, then, with Aristotle, one might hope that the rest of science could be rigourously explained by deduction from necessary principles. Descartes, in fact, hoped to retain just this privelege by identifying the space of pure geometry with the extended plenum of physical matter--so that certainty could still be possible for scientific knowledge. This hope was dashed in the potent writings of Pierre Gassendi (d. 1655), who affirmed that "atoms are extended, but extension is not their essence." In short, man cannot hope for certain knowledge of the essence of things (which only God can know).[32]

THE OTHER TIP: VAN FRAASSEN

In the image of this paper, Western science has traced through its history the figure of a horseshoe. Pythagoras, it was said, and the science he represented could be imagined to exist at one of the horseshoe's tips. The crucial turning point, at the centre of the curve, would lie at about the 17th century during the scientific revolution. Then, with modern science, the other end is reached.

The second stage, as shown above, was a science most characterized by an inner dynamic tension. On the one hand, a view of nature was being promoted which sought to explain all phenomena on the basis of solely mechanical interactions. On the other hand, a Pythagorean confidence remained in the role of abstract mathematical formulations to comprehend and accurately describe the phenomena.

In this picture, the modern era in science and its philosophy can be described as seeking to resolve the paradoxes of that second stage. An interesting representative of this current era is Bas van Fraassen. Like his predecessors from Stage 2, he identifies his concerns with the 'facts'. Not for him is talk of essences, or focus on divine intention. Then, too, he shares their attraction to elegant, and usually mathematically-based, theories which can serve to unify the diverse data. But, to avoid the paradox which befell his forbears when attempting to reconcile these two strains, van Fraassen changes radically the nature of his 'theories': For him, the theory has become relative and provisional; and the interests it serves are largely pragmatic.

In other words, it might be said that the 17th century dilemma was due to their holding simultaneously to two polarized positions, while yet insisting dogmatically on each. Since its science was so largely a study of mechanics, it was presumed that the world itself was literally mechanical in nature. And, since mathematics owed so much to the ancients for its impetus and development, the original prejudices regarding the meaning and nature of mathematical discoveries were adopted. The conflict arose, essentially, from the uneasy juxtaposition of such vying perspectives. But, nonetheless, these viewpoints <u>can</u> converge, if only they are each interpreted less rigidly--which is to say, in the 'agnostic' fashion of van Fraassen.

According to the critics of the so-called mechanical philosophers, the latter abandoned in their method that which they themselves put forward as a guiding principle. Namely, in the interests of disavowing any reliance on occult forces in their explanations (an aim shared in the current era), these older philosophers nonetheless relied on hidden causes and unseen movements of their own. This appears an unacknowledged regression. Some philosophers have tried, as an alternative, to simply remove all reference to the 'hidden'. van Fraassen, in the opinion of this paper, is more truly representative of the direction of modern science when he permits continued theorization involving the unobserved, but holds back from the ontological commitments which, if made, could embroil him in paradox.

What van Fraassen presents as the definitive summary of his own position is the following (italicised in his original text): "Science aims to give us theories which are empirically adequate; and acceptance of a theory involves as belief only that it is empirically adequate." To say that a theory is empirically adequate is to say that it "saves the phenomena"; i.e. that "what it says about the observable things and events in the world is true."[33] The bulk of his text The Scientific Image is an elaboration and a defence of that position.

van Fraassen thus continues the tradition, begun with the scientific revolution, of focusing on the predictive aspect of theories, which enable control and power. The only predictions which could possibly be confirmed or denied are those whose ourcomes are seen to be at least partially observable. What goes beyond the observable, from this perspective, may contribute perhaps to a 'good story' about what may happen in some predicted situation; yet, ultimately, it is the observable portion alone which can be explicitly 'watched for'--to discover either that it does in fact occur, or that it fails to.

Therefore, many criticisms which have been leveled against van Fraassen's dependence on the concept of observability can be countered by recalling this purpose, just described, for his employment of the notion. Musgrave, for instance, in his review of van Fraassen, questions whether such "a distinction [between what is or is not observable by humans, in general], which is admittedly rough and ready, species specific, and of no ontological significance, [can] really bear such an epistemological burden?"[34] That is, he questions whether we should base our inferences regarding what exists on the almost arbitrary consideration of what the physical human species happens to be capable of observing. But in response to this, it must be emphasized that van Fraassen is hardly suggesting that existence is conditional on our ability to observe it; only that if and where existence does go beyond our ability to observe it, then the best one can hope for is speculation--not knowledge.

