DIFFERENCES IN INDIVIDUATION AND VAGUENESS

W. Grafe

Philosophisches Seminar der Universität
Nikolausberger Weg 9c
3400 Göttingen
Federal Republic of Germany

I. EPISTEMOLOGICAL SUGGESTIONS

From an epistemological view, classifying a statement as 'vague'[0] means to judge the statement in question to be a mixture from partial knowledge and partial ignorance. Accordingly it seems desirable to describe the boundary between knowledge and ignorance hidden in the vague statement.

Ludwig discusses [1] vagueness in physics, especially vagueness in measuring statements. The example he uses is 'measurement of Euclidean distance', i.e. the meaning of statements which are often written as "$d(x,y) = \alpha \pm \varepsilon$", where vagueness is expressed by "$\pm\varepsilon$" indicating the so-called "error of measurement". Ludwig maintains that physicists have come to refrain from supposing that physical objects have exact properties which cannot be measured exactly (but only within the indicated 'error of measurement'). The argument substantiating this attitude is obviously that the ascription of precise properties to physical objects is beyond the reach of physical theorizing. But what is the alternative? At first it seems that Ludwig would accept a rivalizing supposition to the effect that physical objects do have 'vague' physical properties. This indeed would be a rather puzzling point of view, not in accord with the course adopted in the beginning. Vagueness of statements there was taken to represent some deficit of knowledge but not a knowledge of a special kind, viz. that some objects does have a 'vague' property or that _n_ objects (in a certain ordering) stand in a vague' _n_-place relation. Now, nothing in Ludwig's formal treatment of 'vagueness' must be understood to imply this 'puzzling view', but on the other hand nothing in Ludwig's exposition is apt to exclude it. Are we then to take the well defined Euclidean distance function d(x,y), which is as precise a

A. Hartkämper et al. (eds.), *Structure and Approximation in Physical Theories*

term as you could imagine and pervert it into a 'vague three place predicate' when applied to empirical objects? And what does such application consist in? Or are we to hold the 'naive view' that physical objects x,y are (at one instant of time) separated by a precise distance, which simply cannot be measured exactly? My way out of this dilemma is to give a rather detailed description of how vagueness gets in, without assuming any predicates to be 'vague'.[2] This description amounts to pointing out different means of individuation of objects on different levels of argument and reducing statements of vagueness to statements of difference in individuation.

II. LUDWIG'S EXEMPLIFICATION OF 'VAGUENESS'

Let me indicate what I hold to be the main features of Ludwig's account of vagueness in the case of measuring d(x,y) and then scetch my point of departure.

To restate the example, we need a rough restatement of the MTA-component of Ludwig's structuring PT of the intuitive concept of "physical theory":

MT is a mathematical theory, containing some (rather weak) set-theory as a subtheory and, in addition to this, 'special axioms'. A suitable two-valued logic is assumed to be incorporated in the language of MT.

G is to be conceived as a kind of 'open collection' (not to be taken as a well defined set), which comprises among other things 'pre-theories' and labelled empirical objects a_i ("genormte Realtexte").

"(——)" is a set of correspondence rules which

1. single out terms Q_j of MT;
2. single out some relations R_n of MT (R_n has K_n places);
3. establish rules how to
   a) state sentences of the form $\ulcorner a_i \in Q_j \urcorner$
   b) state sentences built up from the relations R_n given in 2. taking as arguments the a_i and perhaps specific real numbers and negations of such sentences.

In order to cope with vagueness, 3.b) has to be modified by replacing the R_n by weaker $\tilde{R}_n$.

MTA may then be understood as the extension of MT resulting from the application of the correspondence rules (——) to the a_i in G. To restate the example, MT is supposed to contain metrical Euclidean geometry with the distance function d(x,y) defined as a map: $\mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}_{0+}$ which satisfies the usual equation[3].

The a_i in G are supposed to label marked places in physical space, the only Q_j needed is $\mathbb{R}^3$, the only R_n is $R(x,y,\alpha)$, defined as $d(x,y) = \alpha$. Thus the statement obtained by 3. of (——) are:

a) $a_i \in \mathbb{R}^3$;[3]
b) the measurement-results from G reported in terms of $R(x,y,\alpha)$.

