



Grimau Roca, Berta (2018) From plurals to superplurals: in defence of higher-level plural logic. PhD thesis.

<https://theses.gla.ac.uk/30780/>

Copyright and moral rights for this work are retained by the author

A copy can be downloaded for personal non-commercial research or study, without prior permission or charge

This work cannot be reproduced or quoted extensively from without first obtaining permission in writing from the author

The content must not be changed in any way or sold commercially in any format or medium without the formal permission of the author

When referring to this work, full bibliographic details including the author, title, awarding institution and date of the thesis must be given

Enlighten: Theses

<https://theses.gla.ac.uk/>  
[research-enlighten@glasgow.ac.uk](mailto:research-enlighten@glasgow.ac.uk)

**From Plurals to Superplurals:  
In Defence of Higher-Level Plural Logic**

Berta Grimau Roca

Submitted in fulfilment of the requirements for the degree of  
Doctor of Philosophy (Arts)

School of Humanities  
College of Arts  
University of Glasgow

July 2018



# Abstract

Plural Logic is an extension of First-Order Logic with plural terms and quantifiers. When its plural terms are interpreted as denoting more than one object at once, Plural Logic is usually taken to be ontologically innocent: plural quantifiers do not require a domain of their own, but range *plurally* over the first-order domain of quantification. Given that Plural Logic is equi-interpretable with Monadic Second-Order Logic, it gives us its expressive power at the low ontological cost of a first-order language. This makes it a valuable tool in various areas of philosophy.

Some authors believe that Plural Logic can be extended into an even more expressive logic, Higher-Level Plural Logic, by adding higher-level plural terms and quantifiers to it. The basic idea is that second-level plurals stand to plurals like plurals stand to singulars (analogously for higher levels). Allegedly, Higher-Level Plural Logic enjoys the expressive power of type theory while, again, committing us only to the austere ontology of a first-order language. Were this really the case, Higher-Level Plural Logic would be a very useful tool, extending and strengthening some of the applications of Plural Logic. However, while the notions of plural reference and quantification enjoy widespread acceptance today, their higher-level counterparts have been received with scepticism. The main objection raised against them is that higher-level plural reference is unintelligible. This has been argued, among others, on the grounds that there are no higher-level plurals in natural language and that, if there were any, they could be eliminated.

In this thesis, after introducing the debate on plurals in Chapters 1 and 2, I turn to defending the legitimacy of the notion of higher-level plural reference. To this end, in Chapter 3, I present and elucidate the notion. Next, in Chapter 4, I show that some natural languages clearly contain these expressions and that they do so in an ineliminable manner. Finally, in Chapters 5 and 6, I develop a semantics for higher-level plurals that employs only devices previously well-understood by English speakers. To finish, in Chapter 7, I describe an application of Higher-level Plural Logic: a strengthening of the neo-Fregean programme. After describing my proposal, I turn to the issue of the logical status of this formalism and defend an optimistic take on the matter.



# Contents

<b>Introduction</b>	<b>15</b>
<b>Part I: Plurals</b>	<b>23</b>
<b>1 E Pluribus Unum</b>	<b>23</b>
1.1 Introduction . . . . .	23
1.2 $PL^-$ and $PL$ . . . . .	24
1.2.1 The language of $PL^-$ : $\mathcal{L}_{PL^-}$ . . . . .	24
1.2.2 The language of $PL$ : $\mathcal{L}_{PL}$ . . . . .	25
1.2.3 Proof theory . . . . .	26
1.3 Singularism: an overview . . . . .	28
1.3.1 Set-theoretic singularism . . . . .	29
1.3.2 Property singularism . . . . .	30
1.3.3 Mereological singularism . . . . .	33
1.4 Recap . . . . .	38
<b>2 E Pluribus Plures</b>	<b>39</b>
2.1 Introduction . . . . .	39
2.2 Pluralism: a defence . . . . .	39