Perhaps an example can make this clearer. Suppose there is a theory which says that in ten hours, the 17th dimension will collapse into the 16th. Since, so far as we know, there is no way in which all, or even some part of this event could be observed by humans, then van Fraassen would need to say that adherence or non-adherence to such a theory is wholly optional. This is not because these dimensions depend on man for their existence, but, rather, since there is no prediction which touches man that can be affirmed or denied on the basis of such speculation about these esoteric states.

Contained within the above example is also a clue to van Fraassen's second claim regarding 'observables', which has caused some upset among his critics. It would seem that the line which separates the 'observable' from the 'non-observable' is far from clear. Is the image of someone's knee on an X-ray plate, for example, an 'observation' of that kneecap; or is the only true observation involved that of the plate, itself--while the kneecap remains unobserved? van Fraassen, clearly, would favour the first interpretation; and he would say that what counts as an observation is determined by accepted the current body of theory. Since, in the present case, the accepted theory of X-rays confirms the correspondence between its images and that which is imaged, it is therefore sufficient to see the exposed plate in order to say that one has observed the features shown therein, as well.

The complaint about van Fraassen's answer is that it seems to involve one in a vicious circle: On the one hand, as already described, van Fraassen seeks to tie all theories back to tangible observations. Now, in turn, it seems that what is 'observable' is itself determined by some given theory. Surely, say some, this displays a basic circularity in van Fraassen's account.[35]

This paper would argue, however, that this second objection to van Fraassen's use of observables is, like the first, somewhat off the mark. For consider again that image of a theory which predicts the collapse of higher dimensions. As originally expressed, the theory was unsupportable in van Fraassen's terms, since "so far as we know," it was said, the claims the theory makes are not subject to observation. But, for sake of the present argument, let us now suppose that the theory includes also the following assumptions:

> 1. The 16th and 17th dimensions, though not subject to direct experience as such, do nonetheless have observable effects.
>
> 2. For instance, they do determine the relationships that hold between our phenomenological perceptions of colour and the corresponding wavelengths of light, which can be independently measured.
>
> 3. Thus, when the 17th dimension collapses into the 16th, as is predicted, an observable effect will be that when the familiar colour 'red' is perceived, the measured wavelength of light which corresponds will be increased times pi (compared with the previously expected measurement); and so, too, for all the colours, the corresponding frequencies will be increased times pi.

The addition of such further assumptions to our imagined theory draws attention to these two important points, which bear on problems raised above:

(1) Let us assume that, at the predicted time, the change in correspondences between perceived colours, on the one hand, and the measured wavelengths of these same colours, on the other, actually occurs; or, rather, let us assume that the holders of our imagined theory assert this change to have been made. What could they point to in order to support their claim? Clearly, they would describe the work of those experimenters who have

actually seen certain colours, and read certain numbers from the dials and monitors of their test apparatus, in some specified order. To be sure, it is their theory itself which has said <u>which</u> numbers the researchers should expect to see on their displays (namely, in this example, those numbers which are roughly 3.14 times larger than the numbers which they would formerly have expected from otherwise similar experiments made prior to the dimensional collapse). This, then, expresses the sense in which the theory determines the role for observation: It says where to look, and what to be watching for. Yet there is nothing in this which should be problematic for van Fraassen. If, for whatever reason, theory predicts the appearance of certain numbers, at certain times, on particular display devices, the bottom line of confirmation or disconfirmation still rests with the very human-dependent question: "But did the researchers actually <u>see</u> those expected numbers, or did they not?"

(2) The same example helps, also, to show the way in which the realms of 'observation' can expand in theory-related contexts; though, always, it is bound to the final criterion (vague as it may sometimes be to define) of what the actual human being can confront and recognize. As of now, we presume, there corresponds with each colour we perceive a certain wavelength of light. To affirm this is to embrace a certain theory--a theory which is supported every time the 'seeing of some colour' and the measurement of its anticipated wavelength are conjoined. Once we accept this theory that affirms a certain constant conjunction of

what is (or could be) perceived with what is (or could be) measured, there is a readily understood sense of 'observe' in which the direct experience of the one conjunct can be taken automatically as a case of 'observing' the presence of the as-yet unseen member of the pair.

So, for instance, we may say a star is 'red', based on a direct observation only of a reading of its wavelength from a meter, though perhaps no one has actually ever perceived its alleged 'red' colour in their visual field. Or again, we say we have observed a kneecap upon inspection only of its X-ray exposure without (fortunately) feeling the need, every time, to first cut through the flesh and look directly. Have we really observed the 'red' star or the kneecap? Well, what we have done is 'as good as' having observed it, provisional upon our continued acceptance of the theories which conjoin these phenomena to those we have literally experienced in a direct sense. But our example shows what can happen when one of these provisionally accepted theories is overturned (as when the expected relationship between colours and wavelengths is allegedly altered): Clearly, this reveals the tentative nature of all such indirect observations; and shows, once more, how consistency with strictly direct observation is the more fundamental test.