To avoid inconsistency of MTA only due to the limited accuracy of measurements, the statements obtained by 3.b) of (——) have to be replaced by analogous statements, in which $R(x,y,\alpha)$ has to be replaced by $\tilde{R}(x,y,\alpha)$, which is defined as follows: First define some imprecision-set U for some fixed real number $\varepsilon > 0$ by $U = U_\varepsilon = \{\langle\alpha,\beta\rangle \mid \alpha,\beta \in \mathbb{R} \wedge |\alpha-\beta| \leq \varepsilon\}$. Then $\tilde{R}$ is defined by

$$\tilde{R}(x,y,\alpha) \Leftrightarrow \bigvee_\beta[\beta \in \mathbb{R}_+ \wedge R(x,y,\beta) \wedge \langle\beta,\alpha\rangle \in U] \quad \text{for } x,y \in \mathbb{R}^3 \text{ and } \alpha \in \mathbb{R}_+.$$

Here I conclude my summary of Ludwig's account, because the point I am interested in does not depend on the further refinement of the method of imprecision-sets. Two points seem to me indisputable and I think they are held by Ludwig too:

1. $\tilde{R}$ is a term of MT, i.e. a purely mathematical term just as R, because it is explicitely defined in MT without any reference to G or (——). So $\tilde{R}$ is as exact or precise a relation as R.
2. The claim that $\tilde{R}(x,y,\alpha)$ is needed to replace $d(x,y) = \alpha$ in the statement according to 3.b) of (——) implies that not all of the a_i are uniquely represented in MTA by single elements of $\mathbb{R}^3$ (because $d(x,y)$ is a function, taking a unique value for uniquely identified arguments).

These are the features in Ludwig's presentation of the example "measurement of $d(x,y)$", which I found in need of analysis. The observations 1., 2. above do not decide anything with respect to the 'puzzling view' sketched in section (I), because from Ludwig's treatment it is not clear what is meant by the application of $d(x,y)$ to empirical objects. The source of the trouble is that the statements obtained by 3.a) of (——), which assert the a_i to be elements of $\mathbb{R}^3$ are unacceptable as they stand. Instead of statements of the form $a_i \in \mathbb{R}^3$, we should concentrate on statements of the form $\varphi(a_i) \in \mathbb{R}^3$, where $\varphi(a_i)$ denotes some mathematical objects, while keeing in mind that in order to cope with 'vagueness', we shall have to consider not one single map φ, but some set of such maps. Now Ludwig may well maintain that a suitable variety of such maps is assumed to be supplied by (——). My point is merely that we do not understand what is meant by "MTA contains vague statements" without information about what such mappings look like. Thus, in a sense, the rest of this paper is concerned with the task of understanding what is meant by the statement 3.a) of (——), or better: by what sort of statements they should be replaced.

III.1. CONCEIVING 'VAGUENESS' AS RESULTING FROM DIFFERENCES IN INDIVIDUATION

Before specifying the example, I must list some properties of what I propose to call a "pragmatically controlled (data-) language", conceived as a first order language. The next step then is to conceive a set of data formulated in such a language as a first order theory, say: the 'data-theory'. As we may suppose that such a (finite) set of data has a truth evaluation (is truth-value definite for any datum in the set) on empirical grounds, we may easily fix even the set theoretic interpretation of this data-set. The next, and I hope, the decisive step consists in confronting the doubly interpreted data-set (empirically and set-theoretically interpreted) with the corresponding "idealized measurement structure". This structure is well known from the literature and I take it to be expressed as a first order theory plus set theoretic interpretation (which I choose to be model-theoretic). The vagueness statement then is expected to result from the combination of the two clearly different set-theoretical interpretations. To summarize not only the procedure but also the thesis: A prominent property of a "pragmatically controlled (data-) language" will be that empirical objects, which figure as the denotata of the individual constants of the language, are identified operationally, and consequently by finitary means. (The truth-value decisions of the respective sentences are also approached according to finitary methods.) This, in my understanding of theorizing in physics, is in sharp contrast to any means used by some physical theory in describing (individuating) its objects. Restated: theoretical characterization of objects is non-finitary, e.g. by means of a theory admitting only infinite models (in the model-theoretic sense). Accordingly we should not be astonished, if the difference between theoretical and operational individuation is expressed as some 'vagueness'. (There is no reason to suppose such vagueness of physical assertions as representing a problem 'not yet' solved by physical theory. Of course the possibility of varying the sort or degree of vagueness will depend on the physical knowledge available at the time. But it seems quite plain that such vagueness is rather a consequence of theory-construction in physics, viz. that the structure explained are 'idealized structures' only, which results from the use of mathematical methods.) I do not claim that the contrast "individuation by finitary means" vs. "individuation by non-finitary means" is the only noteworthy thing about "operational" and "theoretical" individuation; but I claim that it suffices as a first attempt to describe how 'vagueness' gets in, as far as our example is concerned.