2.2.1	Pluralist semantics for PL . . . . .	41
2.2.1.1	Metatheory . . . . .	43
2.2.2	The semantic value of plural terms . . . . .	44
2.2.2.1	Rationales for pluralism . . . . .	44
2.2.2.2	Objections and replies . . . . .	45
2.2.3	The semantic value of plural predicates . . . . .	56
2.2.3.1	Mixed pluralism . . . . .	57
2.2.3.2	Superpluralism . . . . .	57
2.2.4	Taking stock . . . . .	60
2.3	Some remarks on ordinary plurals . . . . .	61
2.3.1	Order and repetition . . . . .	61
2.3.2	Pseudo-singularity . . . . .	66
2.3.3	Empty and singular pluralities . . . . .	69
2.4	Conclusions . . . . .	72

## **Part II: Superplurals** **77**

### **3 Superplurals: An Elucidation** **77**

3.1	Introduction . . . . .	77
3.2	Higher-Level Plural Logic . . . . .	80
3.2.1	The language of $\text{HLPL}^-$ : $\mathcal{L}_{\text{HLPL}^-}$ . . . . .	83
3.2.2	The language of $\text{HLPL}$ : $\mathcal{L}_{\text{HLPL}}$ . . . . .	84
3.2.3	Proof theory . . . . .	84
3.2.4	Semantics for $\mathcal{L}_{\text{HLPL}^-}$ and $\mathcal{L}_{\text{HLPL}}$ . . . . .	86

---

3.2.5	The hierarchy of higher-level pluralities . . . . .	89
3.3	Against Higher-Level Plural Logic . . . . .	93
3.4	Elucidating higher-level plural reference . . . . .	97
3.4.1	The Iteration Principle . . . . .	97
3.4.2	Inherited features . . . . .	98
3.4.3	Elucidating the Iteration Principle . . . . .	99
3.5	Recap . . . . .	102
<b>4</b>	<b>Superplurals: a Defence</b>	<b>103</b>
4.1	Introduction . . . . .	103
4.2	Higher-level plurals in natural language . . . . .	104
4.2.1	In English . . . . .	104
4.2.2	In other languages . . . . .	106
4.3	Indispensability . . . . .	113
4.3.1	Against paraphrasing away . . . . .	114
4.3.1.1	Strategy I: Flattening . . . . .	116
4.3.1.2	Strategy II: Reification . . . . .	116
4.3.2	Against analyzing away . . . . .	119
4.3.2.1	Multigradedness . . . . .	121
4.3.2.2	Articulated reference . . . . .	122
4.3.2.3	Cover-based semantics . . . . .	124
4.3.2.4	Group-based semantics . . . . .	128
4.4	Conclusions . . . . .	133



<b>Part III: Aspects</b>	<b>137</b>
<b>5 Reference and Aspects</b>	<b>137</b>
5.1 Introduction . . . . .	137
5.2 Restricted plural reference: groups to the rescue? . . . . .	139
5.3 A Step Back: Restricted Singular Reference . . . . .	143
5.3.1 Revisiting the Logophor View . . . . .	147
5.3.2 My proposal: taking aspects seriously . . . . .	152
5.3.3 Extending the account to plurals . . . . .	156
5.3.3.1 Pseudo-singularity and restricted reference . . . . .	158
5.4 HLP reference as a species of restricted reference . . . . .	159
5.4.1 Rationale . . . . .	160
5.4.2 Pragmatics? . . . . .	163
5.5 Recap . . . . .	166
<b>6 Cluster Semantics for Higher-Level Plurals</b>	<b>167</b>
6.1 Introduction . . . . .	167
6.2 Higher-level plural terms and predicates . . . . .	167
6.2.1 Cluster types and their aspects . . . . .	167
6.2.2 The hierarchy of cluster-restricted terms . . . . .	169
6.2.3 Order . . . . .	171
6.2.4 Repetition . . . . .	173
6.2.5 Restricted components . . . . .	175
6.2.6 A background theory of clusters . . . . .	176

---

6.2.7	Distributivity and collectivity as two sides of the same coin . . . . .	177
6.3	Higher-level plural sentences . . . . .	178
6.3.1	The emergence of higher-level plural interpretations . . . . .	178
6.3.2	Selection restrictions . . . . .	183
6.4	Overcoming the problems of other accounts . . . . .	185
6.5	Conclusions . . . . .	188
 <b>Part IV: Numbers</b>		<b>193</b>
 <b>7 Superplural Logicism</b>		<b>193</b>
7.1	Introduction . . . . .	193
7.2	Plural Logicism via Pair Abstraction . . . . .	195
7.3	Superplural Logicism . . . . .	201
7.3.1	Superplural Frege's Theorem . . . . .	203
7.3.2	Evaluation . . . . .	206
7.4	The Prospects of Logicality . . . . .	207
7.4.1	Universal Applicability . . . . .	208
7.4.2	Ontological Innocence . . . . .	218
7.4.3	Cognitive Primacy . . . . .	223
7.5	Conclusions . . . . .	225
 <b>Conclusions</b>		<b>227</b>
 <b>References</b>		<b>233</b>