The essence, then, of van Fraassen's case is this: The crucial test for any theory is its compatability over time with observed phenomena. In dispute, 'observation' must be taken in its crudest (though vague) sense, as what a human can actually perceive (such as e.g. 'red' or 'the displayed representation of the number 112'). In practice, however, a far more expansive sense of observation is permitted; provided only the theories and presumed associations on which this observational method is based are not, in the given context, being questioned. If these premisses are questioned, the disputants must fall back to observations they can agree on--with direct reports of literal experiences comprising the last resort. (It should not matter that what, exactly, a 'literal experience' is, is unclear; provided the disputants themselves can reach a tentative agreement on the subject.)

Having thus summarized van Fraassen's position at some length, it next remains for this paper to relate it more fully to the general flow of the history of philosophy of science. In particular, it must be shown that his views can be appropriately mapped onto the 'second tip' of the horseshoe image--the position reserved for the thinkers of the 'modern era'.

SUPPORTING MODERN TRENDS

As the reader will recall, this paper has suggested that the emergence of Western science involved at its outset a tendency towards 'secularization'. That is, the pre-scientific emphasis on ritual interaction with nature was progressively diminished in favour of an increased attempt to stand back from it for understanding, and, eventually, for control. This trend, described also as a tendency towards dualism, reached its climax with the scientific revolution, when the mind of the observer and the matter under study were seen as rigidly distinct.

Yet, since ultimately man, his culture, and his reason are themselves <u>also</u> a part of nature--i.e. of that which is under study--this strict dichotomy of mind vs. matter, of scientist vs. his subject, could never really be supported. Once the divinity of mathematics passed from favour as part of this dualistic tendency, it was never clear exactly where or how to classify its content. Who could deny the close kinship of mathematical and deductive thinking with the activity of the mind? Yet, if mathematics were <u>simply</u> of the mind, would this not abandon nature to randomness and chance disorder? Once Descartes had failed in his attempt to enforce a strict and conveniently necessary parallel between mathematics and the actual geometrical properties of the extended universe, it became the pressing problem for all future philosophy of science to resolve this unpleasant dilemma.

It is thus that van Fraassen has been chosen in this study to represent the current stage in this historical development. In him, mathematics has completed its transition from its divine station with Pythagoras, through its uneasy period of coexistence with mechanical philosophy, to, finally (and in answer to the post-cartesian problems), an essentially pragmatic role. But, in the process, a new sort of unity has been restored to science from the forceful dualism of its past: For, since, in current science, both mathematical formalas and physical particles, alike, are postulated--not as certainly existing, but as being <u>effective</u> for certain specific explanations--the need for holding rigidly to the old divisions between their realms is breaking down. And, in the age of science where particles have also become waves, perhaps the only supportable approach is to reduce, along with van Fraassen, one's commitment to the dualistic view.

But even if van Frassen's views can be plausibly related to an historical pattern of development in the philosophy of science, many readers may yet object to this paper's selection of van Fraassen as the special 'representative' of the 'current position' in that field. Unfortunately, there is not the space in this article to fully justify this choice. (At least a book would be required to fully elaborate all the issues and debates which have occurred in the modern philosopy of science, and to clearly demonstrate convergence on a single view such as van Fraassen's.) Instead, what will be offered here is a somewhat eclectic comparison of van Fraassen's position with a few other modern views, in order to show their common, basic similarity in key features.

Perhaps the earliest expression of the theme here attributed to the modern era in science was provided by David Hume in the 18th century. Famous for his arguments that no one has ever seen a 'cause', but only the "constant conjunction of two objects," he calls it merely the result of "custom or habit" "to expect the one from the appearance of the other."[36] What this leaves as the role for sound philosophy is to avoid dogmatism, and to confine itself "to such subjects as fall under daily practice and experience."[37] Such a view is clearly compatible with van Fraassen's more recent injunction that a scientific theory can only be judged on the basis of how it tallies with experience; van Fraassen, like Hume, takes a pragmatic attitude to all that exceeds this limit.

