A Consistent Quantum Ontology

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Abstract

The (consistent or decoherent) histories interpretation provides a consistent realistic ontology for quantum mechanics, based on two main ideas. First, a logic (system of reasoning) is employed which is compatible with the Hilbert-space structure of quantum mechanics as understood by von Neumann: quantum properties and their negations correspond to subspaces and their orthogonal complements. It employs a special (single framework) syntactical rule to construct meaningful quantum expressions, quite different from the quantum logic of Birkhoff and von Neumann. Second, quantum time development is treated as an inherently stochastic process under all circumstances, not just when measurements take place. The time-dependent Schrödinger equation provides probabilities, not a deterministic time development of the world.

The resulting interpretive framework has no measurement problem and can be used to analyze in quantum terms what is going on before, after, and during physical preparation and measurement processes. In particular, appropriate measurements can reveal quantum properties possessed by the measured system before the measurement took place. There are no mysterious superluminal influences: quantum systems satisfy an appropriate form of Einstein locality.

This ontology provides a satisfactory foundation for quantum information theory, since it supplies definite answers as to what the information is about. The formalism of classical (Shannon) information theory applies without change in suitable quantum contexts, and this suggests the way in which quantum information theory extends beyond its classical counterpart.

Contents

1 Introduction

Scientific advances can significantly change our view what the world is like, and one of the tasks of the philosophy of science is to take successful theories and tease out of them their broader implications for the nature of reality. Quantum mechanics, one of the most significant advances of twentieth century physics, is an obvious candidate for this task, but up till now efforts to understand its broader implications have been less successful than might have been hoped. The interpretation of quantum theory found in textbooks, which comes as close as anything to defining "standard" quantum mechanics, is widely regarded as quite unsatisfactory. Among philosophers of science this opinion is almost universal, and among practicing physicists it is widespread. It is but a slight exaggeration to say that the only physicists who are content with quantum theory as found in current textbooks are those who have never given the matter much thought, or at least have never had to teach the introductory course to questioning students who have not yet learned to "shut up and calculate!"

On all sides it is acknowledged that the major difficulty is the *quantum measurement problem*. Significantly, it occupies the very last chapter of von Neumann's 1932 *Mathematical Foundations of Quantum Mechanics* [1], and forms what many regard as the least satisfactory feature of this monumental work, the great-grandfather of current textbooks. The difficulties in the way of using measurement as a fundamental component of quantum theory were summed up by Wigner in 1963 [2], and confirmed by much later work; see, e.g., the careful analysis by Mittelstaedt [3]. A more recent review by Wallace [4] testifies both to the continuing centrality of the measurement problem for the philosophy of quantum mechanics, and to the continued lack of progress in resolving it; all that has changed is the number and variety of unsatisfactory solutions.

Actually there are two distinct measurement problems. The *first measurement problem*, widely studied in quantum foundations, comes about because if the time development of the measurement apparatus (and its environment, etc.) is treated quantum mechanically by integrating Schrödinger's equation, the result will typically be a *macroscopic quantum superposition* or "Schrödinger cat" in which the apparatus pointer—we shall continue to use this outdated but picturesque language—does not have a definite position, so the experiment has no definite outcome. Contrary to the belief of experimental physicists.

If this first problem can be solved by getting the wiggling pointer to collapse down into some particular direction, one arrives at the *second measurement problem*: how is the pointer position related to the *earlier* microscopic situation which the apparatus was designed to measure, and which the experimental physicist believes actually caused the pointer to point this way and not that way? When one hears experimental particle physicists give talks, it sounds as if they believe their detectors are triggered by the passage of real particles zipping along at enormous speed and producing electrical pulses by ionizing the matter through which they pass. No mention of the sudden collapse of (the modern electronic counterpart of) the apparatus pointer at the end of the measurement process. Instead, these physicists believe not only that the apparatus is in a well-defined macroscopic state—each bit in the memory is either 0 or 1—after the measurement, but in addition that this outcome is well correlated with a prior state of affairs: one can be quite confident that a negative muon was moving along some specified path at a particular moment in time. Have they forgotten what they learned in their first course in quantum theory?

It this connection note that laboratory measurements in particle physics are, in the vast majority of cases, not appropriately modeled by the scheme proposed by von Neumann in which a seemingly arbitrary wave function collapse leads to a correlation between the measurement outcome and a property of the measured system at a time *after* the measurement has taken place. In a typical particle physics experiment the particle will either have disintegrated, escaped from the apparatus, or been absorbed long before the measuring apparatus has registered its behavior, and in any case the experimentalist is interested in what was going on *before* detection rather than afterwards.

The absence of a satisfactory solution to this second measurement problem has led to the development of a certain quantum orthodoxy which affirms that the only task of quantum theory is to relate the *outcome* of a macroscopic *measurement* to an earlier and equally macroscopic (i.e., describable in ordinary language) *preparation*.[1] The quantum wave function $|\psi(t)\rangle$ associated with the prepared system, regarded as isolated from its environment until it interacts with the measuring apparatus (the uniwave in the terminology of Sec. 3.2 below) does not have any ontological reference; it is simply a mathematical device for calculating the probabilities of measurement outcomes in terms of the earlier preparation procedure. Stated slightly differently, it provides an abstract representation of the preparation process, or at least as much of, or that part of, the preparation process which is needed to calculate probabilities of outcomes of future measurements.

In this approach "preparation" and "measurement" can be thought of as part of the real world, but what happens in between occurs inside a "black box" which cannot be opened for further analysis and is completely outside the purview of quantum theory—although some future theory, as yet unknown, might allow a more precise description. One can be sympathetic with strict orthodoxy in that it is intended to keep the unwary out of trouble. Careless thinkers who dare open the black box will fall into the quantum foundations Swamp, where they risk being consumed by the Great Smoky Dragon, driven insane by the Paradoxes, or allured by the siren call of Passion at a Distance into subservience to Nonlocal Influences. Young scientists and philosophers who do not heed the admonitions of their elders will, like the children in one of Grimms' fairy tales, have to learn the truth by bitter experience.

The measurement problems and the associated lack of a clear conceptual foundation for quantum theory have not only been a stumbling block in quantum foundations work. They have also slowed down, though fortunately not stopped, mainstream physics research. In older fields such as scattering theory, the pioneers spent a significant amount of time working through conceptual issues. But once the accepted formulas are in place and yield results consistent with experiment, their intellectual descendents have the luxury of calculating without having to rethink the issues which confused their predecessors. In fields in which quantum techniques are applied in fresh ways to new problems, such as quantum information (the technical specialty of the author of this paper), conceptual issues that have not been resolved give rise to confusion and wasted time. Both students and researchers would benefit from having the rather formal approach to measurements found in, e.g., Nielsen and Chuang [6], to mention one of the best known books on the subject, replaced by something which is clearer and more closely tied to the physical intuition needed to guide good research, even in a field heavily larded with mathematical formalism. Black boxes can be a useful approach to a problem, but can also stand in the way of a good physical understanding.

At one time it was optimistically supposed that quantum information would provide a new key to resolving the problems of quantum foundations [7, 8]. However, later developments have not confirmed this earlier optimism, and Timpson's 2008 review [9] and his more recent [10] provide a clear indication of where the trouble lies. Bell's question, "Information about what?" [11], has not been answered. And why not? Timpson realizes that quantum information cannot simply be about outcomes of measurements (assuming the first measurement problem has been solved), for this fails to connect these outcomes with properties of the system being measured. And he rejects the idea that measurements can reveal microscopic quantum properties, for this leads, in his opinion, to hidden variables and all the insuperable difficulties associated therewith. Clearly the problem is a lack of a suitable quantum ontology, something which quantum information could be about. (The possibility that quantum information could be about nothing at all is also discussed by Timpson [10] under the heading of "immaterialism," which he does not find satisfactory. For comments on his own proposal in [9] for defining quantum information see Sec. 7.2.)

The thesis of this paper is that both measurement problems can be, and in fact have been, resolved: the motion of the pointer stilled and the black box opened, by a consistent quantum ontology that builds upon two central ideas. The first is a system of logic that addresses the question of how to reason about a quantum system described mathematically by a Hilbert space. The second is a system of stochastic or

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[1] See, for example, [5].

random dynamics that applies to all quantum dynamical processes, not just measurements. These ideas were brought together for the first time in the author's "consistent histories" interpretation [12]. Subsequently they were developed by Omnès, whose work has appeared in numerous papers and two books [13, 14], and further developed, to some extent independently, by Gell-Mann and Hartle [15] using the name "decoherent histories." The differences between decoherent and consistent histories are not sufficient (in the author's opinion) to merit separate discussions, so the single term "histories" will be employed below; anyone who disagrees is welcome to prepend "consistent" wherever desired. The most complete discussion of histories ideas currently available is the author's [16], hereafter referred to as CQT; for more compact treatments see [17, 18] and the first part of [19].[2]

As is often the case for new ideas, the histories approach was subject so serious criticisms by (among others) d'Espagnat [20, 21], Dowker and Kent [22], Kent [23–25], and Bassi and Ghirardi [26, 27], during the decade and a half that followed the original publications. Responses were published in [28–31]; in some cases a further reply to the response will be found immediately after the reply. While these criticisms were (in the author's opinion) largely based upon misunderstandings of the histories program, they had the good effect of leading to a better and clearer formulation of its basic concepts. Vigorous scientific debate is often beneficial in this way, though it becomes ineffective if criticism are cited and responses thereto ignored. To the extent that a lack of clarity on the part of the advocates of the histories approach may be one source of misunderstanding, it is hoped that the present paper provides some remedy. An approach to quantum foundations that not only resolves both measurement problems but in addition a whole host of quantum paradoxes—six chapters are devoted to such paradoxes in CQT—and is consistent with special relativity (no mysterious nonlocal influences) deserves careful attention.

The approach to quantum ontology presented here starts by assuming that classical mechanics, with its phase space and Hamiltonian equations of motion, embodies much of what one might hope would be true of quantum mechanics: a clean mathematical structure, an intuitive but reasonably plausible way to associate the mathematics with (what realists believe to be) the "reality out there," and a system of interpretation in which human beings can seen as part of, but not an essential component in, the physical world when described in physical terms. Of course, quantum mechanics must be different from classical mechanics in some important way, as otherwise we quantum physicists have been wasting our time. But whatever differences there are at the microscopic level, the older classical ontology should be seen to emerge from, or at least be consistent with, the more fundamental quantum perspective.

Focusing on the changes needed when moving from the classical to the quantum world has two fundamental advantages. First, classical mechanics has been around for a long time, and we can claim to understand it, and the associated realistic ontology, reasonably well. So our journey begins at a well-defined location, rather than with complete ignorance. Second, this route avoids getting entangled in various philosophical issues, such as the ultimate (un)reliability of human knowledge, which beset both classical and quantum ontology. Putting them aside will allow a focus on a few central issues, and the author to stay within areas where he can claim some competence.

The quantum ontology presented below has the following features and consequences. First, it has no measurement problem; equivalently, it resolves both measurement problems. Second, the results are fully consistent with textbook formulations of quantum theory, once one comes to see that the textbook approach provides a set of very successful and reasonably efficient calculational tools, rather than a basic conceptual understanding of the quantum world. It is now possible to see how these calculational tools arise out of a fully consistent quantum perspective. Third, quantum mechanics is a *local* theory in which mysterious nonlocal influences no longer play a role [32]. Thus quantum mechanics is fully consistent with special relativity [33], contrary to claims made in some quarters. Fourth, the entire world of classical physics emerges, is consistent with, quantum physics: classical mechanics in appropriate circumstances is an approximation, sometimes an excellent approximation, to the underlying and more exact quantum mechanics which encompasses all mechanical processes at whatever length scale. In circumstances where classical mechanics applies, ordinary logic suffices for discussing physics, and can be seen to be consistent with the more general mode of reasoning required in the quantum domain. Fifth, quantum mechanics is compatible with the traditional idea of an *independent* physical reality whose fundamental properties can be discussed without needing to make reference to human observers or human consciousness. Sixth, this ontology provides a foundation for quantum information; it supplies a specific answer to Bell's question as to what quantum information is about.

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[2]It is somewhat unfortunate that the discussion of consistent histories presented in [4] bears little resemblance to what is found in [16], despite the latter being listed in the bibliography.

All of this at what price? First, the quantum world must be understood using an appropriate form of reasoning with features which differ not only from ordinary propositional logic, but also from the quantum logic proposed by Birkhoff and von Neumann. Second, determinism must be abandoned: quantum time development is irreducibly stochastic in all circumstances, not just when measurements occur.

The remainder of the paper is organized as follows. The ontology of a quantum system at a single time is the topic of Sec. 2, where it is developed in analogy with classical phase space. In particular, the ontology of Hilbert space quantum mechanics necessarily differs from classical mechanics if one follows (at least part way) von Neumann's interpretation of the basic quantum formalism. The logical problem this poses is discussed, along with the solution proposed by the author and Omnès.

Quantum time development is the topic of Sec. 3. Here the fundamental idea goes back to Born [34], but is further developed: the proper use of Schrödinger's time dependent equation, unitary time development, is to compute probabilities. The notion of a family of histories, needed for a proper probabilistic framework of quantum dynamics, is introduced, along with the technique needed for assigning probabilities in a consistent way to histories inside a closed quantum system.

Following Secs. 2 and 3, which form the heart of the paper, some additional topics are discussed in a more cursory manner. Any viable quantum ontology must be able to make sense of the everyday "classical" world of our ordinary experience, and the strategy used to do this in the histories approach is described in Sec. 4. That program is not yet complete, but nothing known at present seems to stand in the way of its full realization, once a misunderstanding of the histories approach going back to Dowker and Kent [22] has been disposed of. Preparations and measurements are the subject of Sec. 5, which indicates the essentials needed to resolve both measurement problems. Quantum locality, including the validity of what is often referred to as Einstein locality, is treated briefly in Sec. 6 (details will be found in [19, 32]). Section 7 indicates in broad strokes how the ontology presented here provides a foundation for quantum information.

