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THE RIGHT TO REMAIN SILENT

ABSTRACT. The paper points out that in dynamic games a player may be better-off if other players do not know his choice of strategy. That is, a player may benefit by *not revealing* (or not pre-determining) the choice of his action in an information set he (thereby) hopes will not be reached. He would be better-off by exercising his "right to remain silent" if he believes — as the empirical evidence shows — that players display aversion to "Knightian uncertainty". In this case, a player who behaves *strategically*, may wish to avoid revealing his strategy. This is true under various interpretations of the notion of "strategy profiles".

KEY WORDS: (Nash) equilibrium in Strategies, dynamic games, ambiguity, Knightian uncertainty

1. INTRODUCTION

Over the last two decades economic theorists (in fields that include micro-economics, macro-economics, industrial organization, and labour economics) have extensively studied dynamic situations in which players move sequentially. Most of the formal analysis is done by representing the model under consideration as a "game tree", and then employing the notion of "equilibrium in strategy profiles", notably Nash equilibrium (or any one of its many refinements).

In recent years economists and game theorists have come to recognize many shortcomings of Nash equilibrium. In a narrow sense, the contribution of this short paper is pointing out another deficiency of Nash equilibrium in dynamic games. Much more ambitiously, I hope to convince (at least some of) the readers that "equilibrium in strategy profiles" is not the appropriate notion that ought to be used in the analysis of dynamic games. This is true both conceptually and empirically: it is very hard to interpret a strategy profile (viewed as either a choice of actions or as beliefs), and neither introspection nor observed behavior suggest that players consider strategy pro-

Theory and Decision **48:** 193–204, 2000.

files. Moreover, I shall show that the use of "equilibrium in strategy profiles" does not allow players to use ambiguity to their advantage.

In "normal form games" the strategy sets constitute part of the given data. Indeed, such games are now known as "games in *strategic form*". Thus, in any analysis of a normal form game, strategies must constitute the basic (in fact, the only!) "building block". Such is not the case with dynamic games. It was an ingenious idea to *invent* the notion of strategy in dynamic games, enabling to transform them into normal form games. [1]This transformation is not trivial; a "strategy" becomes a function that assigns to every information set an action available at this information set. In particular, a strategy profile specifies the *precise* (perhaps probabilistic) actions to be taken in *every* possible contingency (information set). Clearly, this notion is both complex and unintuitive. It is also very difficult to interpret this notion. But more importantly, I shall argue that rarely do we observe players employing strategies. A more plausible building block in the analysis of dynamic games is, perhaps, *a path* or *a play*, that is, the course of action that is to be followed.

I also contend that in many social interactions in "real life" players communicate and discuss their choice of actions, (even if no agreement can be signed or trusted). Players, typically, negotiate over the "paths" to be taken, not over strategies. The Example in Section 3 illustrates that when players are involved in "open negotiations", it may be disadvantageous for a player to choose a strategy. That is, a player may benefit by *not revealing* (*or not pre-determining*) the choice of his action in an information set he *thereby* hopes will not be reached. He would be better-off to "cross the bridge if and when he gets to it". A player might benefit from exercising his "right to remain silent" if he believes – as the empirical evidence shows – that players display aversion to "Knightian uncertainty". In that case, a player who behaves *strategically*, may wish to avoid revealing/choosing his strategy.

Section 4 concludes the paper with a discussion of some related literature, and with a modification of the example of Section 3 demonstrating that by "remaining silent", *all* players can be made better-off, relative to the (unique) Nash equilibrium.

2. STRATEGIES IN A DYNAMIC GAME

In this section I shall argue that the analysis of a dynamic game should not be based on "equilibrium in strategy profiles". This is true both conceptually and empirically.

On the conceptual level, it is by no means clear what it is that a strategy profile, in dynamic games, represents, because a crucial feature of a dynamic game is that (some of) players' actions are revealed along the play of the game.

Ever since Cournot, a strategy in a normal form game typically represents a *choice of action(s)*. It is in this way that game theorists have, for a long time, interpreted the notion of a strategy also in extensive form game. But then, how are we to interpret, for example, the action player i's strategy specifies in some information set h, if that information set cannot be reached (because of i's own previous choice of actions) if i were to follow this strategy?