# Acknowledgments

I owe the greatest acknowledgment to my supervisors: Fraser MacBride, Adam Rieger, Bruno Whittle and Stephan Krämer. Without their guidance, knowledge and encouragement this thesis would not have been possible. I am also very grateful to Stewart Shapiro for his generous supervision during my visit to Ohio State University. For his support and especially for his help with the last chapter of this thesis, I wish to thank Alan Weir.

For providing me with useful comments and discussion, I am grateful to Simon Hewitt, Eric Snyder, Kevin Scharp, Michele Palmira and Øystein Linnebo.

I presented parts of this thesis in Hejnice, Łódź, Barcelona, Glasgow, Munich, Leeds and Melbourne. I am grateful to the members of those audiences for their valuable feedback.

For their help with various linguistic queries, I would like to thank Mirka Kilpijärvi, Riikka Mäkelä and Rabia Ergin.

For accompanying me along the path that led to this thesis, I am grateful to Carlota Serahima.

I am also very thankful to all my colleagues in Glasgow for their valuable feedback and encouragement as well as for making me feel at home in Glasgow.

For her unwavering support in this and all my endeavours, I owe a huge debt of gratitude to Maite Roca. For his generosity and enthusiasm, for quite a lot of proofreading and for being the LaTeX savvy that he is, I am profoundly indebted to Armand Adroher.

Finally, I would like to thank Francesc Vélez, Ignasi Jané, Calixto Badesa and Hannes Leitgeb for their dedicated teaching and their encouragement. They all share a certain degree of responsibility for the existence of this thesis.

\*\*\*

I am grateful for the financial support provided by the University of Glasgow's College of Arts and by the AHRC (grant number AH/L503915/1).



I declare that this work is the result of my own investigations,  
except where identified by references.



# Introduction

After being a subject of debate for more than three decades, Plural Logic has now been widely adopted as part of the philosophical toolbox. Plural Logic is an extension of First-Order Logic which, as well as the usual singular terms, predicates and quantifiers, has their plural analogues. Plural terms are terms which refer plurally to the objects which first-order singular terms refer to singularly. In other words, they refer to more than one object at once. Plural quantifiers bind plural variables. And plural predicates take plural terms as arguments. In particular, Plural Logic has a logical relation of plural membership which holds between an object and some objects whenever the former is among the latter.<sup>1</sup>

Plural Logic is a first-order logic in the sense that it only has quantification into term position.<sup>2</sup> However, it goes well beyond First-Order Logic in the sense that plural first-order quantification gives it the expressive power of Monadic Second-Order Logic – with Plural Logic we can express the idea that something holds of a certain property by saying that something holds of some objects (intuitively, the objects which fall within the extension of said property).<sup>3</sup> This was famously observed by Boolos (1984, 1985) and it is indeed one of the key reasons why Plural Logic has been seen as a promising device: it gives us the expressive power of Monadic Second-Order Logic while, arguably, avoiding some of its most objectionable putative features, such as its high ontological costs (i.e. to properties or classes).

In fact, many of the applications of Plural Logic turn on its alleged ontological innocence. For example, some philosophers have employed Plural Logic to support their nominalist views. Thus, Lewis (1991) and Black (1971) use it to support their eliminativist views about

---

<sup>1</sup>I refer to this relation as ‘plural membership’ to make explicit the fact that it is a relation between an object and some objects. It should not be confused with a membership relation holding between two objects, such as set membership.

<sup>2</sup>This is why I will often refer to First-Order Logic as ‘singular First-Order Logic’.