Following a brief overall summary in Sec. 8.1, Sec. 8.2 of the concluding Sec. 8 is devoted to a discussion of the logical issues which seem to be at the center of most criticisms of the histories approach, and which need to be clearly understood, whatever conclusion the reader may eventually wish to draw. Section 8.3 contains a few additional remarks about probabilistic dynamics. Finally some open issues, two referring to the histories approach itself and two to its wider applications to problems in the philosophy of science, are mentioned briefly in Sec. 8.4
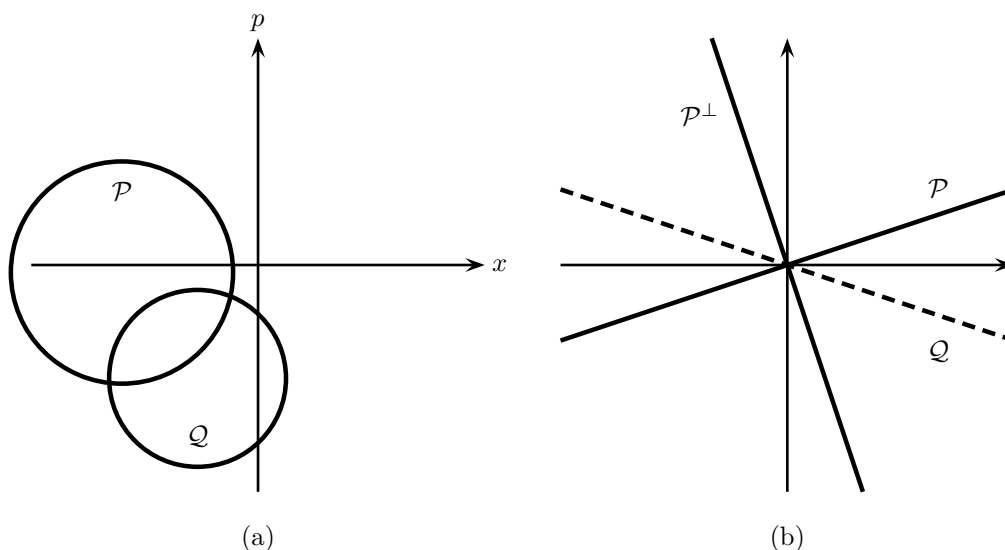
2 System at One Time

Figure 1: (a) Classical phase space; (b) Hilbert space

2.1 Phase Space and Hilbert Space

Starting with classical phase space and classical dynamics, what changes are needed to arrive at the corresponding concepts for quantum theory? The phase space (position x, momentum p) of a particle moving in one dimension is shown in Fig. 1(a), while (b) is a somewhat schematic representation of a two-dimensional (complex) Hilbert space,[3] the closest quantum counterpart to phase space, for the simplest of quantum systems: the spin angular momentum of a spin-half particle (a single qubit in the jargon of quantum information theory).

Following von Neumann [1] we assume that a single point in classical phase space corresponds to a ray or one-dimensional subspace in the Hilbert space: all multiples of a nonzero $|\psi\rangle$ by an arbitrary (complex) number. A ray is a single line through the origin in part (b) of the figure. In addition, the counterpart of a *classical property* P, such as "the energy is between 1.9 and 2.0 Joules", represented by some (measurable) collection of points $\mathcal{P}$ in the classical phase space, is a *quantum property* represented by a (closed) *subspace* of the quantum Hilbert space. The classical property can be represented by an *indicator* function $P(\gamma)$, where γ denotes a point in the classical phase space, and $P(\gamma)$ is 1 at all points where the property holds or is true, and 0 at all other points. The quantum counterpart of an indicator function is a *projector*, a Hermitian operator on the Hilbert space that is equal to its square, so its eigenvalues are 0 and 1. It projects onto the subspace corresponding to the quantum property; any ket in this subspace is an eigenvector with eigenvalue 1. The negation $\neg P$, NOT P, of a classical property corresponds to the set-theoretic complement $\mathcal{P}^c$ of $\mathcal{P}$, with indicator $I - P$; I is the function whose value is 1 everywhere on the phase space. Following von Neumann we assume that the negation of a quantum property is the *orthogonal complement* $\mathcal{P}^\perp$ of the corresponding subspace $\mathcal{P}$, with projector $I - P$, where I is the identity operator on the Hilbert space.

Von Neumann's proposal for associating the negation of a quantum property with the complementary subspace is central to our construction of a consistent quantum ontology, and hence it is appropriate to discuss why this approach is reasonable and to be preferred to the alternative of letting the negation of a property consist of the set-theoretic complement of the ray or subspace. First, the von Neumann approach has been widely accepted and is not (so far as the author is aware) contested by physicists, though it is not always accepted in the quantum foundations community. (We are dealing with an example of the eigenvalue-eigenvector link.) It is consistent with the way students are taught in textbooks to make calculations, even though the idea itself is (alas) not always included in textbook discussions. Second, it is mathematically "natural" in that it makes use of a central property, the inner product, of the Hilbert space, which is what distinguishes such a space from an "ordinary" complex linear vector space. (The inner product defines what one means by "orthogonal.") Third, the orthogonal complement of a subspace is a subspace, whereas the set-theoretic complement is not a subspace; see, e.g., Fig. 1(b).

Fourth, in the case of a spin-half particle the von Neumann proposal asserts that the complement or negation of the property $S_z = +1/2$ (in units of $\hbar$) is $S_z = -1/2$, which is to say one of these properties is the negation of the other, in accord with the Stern-Gerlach experimental result, which showed, somewhat to the surprise of the physics community at the time, that silver atoms passing through a magnetic field gradient come out in *two* distinct beams, not the infinite number which would have been expected for a classical spinning particle. Or might still be expected for a spin-half quantum particle, were one to assume that all the other rays in the two-dimensional Hilbert space represent alternative possibilities to $S_z = +1/2$. However, the Stern-Gerlach result is in complete accord with von Neumann's approach: the negation of the property corresponding to a particular ray in a two-dimensional Hilbert space is the unique property defined by the orthogonal ray, and a measurement determines which of these two properties is correct in a given case.

Fifth there is a sense in which all of chemistry is based on the idea that the electron, a spin-half particle, has only two possible spin states: one "up" and one "down." Granted, students of chemistry find this confusing, since it is not clear how "up" and "down" are to be defined, though they eventually are bludgeoned into shutting up and calculating. Among the calculations which lead to the wrong answer if 2 is replaced by some other number is that of finding the entropy of a partially ionized gas, where it is important to take into account all degrees of freedom, and log 2 is the correct contribution from the electron spin. In addition, the modern theory of quantum information is consistent with the idea that a single qubit with its

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[3]We follow the convention of modern quantum information theory, where the term "Hilbert space" is not restricted to infinite-dimensional spaces, but also includes any finite-dimensional complex linear vector space equipped with an inner product. By restricting the following discussion to finite-dimensional Hilbert spaces we avoid various technical issues that are not pertinent to the conceptual issues we are concerned with.

two-dimensional Hilbert space can contain at most one bit of information; see [33].

It is important to stress this point, for if one accepts von Neumann's negation a wide gap opens between classical phase space and quantum Hilbert space, one clearly visible in Fig. 1. Any point in classical phase space, any possible mechanical state of the system, is either inside or outside the set that defines a physical property, which is therefore either true or false. In the quantum case there are vast numbers of rays in the Hilbert space that lie neither inside the ray or, more generally in higher dimensions, the subspace corresponding to a property, nor in the complementary subspace that corresponds to its negation. For these the property seems to be neither true nor false, but somehow undefined. How are we to think about this situation? This is in a sense *the central issue* for any ontology that uses the *quantum Hilbert space* as the physicist's fundamental mathematical tool for representing the quantum world. One approach, typical of textbooks, is to ignore the problem and instead take refuge in "measurements." But problems do not go away by simply being ignored, and ignoring this particular problem makes it re-emerge under a different name: the measurement problem.

Before going further, let us restate the matter in the language of indicator functions and projectors introduced earlier. The product of two (classical) indicator functions P and Q is itself an indicator function for the property P AND Q, the intersection of the two sets of points $\mathcal{P}$ and $\mathcal{Q}$ in the phase spaces, Fig. 1(a). However, the product of two (quantum) projectors P and Q is itself a projector *if and only if $PQ = QP$*, that is if the projectors *commute*. Otherwise, when PQ is not equal to QP, neither product is a projector, and thus neither can correspond to a quantum property. A ray $\mathcal{Q}$ in a two-dimensional Hilbert space, Fig. 1(b), that coincides neither with a ray $\mathcal{P}$ nor its orthogonal complement $\mathcal{P}^\perp$ is represented by a projector Q that does not commute with P, and thus it is unclear how to define the conjunction of the corresponding properties.

Von Neumann was not unaware of this problem, and he and Birkhoff had an idea of how to deal with it. Instead of using the product of two noncommuting projectors it seemed to them sensible to employ the set theoretic *intersection* of the corresponding subspaces of the Hilbert space, which is itself a subspace and thus corresponds to some property, as a sort of quantum counterpart of AND. The corresponding OR can then be associated with the direct sum of the two subspaces. When projectors for the two subspaces commute these geometrical constructions lead to spaces whose quantum projectors, PQ and $P + Q - PQ$, coincide precisely with what one would expect based on the analogy with classical indicators.

The resulting structure, known as *quantum logic*, obeys some but not all of the rules of ordinary propositional logic, as Birkhoff and von Neumann pointed out. If one naively goes ahead and applies the usual rules of reasoning with these definitions of AND and OR it is easy to construct a contradiction; for a simple example see Sec. 4.6 of CQT. While quantum logic has been fairly extensively studied it seems fair to say that this route has not made much if any progress in resolving the conceptual difficulties of quantum theory. It is not mentioned in most textbooks, and is given negligible space in [4]. Perhaps the difficulty is that physicists are simply not smart enough, and the resolution of quantum mysteries by this route will have to await the construction of superintelligent robots. (But if the robots succeed, will they be able to, or even interested in, explaining it to *us*?)

The histories approach follows von Neumann in letting subspaces represent properties, with the negation of a property represented by the orthogonal complement of the subspace. But it takes a very different and much more conservative attitude than Birkhoff and von Neumann in the case of properties P and Q represented by noncommuting projectors. Since neither PQ nor QP is a projector, let us not try and talk about their conjunction. Let us adopt a language for quantum theory in which PQ makes sense and represents the conjunction of the two properties in those cases in which $QP = PQ$. But if $PQ \neq QP$ the statement "P AND Q" is *meaningless*, not in a pejorative sense but in the precise sense that this interpretation of quantum mechanics can assign it no meaning. In much the same way that in ordinary logic the proposition $P \wedge \vee Q$, "P AND OR Q," is meaningless even if P and Q make sense: the combination $P \wedge \vee Q$ has not been put together using the rules for constructing meaningful statements. Likewise, in quantum mechanics both the conjunction and also the disjunction P OR Q, must be excluded from meaningful discussion if the projectors do not commute.

The two-dimensional Hilbert space of spin-half particle can serve as an illustration. Let

$$[z^+] := |z^+\rangle\langle z^+|, \quad [z^-] := |z^+\rangle\langle z^+|, \tag{1}$$

where we shall (as in CQT) hereafter employ the $[\cdot]$ notation for this type of dyad, be projectors for the properties $S_z = +1/2$ and $S_z = -1/2$; similarly $[x^+]$ and $[x^-]$ are the corresponding properties for S_x. The

product of $[z^+]$ and $[z^-]$ is zero (i.e., the zero operator) in either order, so they commute and their conjunction is meaningful: it is the quantum property that is always false (the counterpart of the empty set in the case of a phase space). However, neither $[z^+]$ nor $[z^-]$ commutes with either $[x^+]$ or $[x^-]$, so the conjunction "$S_x = +1/2$ AND $S_z = +1/2$" is meaningless. In support of the claim that it lacks meaning one can note that every ray in the spin-half Hilbert space has the interpretation $S_w = +1/2$ for some direction in space w. Thus thus there are no spare rays available to represent "$S_x = +1/2$ AND $S_z = +1/2$"; there is no room in the Hilbert space for such conjunctions. It is very important to distinguish "meaningless" from "false". In ordinary logic if a statement is false then its negation is true. But the negation of a meaningless statement is equally meaningless. Thus the statement "$S_z = +1/2$ AND $S_z = -1/2$" is meaningful and always false, whence its negation "$S_z = -1/2$ OR $S_z = +1/2$" is meaningful and always true, and is consistent with the Stern-Gerlach experiment. On the other hand, "$S_x = +1/2$ AND $S_z = -1/2$" is meaningless, and its formal negation, "$S_x = -1/2$ OR $S_z = +1/2$," is equally meaningless.

The student who has learned quantum theory from the usual courses and textbooks may well go along with the idea that "$S_x = +1/2$ AND $S_z = +1/2$" lacks meaning, since he can think of no way of measuring it (and has probably been told that it cannot be measured). However he will be less likely to go along with the equally important idea that the disjunction "$S_x = +1/2$ OR $S_z = +1/2$" is similarly meaningless. Granted, there is no measurement which can distinguish the two, but he has a mental image of a spin-half particle in the state $S_z = +1/2$ as a little gyroscope with its axis of spin coinciding with the z axis. Such mental images are very useful to the physicist, and perhaps indispensable, for they help organize our picture of the world in terms that are easily remembered; they provide "physical intuition." But this particular mental image can be quite misleading in suggesting that when $S_z = +1/2$ the orthogonal components of angular momentum are zero. However, this cannot be the case since, since as the student has been taught, $S_x^2 = S_y^2 = 1/4$. Now any classical picture is bound to mislead to some extent when one is trying to think about the quantum world, but in this case a slight modification is less misleading. Imagine a gyroscope whose axis is oriented at random, except one knows that the z component of angular momentum is positive rather than being negative. Among other things this modified image helps guard against the error that the spin degree of freedom of a spin-half particle can carry a large amount of information, when in fact the limit is one bit ($\log_2 2$).

2.2 Frameworks

Ordinary probability theory uses the concept of a *sample space*: a collection of mutually exclusive alternatives or events, one and only one of which occurs or is true in a particular realization of some process or experiment. One way of introducing probabilities in classical statistical mechanics is to imagine the phase space divided up into a collection of nonoverlapping cells, a *coarse graining* in which each cell represents one of the mutually exclusive alternatives one has in mind. Let $P^j(\gamma)$ be the indicator function for the j'th cell: equal to 1 if γ lies within the cell and 0 otherwise. (We are using the superscript of P for a label, not an exponent; as the square of an indicator is equal to itself, this need not cause confusion.) Obviously the product $P^j P^k$ of the indicator functions for two different cells is 0, and the sum of all the indicator functions is the identity function $I(\gamma)$ equal to 1 for all γ:

$$P^j P^k = \delta_{jk} P^j; \quad \sum_j P^j = I. \tag{2}$$

Next, probabilities are assigned to the events making up a Boolean *event algebra*. If one coarse grains the phase space in the manner just indicated, an event algebra can be constructed in which each event is represented by the union of some of the cells in the coarse graining; equivalently, the event algebra is the collection of all indicator functions which are sums of some of the indicators in the collection $\{P^j\}$, including the functions $\emptyset$ and I which are everywhere 0 and 1, respectively. The probabilities themselves can be specified by a collection $\{p_j\}$ of nonnegative real numbers that sum to 1, with the probability of an event E being the sum of those p_j for which the corresponding indicator functions P^j appear in the sum defining E. To be sure, classical statistical mechanics is usually constructed without using a coarse graining, employing the Borel sets as an event algebra, and then introducing an additive positive measure to define probabilities. There is nothing wrong with this, but for our purposes a coarse graining provides a more useful classical analog.