To rescue the usefulness of the notion of "equilibrium in strategy profiles" (and hence, of Nash equilibrium or its refinements) in dynamic games, it was then suggested to interpret a strategy of player i as representing the *beliefs* other players have over the actions i would take. But, again, because in a dynamic game (some of) players' actions are revealed along the play of the game, the beliefs other players have over the actions i would take should be modified as the game unravels and i's past actions are revealed. Beliefs, therefore, ought to depend on the subgame reached.

In addition to the conceptual difficulties, strategy profiles fail also to be descriptive. Typically, individuals do not consider all possible contingencies. Rather, players often "negotiate openly", trying to "convince", "influence", "coordinate", and "agree" on *a course of action* that is to be followed. Sometimes, such agreements include clauses that prescribe the precise consequences (sanctions/punishments) for *some* deviations. But rarely, if ever, are all possible deviations covered. Almost no contract is "complete". The same is true for any "social norm" or "legal system". They specify the "appropriate/legal/acceptable behavior", but neither the social norm nor any legal system pins down the *precise* actions ("punishments") to be taken in *all* contingencies that might possibly arise when the prescribed behavior is not followed.

To conclude, any notion that uses “equilibrium in strategy profiles” considerably limits the relevance of the analysis. This is true when strategies are interpreted as the actual choice of actions by the players or as players’ beliefs, or as representing the legal system or players’ “thought processes”.

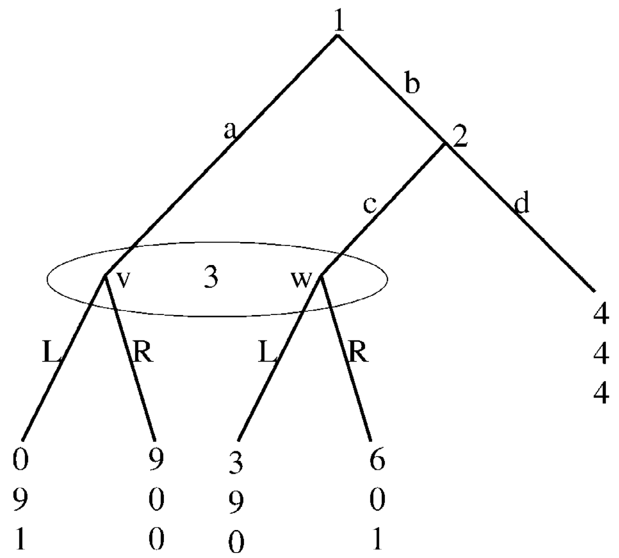

Figure 1.

3. AN EXAMPLE

Consider the following diplomatic “peace-negotiation” scenario, which is represented by the game tree in Figure 1.

Each of the two warring countries, 1 and 2, has to decide whether or not to reach a peace agreement, represented by the path (*bd*). Failing to reach an agreement, country 3 would “re-evaluate” its policy, a decision that will affect both countries 1 and 2. Assume that country 3 has no way to know which of the two countries caused the breakup of the negotiations (otherwise, it could threaten to retaliate against that country). All it observes is whether or not the negotiations were successful. As the payoffs in Figure 1 indicate, it is in the best interest of country 3 that the two warring countries sign the peace agreement. Since country 3 cannot know who is responsible for the breakup of the peace negotiations, both policies L and R are "rational". Both countries 1 and 2 (correctly) anticipate this set of "plausible/rational" re-evaluated policies. Therefore, unless country 3 *pre-determines, or reveals in advance* the policy it is going to adopt should the peace treaty not be reached, countries 1 and 2 have no way to *know* (*even probabilistically*) which policy would be adopted by country 3. It is, then, conceivable that each country will follow the path (bd), but *each because of different reasons*: country 1 for believing that policy L is more likely to be adopted than policy R, and country 2 for believing that policy R is more likely to be adopted than policy L. It is important to observe that if both countries held the same beliefs on the precise likelihood of the adoption of policies L and R, at least one of these two countries would find it in its best interest to jeopardize the peace talks. Nevertheless, by remaining silent, player 3 can create some uncertainty in the other players' minds, thereby accomplishing his goal (that his information set is not reached).