In reaction to this initial statement of a position akin to van Fraassen's, the constructivist schools (including e.g. Kant, and the 20th century positivists) attempted to restore certainty to reason and to mathematics, and, thus, to rational deductions about the world. The key to this attempt was to consider the world of experience, itself, a construction, built up by the mind on its own mathematical, deductive principles. Therefore, it was felt, these latter principles could be relied on just as surely as for Pythagoras or Descartes. That is, we can <u>know</u> the world confirms to these rational principles precisely because, in the final analysis, it has in fact been <u>constructed</u> from them.

Perhaps, it might seem that <u>these</u> schools, if any, are antithetical to the viewpoint of van Fraassen. How, then, can van Fraassen be said to speak on behalf of modern philosophy of science in general? The reply is that, although the the constructivists definitely <u>aim</u> to restore a place for certainty, which van Fraassen himself has finally abandoned, they nonetheless contain within themselves the seeds for their own collapse. Once this occurs, they tend to merge within the general 'modern' perspective, represented by van Fraassen.

Kant, himself, acknowledged that, barring total skepticism (that is, avoiding the premiss that nothing external really exists), there must remain a certain externally existing 'stuff', the 'noumena', about which nothing specific can be known. All descriptions of the known world are already constructions built up from man's own logical framework; one can therefore never know the thing itself. This suggests that the accounts we give of our experienced environment are provisional upon our own adoption a certain logico-mathematical framework. But, in the current era, when there have been shown to be possible alternative mathematics and logics, the relativity which was inherent but hidden in this constructivist outlook has now become apparent. For, which mathematics is to be chosen for our constructions? That is, with which mathematical or logical framework should we model the world; since, by definition of the case, we cannot know the world itself apart from the models we choose? Once the constructivist viewpoint is confronted with such questions, then it soon becomes obvious that even their views tend to merge within the van Fraassenean 'agnosticism' of our time regarding the possible 'truth' of accepted theories.

Ironically, van Fraassen sees his second main adversary, after the positivists, as the so-called realist schools. Perhaps the classical spokesman for such schools is C.G. Hempel. In this author's view, however, van Fraassen's attempt to maintain a polarity between his own view and that of his other contemporaries seems essentially shallow, and hinges on trivialities. Hempel, no less than van Fraassen, acknowledges that "we can never establish with certainty that a given theory is true, that the entities it posits are real. But," he continues, "that is not to disclose a peculiar flaw in our claims about theoretical entities, but to note a pervasive characteristic of all empirical knowledge."[38] In other words, Hempel is in full agreement with van Fraassen that, regarding things empirical, certainty is simply not possible. But, once this is acknowledged, the so-called distinction between Hempel's realism and van Fraassen's view collapses to the following:

> HEMPEL: If my theory says that unobservable entities, 'A', exist, then, if that theory is true, I would of course be committed to believing also in the existence of A's.[39] But since, of course, I can never be certain that this theory is correct, I am never quite certain about the existence of A's, either.
>
> van FRAASSEN: If my theory says that unobservable entities, 'A', exist, then, even if I totally accepted the theory, I would not consider myself committed to believing also in the existence of A's. So, even if I accept the theory, I am never quite certain about the existence of A's.

So where is the great distinction between these two positions? Clearly, neither van Fraassen nor Hempel would acknowledge the existence of entities which are posited only in theories which they do not accpt. But, suppose they share the acceptance of some given theory which happens to purport the existence of unobservables. Would one of these thinkers be more committed to the existence of these objects than the other? No: Hempel would hold back, because no such theory can be believed with certainty; while van Fraassen would likewise refrain from belief, in his case saying that theory-acceptance does not entail such ontological committment. The outcome, in terms of what is or is not believed on the basis of accepted theories, is essentially the same for both thinkers.

Further parallels with van Fraassen's ideas can be found in the work of Israel Scheffler. In his Anatomy of Inquiry, for instance, he speaks in ways which complement van Fraassen's notion that a theory itself determines what are its observables. Says Scheffler: the events we select to confirm or explain a theory cannot be looked upon as simply 'raw' happenings, but always too as events-described-as-P. That is, the theory itself must indicate which features must be present in some event, if it is to be characterized as such an instance of P).[40]

In short, although these few examples could hardly be called a 'proof', they do help to illustrate how the current views in the philosophy of science tend to have a great deal in common. If van Fraassen has been chosen as the spokesman for this era, this has only been to provide a point of focus for this study. As illustrated above, many other thinkers have had valid points to add to this essentially common view.

FIELD, AND THE HORSESHOE MODEL

Throughout, this paper has tried to present the history of the philosophy of science as a smooth transition from Pythagorean towards van Fraassenean perspectives. Hopefully, at least the nature of an overriding trend has been expressed, though the variety and richness of published opinion on the subject can hardly be captured in a single such account. For instance, there has been in each period a diversity of schools and doctrines---which, even if they expressed some common themes, had nonetheless some sharp disagreements with each other.