The quantum counterpart of a classical sample space is referred to in the histories approach as a *framework*. It is a *projective decomposition* (PD) of the identity operator I: a collection $\{P^j\}$ of mutually orthogonal

projectors which sum to the identity operator I, and thus formally satisfy exactly the same conditions (2) as a collection of classical indicators. The fact that $P^j P^k$ vanishes for $j \neq k$ means that the corresponding quantum properties ("events" is the customary term in probability theory) are mutually exclusive: if one is true the other must be false, and the fact that they sum to the identity operator I means that at least one, and therefore only one, is true or real or actual. The corresponding event algebra is the Boolean event algebra of all projectors which can be formed by taking sums of projectors in $\{P^j\}$ along with the 0 operator, which plays the same role as the empty set in ordinary probability theory. As long as the quantum event algebra and the sample space are related in this way, there is no harm in using the somewhat loose term "framework" to refer to either one, as we shall do in what follows.

Two frameworks $\{P^j\}$ and $\{Q^k\}$ are *compatible* if all the projectors in one commute with all of those in the other: $P^j Q^k = Q^k P^j$ for all values of j and k. Otherwise they are *incompatible*. One says that $\{P^j\}$ is a refinement $\{Q^k\}$ if the projectors in the PD of the latter are included in the event algebra of the former; equivalently, $\{Q^k\}$ is a coarsening of $\{P^j\}$. Obviously two frameworks must be compatible if one is to be a refining or coarsening of the other. In addition, two compatible frameworks always possess a *common refinement* using the PD consisting of all the nonzero products $P^j Q^k$; its event algebra includes the union of the event algebras of the separate frameworks which it refines. Note that a given framework $\{P^j\}$ may have various different refinements, and two refinements need not be compatible with each other. Therefore when discussing quantum systems one must keep track of the framework being employed in a particular argument. This is not important in classical physics, where one can either adopt the finest framework possible at the outset, or else refine it as one goes along without needing to call attention to this fact. In quantum mechanics one does not have this freedom, and carelessness can lead to paradoxes.

2.3 The Single Framework Rule

A central concept of the histories approach is the *single framework rule*, which states that probabilistic reasoning that starts from data (observed or simply assumed) about a quantum system and leads to conclusions about the same system, typically expressed as conditional probabilities, is invalid unless it is carried out using a *single framework* as defined above. In particular it is not valid when it results from combining *incompatible* frameworks.

As the single framework rule has been frequently misunderstood by critics of the histories approach, it is important to clarify what it does and does not mean. While it rules out improper *combinations* of descriptions, it does not prevent the physicist from employing a variety of different and possibly incompatible frameworks when constructing several distinct descriptions of a quantum system, each of which may provide some physical insight about its behavior. This is the principle of Liberty: the physicist can use whatever framework he chooses when describing a system. All properly constructed individual frameworks are equally acceptable in terms of fundamental quantum mechanics: the principle of Equality. The principle of Incompatibility forbids combining incompatible frameworks. Not all frameworks are equally useful in answering particular questions of physical interest, let us call this the principle of Utility. It is by combining these principles that the single framework rule arrives at a consistent quantum ontology adequate for understanding the quantum world in a realistic way, while at the same time resolving or avoiding (or "taming") the numerous paradoxes or inconsistencies that beset alternative approaches to quantum interpretation.

The physicist's Liberty to choose different frameworks should not be thought of as in any way influencing reality. Choosing a description is choosing what to talk about, and what physicists choose to talk about has very little (direct) influence on what actually goes on in the world. Shall we discuss the location of Jupiter's center of mass, or its rate of rotation? Either is possible, and neither has the slightest influence on the behavior of Jupiter. What about a silver atom approaching a Stern-Gerlach apparatus? Shall we discuss its (approximate) location or the value of S_x or the value of S_z? Any one of these is possible, and the single framework rule allows location to be combined with S_x, or with S_z. But not with both, since it is impossible to put both S_x and S_z, at least when referring to a single particle at a particular time, in the same framework. In this case, no less than for Jupiter, the physicist's choice of framework has not the slightest influence on the silver atom. And there is no law of nature which singles out a framework that includes S_x as somehow "correct" or "true" in distinction to the S_z framework; that would contradict Equality. However, they are incompatible. They cannot be combined. What does this mean?

Incompatibility in this technical sense is a feature of the quantum world with no exact classical analog: in classical physics all the operators commute. But classical analogies and disanalogies, together with applications to various specifically quantum situations can help tease out its intuitive meaning. Let us start

with a disanalogy. A coin can land heads or tails, two mutually exclusive possibilities: if one is true, the other must be false. There is a temptation to think of the relationship of the incompatible S_x and S_z frameworks in this way, and it must be resisted, for it leads to a serious misunderstanding. The properties $S_z = +1/2$ and $S_z = -1/2$ are analogous to heads and tails: they are mutually exclusive. If one is true the other is false, and the combination "$S_z = +1/2$ AND $S_z = -1/2$" is meaningful and false, as discussed earlier. On the other hand the combination "$S_x = +1/2$ AND $S_z = -1/2$" is meaningless, neither true nor false. Statements belonging to incompatible frameworks cannot be compared in any way, which is also why it is meaningless to say that one framework rather than the other is the true or correct way of describing the quantum world. Equality must be taken seriously.

For a positive analogy, think of a framework as something like a coarse graining of the classical phase space as discussed earlier in Sec. 2.2. Many coarse grainings are possible and the physicist is at Liberty to choose one that is convenient for whatever purposes he has in mind. There is no "correct" coarse graining, though some coarse grainings may be more useful than others in discussing a particular problem. The physicist's choice of coarse graining does not, of course, have any influence on the system whose properties he is trying to model. In all these respects the choice of coarse graining is like the choice of a quantum framework. But classical coarse grainings of the same phase space can always be combined: the common refinement is constructed in an obvious way using cells formed by intersections of those taken from the two coarse grainings that are being refined. However, PDs of the same Hilbert space cannot in general be combined, so in this respect the analogy fails. However, it is still helpful in illustrating some aspects of the quantum situation, and in avoiding the misleading idea that the relationship between different quantum frameworks is one of mutual exclusivity.

Similarly, choosing a framework is something like choosing an inertial reference frame in special relativity. The choice is up to the physicist, and there is no law of nature, at least no law belonging to relativity theory, that singles out one rather than another. Sometimes one choice is more convenient than another when discussing a particular problem; e.g., the reference frame in which the center of mass is at rest. The choice obviously does not have any influence upon the real world. But again there is a disanalogy: any argument worked out using one inertial frame can be worked out in another; the two descriptions can be mapped onto each other. This is not true for quantum frameworks: one must employ a framework (there may be several possibilities) in which the properties of interest can be described; they must lie in the event algebra of the corresponding PD.

For a more picturesque positive analogy consider a mountain, say Mount Rainier, which can be viewed from different sides. An observer can choose to look at it from the north or from the south; there is no "law of nature" that singles out one perspective as the correct one. One can learn different from different viewpoints, so there might be some Utility in adopting one perspective rather than the other. But once again the analogy fails in that the north and south views can, at least in principle, be combined into a single unified description of Mount Rainier from which both views can be derived as partial descriptions. Let us call this the *principle of unicity*. It no longer holds in the quantum world once one assumes the Hilbert space represents properties in the manner discussed above.

But how can Utility play a role in quantum physics? Again consider the case of a spin-half silver atom, and suppose it is midway in its trajectory from an apparatus where a competent experimentalist has prepared it in a state with $S_x = +1/2$ to an apparatus, also constructed by a competent experimentalist, which will later measure S_z with the pointer corresponding to $S_z = +1/2$. What can one say about the spin of the atom midway between preparation and measurement, assuming it travels in a region free from magnetic fields that could cause the spin to precess? There is a framework which at the intermediate time includes the possibilities $S_x = +1/2$ and $-1/2$; using this S_x framework and the data about the preparation one can infer that $S_x = +1/2$ with probability 1 and $S_x = -1/2$ with probability 0. There is an alternative S_z framework that at the intermediate time includes the possibilities $S_z = +1/2$ and $-1/2$, and it can be used to infer from the later measurement outcome that at the intermediate time the (conditional) probabilities for $S_z = +1/2$ and $-1/2$ are 1 and 0 respectively. The S_x framework is useful if one is concerned with whether the preparation apparatus was functioning properly, while the S_z framework is useful if one wants to discuss the proper functioning of the measuring device.

3 Time Development

3.1 Quantum dynamics: histories

The histories approach treats the time dependence of a quantum system as a random or stochastic process, one in which the future and past states of the system at different times are not determined by the present state, but only related to it by certain probabilities, which only in very special cases are 0 and 1, corresponding to a deterministic time development. Indeed, most physicists accept that in practice quantum-mechanical time development is probabilistic and not deterministic. Take the case of spontaneous decay. Modern physics possesses tools for calculating decay rates of atoms in excited states, or unstable nuclei, and they work reasonably well. If the quantum world were deterministic one would expect to find somewhere in the theoretical formalism a prescription for predicting the (relatively) precise time of decay of, say a radioactive nucleus. This time can be measured quite precisely, better than a millisecond, for a nucleus with a half life of minutes or hours or even years. Most quantum physicists do not believe that there is some "marker" or "clock" inside the nucleus which before the decay can be used to the time of decay; there is no room for it in the Hilbert space description.[4] Hence the assumption of a probabilistic dynamics does not, in the modern context, represent much of an innovation. But doing it in a consistent way that avoids paradoxes is not altogether straightforward.

The essential mathematical structures needed for introducing probabilities into quantum mechanics were introduced in Sec. 2.2: a sample space constituted by a projective decomposition (PD) of the identity operator I on the Hilbert space, and a Boolean event algebra of projectors generated from the PD in a natural way. What remains is to assign nonnegative probabilities p_j summing to 1 to the elements P^j of the sample space, and thereby to the subspaces (events) that constitute the event algebra in precisely the same way as in other applications of probability theory. If this can be done, one can then carry out probabilistic reasoning in the quantum domain following all of the ordinary rules of probability theory *provided the sample space (and event algebra) remain fixed while various (conditional) probabilities are computed*—the single framework rule. Just as in other applications of probability theory there is no general rule that specifies the probabilities p_j: they enter a probabilistic description of the world as parameters, and some exercise of judgment on the part of the scientist is generally necessary, as well as input data, results of experiments, etc.

However, quantum theory introduces a new element not found in other applications of probability theory. It applies to the time development of a *closed* quantum system, by which we mean either that it is isolated, completely self-contained with no environment with which it interacts, or that its interaction with with its environment is well-enough approximated by assigning to the system itself a (possibly time-dependent) Hamiltonian, which can then be employed in Schrödinger's equation. In this case certain (conditional) probabilities relating states of the system at different times can be assigned through the Born rule and its extensions to more than two times, as discussed in Secs. 3.3 and 3.4. The unitary time evolution induced by Schrödinger's equation is used in calculating these probabilities, but only in exceptional cases, as discussed in Sec. 3.2, can it be used directly (without a probabilistic interpretation) to describe what is really going on in a closed quantum system.

Before assigning probabilities to processes occurring in time, we need to construct an appropriate quantum sample space. How is this to be done? Classical physics provides a useful hint. The sample space for a coin tossed three times in a row consists of the eight possibilities HHH, HHT,... TTT, where HTT means heads on the first toss, tails on the second and third. This is just the Cartesian product of the sample space for a single toss of the coin, and is in fact identical to the sample space needed to describe three different coins all tossed at the same time. Hence in ordinary probability theory sequences of events at successive times in a particular system are formally the same thing as multiple copies of the same system considered at a single time.

In quantum mechanics the mathematics for describing a compound system consisting of a collection of (distinguishable) subsystems is well know: one uses the tensor product of the Hilbert spaces. This immediately suggests that the way to construct a quantum sample space for a system at successive times is to use a tensor product of copies of its Hilbert space, as first proposed in [38]. Thus for a spin-half particle

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[4]A notable exception is the proponents of Bohmian mechanics, who add additional "hidden" variables to the Hilbert space description. See, e.g., [35]. However, in the case of radioactive decay they have no way of accessing this deterministic internal clock other than by observing the actual time of decay. Determinism is also upheld by followers of Everett, see, e.g., [36, 37], but they, too, have no way of accessing the alternative worlds (or minds or whatever) in which the nucleus decays at a time different from that observed in the laboratory.

the 8-dimensional Hilbert space

$$\breve{\mathcal{H}} = \mathcal{H}_0 \odot \mathcal{H}_1 \odot \mathcal{H}_2 \tag{3}$$

is appropriate for describing its properties at three successive times, $t_1 < t_2 < t_3$, where each $\mathcal{H}_m$ is a copy of the two-dimensional Hilbert space needed to describe it at one time, and following CQT we use the symbol $\odot$ in place of the customary $\otimes$ symbol—formally they mean the same thing—in order to emphasize that we are considering a sequence of successive times, rather than several systems at the same time. A spin-half particle that possesses property F_m at time t_m is then described by a *history projector*

$$F_0 \odot F_1 \odot F_2, \tag{4}$$

where each F_m is a projector on the two-dimensional single-time Hilbert space. One might, for example, suppose that each F_m is $[z^+]$ or $[z^-]$. This is rather like the case of flipping a coin three times: think of $[z^+]$, meaning $S_z = +1/2$ as heads, $[z^-]$ as tails. There are eight possibilities, and summing the eight projectors indeed yields $\breve{I}$, the identity operator on $\breve{\mathcal{H}}$. Each element in the sample space is called a *history*, and together they constitute a *family of histories*. Let us call this family $\mathcal{F}_1$. An alternative family can be constructed using the alternatives $[z^+]$ and $[z^-]$ at t_0, but then the alternatives $[x^+]$ and $[x^-]$ at times t_1 and t_2. This second family $\mathcal{F}_2$ corresponds to a different PD of $\breve{I}$, and $\mathcal{F}_2$ is incompatible with $\mathcal{F}_1$ because the two sets of projectors, regarded as operators on $\breve{\mathcal{H}}$, do not commute with each other.

The generalization to an arbitrary but finite collection of $f + 1$ times $t_0 < t_1 < \cdots t_f$ proceeds in an obvious way. First define the *histories Hilbert space*, the obvious generalization of (3) to $f + 1$ copies of the single-time Hilbert space. Next introduce a *sample space* $\{Y^\alpha\}$ of histories of the form

$$Y^\alpha = F_0^\alpha \odot F_1^\alpha \odot \cdots F_f^\alpha, \tag{5}$$

where the superscript α labels the different elements of the sample space, F_m^α is a projector representing the a property of the quantum system at time t_m, and the histories projectors satisfy the condition

$$\sum_\alpha Y^\alpha = \breve{I}, \tag{6}$$

the counterpart of (2), where $\breve{I} = I_1 \otimes I_2 \otimes \cdots$ is the identity operator on the histories Hilbert space $\breve{\mathcal{H}}$. This ensures that $Y^\alpha Y^\beta = 0$ if $\alpha \neq \beta$, i.e., two distinct histories belonging to this sample space are mutually exclusive.