III.2. A DATA LANGUAGE FOR BASIC MEASUREMENTS

First I shall state some conditions (1) on any pragmatically controlled language of a set of data and secondly state conditions (2) on a data-laguage for basic measurements of lengths by rods. This is a considerable simplification of Ludwig's example (Euclidean geometry), but I think it will serve the purpose of demonstration just as well.

(1) L is a first order language with identity and with only finitely many individual and predicate constants. For each indi-

vidual constant ⌜a_i⌝ the object denoted is operationally identified, and this object is denoted by no other individual constant. For any atomic sentence of L the truth-value can be established according to a set of accepted methods. (These methods may, but need not all be operational.) A set D of data in L is a finite set of quantifier-free sentences in L, including for any individual constant ⌜a_i⌝, that occurs in some member of D, the equation ⌜$a_i = a_i$⌝. Moreover D is consistent and the sentences of D have been decided according to the accepted methods.[4)]

(2) The predicates for length-comparison by rods include two two-place predicate constants "R" and "G", which shall be understood as follows: "R" denotes the operational "greater than" and "G" the operational relation "is equal in length to". The accepted methods for assigning truth-values shall include: for no a_i is ⌜$Ra_i\ a_i$⌝ or ⌜$Ga_i\ a_i$⌝ true. The language may or may not contain a further three-place predicate constant "Add" denoting "addition of lengths".

Suppose, the length comparisons lead to results of the kind we expect of such comparisons from earlier experience. Then we may define a finite weak order if D is complete in the sense, that <u>all</u> sentences of the forms ⌜$Ra_i\ a_j$⌝, ⌜$Ga_i\ a_j$⌝, ¬⌜$Ra_i\ a_j$⌝, ¬⌜$Ga_i\ a_j$⌝ have been decided as true or false respectively and D is the set of sentences thus decided as true.

An equivalence relation may be defined by ⌜$Exy \Leftrightarrow Gxy \vee x=y$⌝, and the reflexive order relation may be defined by ⌜$Sxy \Leftrightarrow Exy \vee Rxy$⌝, thus expanding the data-language by two two-place predicate constants "E" and "S". As domain for the finite weak order $\mathcal{E}$ we choose (1-1) a denotation for any ⌜a_i⌝ occuring in some sentence of D. $\mathcal{E}$ is then determined uniquely up to isomorphism as a finite relational structure. Hence the set-theoretical interpretation of D is unique and so is the truth-evaluation by the operational identification of the a_i and the operationally decided predicates "R" and "G".

Keeping in mind the goal of comparing $\mathcal{E}$ with some idealized measurement structure, it seems fair enough to expand $\mathcal{E}$ by adding "Add" to the relations "R", "G", "E", "S", taking its values in accordance with the operational results. But note that the set of a_i's is not enlarged by allowing the operationally interpreted "Add". Of course we may assume D', resulting from D by adding the atomic "Add"-sentences or their negations respectively, to be complete in the quantifier-free sublanguage of L. So D' defines uniquely a finite relational structure $\mathcal{E}'$, which is simply the expansion of $\mathcal{E}$ by "Add".

III.3. AN IDEALIZED MEASUREMENT STRUCTURE

As an idealized structure for basic measurements of lengths we consider the 'positive closed extensive structure' of Krantz et al. (1971). It differs from the finite weak order E' mainly in being closed with respect to "length-addition".

Krantz's idealized addition operation "o" exhibits the Archimedian property. It can be proven (cf. e.g. op.cit., pp. 74 f) that there is a homomorphism into the real numbers, which is unique up to the choice of some positive scale factor. For nearly all finite structures like E', this does not hold[5]. As the idealized structure is closed with respect to length-addition and hence infinite, there is no sense in taking (even some of) its objects to be operationally identified.

I shall now write down as first order formulae the axioms for the 'idealized measurement structure', taken from Krantz et al. (op. cit., p. 73), where $\underset{\sim}{\supset}$ and o are taken as basic predicates.