<sup>3</sup>We can replace talk of properties by plural talk of the objects in their extensions, but how are we to replace talk of relations with plurals? As we will see, plurals do not seem suitable to capture polyadic predication.



sets, Boolos (1984, 1985) and Uzquiano (2003) use it in the framework of an eliminativist view of classes, Hossack (2000) uses it in an eliminative project about complex objects and Rosen and Dorr (2002), to defend eliminativism about mereological sums. Moreover, because of its ontological innocence, Plural Logic would have a better claim than Monadic Second-Order Logic to count as genuinely logical, thus having applications in the logicist field.<sup>4</sup> However, Plural Logic only has these advantages if it is taken at face value, that is, if it is given an interpretation according to which plural terms denote more than one object. The view that this is the correct (or, at least, a correct) interpretation of Plural Logic is what I call ‘pluralism’.

Some logicians believe that Plural Logic can be extended by adding higher-level plural terms and quantifiers to it.<sup>5</sup> Crucially, second-level plurals do not stand in predicate position with respect to plurals, but rather stand to them as the latter stand to singulars. The same holds of third-level plurals. And so on. Higher-Level Plural Logic also has a relation of membership: an extension of our old relation of plural membership, which holds not only between a plurality<sup>6</sup> and its members, but also between a higher-level plurality and its members – members which are possibly pluralities themselves.

Importantly, Higher-Level Plural Logic is supposed to be more expressive than Plural Logic and to inherit some of its key characteristics – most crucially, its ontological innocence. It is because of this alleged gain in expressive power free of additional ontological costs that higher-level plural reference has attracted some attention in the recent literature on plurals. The basic idea is that just as plural language allows us to go from ‘cat’ to ‘cats’, a higher-level plural language takes us even further allowing us to talk of what in an extension of ordinary English would be captured by terms like ‘catses’, ‘catseses’, etc. Informally speaking, just as ‘cats’ denotes a plurality of cats, ‘catses’ would denote a plurality of pluralities of cats. I will come back shortly to this idea and the criticisms it has received, but let me stop to present some reasons why one may be interested in a higher-level extension of Plural Logic in the first place.

There are at least three areas where higher-level plurals have promising applications. To begin with, Higher-Level Plural Logic could be of use in nominalist endeavours in which the use of Plural Logic were insufficient. I can foresee two main applications in this area. On the

---

<sup>4</sup>I will look at some of the proposals to employ plurals for neo-logicist purposes in the last chapter.

<sup>5</sup>In this thesis, I will sometimes use the term ‘superplural’ interchangeably with ‘higher-level plural’.

<sup>6</sup>Hereafter, I shall use the term ‘plurality’ as convenient shorthand for ‘objects’. Accordingly, ‘the plurality’ means ‘the objects’ and ‘a plurality’ means ‘some objects’. This term is a pseudo-singular term – a syntactically singular, but semantically plural term. In Chapter 2, Section 2.3.2, I will argue for the legitimacy of this notion.

one hand, a logic containing all finite levels of plural variables and quantifiers would be equi-interpretable with a simple type theory, thereby showing that we can enjoy the expressive power of the latter while being committed solely to the values of its singular first-order variables. Linnebo and Rayo (2012) consider this application in their comparison of type theory and ZF set theory.<sup>7</sup> On the other hand, one could employ higher-level plural discourse to replace talk of sets. The idea would simply be to replace all singular reference to sets of individuals with plural reference, singular reference to sets of sets with second-level plural reference, and so on. Oliver and Smiley (2016, ch. 15) argue for the legitimacy of Higher-Level Plural Logic and use it precisely to this end. In particular, they suggest that it be used to interpret what they call ‘Cantorian set theory’, a set theory without singletons or the empty set.

Another area where Higher-Level Plural Logic promises to deliver results is in the interpretation of polyadicity. As I mentioned above, Plural Logic can be used to interpret only the monadic fragment of Second-Order Logic. For some applications this is enough, but not for others. For example, an area where we could benefit from a plural treatment of the polyadic fragment of Second-Order Logic is neo-logicism. Plural Logic can be used to develop a strengthened version of neo-Fregeanism by replacing second-order expressions with plural ones. However, if one is to proceed via Hume’s Principle, interpreting the monadic fragment of Second-Order Logic will not be enough. One will need to replace its polyadic fragment as well. There are various ways to go about doing that, but most of them have limitations which make them unsuitable for neo-Fregean purposes. Fortunately, higher-level plural reference lends itself to a simple strategy to tackle polyadicity. Roughly speaking, with the aid of higher-level plural reference, one can encode pairs by employing one of the usual set-theoretic definitions of ordered pair in its higher-level plural version.<sup>8</sup>

Finally, one may want to develop Higher-Level Plural Logic as a tool for natural language analysis. For example, Cotnoir (2013) proposes a semantics which makes use of higher-level plurals for generalized identity claims – including one-many, one-one and many-many identity statements – and Nicolas (2008) argues that higher-level plurals should be employed in the semantic treatment of mass nouns.