Two special cases are worth mentioning. The first is that of a *fixed initial state*: F_0^α at time t_0 is equal to the same projector, call it P_0, for all α with the single exception of a particular history labeled $\alpha = 0$. For the $\alpha = 0$ history, $F_0^0 = (I - P_0)$ at time t_0, and $F_m^0 = I$ at all later times $t_m > t_0$. This special history, whose physical significance is that P_0 is *not* true at t_0, is a throwaway: one sets its probability to zero; it has been introduced simply to ensure that (6) is satisfied. One then fixes attention on cases in which the system is initially in the state P_0, which may be a pure state but may also correspond to a subspace of dimension greater than one. A second special case is the one in which at each time t_m the projector F_m^α is drawn from a fixed PD $\{P_m^j\}$ which can be different for different times t_m. Although these special cases, often combined, receive the most attention in CQT, there are also other interesting examples.

Thus the situation for describing a quantum system at multiple times is closely analogous to describing it at a single time: there are many incompatible alternative frameworks or sample spaces. And the same principles of Liberty, Equality, and Utility apply. The physicist is at Liberty to choose a sample space that is useful for discussing questions he considers interesting; no one is more "fundamental" than another. Two incompatible spaces cannot be combined and, as we shall see in Sec. 3.4, there are circumstances under which this single framework rule must be made more stringent. But not all sample spaces are equally useful. As a trivial example, the family $\mathcal{F}_1$ introduced above is obviously more helpful than $\mathcal{F}_2$ if the physicist is trying to answer a question like: "The silver atom had $S_z = +1/2$ at time t_3; what can one say about S_z at some earlier time?"

Some of the rules needed for assigning probabilities to histories comprising a particular family or framework in a closed quantum system are given in textbooks in the particular case of the Born rule, Sec. 3.3 below. However, the presentation tends to be accompanied by references to measurements and wave function collapse that do not properly reflect how measurements are actually used and interpreted by competent experimental physicists, the topic of Sec. 5 below. As the appropriate way to introduce probabilities in a *closed* system is discussed in considerable detail, with numerous examples, in CQT, the present discussion is

limited to the highlights. We begin by considering the simplest situation, unitary families, in Sec. 3.2; then two-time families, the traditional Born rule, in Sec. 3.3. These will serve to set forth the basic strategy of the histories approach. The extension of these ideas to more complicated situations, which involves additional technical difficulties, is briefly taken up in Sec. 3.4.

3.2 Unitary families and the uniwave

The simplest type of history family for a closed system from the perspective of assigning probabilities is one in which they are all either 1 or 0. Such a family is a *unitary* family. The simplest way to construct such a family is to assume a particular pure state $|\psi(t_0)\rangle$ for the system at an initial time t_0, and integrate Schrödinger's equation to obtain $|\psi(t_m)\rangle$ at the later times of interest, which we assume constitute a finite collection $t_0 < t_1 < ....t_f$. At time t_m introduce a PD with just two projectors: $P_m = [\psi(t_m)]$ and its negation $I - P_m$, and construct the history sample space by choosing one of these for each t_m. This family satisfies the consistency conditions in Sec. 3.4, and if P_0 at t_0 is assigned probability 1, the single history $P_0 \odot P_1 \odot \cdots P_f$ has probability 1 and all other members of the sample space have probability 0. Consequently, one can conclude that if the system has property P_0 at t_0 then it has the property P_m at each later time t_m. Alternatively, one can reach the corresponding result by conditioning on P_n at some particular time t_n. Hence for a unitary family the histories approach yields a deterministic quantum dynamics.

Let us introduce the technical term *uniwave* for a a wave function $|\psi(t)\rangle$ that satisfies Schrödinger's equation, thus undergoes a *unitary* time evolution, for a closed system, the *"universe."* In many interpretations of quantum mechanics the uniwave, often referred to simply as "the wave function," plays a central role. But not in the histories approach; with the advent of Equality the uniwave is only a common citizen and no longer king. It describes reality for a unitary family, but not in more general probabilistic situations where, as we shall see in Sec. 3.3, it is further demoted to a mere mathematical tool or pre-probability for calculating probabilities which can, if one wants, be obtained using different solutions to Schrödinger's equation. This is in contrast to some other interpretations of quantum mechanics. The uniwave is an absolute monarch in the Everett (many worlds) interpretation [36, 37], where it constitutes the official ontology: at each time it represents all of reality. Bohmian mechanics [35] adds additional (hidden) variables, but their motion in time is then determined by the the uniwave.

3.3 Born rule

After unitary families, in which the probabilities are trivial, the simplest situation for a closed system is a family that involves only two times t_0 and t_1. One typically assumes that t_0 is earlier than t_1, but this is not essential. Suppose the histories framework is provided by an orthonormal basis $|\psi_0^j\rangle$ at t_0 and another, in general different, orthonormal basis $|\phi_1^k\rangle$ at t_1. Let $T(t_1, t_0)$ be the corresponding unitary time development operator, given by $e^{-i(t_1-t_0)H/\hbar}$ in the case of a time-independent Hamiltonian H. The Born rule assigns a weight

$$|\langle\phi_1^k|T(t_1,t_0)|\psi_0^j\rangle|^2 = |\langle\psi_0^j|T(t_0,t_1)|\phi_1^k\rangle|^2 \tag{7}$$

to the history $[\psi_0^j] \odot [\phi_1^k]$, which can then be interpreted as the conditional probability of $[\phi_1^k]$ at time t_1 given $[\psi_0^j]$ at time t_0, or of $[\psi_0^j]$ at t_0 given $[\phi_1^k]$ at t_1.

The complete formal symmetry between the two times is not so obvious if one writes the weight in (7) in the equivalent form

$$|\langle\phi^k|\psi(t_1)\rangle|^2, \quad |\psi(t)\rangle := T(t,t_0)|\psi(t_0)\rangle, \quad |\psi(t_0)\rangle := |\psi_0^j\rangle \tag{8}$$

for some particular choice of j. That is, one starts with an initial state at t_0, constructs the corresponding uniwave, and uses it to compute the weight. In the histories approach there can be no objection to (8) as a mathematical formula, but since $|\psi(t_1)\rangle$, or more precisely its projector $[\psi(t_1)]$, will in general be incompatible with the chosen PD $\{[\phi_1^k]\}$ at time t_1, one cannot consistently speak of it as a property of the system at t_1: it does not make sense within the family of histories whose probabilities are being calculated by means of the Born rule. Instead, $|\psi(t_1)\rangle$ is best thought of as a *pre-probability* in the notation of CQT: a mathematical tool used for computing probabilities, but which need not have any counterpart in physical reality.

A further indication that in this situation the uniwave is playing a subsidiary role and does not represent a physical property is that the Born weights in (7) can be calculated by an alternative route that makes no

reference to it. Thus let us define the kets

$$|\phi_0^k\rangle = T(t_0, t_1)|\phi_1^k\rangle, \tag{9}$$

i.e., for each k integrate Schrödinger's equation "backwards" from t_1 to t_0, starting with $|\phi_1^k\rangle$ as the "initial" state. Then it is obvious that

$$|\langle\phi_1^k|T(t_1, t_0)|\psi_0^j\rangle|^2 = |\langle\psi_0^j|\phi_0^k\rangle|^2, \tag{10}$$

so we have obtained the weights in (7) without the help of the uniwave defined in (8), by employing the $|\phi_0^k\rangle$ as pre-probabilities.

Dethroning the uniwave from king to the very subsidiary role of serving as a pre-probability, with even less reality than a probability, solves the first measurement problem. Instead of declaring that the pointer is in some macroscopic superposition, use a framework in which the projectors in the PD refer to different pointer positions, and the (first) measurement problem has disappeared. Equivalently, the Schrödinger cat paradox vanishes once the physicist is given Liberty to choose an appropriate framework of the quasiclassical form, Sec. 4, for events at later times.

Two additional remarks. First, it is not necessary to use orthonormal bases at t_0 and t_1; coarser PD's are acceptable, e.g., $\{P^j\}$ at t_0 and $\{Q^k\}$ at t_1, with the weight formula (7) replaced with the more general

$$\mathrm{Tr}(Q^k T(t_1, t_0) P^j T(t_0, t_1)) = \mathrm{Tr}(P^j T(t_0, t_1) Q^k T(t_1, t_0)), \tag{11}$$

which reduces to (7) when $P^j = [\psi^j]$ and $Q^k = [\phi^k]$. The only thing one needs to be careful about is normalization when turning weights into probabilities, thus

$$\mathrm{Pr}(Q^k \,|\, P^j) = \mathrm{Tr}(Q^k T(t_1, t_0) P^j T(t_0, t_1))/\mathrm{Tr}(P^j). \tag{12}$$

Second, our discussion of the Born rule made no references whatever to measurements: it applies to any family of histories in a closed system as long as they involve just two times. Naturally, the closed system may include a measuring apparatus, and the Born rule applies to the whole system. For more on measurements, see Sec. 5.

3.4 Multiple times: consistency

The Born rule suffices for assigning probabilities or weights to histories inside a close quantum system when *only two times* are involved. With three or more times an additional *consistency condition* is needed. To see why, consider a family of histories involving the three times $t_0 < t_1 < t_2$. The appropriate version of (7), or more generally (11), can be used to calculate weights for histories involving just the two times t_0 and t_1, or just t_0 and t_2, or t_1 and t_2, but there is no guarantee that these can be turned into a corresponding joint probability distribution for events at all three times. This resembles a situation in classical stochastic processes: a rule which relates probabilities of events at only two times cannot be used to generate a multitime probability distribution without making additional assumptions; e.g., that the process is Markovian. The quantum situation is more subtle, and resembles the problem faced when deciding what to do with incompatible PDs at a single time, Sec. 2.

In the histories approach one first postulates a reasonable mathematical form for weights appropriate to a family of histories of a closed quantum system involving an arbitrary (finite) collection of times, and then only applies it to families which satisfy a fairly stringent *consistency condition*. Both weights and consistency can be obtained from the *decoherence functional*

$$\mathcal{D}(\alpha, \beta) = Tr[K^\dagger(Y^\alpha)\, K(Y^\beta)] \tag{13}$$

defined in terms of a *chain operator*

$$K(Y^\alpha) := F_f^\alpha T(t_f, t_{f-1}) F_{f-1}^\alpha T(t_{f-1}, t_{f-2}) \cdots F_1^\alpha T(t_1, t_0) F_0^\alpha, \tag{14}$$

with the analogous definition for $K(Y^\beta)$. The consistency condition is then the requirement

$$D(\alpha, \beta) = 0 \text{ for } \alpha \neq \beta, \tag{15}$$

and when it is satisfied probabilities can be computed from the collection of nonnegative weights

$$W(\alpha) = \mathcal{D}(\alpha, \alpha). \tag{16}$$

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When $f = 1$, so the family of histories involves only the two times, the consistency condition is always fulfilled, so it did not have to be considered in our earlier discussion of the Born rule in Sec. 3.3, and the Born weights coincide with those in (16). In this sense the histories approach is a generalization of the Born rule.

For further details the reader is referred to Chs. 10 and 11 of CQT, and to the chapters that follow for various applications that draw out the physical significance of the consistency condition and probabilities which can be assigned when it is satisfied. That treatment needs to be updated in two respects. First, the consistency condition used throughout CQT is the one stated above: the "medium decoherence" condition in the terminology of [39]. A weaker condition mentioned in Sec. 10.2, but (fortunately!) never employed there or elsewhere in CQT, has been shown to be unsatisfactory by Diosi [40]. Second, anyone who wants to understand the consistency conditions, or teach them to students, would be well advised to start with simpler situation represented by chain kets and discussed in Sec. 11.6, rather than the very general form given in Ch. 10.

The use of consistency conditions extends the cautious and conservative approach employed in Sec. 2.1: restrict one's discussion to cases that make sense. In particular, use only those sample spaces of histories of a closed quantum system for which the assignment of probabilities based upon the dynamical laws can be done in a consistent fashion. If the family does not satisfy the consistency conditions its discussion is deferred to the robots, Sec. 2.1. In the meantime the families which do satisfy the consistency conditions seem adequate: what is allowed covers what physicists need to talk about, and what is excluded includes things like the mysterious instantaneous nonlocal influences that have long been an embarrassment to quantum foundations and are often thought to be an impediment to connecting quantum theory with special relativity; see Sec. 6. Though quite restrictive, consistency conditions permit a much wider discussion than found in textbooks, or allowed by the black box strictures of quantum orthodoxy. But see the further comments in Sec. 8.4.2.

For the reasons just mentioned it has become customary when discussing the stochastic time dependence of closed quantum systems to limit the concept of "framework" to families for which the consistency conditions are satisfied. This means both a restriction on the form of the history space PD allowed for consistent probabilistic reasoning about what is going on in a closed quantum system, and an extension of the single framework rule to exclude combining two consistent families when the resulting common refinement will not satisfy the consistency conditions. If they cannot be combined they are said to be incompatible.

Thus it seems reasonable to include in a consistent quantum ontology only those families of histories for a closed system which satisfy consistency conditions. The physicist will have many different incompatible families from which to choose, and can approach this multiplicity with the same attitude adopted in Sec. 2 for a system at a single time. In a given family one and only one history actually occurs: they are mutually exclusive. The choice of family is up to the physicist, and will generally be made on the grounds of utility, e.g., how to model a particular experimental situation with apparatus put together by a competent experimentalist. This choice has no influence upon what really goes on in the world, and alternative choices applied to the same set of data and conclusions will yield consistent results. The quantum world is of such a nature that it can be described in distinct ways which (in general) cannot be combined into a single all-encompassing description.

3.5 Which history occurred?

Mixing up the concepts of quantum incompatibility and mutual exclusivity has led to a serious misunderstanding of the ontology of stochastic histories which is a reflection of the problem of understanding what is real about a quantum system at a single time. Here the issue is: which history actually occurred?

The first step in the reply is to say that if a single framework, a single consistent family, of histories is in view, the sample space, represented mathematically by an appropriate PD of the history identity, is a collection of mutually-exclusive possibilities, one and only one of which actually occurs. The same as in the case of three tosses of a coin: one and only one of the eight possible sequences occurs on the particular occasion in which the experiment is carried out. (Naturally, members of the corresponding event algebra need not be mutually exclusive: the event of tails on the first two tosses is distinct from that of tails on the last two tosses, but the two are not mutually exclusive.) The same is also true for any ordinary (not quantum) scientific application of probability theory: only one of the potential possibilities represented in the sample space of histories actually occurs. The notion that there are separate universes in which each possibility in the sample space is realized seems rather bizarre when one is considering, e.g., whether it will

snow in Pittsburgh on Christmas day in the year 2050. In this respect quantum ontology as understood using the histories approach is much closer to other applications of probability theory than is the "many worlds" [36, 37] understanding of Everett's ideas.

The second step in the reply addresses the issue of two or possibly more incompatible families of histories. What has just been said about one and only one history *applies separately to each family*. But *incompatible families* cannot be *combined*. Hence it is meaningless (histories quantum mechanics assigns it no meaning) to assert that there must be some pair of histories (Y_1, Y_2), such that the first occurred in family $\mathcal{F}_1$ and the second in the incompatible family $\mathcal{F}_2$. Assertions of this sort are just what the single framework rule is designed to exclude. For an example of how this works in a particular gedanken experiment see Sec. 18.4 of CQT.