[O.1] $\bigwedge xy[x \supset y \Leftrightarrow x \underset{\sim}{\supset} y \wedge \neg\, y \underset{\sim}{\supset} x]$; Def. of "$\supset$" in terms of "$\underset{\sim}{\supset}$"

[O.2] $\bigwedge xy[x \sim y \Leftrightarrow x \underset{\sim}{\supset} y \wedge y \underset{\sim}{\supset} x]$; Def. of "$\sim$" in terms of "$\underset{\sim}{\supset}$"

[1.1] $\bigwedge xyz[x \underset{\sim}{\supset} y \wedge \underset{\sim}{y} \supset z \Rightarrow x \underset{\sim}{\supset} z]$; Transitivity of "$\underset{\sim}{\supset}$"

[1.2] $\bigwedge xy[x \underset{\sim}{\supset} y \vee y \underset{\sim}{\supset} x]$; "$\underset{\sim}{\supset}$" is connected

[2] $\bigwedge xyz[x o (y o z) \sim (x o y) o z]$; "o" is weakly associative

[3] $\bigwedge xyz[x \underset{\sim}{\supset} y \Leftrightarrow x o z \underset{\sim}{\supset} y o z \Leftrightarrow z o x \underset{\sim}{\supset} z o y]$; monotonicity

[4.1] $\bigwedge x[1x = x \wedge \bigwedge_n (n+1)x = nx o x]$; "nx" recursively defined

[4.2] $\bigwedge xy[x \supset y \Rightarrow \bigwedge_{zu} \bigvee_n (nx o z \underset{\sim}{\supset} ny o u)]$; 'Archimedian property"

[5] $\bigwedge xy[x o y \supset x]$; positivity.

[O.2] implies what is sometimes called the "weak antisymmetry of $\underset{\sim}{\supset}$" and so any model of [O.2] $\wedge$ [1.1] $\wedge$ [1.2] is a weak order. Of course [1.2] implies the reflexivity of "$\underset{\sim}{\supset}$". Infinity comes in by axioms [4.1] - [5].

"$\underset{\sim}{\supset}$" is the idealized counterpart of the reflexive order-relation "S" defined in III.2. "$\supset$" corresponds to "R", "$\sim$" to the equivalence relation "E" and "o" corresponds to "Add". As noted above it is via the properties of "o" that infinity comes in. This is due to the metalogical requirement, that any model of the axioms has to be closed with respect to the composition "o". Nothing like this is required for model-theoretic interpretations of predicate-constants. This is why for the empirical (data-)language, we had to choose "Add" as a predicate-constant rather than an operation symbol[6].

Axiom [4.2] (together with [4.1] asserts the existence of lengths transcending any given length. This is what I call 'idealized rod production', having no counterpart in an empirical domain.

III.4. THE STRUCTURE $\mathcal{U}$

The set of (first order) logical consequences of the axioms [0.1] - [5] I call $T^+_<$. The consistency of $T^+_<$ is assumed. Accordingly, $T^+_<$ is assumed to have a model, and as $T^+_<$ only admits infinite models, $T^+_<$ has a countable model. We single out one such countable model and refer to it as "$\mathcal{U}$", $\mathcal{U}$ is a set-theoretical entity of the same sort as $\mathcal{E}'$, our uniquely determined model of the set of data D'. We may now choose an object from $\mathcal{E}'$ (i.e. some set-theoretic picture of some a_i) to be the unit element. The unit of $\mathcal{E}'$ of course must be uniquely pictured in $\mathcal{U}$. What about the pictures of the remaining a_i in $\mathcal{E}'$? The sentences of D' and the choice of the unit in the empirical domain and thus in $\mathcal{E}'$ do not in the least determine a unique embedding of $\mathcal{E}'$ in $\mathcal{U}$. The only exception is: $\mathcal{E}'$, and thus the empirical domain, contains only objects which belong to some "initial segment of a standard series". A standard series is a sequence of objects such that its n-th element equals in length n times the unit length. I refer to this exception as the "ruler-case".

IV. CONCLUSION

In order to get a description of how mathematics is applied to empirical objects in basic measurement, I gave a description of basic measurement which is sumarized in the scheme below.

(a) Operationally identified a_i and operationally decided predicates "R", "G" and "Add";

↓ determine empirically

(b) the set of data D', formalized in the quantifier-free sublanguage of the indicated language L

↓ determines logically, up to isomorphism,

(c) the finite weak order $\mathcal{E}'$, containing a value for "Add" too; set-theoretic entity of the type 'relational structure'

↓ isomorphic embedding, not unique, except in the ruler-case

(d) idealized measurement structure $\mathcal{U}$, which is a model of $T^+_<$ (of § III.3.), a countably infinite relational structure

↓ homomorphic embedding, unique up to a scale factor

(e) the set of positive real numbers $\mathbb{R}_+$ with $\leq$, $=$ and $+$, $\cdot$.