However, no Nash equilibrium for this game supports the path (bd). In fact, this game possesses a *unique Nash equilibrium*, which is given by: player 1 chooses actions a and b with equal probabilities [i.e., he uses the mixed strategy $(\frac{1}{2}a, \frac{1}{2}b)$], player 2 chooses c (with probability 1), and player 3 chooses actions L and R with equal probabilities [i.e., he uses the mixed strategy $(\frac{1}{2}L, \frac{1}{2}R)$]. The resulting equilibrium payoff vector is (4.5,4.5,0.5).

The success of the peace talks between Israel and Egypt (players 1 and 2) mediated by the USA (player 3) following the 1973 war, may be, at least partially, attributed to such a phenomenon. Egypt and Israel were each afraid that if negotiations broke down, she would be the loser.

"And once a negotiation is thus reduced to details, it has a high probability of success – unless one party has consciously decided to make a show of flexibility simply to put itself in a better light for a deliberate breakup of the talks. Egypt was precluded from such a course by the plight of the Third Army, Israel by the fear of diplomatic isolation." (Kissinger 1982: 802).

I shall now show how player 3 can implement the path (bd) when players are allowed to openly communicate. Were I player 3, I would suggest players 1 and 2 to follow the path (bd). I definitely would choose *not* to disclose the choice of my action if my information set were to be reached. By “remaining silent”, players 1 and 2 would no longer have a *single* common belief about my choice of action. It is then conceivable that player 1 might fear that I would choose L (with probability greater than 5/9), and that player 2 might fear that I would choose R (with probability greater than 5/9). In this case, each of the two players would be happy with the payoff of 4, thus, they would accept my suggestion to follow the path (bd). And I shall get a payoff of 4 instead of my Nash equilibrium payoff of 1/2. That is, by *deferring or concealing* the choice of my strategy, I may well deter the players from employing the Nash strategies, thereby considerably increasing my own payoff.

The unique Nash equilibrium may not be acceptable even if it is interpreted as a *recommendation*. Indeed, if either an outside recommender or one of the two players were to suggest that we follow the unique Nash equilibrium rather than the path (bd), I, as player 3, would openly reject this recommendation. Instead, I would tell the other two players that I am not yet sure which probability distribution over my actions L and R I will choose, but in any case, I can assure them that I shall *not* follow their (Nash) recommendation. Note that this threat of mine is “credible”. For, if players 1 and 2 would follow their Nash strategies, then my (expected) payoff is 1/2 *no matter what action I choose*. I stand to lose nothing by adhering to my threat. It is, therefore, likely that players 1 and 2 would reconsider and agree to follow that path (bd), instead.

4. CONCLUDING REMARKS

REMARK 1. The following simple modification of our example shows that the strategic employment of Knightian uncertainty might yield an outcome that is Pareto superior to the (unique) Nash outcome.

Since the game in Figure 1 has a unique Nash equilibrium that passes through player 3’s information set, the only Nash payoff in the game depicted in Figure 2 is (2, 2, 2, 2). But, if player 3 does not specify his strategy, then the players may well agree to follow the path (Dbd) which yields the Pareto superior payoff of (4, 4, 4, 4). [Note that in this example player 4 need not worry that player

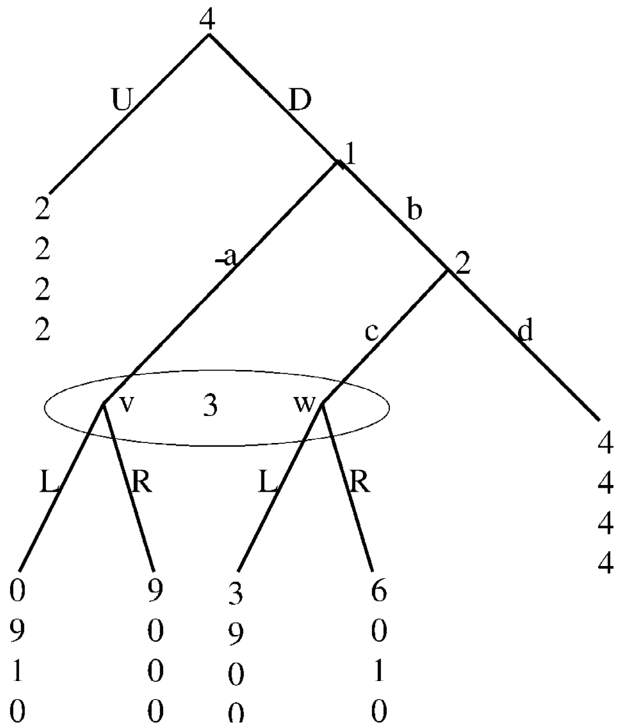

Figure 2.