4 Classical Limit

4.1 Quasiclassical frameworks

An important feature of a satisfactory quantum ontology is that it should explain how classical mechanics at the level of macroscopic objects, such as dust particles or pennies or planets, emerges from a fundamental quantum description of the world. This problem has been addressed from the histories perspective, and at of the present time, while the task is still incomplete, there are no outstanding problems that indicate difficulties in the way of an approach pursued in somewhat different ways by Omnès [14, 41] and by Gell-Mann and Hartle [39]. The following remarks provide only a fairly elementary explanation of the basic approach.

The first important idea is that of *coarse graining* the Hilbert space by using an appropriate PD in which the subspaces correspond to ordinary macroscopic descriptions of the world. Properties associated with a pointer pointing in a particular direction, or its modern counterpart, a ("classical") bit represented by some macroscopic state of an electronic memory device, are represented by subspaces of the Hilbert space of exponentially large dimension 10^ν, where where ν is itself an enormous number, e.g., 10^{10}. In such cases many different coarse-grained projectors could be used to represent what a physicist would be say is essentially the same macroscopic property. What seems plausible is that a suitably chosen coarse-grained *quasiclassical* PD can represent the different possibilities in a macroscopic situation, such as the outcome of a laboratory measurement, with sufficient accuracy "for all practical purposes". A quasiclassical PD both cannot and should not be precisely defined; there will be many different PDs that provide very similar results. If this seems sloppy by the standards of pure mathematics or philosophy, the appropriate response is that approximations are in practice essential in all of physics, and which approximations are appropriate in any particular case are to some extent a matter of judgment, and depend upon the problem being addressed.

The next step is to choose a family of quasiclassical histories, a *quasiclassical framework*, in which events at each time correspond to a quasiclassical PD. In constructing this framework the time intervals should not be made too small, as a judicious choice will make it easier to satisfy the consistency conditions, Sec. 3.4 for the probabilistic dynamics of a closed system. What one expects is that given a suitable coarse graining of both phase space and time, classical dynamics will emerge as an approximation to the more exact, but extremely difficult to calculate, quantum dynamics. This has not been proved, but various calculations and arguments make the existence of such a framework (or frameworks, since the choice is not unique) plausible. In this connection several comments are in order.

First, quantum dynamics, as already noted in Sec. 3, is not to be thought of as unitary time development, even though the latter is always a possible description for a closed system. Instead it is fundamentally stochastic. How can a deterministic classical mechanics emerge from an underlying stochastic quantum dynamics? This depends on the magnitudes of the probabilities. It is easy to envisage situations in which given a suitable coarse graining the probability of one macro property being followed by another a short or even much longer time later is almost equal to 1, with the deviation being very small. This then amounts in practice to a deterministic dynamics.

Second, classical mechanics is formulated in terms of a continuous, not a discrete time development. How can this be reconciled with a quantum stochastic dynamics in which time intervals need to remain finite? Again, the time intervals of interest when considering the flight of a golf ball are quite large compared with the time scales which enter into the quantum physics of matter. What one expects is that a discrete time development consistent with quantum consistency conditions will in many circumstances lead to a time dependence which can be satisfactorily *approximated* by the differential equations of classical mechanics. This seems a reasonable hope given what we understand at present about the quantum mechanics of large systems, even if much is still waiting to be confirmed by serious and systematic studies.

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Third, in addition to situations in which a continuous and deterministic classical dynamics should be a good approximation to the quantum world, there will also be situations in which the intrinsic uncertainties associated with quantum dynamics manifest themselves as genuinely stochastic behavior at the macroscopic level, behavior that is quite unpredictable in terms of some earlier state of the universe. Radioactive decay, detected by devices whose later behavior can be fitted within a quasiclassical framework, is an obvious example. Another is chaotic motion of systems in which, according to classical mechanics itself if taken literally, any small change in the starting point in the system's phase space is rapidly magnified until for all practical purposes the motion is no longer deterministically linked to an initial state. One would hardly expect a quasiclassical quantum description applied to such a situation to yield a deterministic outcome, and the stochastic quantum dynamics of the histories approach provides a plausible structure in which such a probabilistic time development can be seen to be consistent with a fully quantum mechanical world. Calculations to verify these ideas face formidable technical problems, but nothing in our present state of knowledge suggests there is any difficulty in principle in describing such situations in quantum terms.

Fourth, it is reasonable to expect that the consistency conditions, Sec. 3.4, will be satisfied in a quasiclassical framework. Although when expressed mathematically the conditions (15) are quite strict, the attitude of the physicist is that as long as the violations of these conditions are small compared to the diagonal weights (16) one is interested in, it does not matter. That this attitude is not unreasonable is supported by the work of Dowker and Kent [22], who on the basis of a parameter counting argument concluded that in the vicinity of an almost-exactly consistent family there is another one, obtained by slightly adjusting the properties under discussion, that is to say the projectors representing them, which is fully consistent with the mathematical conditions exactly satisfied. Since in the case of quasiclassical coarse grainings there is enormous room (as viewed microscopically) to make adjustments which leave macroscopic properties the same for all practical purposes, consistency conditions do not seem to pose a significant problem.

Fifth, what is the role of decoherence? Up till now most research on this topic, see for example [42, 43] has been carried out without an adequate ontological framework. Hence there are doubts as to the nature of the appropriate concepts, and what the calculations actually mean. Simple models suggest that decoherence in the form of a fairly weak interaction with a suitable environment can render certain histories of a system consistent when they would otherwise, in the absence of the environment, be inconsistent. This supports the plausibility of classical physics emerging from quantum physics in an appropriate quasiclassical framework. In the histories approach decoherence is *not* needed to resolve the measurement problem; see the following section.

4.2 Persistence of quasiclassical behavior

Since it has been frequently cited, e.g. [4, 44], in work which makes no reference to the detailed rebuttal found in [28], it seems worthwhile discussing a criticism of the histories approach found in [22], and also in [23, 25], having to do with persistence in time of quasiclassical behavior: Suppose we know (or suppose) that the world has been quasiclassical up to now. How do we know that quasiclassicality will continue to be the case tomorrow? There is no guarantee (so the criticism goes) that this will be the case, because the histories approach contains no principle which singles out the correct family for describing the world, and it is easy to construct examples of consistent families which employ quasiclassical PDs up to some point in time, but then at later times use PD's which are incompatible with the quasiclassical form.

The basic misunderstanding arises from supposing that "quasiclassical" is an adjective that somehow describes, within histories quantum mechanics, some property of the world. But it is not a property of the world; instead it is a property of a certain type of *description* of the world. The analogy of coarse graining a classical phase space in order to derive, say, hydrodynamics from classical statistical mechanics can help show where the error lies. This coarse graining has worked successfully for deriving the hydrodynamic laws that apply up to time t_1. But what guarantees that the same coarse graining will apply at times later than t_1? Only the fact that the physicist who has carefully done the work is not likely to throw away an idea that has succeeded and replace it with something that doesn't work as well. This, of course, has nothing to do with the flow of water through real pipes; the laws of hydrodynamics do not change because the physicist trying to relate them to microscopic (classical) mechanics has adopted an ineffective approach.

To put it in a different way, the relationship between different frameworks is not one of mutual exclusivity, where one is right and the other is wrong. Once one appreciates this fact, there is no reason to look for a law of nature that one single out a particular framework. The existence of alternative frameworks in no ways invalidates the conclusions drawn using a particular framework. Had Dowker and Kent been able to

conclude that *no* quasiclassical framework could be consistently used over the long period of time required to get from, let us say, the big bang to a few billion years into the future, that would have counted as a serious objection to the histories program. But that is not not the case.

5 Preparation and Measurement

The black box approach to quantum theory mentioned in Sec. 1 makes reference to "preparation" and "measurement" as unanalyzable (from a quantum perspective) macroscopic processes to be understood in classical terms, while talk about what it is that is actually prepared, or what a measurement actually measured, is forbidden, at least by the strict rules of this particular orthodoxy. The consistent quantum ontology that is the subject of this paper makes it possible to open the black box without falling into the quantum foundations swamp, and one can "watch" what is going on inside. But in addition it provides the tools needed to treat both preparation and measurement as examples of *quantum mechanical* processes governed by exactly the same principles that apply to all other quantum processes, microscopic or macroscopic, whether or not they have anything to do with either preparation or measurement.

Measurements have been extensively discussed in CQT Chs. 17 and 18, to which we refer the interested reader for details on topics such as macroscopic description of measurements in which one does not assume the apparatus is in a pure state prior to measurements, and the role of thermodynamic irreversibility. These are omitted from the following discussion in order to focus on the essentials. We also omit discussions of delayed choice experiments, Ch. 20 of CQT, which have sometimes been misinterpreted to mean that in the histories approach the future can influence the past.

In what follows we shall use a simple model, a slight modification of the one introduced by von Neumann, in order to discuss both preparations and measurements, starting with the latter. Then we shall argue that when the appropriate analyses are put together one arrives at a realistic quantum description of at least some aspects of what goes on at a microscopic level between a preparation and a measurement, thus opening the black box.

5.1 Measurement model

Here is the modification of von Neumann's model. Let $\mathcal{H}_s$ be the Hilbert spaces associated with the system to be measured—hereafter referred to as the *particle*—and $\mathcal{H}_M$ that of the measuring apparatus, including its environment if that is of interest. The two taken together form a closed quantum system, so unitary time development of the combination makes sense. We assume that at time t_0 the particle is in a normalized state

$$|\psi_0\rangle = \sum_j c_j |s^j\rangle, \tag{17}$$

a linear combination of orthonormal basis states $\{|s^j\rangle\}$ of $\mathcal{H}_s$, and the apparatus is in the state $|M_0\rangle$. Let $t_0 < t_1$ be two times preceding the time interval from t_1 to t_2 during which the particle interacts with the apparatus, and let the unitary time development from t_0 to t_1 to t_2 be given by

$$|s^j\rangle \otimes |M_0\rangle \rightarrow |s^j\rangle \otimes |M_1\rangle \rightarrow |s^1\rangle \otimes |M^j\rangle. \tag{18}$$

Note that in (18) at the final time t_2 the particle is always in the *same* state $|s^1\rangle$ whatever may have been the earlier state $|s^j\rangle$. In words, during the time interval from t_0 to t_1 the particle state does not change, whereas the apparatus undergoes a time evolution $|M_0\rangle \rightarrow |M_1\rangle$, the nature of which is unimportant for the present discussion as long as $|M_1\rangle$ is an appropriate "ready" state that leads at time t_2 to a state $|M^j\rangle$ corresponding to the pointer being in a definite direction that is determined by the earlier state $|s^j\rangle$ of the particle at time t_1. One could, for example, think of a particle with a spin degree of freedom moving towards an apparatus where some component of the spin will be measured. In this case the $\{|s^j\rangle\}$ would refer to the spin states, whereas the center of mass motion of the particle should be thought of as part of the apparatus, and the change from $|M_0\rangle$ to $|M_1\rangle$ could incorporate the center of mass motion. As noted in Sec. 4, distinct macroscopic states always correspond to orthogonal subspaces (in an appropriate quasiclassical framework) of the apparatus Hilbert space, so there is no problem in supposing that the $\{|M^j\rangle\}$ in (18) are orthogonal, as required by unitary time evolution.

By linearity the dynamics in (18) applied to the initial state in (17) results in a unitary time evolution

$$|\Psi_0\rangle = |\psi_0\rangle \otimes |M_0\rangle \rightarrow |s^1\rangle \otimes \left(\sum_j c_j |M^j\rangle\right), \tag{19}$$

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from t_0 to t_2, where Schrödinger's fearsome cat, equivalently the first measurement problem, appears on the right side. Its resolution is straightforward: in order to have something useful for comparison with the work of a competent experimentalist, we should employ a framework in which the different pointer positions make sense, a quasiclassical framework of the type mentioned in Sec. 4. Let its projectors $\{P^j\}$ form a PD on $\mathcal{H}_M$ chosen in such a way that

$$P^j|M^j\rangle = |M^j\rangle. \tag{20}$$

(As usual the symbol P^j can also stand for $I_s \otimes P^j$.)

Then a straightforward application of the Born rule, (11) and (12) in Sec. 3.3, to this situation results in

$$\Pr(P^j \text{ at } t_2) = |c_j|^2, \tag{21}$$

the standard result found in every textbook. This is actually a probability conditional upon the initial state $|\Psi_0\rangle$ at $t = 0$, but in this and the what follows we simplify the notation by omitting this condition, which is always the same. Note that there has been no reference to a distinct type of time evolution, as one finds in von Neumann's original treatment. Instead we have simply applied the principles of stochastic quantum time evolution discussed in Sec. 3 to a particular situation involving a closed quantum system that includes the measuring apparatus along with the measured particle. The first measurement problem has vanished, or one might say that it never makes its appearance once Equality has terminated the absolute reign of the uniwave. There is, to be sure, an alternative unitary framework in which the fearsome cat, the right side of (19), is present at t_2, and physicists, philosophers and science fiction writers are at Liberty to contemplate it as long as they keep in mind its incompatibility with (and thus irrelevance to) the sorts of descriptions commonly employed by competent experimental physicists when describing work carried out in their laboratories.

Now that we have a description in which the pointer has stopped wiggling, so its position makes sense, we can address a second concern of the competent experimentalist: how are these pointer positions related to the state of the particle just before the measurement took place? He has, after all, designed the apparatus so that a pointer position P^j corresponds to a prior state $|s^j\rangle$ of the particle; is this consistent with a proper quantum description of what is going on? To address this question we need the more detailed description provided by the family

$$[\Psi_0] \odot \{[s^j]\} \odot \{P^k\}, \tag{22}$$

where the alternatives at times t_1 and t_2 are enclosed in braces $\{\}$. These histories involve three times, so consistency is important, but that is easily checked, see CQT Ch. 17 for details, and one arrives at

$$\Pr(s^j \text{ at } t_1 \text{ AND } P^k \text{ at } t_2) = \delta_{jk}|c_j|^2, \tag{23}$$

whence it follows, assuming that $|c_k|^2$ is positive so that the conditional probability makes sense,

$$\Pr(s^j \text{ at } t_1 \,|\, P^k \text{ at } t_2) = \delta_{jk}. \tag{24}$$

In words, given the pointer position is P^k at time t_2, it follows with certainty (conditional probability 1) that the particle was in the corresponding state at t_1, i.e., the apparatus was functioning according to design. Thus the second measurement problem has been solved. This is, to be sure, a very simple measurement setup, but it indicates an approach which can be extended to more complicated situations.

Several comments can be made about this result. First, the solution to the second measurement problem resembles that of the first in that a key role is played by dethroning the uniwave. In this case the uniwave of interest is not the big one describing the very large universe of the particle-plus-apparatus, but the small system of the particle in isolation as it travels through orthodoxy's black box. Strict orthodoxy, let it be noted, does not ascribe reality to this little uniwave; it is simply a calculational tool. The histories approach, by contrast, is willing to treat it as real provided a framework has been adopted in which it fits. However, since at the time t_1 just before the measurement $|\psi_0\rangle$ is incompatible with the PD corresponding to the properties $\{|s^j\rangle\}$ the apparatus has been designed to measure, a framework in which $|\psi_0\rangle$ makes sense will not be useful for discussing the measurement *as a measurement*, i.e., as measuring *something*, so the physicist interested in that aspect of things must use something else.