My thesis is now a comment on the scheme above:
(i) It is an empirical question, whether the operational verification of the sentences of D' leads to results of the kind that we expect, i.e. whether D' turns out to be consistent, determines a weak order, and does not conflict with the axioms for "o" in its

"Add"-sentences. But if this is the case, $\mathcal{E}'$ is determined uniquely by D' by purely logical and set-theoretical means.

So far, there is no chance for 'vagueness' to get in. Though $\mathcal{E}'$ is a set-theoretical construct, it is only a set-theoretical paraphrase of an empirically determined truth-evaluation. The unique truth-evaluation is just the only thing, which is common to D', conceived as operationally interpreted, and to D', conceived as the set of quantifier-free sentences true in $\mathcal{E}'$. This double interpretation is the only link between empirical and set-theoretical semantics in the example. But 'vagueness' did not appear on the stage of (a) - (c).

(ii) Obviously the source of 'vagueness' is located in the set $I(\mathcal{E}',\mathcal{U})$ of isomorphic embeddings φ of $\mathcal{E}'$ into $\mathcal{U}$. This is, because the remaining part of the construction is the homomorphic embedding $\underline{h}$ of $\mathcal{U}$ in $\langle \mathbb{R}_+,<,=,+,\cdot \rangle$, uniquely determined up to a scale factor.

(iii) Now we are in the situation to state some features of 'vagueness' and how it gets in. First, we might conclude, that none (!) of the sentences considered is vague, because to any sentence a set-theoretic truth-condition is assigned, which would e.g. not be the case, if the methods from box (a) did not lead to yes-no-decisions in all cases. Secondly, what is my answer to the question of what the sentences obtained by Ludwig's (—) 3.a) are apt to mean?

Well then, take the whole, consisting of boxes (a)-(e) and their interrelations, to be a part of PT in Ludwig's sense. Define mappings, that transport any a_i to $\mathbb{R}_+$. Part of the definition of any such mapping is some $\varphi \in I(\mathcal{E}',\mathcal{U})$. We know the set $I(\mathcal{E}',\mathcal{U})$ to contain not one, but 'many' elements (except in the ruler case). The elements of $\mathcal{U}$ are pictured in $\mathbb{R}_+$ by $\underline{h}$ uniquely (unit given). Thus the ambiguous representation of our empirical objects a_i within $\mathbb{R}_+$ is determined by their ambiguous representation in $\mathcal{U}$.

What sort of 'vagueness' is characterized by the set $I(\mathcal{E}',\mathcal{U})$? First, it is not 'error of measurement', since we assumed the measurements to be as precise as can be expected at all, namely to lead to definite yes-no-decisions. Secondly, it would be systematically misleading to say that "we do not know, which mathematical statements should be 'equated' with the empirical statements". There is nothing to know here. What is to be explained, is the way in which empirical statements are correlated with mathematical ones. That this correlation is not unique, and in principle cannot be unique, is simply a consequence of theory construction. The (—) 3.a) statements then have to be replaced by statements of the form "$\psi(a_i) \in X$", where both, $\psi(a_i)$ and X are terms of MT. The story to be told is that application of MT terms to empirical objects does not take place.

(iv) Let me close the paper with some remarks concerning 'individuation'. The fact that the embedding of $\mathcal{E}'$ into $\mathcal{U}$ is not uniquely determined, is drawn from the enormous differences between the way

$\mathcal{E}'$ was determined as opposed to the way $\mathcal{U}$ was. But we should refrain from stating this difference in terms of 'operational' vs. 'axiomatic'. The reason is, that on the stage of box (c) only one characteristic of 'operational truth-value-determination' is left: $\mathcal{E}'$ is characterized by finitary means, exemplifying "finite individuation". Only this information is transported to $\langle \mathbf{R}_+, <, =, +, \cdot \rangle$, nothing else can be confronted with the set-theoretic interpretation $\mathcal{U}$ of $T^+_<$. Again, the term 'axiomatic' does not characterize $\mathcal{U}$ as different from $\mathcal{E}'$, since we used a finite set of axioms to characterize $\mathcal{E}'$ uniquely up to isomorphism. This is even more than can be expected of an axiomatic characterization of $\mathcal{U}$, as $T^+_<$ is a theory of first order admitting infinite models. I think the difference is well described by stating, that operational individuation is by finitary means, which feature is preserved on the stage of $\mathcal{E}'$, while the individuation of objects of $\mathcal{U}$ is by non-finitary means, i.e. by means of an operation ("o") which admits only infinite models.