3 might decide to "double cross", i.e., to remain silent in order to induce player 4 to choose *D*, and then to disclose his choice were his information reached. Player 3's interests are best served by remaining silent.]

REMARK 2. Knight (1921) argued for a distinction between uncertainty (a situation in which players are not informed about the "objective" probabilities) and risk (when the "objective" probabilities are known by the players). There is ample evidence that players behave differently under uncertainty and risk. Specifically, most players exhibit aversion to uncertainty. The best known example of this phenomenon is the Ellsberg (1961) Paradox. As Ellsberg (1961: 656) notes:

"The important finding is that, after rethinking all their 'offending' decisions in the light of [Savage] axioms, a number of people who are not only sophisticated but reasonable decide that they wish to persist in their choices. This includes many people who previously felt a 'first-order commitment' to the axioms, many of them surprised and some dismayed to find that they wished, in these situations, to violate the Sure-thing Principle."

Many subsequent studies (see, e.g., Camerer and Weber, (1992)) have found ambiguity premiums which are strictly positive. Observe that for the purpose of this paper, the magnitude of these premiums (which is typically around 10–20% in expected value terms) is irrelevant. The existence of these premiums implies that one can construct examples, similar to the one given in Section 3, in which it would benefit a player not to reveal/pre-determine his choice of actions in some contingencies.

REMARK 3. As was mentioned in the Introduction, there are many other solution concepts that support the path (*bd*). Bernheim (1984) and Pearce's (1984) notion of "rationalizability", which is appropriate if no communication among players takes place, includes this path. Other concepts that include this path emerge from the recent literature on "learning", and are motivated by the fact that "off equilibrium choices" are not observed, and hence the requirement of "commonality of beliefs" cannot be justified. Finally, (*bd*) is also included in the solution concepts that modify the notion of Nash equilibrium to incorporate Knightian uncertainty.

But all of the above are notions of "equilibrium in strategies" (or in "capacities"), and they all extend the notion of Nash equilibrium. The same is true for rationalizable outcomes. Thus, even in our simple example, these notions support *other paths as well* (including the "Nash path"). In contrast, I am not attempting here to come up with an "equilibrium notion" in the absence of commonality of beliefs or in the presence of Knightian uncertainty. Rather, I suggest that players *use* these features to their advantage. In particular, in our example, I suggest that it is the path (*bd*) that would result in that game.

REMARK 4. Of course, just as it might pay a player not to reveal his choice of "credible" action in some of his information sets, (as is the case with player 3 in our example), there are other situations in which a player may wish to reveal the actions he intends to take in the future, thereby attracting players to his information set. I intend to further study the set of paths that is likely to prevail when players behave strategically, but my purpose here is only to suggest that equilibrium in strategies might be inappropriate to study strategic behavior in dynamic games.

APPENDIX: PROOF OF UNIQUENESS

We shall now verify that the game depicted in Figure 1 admits a unique Nash equilibrium, given by: Player 1 uses the mixed strategy $(\frac{1}{2}a, \frac{1}{2}b)$, player 2 uses the pure strategy c, and player 3 uses the mixed strategy $(\frac{1}{2}L, \frac{1}{2}R)$.

It is easy to see that there is no Nash equilibrium in which player 3 employs a pure strategy, since if it is R then player 1 must choose a, in which case, player 3's best response is L. If, on the other hand, player 3's pure strategy is L, then player 1 will choose b, player 2 will choose c, in which case, player 3's best response is R.

Moreover, in every Nash equilibrium player 1 must employ a strictly mixed strategy, since otherwise player 3 would know whether he is in vertex v or in vertex w, and thus employ a pure strategy, contrary to the above argument.

As for player 2, he cannot employ the pure strategy d, since then player 3 would know that he is in vertex v and employ the pure strategy L, contradicting our conclusion that in every Nash equilibrium player 3 does not employ a pure strategy.