However, as one would expect, unlike its plural counterpart, Higher-Level Plural Logic has been received with scepticism by the philosophical community. The very intelligibility of the notion of higher-level plural reference has been questioned, partly on the basis that

---

<sup>7</sup>Even though they ultimately argue against the claim that the higher-level plural reading carries any advantage.

<sup>8</sup>In Chapter 7, I develop this application.

natural language does not contain any such device. Moreover, it has been argued, even if there were ordinary higher-level plural expressions, they would be eliminable in favour of singular or plural ones.

My main aim in this thesis is to make a case for the legitimacy and usefulness of higher-level plural devices. More precisely, I intend to show that

- (i) Higher-level plurals are clear enough to be incorporated into a formal language which is to be taken at face value. This is the view I call ‘higher-levelism’. It is analogous to pluralism, the view that Plural Logic is legitimately interpreted by means of metalinguistic plural expressions.
- (ii) Higher-Level Plural Logic has, at least, one promising application in the foundations of mathematics: a form of logicism.

Most of the thesis (Chapters 1-6) will be devoted to meeting the former aim. I will turn to the latter in the last chapter.

Many of the arguments I provide for (i) and (ii) are based on the assumptions behind the arguments which have been given in favour of plural reference and Plural Logic. Thus my own arguments should be especially appealing to those who favour Plural Logic. In fact, the main conclusion of this thesis (as its title intends to capture) will be that the advocate of Plural Logic who does not embrace Higher-Level Plural Logic finds herself in an awkward position, since the view according to which the former is legitimate and indeed successful in its applications, but the latter is not, is likely to be based on a collection of assumptions which are in tension with one another. In other words, my thesis should convince most friends of plurals to go higher-level and most foes of higher-level plurals to withdraw their allegiance to plurals. For better or for worse, it is an all or nothing matter.

Finally, an underlying objective of my thesis is to contribute to bringing together the linguistic and the philosophical debates around plurals. Despite their common object of study, there has not been much dialectical exchange between the two traditions.<sup>9</sup> Thus one of the aims of this dissertation is to further promote this exchange by finding common themes with respect to the topic of higher-level plural reference.

Although this thesis is, of course, written from an Anglophone perspective, it will be important, given the nature of some points of the debate, that we occasionally go beyond

---

<sup>9</sup>We find recent notable exceptions in the collection Arapinis, Carrara, and Moltmann (2016) and in Oliver and Smiley (2016).

it. To this end, I shall not confine my examination of the linguistics literature to the field of formal semantics, but I shall also help myself to some results of empirical linguistic studies about grammatical number.<sup>10</sup>

\*\*\*

In order to achieve the aims just described I proceed as follows. In Part I, I focus on plural reference. My objective there is two-fold. On the one hand, to contextualize the higher-level plural debate within the wider debate around plurals. And, on the other, to identify the main assumptions of the arguments behind pluralism. These assumptions are the ones I shall take for granted in the remainder of the thesis. More precisely, in Chapter 1 I shall focus on singularist semantics (semantics which assign each plural term a single collectivizing object – a set, a property or a sum – as semantic value) and describe the sort of complaints which have been raised by the pluralist against them. In Chapter 2, I shall turn to the other side of the debate: I shall survey the complaints raised against pluralism and offer some rebuttals thereof. The general principles guiding the pluralist should become clear once we finish this survey.