I do believe that this contrast between $\mathcal{E}'$ and $\mathcal{U}$ is not an accident, but caused by the different purposes they served. The purpose of $T^+_<$ and, accordingly, of $\mathcal{U}$ is to define a certain concept of measurement, that is general enough to subsume all special cases of sets of data like D', and also definite enough to imply the uniqueness of the map into the positive reals.

(v) The last thing I have to explain is, why I call the above mentioned contrast one of 'individuation'. Could one not as well call it a contrast of, say, 'types of predication'?

I do not think so, if one accepts a set-theoretical frame for describing theories, which is common to Ludwig's approach and to model theoretic treatment: The predicates or functions are set-theoretical constructs over the set of 'objects'. Thus, no predicate or function can be well defined unless the base sets are defined, but not vice versa. Thus the answer depends on a methodological decision. But what I hold to be independent of any such decision, is that the contrast of finitary vs. non-finitary means produces the so-called 'vagueness' of statements of basic measurements. Such 'vagueness' is understood to be unavoidable, as far as physics is designed to compare empirical facts with mathematical entities produced by 'non-finitary' methods.

From the epistemological point of view: It turned out that there are no 'vague statements', but contrary to common belief measurement statements are not applications of mathematical terms to empirical objects. There is no such application, and no such application is needed to describe measurement. There is only a correlation of empirical statements with mathematical statements, which is not unique. The (limited) arbitrariness exhibited shows the 'naive view' as well as the 'puzzling view' mentioned in section (I) not only to be badly substantiated, but from the outset unpromising as an attempt to characterize the way, measurement works.

FOOTNOTES

0) single quotation marks ',' are the 'irónic ones), the double ones "," are used to cite (interpreted) expressions in the usual way; Quine's quasi-quotation signs ⌜,⌝ are also occasionally used. No strict treatment of quotation is intended, but hints may seem useful and thus are given.

1) my references to Ludwig's treatment of the example "measurement of d(x,y)" as an example of 'vagueness of measurement' are to: Ludwig (1970), II § 6; (1978a), § 6; (1978b), II §§ 1-2, and III § 5.

2) Thus I reject the description, that 'vagueness of measurement-statements' is resulting from the use of vague predicates in the sense in which "fish" might have been vague once with respect to whales, or "bird" with respect to bats. In fact my thesis might well be understood to imply that 'vagueness' is a misnomer in the cases considered, for if we know anything relevant which is knowable at all, the phenomenon called "vagueness" will still be present.

3) I do not see any relevant modification in taking d(x,y) not to be a function of the sort described, but a function $d : X \times X \to \mathbb{R}_{0+}$, where X is some set constructed from a base set Y that is different from $\mathbb{R}$ but satisfies the same axioms. Thus I take X to be $\mathbb{R}^3$, as Ludwig does in (1978b), p. 10.

4) Note first, that the equation ⌜$a_i = a_i$⌝ are empirical assertions: they state, that the denotations of the ⌜a_i⌝ do not change (during the measurements recorded in D) in the properties expressed by the predicate constants occuring in some member of D. Note further that the condition of the consistency of D is independent of the condition that all members of D are decided as true according to the accepted methods, and so that both are needed.

5) the exception is simply what I term below in section III.4. the 'ruler-case'

6) Similarly stated in Krantz et al. (1971), pp. 81f.

LITERATURE

C.C. Chang, H.J. Keisler, 1973, "Model Theory", Amsterdam.

D.H. Krantz, R.D. Luce, P. Suppes, A. Tversky, 1971, "Foundation of Measurement", Vol. I, New York.

G. Ludwig, 1970, "Deutung des Begriffs 'physikalische Theorie' und axiomatische Grundlegung der Hilbertraumstruktur der Quantenmechanik durch Hauptsätze des Messens", Berlin.

G. Ludwig, 1978a, "Die Grundstrukturen einer physikalischen Theorie", Berlin

G. Ludwig, 1978b, "Einführung in die Grundlagen der Theoretischen Physik", Vol. I, 2nd edition, Braunschweig.