Denote by α, β, and γ, respectively, the probabilities that player 1 chooses a, player 2 chooses c and player 3 chooses L in a Nash equilibrium for this game. By the above discussion, we have that $0 < \alpha, \gamma < 1$, and $\beta > 0$. We shall now show that the only values that α, β and γ can assume are 1/2, 1, and 1/2, respectively.

To see that $\beta = 1$, assume otherwise. Then, since we have established that $\beta > 0$, player 2 employs a strictly mixed strategy and therefore he must be indifferent between c and d. That is, $9\gamma = 4$, i.e., $\gamma = 4/9$. Since $\beta < 1$, player 1's unique best response is a, (guaranteeing himself the payoff of 5), which contradicts our conclusion that player 1 uses a strictly mixed strategy. Thus, $\beta = 1$.

As $0 < \alpha < 1$, player 1 is indifferent between a and b. That is $9(1 - \gamma) = 3\gamma + 6(1 - \gamma)$, implying that $\gamma = 1/2$. Finally, since $0 < \gamma < 1$ player 3 is indifferent between L and R, that is $\alpha = 1/2$.

Thus, in the unique Nash equilibrium in this game, $\alpha = 1/2$, $\beta = 1$, and $\gamma = 1/2$ – as we wished to show.

ACKNOWLEDGMENTS

I thank Daniel Arce, Geir Asheim, Ariel Assaf, Giacomo Bonanno, Faye Diamantoudi, Benyamin Shitovitz, Xiao Luo, and Licun Xue for their useful comments and advice. I also thank the Editor for his support and encouragement. Financial support from the Research Council of Canada [both the Natural Sciences and Engineering (NSERC), and the Social Sciences and Humanities (SSHRC)] and from Quebec's Fonds (FCAR) is gratefully acknowledged.

NOTES

1. My understanding is that the definition of a strategy in dynamic games is due to Kuhn (1953).
2. This is evidenced, for example, by the difficulty almost every student encounters when first exposed to this notion.
3. See Greenberg (1990, 1996), and Greenberg, Monderer, and Shitovitz (1996).
4. This, of course, is in sharp contrast to the social environment envisioned by Nash (1951) where: "each participant acts independently, without collaboration or communication with any of the others".
5. See Remark 4.
6. See Remark 2.
7. Ed Green communicated to me that some of his colleagues in the Federal Reserve System in Minnesota use the term "constructive ambiguity" to describe a policy of being deliberately vague about how far they would be willing to go to bail out a large bank if one were to fail.
8. For a more detailed discussion, see, e.g., Rubinstein (1991).
9. A similar criticism, regarding the notion of subgame perfect equilibrium, was put forward by Binmore (1987), arguing that players cannot hold to their *beliefs* if these beliefs have been proved to be wrong in the past; see, also, Osborne and Rubinstein (1994).
10. Only a very limited set of real life situations is captured in Nash's "complete non-communicative" realm. As I have argued in Greenberg (1990), the description of a normal form game does not provide any information concerning the way in which the game is being played. For example, it provides no information concerning the availability of legal institutions that allow for binding agreements, self-commitments, or coalition formation. Nash equilibrium, in addition to providing a solution concept, also "completes" the

description of the game by assuming that every player takes the actions of the other players as given.

11. For a more detailed analysis and discussion, see Greenberg (1996).
12. The example is reminiscent of the "horse-shaped game" in Fudenberg and Kreps (1995, Example 6.1).
13. Country 3's payoff in that case is 4, while the most it can obtain if the negotiations break up is a payoff of 1.
14. See Remark 2.
15. See proof in appendix.
16. Thus, the USA was unable to know which of the two players is "really" responsible for the breakup of the talks, as is reflected in Figure 1. (My footnote.)
17. See, e.g., Fudenberg and Levine (1993), Kalai-Lehrer (1993), and Rubinstein and Wolinsky (1994).
18. See, e.g., Dow-Werlang (1994), Goes et al. (1998), Hendon et al. (1994), Klibanoff (1993), and Lo (1996). Goes et al. (1998) consider a game similar to our example, and they, too, single out the path (bd), from among the set of *Nash equilibrium in lower probabilities*.

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