With Hartry Field, however, we see evidence of another type of diversion from the pattern here presented. He is distinguished from his other modern contemporaries not so much by holding to another view within a common framework of shared ideas, as because, in a very real sense, his ideas are a sort of throwback to an earlier time. In particular, the thrust of his arguments appear to belong more appropriately to the time of the scientific revolution and Descartes than to the present debates. It is therefore useful to take a special look at Field's proposals to see how they can be mapped onto our present model.

The essence of Fields's arguments in his Science Without Numbers is a claim that mathematics and numbers are expendable (in his words, "conservative.")[41] for the pursuit of science. That is, whatever science needs to express or demonstrate can be accomplished without the employment of numbers. Thus, though mathematics may provide a useful tool, the existence of its entities need not be at all acknowledged.

Ironically, it is Field's strong attitude towards mathematics which places him so out of step with his peers. His general set of beliefs is not particularly distinct from common views. For instance, Kemeny, in his excellent work A Philosopher Looks at Science, virtually paraphrases Field (though writing 20 years in advance of him) when he says of the scientific method: "it starts with facts, ends with facts, and the facts ending one cycle are the beginnings of the next cycle."[42] Where does mathematics fit in? For Kemeny, no less than for Field, mathematics is an extension of pure logic. From facts we induce theories; mathematics, as simply a convenient shorthand for logic, helps us to deduce predictions from these theories; and these predictions are verified or refuted by reference to other facts.[43] The van Fraassenean emphasis on confirmation-by-facts reveals the modernity of this view; and Kemeny has already made clear his own assessment of the ultimate 'conservativeness' of mathematics.

In fact, we are reminded, here, of the modern emphasis--which van Fraassen also employs--on models in scientific theories. Like the mathematics described by Kemeny, models (according to Hesse in her Models and Analysis in Science) are required to make theories predictive.[44] Even if verification depends on discreet observations, a theory must somehow contain within itself a basis for deciding where next to look--for what to expect. Strict logico-mathematical deduction from accepted theory is one such basis, though by no means the only one.[45]

So what is Field's point in insisting that numbers have no existence? In the present phase of science, surely, there is no need to devote a book to such a subject. Since numbers are not observables as such, they must be considered parts of those theories or models which are alleged to connect our observations into some more or less unified picture. In this respect, their status is no better or worse, no more or no less 'real', than that of quarks or the fourth dimension. Since the role they play is provisional, in any case, in agnostically-held theories, there simply seems no point in focusing on them (as opposed to other unobservables) for special exclusion from our ontology.

In the second phase of the 'horseshoe', however, there would have been reason for concern; and it seems that Field's attitude is largely an inheritance from that period. At that time, when the dichotomy between mind and matter was in its heyday, the role of mathematics was indeed a sore point. If it belonged on the mind side of the barrier--and surely it must--then how could its role in the explanation of the movements of objects be accounted for? When questions such as these were still in vogue, Field's contributions would have been most welcome. (Notice, by the way, that his examples are drawn primarily from Newtonian physics---which is science of the second stage, not today's.) Field's solution: 'Though it is convenient to employ mathematics in scientific explanation, all reference to mathematics can, with effort, be removed; and, thus, the 'matter' side of our rigid dichotomy can remain untainted by mind, in the form of numbers.' This would seem, no doubt, to have been an excellent solution for this problem, for those involved at the second phase of the horseshoe; but, for today, unfortunately, the problem itself seems anachronous.

CONCLUSION AND AFTERWORD

A model for interpreting the history of mathematics's role in the philosophy of Western science has now been drawn. According to this model, the philosophy of science has traced a horseshoe-curve through time. The emergence of this 'horseshoe' with Pythagoras and the other Greek scientists can be seen as the rise not so much of the content of what we now recognize as Western science and mathematics, as (more importantly) the emergence of that rational and deductive orientation which has since been characteristic of the scientific enterprise.

From what has just been said, one of the two interpretations intended for the horseshoe image of this paper can be derived. As mentioned in the introduction, the 'horseshoe' is perhaps the one symbol in logic most characteristic of the deductive mode of thought (since it represents the 'if...then' operator employed in modus ponens). So, from this perspective, to say that the the period in Western science from Pythagoras to the present day has traced out the curve of a horseshoe is to say that, in that period, reliance on the deductive mode of logic has been a central feature in the corresponding science.