Second, it is worth noting that the measurement *outcome* in this model, the final $|M^j\rangle$ or P^j, lacks any interesting connection with the state of the particle *after* the measurement is over. We used $|s^1\rangle$ in the final term in (18), but any other choice would have been equally good, including a final state $|s^k\rangle$ with the relationship of k to the earlier j chosen randomly, or even von Neumann's original version with $|s^j\rangle$

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left unchanged in the time step from t_1 to t_2. In the majority of situations in the laboratory in which microscopic properties are measured by some macroscopic apparatus, the original state of the particle is not preserved, and is in any case entirely irrelevant to the interpretation of the experiment. There is no reason to believe that the spin of a silver atom flying through a Stern-Gerlach remains the same when it becomes attached to a glass plate; indeed, the very concept of "the spin of this silver atom" becomes rather doubtful in that situation. Von Neumann should not be faulted; his proposal came in the early days of quantum mechanics when the role of measurement was not properly understood, and it was valuable to have a model in which a second measurement could confirm the outcome of the first. But it is most unfortunate that quantum textbooks continue to confuse students by treating the von Neumann model together with its mysterious wave function collapse as somehow essential for understanding the measurement process, a full quarter century after the publication of a much better approach.

Third, nowhere in the above discussion has any reference been made to *decoherence*. This is because the appeal to decoherence is neither necessary nor sufficient for resolving the measurement problem(s), despite occasional claims in the literature that it can do so, see e.g. [43, 45], or at least that it plays some essential role. As has occasionally been pointed out by critics, e.g., [46, 47], decoherence does nothing to remove the underlying difficulty caused by the assuming that quantum reality is represented by the uniwave. By contrast, the histories approach, rather than appealing to decoherence, goes right to the heart of the difficulty and resolves it by dethroning the uniwave. This is not to say that decoherence is unimportant for understanding the real world, including real laboratory experiments. But its importance is not in solving the measurement problems—and in any case it can make no pretense of solving the second measurement problem—but rather for formulating and justifying the quasiclassical descriptions and approximations needed to derive ordinary macroscopic properties and classical dynamics from quantum theory, as discussed in Sec. 4.

Finally it is worth emphasizing that the correct description of a quantum measurement *as a measurement* requires that the framework contain both an appropriate PD for the pointer positions at a time after the measurement takes place *and* a (different) PD representing the appropriate particle properties *before* it takes place. Both requirements will in general be incompatible, though in somewhat different ways, with the claim that the uniwave constitutes quantum reality. This helps explain why approaches based on the uniwave have not succeeded in resolving the infamous measurement problem of quantum foundations, and suggests that they are unlikely to do any better in the future.

5.2 Preparation model

Von Neumann's original model, in which the final step in (18) is

$$|s^j\rangle \otimes |M_1\rangle \rightarrow |s^j\rangle \otimes |M^j\rangle, \tag{25}$$

i.e., the particle state $|s^j\rangle$ is left unchanged by its interaction with the apparatus, represents a nondestructive measurement, and can also be viewed as a model for *preparation*. We again assume a starting state $|\psi_0\rangle$ of the particle to be a superposition of the basis states, (17), so unitary time development now leads, in place of (19), to

$$|\Psi_0\rangle = |\psi_0\rangle \otimes |M_0\rangle \rightarrow \sum_j c_j \left(|s^j\rangle \otimes |M^j\rangle \right). \tag{26}$$

Again, unitary time evolution leads to a macroscopic superposition, and again the route to a simple physical interpretation is to employ a PD $\{P^j\}$ of the same type used previously for the apparatus, with (20) satisfied, i.e., each P^j identifies a pointer state. In addition we let $[s^i]$ be the projector onto state $|s^i\rangle$ of the particle, so the total PD on the tensor product of $\mathcal{H}_s \otimes \mathcal{H}_M$ at the final time t_2 is $\{[s^i] \otimes P^j\}$. A simple application of the Born rule then leads to

$$\Pr([s^i] \text{ AND } P^j \text{ at } t_2) = \delta_{ij}|c_j|^2, \tag{27}$$

where, as previously, we omit explicit mention of the condition $|\Psi_0\rangle$. It follows immediately, assuming $|c_j|^2$ is positive, that

$$\Pr([s^i] \text{ at } t_2 \,|\, P^j \text{ at } t_2) = \delta_{ij}. \tag{28}$$

In words, if after its interaction with the particle the apparatus is in the state P^j, i.e., the pointer is in the direction j, then the particle is in the corresponding state $[s^j]$, equivalently $|s^j\rangle$ (the phase does not matter). This is consistent with von Neumann's idea that an outcome P^j "collapses" the wave function to $|s^j\rangle$, but rather than having to invoke some "magical" collapse process the result in (28) is a quite straightforward

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consequence of using standard probabilistic reasoning. Wave function collapse as used in textbooks is thus nothing but a system for calculating conditional probabilities, and the books would be much less confusing and less liable to misinterpretation were it stated as such. Or, given the enormous confusion wave function collapse has caused in quantum foundations studies, it might be better to never mention the idea, and simply teach students the proper use of probability theory, a subject which ought to be included in every introductory quantum textbook.

While this discussion has been limited to a very simple model of a preparation process, it indicates the correct direction to go in more complicated situations. Suppose, for example, that in place of (26) there is a unitary time development

$$|\Psi_0\rangle \rightarrow \sum_j c_j \Big(|r_j\rangle \otimes |M^j\rangle \Big), \tag{29}$$

starting with an initial state $|\Psi_0\rangle$ whose details need not concern us, where the normalized states $\{|r_j\rangle\}$ need not be orthogonal. However, the product states $|r_j\rangle \otimes |M^j\rangle$ are orthogonal for different j; so one has an example of dependent or contextual events or properties as discussed in Ch. 14 of CQT. Once again, given the macroscopic outcome P^j one can conclude that the system is in the state $|r_j\rangle$, regarded as a contextual property which depends on P^j, and there is no need for wave function collapse, though that can be a convenient calculational tool.

Finally, note that a description of a process of preparation followed by a measurement, of course using separate pieces of apparatus, can be constructed by combining the results presented here with those in Sec. 5.1. Doing so effectively pries open orthodoxy's black box and allows the physicist to "see" what has been prepared, and also understand the microscopic state of affairs that gives rise to the final measurement outcome. Events at intermediate times while the particle is isolated can also be discussed, but now one needs to pay attention to consistency requirements in order to arrive at a sensible probabilistic description.

5.3 POVMs

In Sec. 2 it was noted that quantum properties of a system at a single time are associated with a projective decomposition (PD): a set of mutually orthogonal projectors on the Hilbert space that sum to the identity. There is a more general notion of a decomposition of the identity, usually referred to as a POVM (positive operator valued measure): a collection $\{R_k\}$ of positive operators (Hermitian with nonnegative eigenvalues) which sum to the identity. Since the square of a positive operator is in general not equal to itself we shall use the subscript k as a label, while retaining the superscript label for a PD. We assume that both the number of operators in the POVM and dimension of the Hilbert space on which they act are finite.

A POVM is a useful mathematical tool in various situations, so it is natural to ask whether it has some physical interpretation. In the case of a PD, which is a special case of a POVM, the elements are associated with mutually exclusive properties, but for a general POVM such an interpretation is not possible, especially when, as is usually the case, the different operators do not commute with each other. When used in some sort of "measurement situation" the POVM typically serves as a pre-probability: a device for calculating probabilities, but which does not itself have any direct physical interpretation. So when used this way a POVM does not have an ontological reference. A simple example will serve to illustrate this point.

Consider a Hilbert space $\mathcal{H} = \mathcal{H}_s \otimes \mathcal{H}_A$, where $\mathcal{H}_s$ is a system in an unknown state $|\psi_0\rangle$ at t_0, whereas the auxiliary or "ancillary" system $\mathcal{H}_A$ is known to be in a specific state $|A_0\rangle$; thus the initial state of $\mathcal{H}$ is

$$|\Psi_0\rangle = |\psi_0\rangle \otimes |A_0\rangle. \tag{30}$$

Next let $\{P^j\}$ be a PD for $\mathcal{H}$ at a time $t_1 > t_0$, and assume the time development from t_0 to t_1 is trivial, $T(t_1, t_0) = I$. (One could replace this with an arbitrary unitary without changing the following discussion in any essential way.) The probability of P^j at time t_1 is then given by the Born rule:

$$\Pr(P^j \text{ at } t_1) = \text{Tr}(P^j[\Psi_0]) = \text{Tr}_s(R_j[\psi_0]) \tag{31}$$

where

$$R_j := \text{Tr}_A(P^j[A_0]) \tag{32}$$

is an operator on $\mathcal{H}_s$, that is, on the Hilbert space that holds the unknown $|\psi_0\rangle$, and Tr_s and Tr_A denote partial traces. It is easily checked that the operator R_j is positive and that $\sum_j R_j = I_s$, so $\{R_j\}$ is a POVM on $\mathcal{H}_s$.

What (31) tells us is that the probability of a physical property P^j of the joint system represented by tensor product $\mathcal{H}_s \otimes \mathcal{H}_A$ can be calculated by means of a formula, the right side of (32), which only involves the initial state $|\psi_0\rangle$ on the smaller Hilbert space. To be sure the initial state $|A_0\rangle$ of the ancillary system is playing a role, as is evident from (32). Nevertheless, in some circumstances the POVM $\{R_j\}$ provides an efficient way of calculating probabilities, in a way roughly analogous to the uniwave $|\psi(t)\rangle$ when used in the Born formula in (8), or—perhaps a closer analogy—the $\{|\phi_0^k\rangle\}$ in (10). But in none of these cases is one obliged to identify the pre-probability, which is a calculational tool, with some aspect of physical reality; the probabilities of interest can very well be computed by alternative methods in which the pre-probability never appears.

To be sure, sometimes the same mathematical object which within one framework serves only as a (dispensable) pre-probability can in another framework represent a physical property; in particular, the uniwave can be used in this way in unitary families. This might in some circumstances be possible for POVM elements, but it is not obvious, at least in general, how to construct the appropriate framework.

6 Quantum Locality

One frequently encounters the claim that quantum theory is intrinsically nonlocal because of the presence of mysterious long-range influences which can act instantaneously, in contraction to special relativity, but which cannot carry any information, which means they are experimentally undetectable.[5] And sometimes it is asserted in addition that any future theory which yields certain quantum mechanical results that that have been confirmed by experiments must be similarly nonlocal (e.g., [48]). Such claims are often based on observed violations of Bell's inequality, though one can arrive at similar conclusions using Hardy's or the GHZ paradox.

6.1 Genuine nonlocality

The first point to be made is that there is a very genuine sense in which quantum mechanics is nonlocal, already evident in a wavepacket $\psi(x)$ for a particle in one dimension, which for convenience we assume is a continuous function. If one regards the corresponding $|\psi\rangle$ or projector $[\psi]$ as representing a quantum property, it is easy to show that it fails to commute with the projector $X(x_1, x_2)$ defined by

$$(X(x_1, x_2)\psi)(x) = \begin{cases} \psi(x) & \text{if } x_1 \le x \le x_2, \\ 0 & \text{otherwise,} \end{cases} \tag{33}$$

unless the support of $\psi(x)$, the set of points where it is nonzero, falls entirely inside or entirely outside the interval from x_1 to x_2. Since the physical interpretation of the property represented by $X(x_1, x_2)$ is that the particle lies inside this interval, we have here a simple example of a sense in which a quantum particle under certain circumstances can be said to be "nonlocal," meaning that it lacks a precise location.

It is perhaps worth pointing out in this connection an error one frequently encounters in popular expositions of quantum mechanics, though sometimes also in more technical publications, where it is asserted that a quantum particle "can be in two places at the same time." This is quite wrong, or at the least thoroughly misleading, since the product of two projectors corresponding to the particle lying in two nonoverlapping regions will be zero, corresponding to the fact that this property is always false. It would be much better to say that the particle does not have a definite location.

A second and technically simpler example of such nonlocality, since it involves only a finite-dimensional Hilbert space, is provided by the well-known singlet state

$$|\psi_0\rangle = \left(|z_a^+, z_b^-\rangle - |z_a^-, z_b^+\rangle\right)/\sqrt{2} \tag{34}$$

of two spin-half particles a and b. It is a straightforward exercise to show that the corresponding projector $[\psi_0]$ does not commute with any nontrivial projector referring to particle a alone, i.e., of the form $P \otimes I_b$ (the trivial projectors are 0 and the identity), or with one referring to particle b alone. If particles a and b are in different locations then one can say that $[\psi_0]$ is incompatible with any local property. While this is true, it is well to remember that what is involved is the fundamentally quantum idea of incompatibility, which can have local as well as nonlocal manifestations. Thus $|\psi_0\rangle$ is the spin part of the ground state wave function of the hydrogen atom, so one cannot consistently combine the assertion that the hydrogen atom is in its

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[5]For a lengthy list of work by some of the principal advocates and critics, see the bibliography in [32].

ground state with (nontrivial) talk about the spin angular momentum of either the electron or the proton. In this case the electron and the proton are not at separate locations. (Claiming that they are would be inconsistent with the spatial part of the ground state wave function.)

6.2 Spurious nonlocality

Having discussed cases of genuine quantum nonlocality, let us now turn to the source of the mistaken notion that the quantum world is somehow pervaded by nonlocal influences, which even their proponents admit cannot be used to transmit information, and are hence experimentally undetectable. As the whole topic has recently been discussed at some length elsewhere [32], it will suffice for present purposes to focus on the central point where claims of such nonlocality go astray. Let us start by briefly repeating an example from [32]. Charlie in Chicago takes two slips of paper, one red and one green, places them in two opaque envelopes, and after shuffling them so that he himself does not know which is which, addresses one to Alice and the other to Bob, who live in different cities, but know the protocol Charlie is following. Upon receipt of the envelope addressed to her Alice opens it and sees a red slip of paper. From this she can immediately conclude that the slip in Bob's envelope is green. Her conclusion is not based on a belief that opening her envelope to "measure" the color of the paper inside has some magical long-range influence on what is in Bob's envelope. Instead Alice employs statistical reasoning in the following way. From her knowledge of the protocol Alice can assign a probability of $1/2$ to each of the two possibilities: $A\!:\!G$ AND $B\!:\!R$, green slip sent to Alice and red slip to Bob; and $A\!:\!R$ AND $B\!:\!G$. This implies a marginal probability of $1/2$ for each possibility, $B\!:\!R$ or $B\!:\!G$, for the color of the slip in Bob's envelope. However, upon opening her envelope and observing that the slip is red, Alice can replace these with the conditional probabilities $\Pr(B\!:\!G\,|\,A\!:\!R) = 1$ and $\Pr(B\!:\!R\,|\,A\!:\!R) = 0$.

Now suppose that Charlie at the center of a laboratory pushes a button so that one member of a pair of spin-half particles initially in the singlet spin state (28) is sent towards Alice's apparatus at one end of the building, while the other is simultaneously sent towards Bob's apparatus at the other end. If Alice measures the x component of spin of particle a and the outcome corresponds to $S_{ax} = 1/2$, what can she say about S_{bx} for particle b traveling towards Bob, assuming that both particles have been traveling in field-free regions? By applying the Born rule, Sec. 3.3, using $|\psi_0\rangle$ as a *pre-probability* to a framework that includes both S_{ax} and S_{bx}, Alice, whom we assume is both a competent experimentalist and has had an up-to-date course in quantum mechanics, can conclude that $S_{bx} = -1/2$. And this conclusion is reached by *precisely the same* sort of statistical reasoning that applies in the case of colored slips of paper. Nonlocal influences are involved to no greater extent than in the case of the colored slips of paper discussed earlier.

Suppose on the other hand that Alice decides at the very last minute, after the two particles are already on their way, to measure S_{az} in place of S_{ax}. How will this change things? In particular does it somehow alter the spin of particle b, on its way to Bob? Not at all, as demonstrated by the detailed analysis in Sec. 23.4 of CQT. What happens is that Alice learns something different about her particle, namely the value of S_{az} just before the measurement takes place. By employing a different framework, incompatible with the previous framework, with S_{az} in place of S_{ax}, Alice can now on the basis of the measurement outcome make an inference with probability 1 about S_{bz}, but she loses the ability to say anything about S_{bx}.

The reader may object that *if* Alice *had* measured S_{ax} rather than S_{az}, she would have gotten a definite value, and from this she *could have* inferred the value of S_{ax} before the measurement. And therefore there must have been both a definite value for S_{az} and a definite value for S_{ax} just before the measurement took place. The words in italics indicate that the preceding is a counterfactual argument of the sort philosophers have trouble analyzing. Thus one must approach its use in a quantum context with particular care; see the discussion in Ch. 19 of CQT. For present purposes it suffices to note that the conclusion makes assertions about both S_x and S_z for the same particle at the same time, and thus violates the single framework rule: we have reached statements which cannot both be simultaneously embedded in Hilbert space quantum mechanics. Note that the issue has to do with *local* properties determined by local measurements. Particle b has never been mentioned; it need not even exist.

A more detailed study of derivations of Bell's inequality—we refer the reader to Ch. 24 of CQT as well as [19, 32]—shows that it is the matter just referred to, the attempt to ascribe incompatible properties to a single quantum system, that invalidates derivations of Bell's (or, to be more precise, the CHSH) inequality when it is applied to the microscopic quantum world, rather than the macroscopic world of ordinary experience, where the assumptions needed to justify it can be satisfied, at least to a very good approximation. To be sure there is by now an enormous literature devoted to Bell's inequality, and someone

trying to refute its applicability to the quantum world is in somewhat the same position as the professor challenged by a student to point out the error in reasoning in an examination paper that has by a circuitous route arrived at a result which is clearly wrong. Rather than seek to identify the error in each and every (supposed) derivation of Bell's inequality, it seems better to throw the challenge back to the other side. The nonlocal influence claim violates the principle of Einstein locality summarized briefly below. The complete and relatively simple proof is given in Sec. 6 of [32]. Can the reader find a flaw in it?

6.3   Einstein locality

By applying quantum mechanical principles including the single framework rule one can establish the following principle of *Einstein Locality*.[6]

> Objective properties of isolated individual systems do not change when something is done to another non-interacting system.

The proof is given in [32]; what is useful here is a precise definition of the terms. Let A, Hilbert space $\mathcal{H}_A$, be the isolated individual system whose properties are under discussion. Let B and C, Hilbert spaces $\mathcal{H}_B$ and $\mathcal{H}_C$, be systems which do not interact with A during the time interval t_0 to t_1 of interest in the sense that the time-development operator factors:

$$T_{ABC}(t,t') = T_A(t,t') \otimes T_{BC}(t,t') \tag{35}$$

The role of the ancillary system C is to "do something" to B; this can be modeled by varying the initial state $|\phi_0\rangle_C$ of C at t_0 while keeping the initial state of $|\Phi_0\rangle_{AB}$ of system AB remains fixed. Note that C can interact with, and thus "do something" to, B during the interval from t_0 to t_1. The claim that the properties of A do not change when something is done to B is then established by an argument that shows that the probabilities associated with a consistent family of histories of A, involving projectors on $\mathcal{H}_A$ and thus referring to its properties alone, are independent of the choice made for $|\phi_0\rangle_C$. In addition, the consistency of that family does not depend upon $|\phi_0\rangle_C$.

While the mathematical argument is straightforward, the issue of whether the English words in the above statement of Einstein locality have been correctly translated into the mathematics of quantum theory is less so. Essential to the argument is, of course, the assumption that any objective properties of A must be represented by projectors on its Hilbert space, and if the family of histories contains a sequence of such properties at three or more times (including the initial time) then the consistency conditions are satisfied. The properties of A are *objective* in the sense that the whole situation is modeled from the perspective of a physicist who is *outside* the system being described. As always, the sequence of properties under discussion is determined by the physicist, but any other physicist who uses the same framework, or a more general one that contains this framework, will come to exactly the same conclusion about the probabilities. The next subtlety concerns how the phrase *something is done*, with its implicit but not insignificant association with the concept of free choice by a conscious agent, is modeled in quantum terms. The approach used here, where a third system C is employed to *do* something to B, is at least consistent with discussions in contemporary quantum information theory where Alice is said to "do" something to a quantum system. A final issue has to do with using *initial conditions*. Again, this is consistent with the approach used in quantum information theory, but buried here is an important and unresolved problem related to thermodynamic irreversibility; see Sec. 8.4.

Assuming the arguments used to justify Einstein locality are correct, the result is an extremely simple explanation of why the mysterious nonlocal influences can carry no information: they do not exist. They are the residue of a faulty analysis that is inconsistent with the principles of quantum reasoning needed to resolve the measurement problem(s).

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[6]The wording is essentially the same as in Sec. 2 of [49], where Mermin refers to it as "generalized Einstein locality." For Einstein's own statement see p. 85 of [50]: "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former."

7   Quantum Information

7.1   Histories approach

The histories approach provides a quite definite answer to Bell's [11] query directed at attempts to explain quantum mysteries by appeal to information: "*Whose* information? Information about *what*?" The second question belongs to the realm of ontology, and the ontology which is the subject of the present paper gives a definite answer: the information is about *quantum properties*, represented mathematically as subspaces of, or the corresponding projectors on, the quantum Hilbert space. More generally, information can be about a time sequence of such properties, a quantum history.

The modern theory of information, see [51] is formulated in terms of probabilities and probabilistic reasoning, the sort found in probability theory textbooks, and one hopes will someday be used in quantum textbooks. Thus it is natural to expect that "classical" information-theoretic concepts will appear in quantum information theory in much the same form *as long as attention is confined to a single framework*. This is indeed the case; e.g., [33], and while it by no means exhausts the contents of quantum information theory, it does provide a good beginning, one that is more satisfactory than the approaches discussed in by Timpson in [9], which as he himself points out in his Sec. 4.4.1 cannot provide a satisfactory answer to the question of, as he puts it, "how things are with a system prior to measurement."

Not only does the histories approach provide a good beginning point by supplying a criterion for how classical information theory can be applied without fear of generating contradictions or otherwise falling into the quantum foundations swamp, it also provides a certain perspective on what remains to be done: the part of quantum information that goes beyond the classical theory has to do with comparing results that are obtained if one uses alternative *incompatible* frameworks. But is not such comparison forbidden by the single framework rule? Not at all. What is forbidden is *combining* the probabilities worked out in different incompatible frameworks. Indeed, it is comparison without combining that is resulting in new physical insights in this very active field of current physics research. To be sure, the typical practitioner has learned whatever quantum mechanics he knows from the standard textbooks, and has to make frequent reference to "measurements" without any clear idea of what this means. Research papers are often filled with complicated mathematical formulas and concepts whose physical significance is unknown, to the author as well as to the reader. Bell would not be pleased. One can at least hope that a consistent formulation of quantum ontology will at some future time help bring some order to this conceptual chaos.

As to Bell's first question, "whose information?", the answer has already been suggested by the little scene presented in Sec. 6.2. The outcome of her experiment provides Alice with information about the prior physical state of a particle which her apparatus was designed to measure. It is her information, not Bob's information; his particle is completely unaffected by the distant measurement. By combining the results of her experiment with additional information related to the preparation of the particle pair—see the remarks on preparation in Sec. 5.2—she is then able to infer something about a *microscopic* quantum particle that is, or was, on its way to Bob, and by this means say something about the outcome of a measurement which he has already made, or perhaps will make at some future time. If the latter, then it is correct to say that Alice has information about the outcome of a future experiment. On the other hand, an ontology which says that the *only* information quantum theory provides relates to the outcome of future experiments is seriously inadequate, both for the needs of experimental physicists and for theorists interested in quantum information; it leaves the black box tightly closed.

Readers with the appropriate technical background who are interested in how an approach of the sort just discussed can provide a perspective on some problems in quantum information problems are invited to take a look at [33, 52–54].

7.2   Sources and information

In Sec. 4.2 of his [9] Timpson asserts that he can make a "perfectly precise and adequate definition of quantum information" based upon coding ideas. The strategy is to imagine a source that produces a sequence of quantum states—it will suffice to consider the case where these are pure states—analogous to a classical source producing a sequence of symbols drawn from some alphabet. A sequence of quantum states is in effect a history of the sort discussed above in Sec. 2 and could be described in the language of histories (to which Timpson makes no reference). However, there is a serious difficulty with this proposal: a particular quantum state in the sequence is drawn from an alphabet of states which are *not* required to be orthogonal: see the beginning of his Sec. 4.2.1.3 (p. 225). But in this case the history projectors making up

the corresponding family will not in general be orthogonal to each other, so they will not form a quantum sample space, and considering them to be the referents of quantum information, what quantum information is *about*, leads into the great swamp.

This has long been appreciated by the orthodox, who, while they would not state it precisely this way, in effect use the following strategy. Every time Alice prepares a state she records the settings on her preparation device. Distinct macroscopic records correspond to orthogonal quantum states or projectors, and by tying the records to the not-necessarily-orthogonal microscopic states of particles prepared by the apparatus one arrives at what in Ch. 14 of CQT are referred to as dependent or contextual events. Holevo's famous bound (see, e.g., Sec. 12.1.1 of [6]) then tells us the maximum amount of information about Alice's notebook that Bob's notebook can contain, following whatever measurement procedure he may employ. The orthodox regard this as perfectly sensible, since it makes no reference to what may be happening inside the black box. If Timpson wants to say that quantum information is simply information about a macroscopic preparation procedure his proposal cannot be faulted. However, by his own standards this amounts to the sort of instrumentalism he does not find satisfactory as a solution to the quantum information problem.

8 Conclusions

8.1 Summary

The consistent ontology for quantum mechanics described in Secs. 2 and 3, with applications to the classical limit, preparation and measurement, locality, and quantum information in Secs. 4 to 7, resembles the ontology of classical mechanics, as represented mathematically by a phase space and deterministic Hamiltonian dynamics, in several important respects. It is realistic: the real world is "out there", not just a part of some observer's consciousness, and its structure is reflected in the mathematical theory constructed by physicists for describing it. But of course it differs from its classical predecessor in important ways, which can be conveniently summarized under two headings. First, the Hilbert space description of a system, in which its properties are represented by subspaces, requires a new logic in the sense of a mode of reasoning about the world, with rules somewhat different from those familiar in classical physics, where ordinary propositional logic fits very comfortably onto an algebra of physical properties corresponding to subsets of the phase phase. Second, quantum dynamics is intrinsically stochastic or probabilistic: probabilities are present in the basic axioms that apply without exception to all quantum processes, not just to those associated with some form of "measurement."

By using the new logic and the new dynamics one arrives at an interpretation of quantum mechanics in which the measurement problems, long the bane of quantum foundations research, disappear. Not only does one know how to make sense of the pointer position at the end of the measurement, one also knows how to relate it to a property the microscopic system possessed *before* the measurement took place. Human consciousness need not be invoked in order to address or solve these problems, thus getting rid of a difficulty which has plagued quantum foundations ever since the days of von Neumann's "psycho-physical parallelism" [1]. Psychology can be cleanly split off from physics (also see Sec. 8.4) to the benefit of both disciplines. One can speak of an objective quantum reality: different observers can agree because there is something "out there" in the world, external to themselves, about which agreement is possible; it is not just a matter of making bets about the outcomes of future experiments. Wave function collapse is unnecessary; it can be (and probably should be) replaced with conditional probabilities, Sec. 5.2. Instantaneous nonlocal influences have been cleaned out of the quantum foundations swamp, Sec. 6, removing an apparent conflict between quantum theory and special relativity. It is now possible, Sec. 7, to provide a reasonable ontology for quantum information. Its probabilities refer to quantum properties, and it includes in a natural way, in each single framework, the standard ideas and intuition of ordinary (Shannon) information theory. The remaining specifically "quantum" problems of information theory involve comparison of, not combinations of, incompatible frameworks.

In addition to all of this, a whole series of quantum paradoxes (Bell-Kochen-Specker, Einstein-Podolsky-Rosen, Hardy, . . .) are resolved by the histories approach. They have not been discussed in this paper because a large number have been treated at considerable length in Chs. 20 to 25 of CQT, and more could be treated by the same methods. Such paradoxes can be understood using a consistent set of underlying quantum principles, and not left as unresolved conundrums, as in [55]. Instead they are interesting, one might even say beautiful, illustrations of ways in which the quantum world differs from the classical world of our everyday experience, in much the same way as the twin paradox provides a striking illustration of the principles of special relativity.

The new logic and the new (relative to classical mechanics) dynamics, which form the heart of this paper, are summarized below in Secs. 2 and 3. The final Sec. 8.4 indicates some open problems.

8.2   The new logic

The new logic, in which conjunctions and other combinations of incompatible quantum propositions, represented by noncommuting projectors, are ruled out of acceptable quantum descriptions by the single framework rule, represents a radical break with classical physics. Indeed, one might say it is the central feature that distinguishes the quantum world from the pre-quantum world of everyday experience. It is as radical as Copernicus' shifting the earth out of the center of the universe, or of Einstein's relativizing time. The assumption that if it makes sense to talk about A with reference to some system, and to talk about B for the same system, then it also makes sense to talk about A AND B, is essentially automatic in everyday reasoning as well as in classical physics. A proposal that this "obviously true" fact should not be correct in general for quantum physics tends to arouse the immediate reaction of "that cannot make any sense." So it is not surprising that the new logic with its single framework rule has encountered considerable resistance from those who think it much too radical to be an acceptable part of good science, even though they themselves are not able to provide a solution to quantum mysteries by other means. Thus it is important to understand in a clear way where the single framework rule comes from, what it affirms, and what it does *not* say.