But, again, as mentioned in the introduction, the horseshoe image can be seen in another way. For, to trace a horseshoe pattern is to begin a return, at some point, towards the place of origin. We have seen, in this paper, a number of respects in which science has, indeed, curved 'back' in this way:

1) With Western science began a trend towards dualism. By the time of Descartes, this tendency had reached its extreme possibility, with the rigid mind/body distinction. By van Fraassen, a return has indeed begun, as theories blend waves with particles, and define to large extents their own observables.

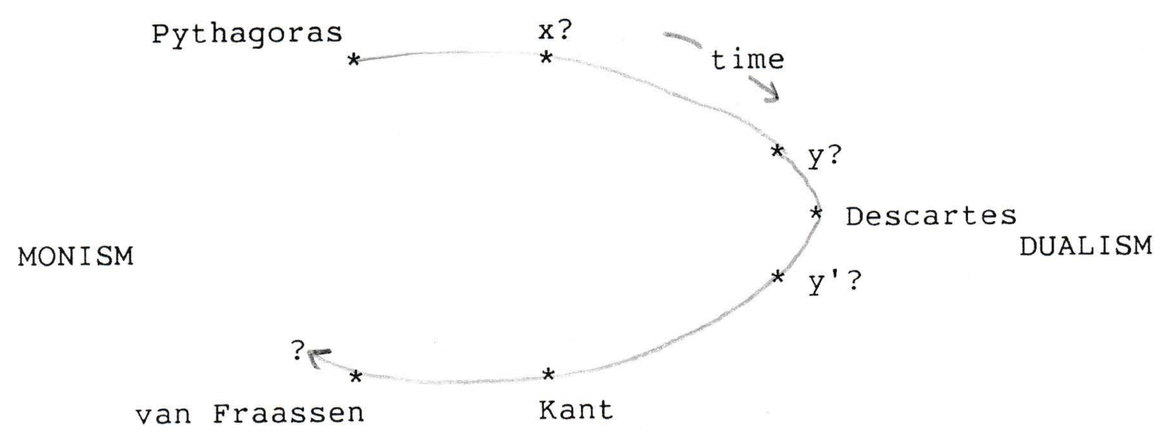
2) Also, science began with a search for certainty. Pytha-

goras, indeed, focused nearly all his attention on the certain numerical relations which underlie the manifestations of nature. With the scientific revolution, the grasp on certainty became more tenuous, as matter itself was seen as mechanical and devoid of its own intelligence. In our own era, a great deal of this hope for certainty has been simply abandoned. Thus, it might be said, there has been somewhat of a return to the attitude of Heraclitus--according to which it might be preferable, in some respects, to acknowledge one's limits by employing paradoxical, but useful and non-dogmatic, models, rather than insisting always on a literal adherence to some one perspective.

In short, this paper has completed its attempt to briefly trace the history of philosophy of science in the image of a horseshoe's curve, and to clarify and support that model. In the course of this discussion, a number of philosophers have been quoted or described to discover how their contributions reflect the general theme of some stage within the 'horseshoe'.

Of course, the presenting of this model proposes at least as many questions as it (hopefully) solves. For, let the model be drawn as shown below:

Figure 1: The Horseshoe of Western Science



The philosophers indicated are taken to hold increasingly dualistic views the further to the right they appear on the figure. Time is represented as the progression of the horseshoe curve (approximately clockwise) from the emergence of Western science. Where is the curve heading next? How--more explicitly--should individual thinkers be placed on the curve? What does the vertical distance between points represent (if anything)? Is there a philosopher, or at least a general philosophical period or position, corresponding to point x (which is 'opposite' from Kant)? What is x's precise relation to Kant's own views? These, and many like these, are the sorts of questions for possible research which the model is intended to suggest.

To close this paper, the reader will be left with a question: If indeed the horseshoe image of the philosophy of science provides a useful model for the tendencies and progress of this field, then what can we expect with regard to its future? Will it, perhaps, complete its cycle in the direction of its origin; and, if so, what would this mean?

To answer such questions, even if possible, would be clearly beyond the scope of this paper. What follows is not intended as a rigorous argument. Since the case for this paper's claims has already been presented and defended, above, all that remains is to offer, as an afterword, the following two quotes, each of which suggests a vision of where the next steps of science may lie. Whether these contain some truth, or whether some more traditional path will be followed, it is perhaps too soon to say. But since the trends revealed in this paper suggest that science is not likely to remain stagnant, but is changing in its form and content, it is felt a fitting close to present these thoughtful views.