First it should be stressed that the single framework rule arises from the effort to make physical sense of the Hilbert space structure that underlies all of modern quantum mechanics, as interpreted by von Neumann. Once the association of properties with subspaces and their negations with orthogonal complements of these subspaces has been made, there are serious logical issues, as Birkhoff and von Neumann pointed out. How are they to be dealt with? Broadly speaking, there have been three approaches. First, that of quantum logic, which, as noted in Sec. 2.1, has not resolved the conceptual difficulties of quantum theory, though smarter physicists or even smarter robots, may someday make more progress. Second, that of standard quantum mechanics as embodied in the textbooks, where the strategy is not to discuss the logical issues but instead appeal to measurements. This reaches its extreme in the unopenable black box which quantum orthodoxy inserts between preparation and measurement. The field of quantum foundations can be thought of as a protest against this approach of invoking measurement as a way out of quantum conceptual difficulties.

Third is the histories strategy, in which quantum descriptions of the world are split off into families or frameworks that cannot be combined with each other, but within each framework ordinary propositional logic applies, along with the usual rules and intuition associated therewith. The histories formulation has some things in common with the other two approaches. It is a form of quantum logic in the sense of a scheme for correct reasoning in the quantum domain that differs from the logical scheme of classical physics. However, its single framework rule is not part of what is usually referred to as quantum logic. And it shares with textbook quantum mechanics and quantum orthodoxy the refusal to talk about certain things. But what it does allow the physicist to talk about is now greatly enlarged: it includes a whole series of microscopic properties and events. Including a significant set of things that experimental physicists think they are able to detect with their instruments. For these reasons the logic used in histories is perhaps not quite as radical or as innovative as might at first be supposed, though it obviously includes some very new features.

There have, nonetheless been some severe misunderstandings of the histories approach to this problem by means of its single framework rule. A first type of misunderstanding arises from supposing that the single framework rule, perhaps because it is both unfamiliar and not presented in the textbooks, does not have to be taken seriously. Claims that the histories approach leads to contradictions, as in [25–27], can be refuted, as in [28–31], by working through the argument and seeing at which point the claimant has, perhaps unwittingly, strayed from one framework onto a different, incompatible one in the course of constructing a logical argument. While what is going on is clearest in situations like the Bell-Kochen-Specker paradox, there are also situations in which the consistency conditions of quantum dynamics play an important role, as in the case of the Aharonov and Vaidman three box paradox [56] which forms the basis of Kent's criticism in [25]; see the detailed discussion of this paradox and the associated incompatible frameworks in Sec. 22.5 of CQT.

It is worth noting at this point that there is a a very general argument, Ch. 16 of CQT, that the histories approach will not result in contradictions, and critics have yet to find any flaws in it. To be sure there may be flaws that lie undiscovered because the logical structure employed in the histories approach has yet to be subjected to sufficiently severe scrutiny by those who have first taken the trouble to carefully understand what it is all about. The author hopes the present paper, by addressing various misunderstandings in a

direct way, may provoke that sort of serious study of a system of quantum interpretation which, on the basis of the problems and paradoxes it resolves, has some claim to being the best and most consistent approach to quantum ontology currently available.

A second type of misunderstanding is to suppose that the incompatibility of frameworks, which prevents their being combined, is the same as their being *mutually exclusive*, one is true and the other is false, in the same sense that the events of a coin landing heads or tails are mutually exclusive. This leads among other things to the erroneous idea that histories quantum mechanics is incomplete unless it includes some "law of nature" that specifies the correct framework that obtains in a particular physical situation. A particular case is the Dowker and Kent critique of the notion of a quasiclassical framework discussed in Sec. 4.2. Another instance is provided by Wallace's assertion (p. 39 in [4]) that the histories approach leads to a view of reality that only makes sense "when described from one of indefinitely many contradictory perspectives." Presumably these "perspectives" are frameworks; see the refutation in [29] of a similar claim in [25].

One possible source for the mixup between the relationship of quantum incompatibility and that of being mutually exclusive is the unfortunate reliance in textbooks upon an unanalyzed (and, for the orthodox, unanalyzable) measurement process when presenting a physical interpretation of quantum mechanics. The books state, correctly, that there is no measurement which can simultaneously determine S_x and S_z for a spin half particle. The measurement setups required in these two cases *are* mutually exclusive: they correspond to macroscopically distinct arrangements, so the subspaces of the quantum Hilbert space required to represent them are necessarily orthogonal to each other. That does not of course mean that the microscopic properties measured by these distinct apparatus setups are also mutually exclusive.

If the fundamental logic needed to describe the quantum world is so different from the logic of everyday affairs, why is the latter so successful and widespread? The histories approach answers this question in the manner sketched in Sec. 4: to describe the macroscopic world one only needs a single quasiclassical framework or, to be more precise, all frameworks of this type give the same results "for all practical purposes." Until one arrives at situations in which quantum effects (those associated with noncommuting operators on the Hilbert space) become important, classical mechanics is adequate, and one would expect that whenever classical mechanics is adequate the logic that corresponds so well to subsets of the phase space will also be adequate.

Obviously the single framework rule is not something easy to understand. The author hopes that the various analogies and comparisons given above in Sec. 2.3 will assist the reader in gaining some intuitive grasp on an unfamiliar concept. However, for the serious student of quantum mechanics there is no substitute for working through various specific quantum examples, such as the those scattered throughout CQT.

8.3   Indeterministic dynamics

The histories approach assumes that quantum time development is basically stochastic or probabilistic. Always, not just when measurements are being carried out. One starts with a history Hilbert space $\breve{\mathcal{H}}$, a tensor product of the sample space of a single system at each of the (discrete, and, for convenience, finite in number) times of interest. Elements of the sample space, referred to as histories, are then projectors forming a PD of the identity on $\breve{\mathcal{H}}$. As in other applications of probability theory to stochastic processes, the physical (ontological) interpretation is that just one of the possible histories in a given sample space actually occurs. In this sense the histories approach is, as noted in Sec. 3.5, quite distinct from many-worlds versions of quantum theory in which things that do not occur here are supposed to take place in some alternative but inaccessible universe.

In the case of a closed quantum system weights can be assigned to elements of the sample space provided a consistency condition, vanishing of the off-diagonal elements of the decoherence functional, are satisfied, and a consistent set of probabilities defined using the weights and additional data (such as an initial state). The Born rule is a particular example of this scheme when only two times are involved, in which case the consistency conditions are always satisfied. Provided the consistency conditions are satisfied such a sample space, and its associated event algebra, forms a consistent family or framework. Two such frameworks can be combined only if the different projectors commute (the same rule as for systems at a single time) and if the common refinement resulting from products of these projectors also satisfies the consistency condition. Otherwise they are incompatible, and the single framework rule forbids combining them for purposes of calculating probabilities or carrying out probabilistic reasoning of the sort that reduces to ordinary propositional logic when probabilities are 0 and 1.

Because stochastic models are widely used in many branches of science, including physics, and because

probabilities are already found, though not very well explained, in quantum textbooks, the second major way in which the histories quantum mechanics departs from classical mechanics, its use of probabilistic time development as a fundamental law, not just for measurements or macroscopic systems, may be a bit easier to accept than the new logic. Nonetheless, the notion that time development is fundamentally indeterministic is one which many physicists find unappealing, even if not completely unacceptable. The objections by Einstein are well known, and the continuing popularity, at least in some quarters, of the Everett and Bohm interpretations indicates something of the appeal of classical determinism. To be sure, one can speculate that even Einstein might have found stochastic quantum time development acceptable if one consequence would be to make it consistent (in a sense he might not have anticipated) with his notion of locality, Sec. 6.3.

Note that the general strategy involved in the new logic, that of restricting the domain of logical or, in this case, probabilistic reasoning by a rule that prevents combining incompatible frameworks, is central to the histories discussion of time dependence. When assigning probabilities to a closed quantum system not only must one use a PD of the history identity operator, but an additional consistency condition must be fulfilled for histories involving more than three times, Sec. 3. Whereas these consistency conditions can be stated in a very simple and clean mathematical form, (15), they still look a bit strange, and have no (obvious) counterpart in the notions needed to describe a quantum system at a single time. While no inadequacy has been identified in the current histories formulation of stochastic time development, there may be something more interesting lurking there; see below.

8.4   Open issues

The quantum ontology introduced in this paper seems satisfactory in a number of respects. It is based upon the quantum Hilbert space so has no need for additional (hidden) variables. Time development is stochastic, and calculating the probabilities requires only the use of unitary time development induced by Schrödinger's equation, not some modification thereof. There is no measurement problem: measurements (and preparations) can be consistently understood using the same tools employed for all other quantum processes, and measurement apparatus designed by competent experimentalists does what it was designed to do: reveal properties microscopic systems had *before* the measurement took place. The quantum world is *local* when allowance is made for the width of quantum wave packets. There is neither action nor passion at a distance. And one has a preliminary, but thus far satisfactory, ontology for quantum information.

That is not to say all problems in the domain of quantum interpretation have been solved. Some of the work that still needs to be done is quite technical; e.g., developing appropriate conceptual tools to describe electronic transport in nanostructures, in order to replace the present quasiclassical descriptions adequate for larger structures but inadequate at the smallest scales. Whatever the practical importance of such work, it is unlikely to appeal as a subject of research to philosophers and physicists concerned with foundations of quantum theory. But there are other topics they should find more interesting. Here are four of them.

8.4.1   Entangled histories

Whereas the principles of quantum stochastic dynamics summarized in Sec. 3 are both consistent and provide what seems to be a quite adequate foundation for all the sorts of calculations taught in textbooks and used in current research papers, they are incomplete in the following sense. Most discussions of histories and all discussions of consistency conditions known to the author employ a sample space of *product* histories: at each time in the history tensor product space a projector represents a property of the system *at that particular time*. But the tensor product space representing a composite quantum system—two or more subsystems—at a single time also contains what are called *entangled states*, which cannot be thought of as assigning a particular property to each subsystem; e.g., the singlet state (34). Consequently, the tensor product space of histories also includes states which are, so-to-speak, entangled between two or more times. What is their physical significance? Could they serve a useful role in describing some sort of interesting time development? And how, assuming it to be possible, are probabilities to be assigned in the case of a closed quantum system? It is not obvious how consistency conditions as presently formulated, see (15), can be extended to this case, since the temporal ordering of events plays a crucial role. Thus this is thus an open question.

8.4.2   Sufficiency of the language

There is an obvious question about an approach to quantum interpretation which employs the histories strategy as embodied in the single framework rule, and thus declares various subjects out of bounds because they involve meaningless combinations of incompatible frameworks. Are there important topics, topics central to understanding the quantum mechanical world, which are thereby excluded? Or, to put the matter a different way: is there sufficient flexibility in the language of quantum mechanics, when subject to the single framework rule, to allow the sorts of descriptions and modes of reasoning which in practice quantum physicists need in order to pursue their discipline? At the present time the answer seems to be "yes." But this is necessarily tentative, for quantum mechanics is by now a vast discipline with an enormous range of applications, and no individual can be expected to be familiar with all of them. The fact that no failure has come to light in the decade following the publication of CQT lends support to the idea that the histories approach is sufficient. Its success in terms of treating numerous toy models and resolving numerous paradoxes suggests that exceptions to its rules, if they exist, may be hard to find.

Indeed, the plethora of examples studied thus far would suggest that the single framework rule "draws the line" rather much in accord with the quantum physicist's intuition. The mysterious nonlocal influences that cannot carry information, and are thus forever beyond the reach of experimental evidence, disappear when the single framework rule is enforced, Sec. 6. On the other hand, the black box of quantum orthodoxy has been pried open, and the belief of experimental physicists that the apparatus they build really measures something has been confirmed: they have been right all along to ignore the strictures found in the books from which they first learned quantum theory. But there are also experiments involving interference effects in which one must be more careful—and the histories approach not only indicates which these are, but suggests useful ways to think about them. Nevertheless, the question of adequacy remains open, and one can certainly imagine the possibility that further research will uncover defects in the histories approach which have hitherto escaped the attention of its advocates. A critical examination of histories ideas and conclusions by those who have taken the trouble to try and understand them would be most welcome; one aim of the present article is to encourage such.

8.4.3   Thermodynamic irreversibility

A problem belonging to a somewhat different category is that of understanding thermodynamic irreversibility. Frigg's discussion in [57] is limited to *classical* statistical mechanics, and this for a very good reason. Little progress can be expected in studying thermodynamic irreversibility from a fundamental quantum perspective as long as one has to depend upon the textbook formulation using measurements. Everyone agrees that measurements are thermodynamically irreversible processes. Thus a discussion that relies on them must introduce at the outset, in an arbitrary and totally uncontrolled manner, the very phenomenon one is trying to understand. That is not very appealing.

By contrast, the histories formulation has no measurement problem. But it does introduce probabilities at a fundamental level; is this equally bad? No, for even classical probabilistic descriptions *need not* result in assigning a direction to time; see [57,58]. That the histories probability assignment in fact does *not* introduce a time direction is evident in its fundamental formulas used to check for consistency and assign probabilities: (15) and (16) in Sec. 3.4. (Note that taking the adjoint of the chain operator $K(Y^\alpha)$ amounts to reversing the order of the times.) It is necessary at this point to clear up a matter that perennially causes confusion. Unitary time development means that $T(t',t) = T(t,t')^\dagger$ is the inverse quantum operator to $T(t,t')$, and this is what is behind the assertion that the histories approach does not single out a direction of time. However, that is *not* the same as saying that the (closed) quantum system under discussion is invariant under the symmetry operation of *time reversal*, which amounts to imposing an *additional* condition connecting $T(t',t)$ and $T(t,t')$. (That time reversal invariance in this sense is not relevant to understanding the second law of thermodynamics follows from the observation that systems placed in magnetic fields remain irreversible.)

With the measurement problem disposed of, the way is now open to develop an understanding of thermodynamic irreversibility in consistent *quantum* terms. This may or may not clear up any of the conceptual difficulties which arise in the classical case; that remains to be seen. But it has become a a reasonable and interesting topic for research.

8.4.4 Epistemology

A fourth problem concerns epistemology: how do human beings know things and what is the relationship of this (supposed) knowledge to the real world, especially the microscopic quantum world? The histories ontology does not automatically provide an answer to this question. It does provide a framework (now using that word in a nontechnical sense) for rational discussion and exploration, one that is different from classical mechanics, but not so different as to make useless the insights provided by classical physics. Sensible quantum descriptions can be constructed from the perspective of someone outside the system being considered. It makes quantum sense to speak of neurons in the brain in the manner of physiologists, and to imagine memories and consciousness itself as somehow represented or expressed in an appropriate quantum framework, whether or not things are being measured or observed in some way. Consciousness, brain function, etc. remain problems to be solved, but quantum mechanics need not be essential to their formulation or discussion, provided there is a suitable quasiclassical framework within which the quantum world can exhibit itself in the familiar classical terms of signals, perhaps with stochastic corrections, traveling along neurons, etc. If nothing else, clearing away the cobwebs of wave function collapse, many minds, nonlocal influences, and the like may free up for the serious study of important epistemological problems some of the intellectual power that might otherwise be dissipated in the quantum swamp.

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