> DR. FRITJOF CAPRA (The Tao of Physics): The age-old tradition of exploring complex structures by breaking them down into simpler constituents is so deeply ingrained in Western thought that the search for these basic components is still going on.
>
> There is, however, a radically different school of thought in particle physics which starts from the idea that nature cannot be reduced to fundamental entities, such as elementary particles or fundamental fields. It has to be understood entirely through its self-consistency, with its components being consistent both with one another and with themselves. This idea ... is known as the 'bootstrap hypothesis.'[46]

CARL JUNG (Preface to Wilhelm's Translation of the I Ching): My position in these matters is pragmatic, and the great disciplines that have taught me the practical usefulness of this viewpoint are psychotherapy and medical psychology. Probably in no other field do we have to reckon with so many unknown quantities, and nowhere else do we become more accustomed to adopting methods that work even though for a long time we may not know why they work. ...The irrational fulness of life has taught me never to discard anything, even when it goes against our theories (short-lived at best) or otherwise admits of no immediate explanation. It is of course disquieting, and one is not certain whether the compass is pointing true or not; but security, certitude, and peace do not lead to discoveries.[47]

1. Regarding this interpretation for the usefulness of models in suggesting further questions for research, compare Mary B. Hesse, *Models and Analogies in Science* (Notre Dame, Indiana: University of Indiana Press, 1966), p. 8.

2. Edward A. Maziarz and Thomas Greenwood, *Greek Mathematical Philosophy* (N.Y.: Frederick Ungar, 1968), p. vii

3. Warren A. Shibles, *Models of Ancient Greek Philosophy* (London: Vision Press, 1971), p. 44

4. Heraclitus (49aFr), in Walter Kaufmann, ed., *Philosophic Classics: Thales to St. Thomas* (Englewood Cliffs, N.J.: Prentice-Hall, 1961), p. 19.

5. O. Neugebauer, *The Exact Sciences in Antiquity*. (Princeton, N.J.: Princeton University Press, 1952), p. 142.

6. *Ibid.*, p. 35.

7. Maziarz and Greenwood, pp. 7,9.

8. *Ibid.*, p. 7.

9. F.M. Cornford, *From Religion to Philosophy: A Study in the Origins of Western Speculation* (N.Y.: Harper & Row, 1957), pp. 198ff.

10. *Ibid.*

11. Maziarz and Greenwood, p. 12.

12. *Ibid*., p. 17.

13. Aristotle (Metaph. 987a) in Kaufman, p. 389.

14. Cornford, p. 207.

15. *Ibid*., p. 209.

16. *Ibid*., pp. 212f.

17. *Ibid*., p. 213.

18. Edward Caird, *The Evolution of Theology in the Greek Philosophers* (Glasgow: James MacLehose and Sons, 1904), p. 20.

19. Cornford, pp. 194ff.

20. John H. Finley, *Four Stages of Greek Thought* (Stanford, Cal.: Stanford University Press, 1966), pp. 84f.

21. Cornford, p. 255.

22. Maziarz and Greenwood, p. 158.

23. Richard S. Westfall, *The Construction of Modern Science* (N.Y., Toronto, *et al* : John Wiley and Sons, 1971), p. 30.

24. *Ibid*., p. 31.

25. *Ibid*., pp. 31f.

26. F.M. Cornford, _The Laws of Motion in Ancient Thought_ (Cambridge: Cambridge University Press, 1931), p. 17.

27. _Ibid._, pp. 15f, 20.

28. Westfall, p. 1.

29. _Ibid._, p. 12.

30. _Ibid._, p. 18.

31. Cornford, _Laws of Motion_, pp. 22f.

32. Westfall, p. 40.

33. Bas C. van Fraassen, _The Scientific Image_ (Oxford: Clarendon, 1980), p. 12.

34. Alan Musgrave, "Constructive Empiricism vs. Scientific Realism," _Philosophical Quaterly_ 32 (July 1982) :265.

35. Michael Friedman, "Bas C. van Fraassen: _The Scientific Image_," _Journal of Philosophy_ 79 (May 1982) :278.

36. David Hume, _Enquiries Concerning the Human Understanding and Concerning the Principles of Morals_, 2nd ed., edited by L.A. Selby-Bigge (Oxford: Clarendon Press, 1902, 1970), p. 43.

37. _Ibid._, p. 162.

38. C.G. Hempel, The Philosophy of Natural Science (Englewood Cliffs, N.J.: Prentice-Hall, 1966), pp. 80f.

39. For this notion of ontological commitments being a function of the variables which are bound within an accepted theory, see also Willard Van Orman Quine, From a Logical Point of View (Cambridge, Mass. and London, England: Harvard University Press, 1953), pp. 12ff.

40. Israel Scheffler, The Anatomy of Inquiry (N.Y.: Alfred A. Knopf, 1963), p. 59.

41. Hartry H. Field, Science Without Numbers (Princeton, N.J.: Princeton University Press, 1980), p. 13.

42. John G. Kemeny, A Philosopher Looks at Science (Toronto, et al: D. Van Nostrand, 1959, 1961), p. 85.

43. Ibid., p. 86.

44. Hesse, p. 19.

45. For more on models, and their potential employment, see also: (a) Martin H. Cundy and A.P. Rollett, Mathematical Models (Oxford: Clarendon, 1951-1957), p. 11; and (b) Philip J. Davis and Reuben Hersh, The Mathematical Experience (Boston: Houghton, Mifflin, 1981), p. 78.

46. Fritjof Capra The Tao of Physics (U.K.: Fontana/Collins, 1976), pp. 301f.

47. Carl Jung, Foreword to The I Ching: or Book of Changes, Translated into English by Cary F. Baynes, from the Translation of Richard Wilhelm (Princeton, N.J.: Princeton University Press, 1950, 1967), p. xxxiv.

BIBLIOGRAPHY

Caird, Edward. The Evolution of Theology in the Greek Philosophers. Glasgow: James MacLehose and Sons, 1904.

Capra, Fritjof. The Tao of Physics. U.K.: Fontana/Collins, 1976.

Cornford, F.M. From Religion to Philosophy: A Study in the Origins of Western Speculation. N.Y.: Harper and Row, 1957.

Cornford, F.M. The Laws of Motion in Ancient Thought. Cambridge: Cambridge University Press, 1931.

Cundy, H. Martin, and Rollett, A. P. Mathematical Models. Oxford: Clarendon, 1951-1957.

Davis, Philip J., and Hersh, Reuben. The Mathematical Experience. Boston: Houghton, Mifflin, 1981.

Field, Hartry H. Science Without Numbers. Princeton, N.J.: Princeton University Press, 1980.

Finley, John H. Four Stages of Greek Thought. Stanford, Cal.: Stanford University Press, 1966.

Friedman, Michael. "Bas C. van Fraassen: The Scientific Image." Journal of Philosophy 79 (May 1982): 274-283.

Hempel, C.G. The Philosophy of Natural Science. Englewood Cliffs, N.J.: Prentice-Hall, 1966.

Hesse, Mary B. Models and Analogies in Science. Notre Dame, Indiana: University of Indiana Press, 1966.

Hume, David. Enquiries Concerning Human Understanding and Concerning the Principles of Morals. 2nd ed. Edited by L.A. Selby-Bigge. Oxford: Clarendon, 1902, 1970.

Jung, C.G. Foreword to The I Ching | or Book of Changes|. Translated into English by Cary F. Baynes from the German Translation of Richard Wilhelm. Princeton, N.J.: Princeton University Press, 1950, 1967.

Kemeny, John G. A Philosopher Looks at Science. Toronto, et al: D. Van Nostrand, 1959, 1961.

Kaufman, Walter, ed. Philosophic Classics: Thales to St. Thomas. Englewood Cliffs, N.J.: Prentice-Hall, 1961.

Levi, Isaac. Gambling With Truth. N.Y.: Alfred A Knopf, 1967.

Malament, David. "Hartry H. Field: Science Without Numbers." Journal of Philosophy 79 (September 1982): 523-534.

Maziarz, Edward A., and Greenwood, Thomas. Greek Mathematical Philosophy. N.Y.: Frederick Ungar, 1968.

Musgrave, Alan. "Constructive Empiricism vs. Scientific Realism." Philosophical Quaterly 32 (July 1982): 262-271.

Neugebauer, O. The Exact Sciences in Antiquity. Princeton, N.J.: Princeton University Press, 1952.

Quine, Willard Van Orman. From a Logical Point of View. Cambridge, Mass., and London, England: Harvard University Press, 1952, 1980.

Scheffler, Israel. AThe Anatomy of Inquiry. N.Y.: Alfred A. Knopf, 1963.

Schlesinger, G. Confirmation and Confirmability. Oxford: Clarendon Press, 1974.

Shibles, Warren A. Models of Ancient Greek Philosophy. London: Vision Press, 1971.

van Fraassen, Bas C. The Scientific Image. Oxford: Clarendon Press, 1980.

Westfall, Richard S. The Construction of Modern Science. N.Y., Toronto, et al: John Wiley and Sons, 1971.