In Part II, I turn my attention to higher-level plurals. My objective here is to reply to the main objections raised against them. To this end, in Chapter 3, after describing the formal language, I start by offering an informal elucidation of the notion of higher-level plural reference. The aim is to clarify the notion to some extent and show that, at least *prima facie*, there is nothing problematic about it. Next, in Chapter 4, I argue that higher-level plurals are present in ordinary language in their paradigmatic morphology and that English contains less paradigmatic forms as well. Moreover, I argue that these idioms can neither be paraphrased away (i.e. syntactically reduced to plurals or singulars) nor analyzed away (i.e. semantically reduced to plurals or singulars). The way in which I carry out these tasks is by considering different reductionist syntactic and semantic analyses of higher-level plurals and arguing that they are not satisfactory (especially so by the lights of the pluralist).

In Part III, I follow one last strategy to make a case for higher-levelism. This one is aimed at the sceptic for whom the evidence coming from the observation of natural language and the arguments against dispensability are not enough – the sceptic is someone who thinks that despite all that has been argued, apparent higher-level plurals must be understood as being

---

<sup>10</sup>Moreover, I will occasionally help myself to informal reports by native speakers of other languages (including my own with respect to Catalan and Spanish) – although I have not carried out anything even remotely close to a proper empirical study.

really plurals. The strategy I follow here is that of offering a semantics for ordinary higher-level plurals which makes use only of previously well-understood (by the sceptic) idioms – namely, singular and plural reference. My claim is that, despite being just a model of the direct understanding of higher-level plurals and as such not fully capturing their meaning, it overcomes the problems of the alternative analyses presented in Part II. My aim is to engage with the sceptic by providing a semantics she can fully grasp and which approximates the face value interpretation of higher-level plurals as much as possible. However, ultimately, this semantic theory should be seen as a mere bridge and should eventually be left behind.

Finally, in Part IV, I present an application of the formalism described and endorsed so far. This application is a form of neo-Fregeanism in its Scottish variant. More precisely, in Chapter 7 (the only chapter in this part), I use Third-Level Plural Logic to interpret dyadic Second-Order Logic. This suffices to express a higher-level plural version of Hume's Principle and from it derive a result analogous to Frege's Theorem. I close the chapter by getting into deep waters and looking into the logical status of Higher-Level Plural Logic. I finish on an optimistic note by concluding that it has a good claim to be considered pure logic.

# **Part I: Plurals**



# Chapter 1

## E Pluribus Unum<sup>11</sup>

### 1.1 Introduction

Plural terms are referring expressions which, intuitively, denote<sup>12</sup> various objects at once. For example, ‘the member states of the European Union’, ‘Serena and Venus’, ‘Hogwarts’ graduates’ and ‘the Outer Hebrides’ are all ordinary plural terms. As one can see, plural terms<sup>13</sup> can take various forms: plural definite descriptions, lists of terms, functional terms and plural proper names, among others.

Semantic theories for plural language can be classified according as to whether they adhere to this intuition and assign plural terms more than one referent or whether they depart from it and take plural terms to denote a single object. I shall call the semantics falling within the former category ‘pluralist semantics’ and those falling within the latter ‘singularist semantics’ (both for ordinary and formal plural languages). Analogously, as I said in the Introduction I shall call someone who endorses the use of the former, a ‘pluralist’. Accordingly, I shall call someone who endorses the use of the latter, a ‘singularist’. The pluralist believes that plural expressions can figure in formalisms and be interpreted at face value. By contrast, the singularist accepts the use of plurals in formalisms only insofar as they are analysed away, as denoting a single object. Whether one goes one way or the other turns on whether one accepts or rejects ordinary plurals themselves as legitimate devices to carry out semantic theorizing.

---

<sup>11</sup>This chapter is named after Burgess (2004).

<sup>12</sup>Hereafter, I use ‘denote’, ‘refer’ and other more informal expressions (such as ‘pick out’), indistinctly.

<sup>13</sup>I shall often call plural terms simply ‘plurals’. Analogously for other kinds of terms.



My objective in this and the next chapter is to present pluralism, the standpoint from which the remainder of this thesis departs. My aim is not to convince the reader to align with the pluralist. Rather I aim to identify what reasons those who adopt it typically have to do so. In order to contextualise the pluralist position, I need to give a brief overview of its main rival, singularism, in its various forms. Again, although I will raise some objections to the singularist proposals, my aim in doing so is to show what sort of concerns have driven pluralism, rather than showing that singularism is to be rejected.

In this chapter, after presenting the language of Plural Logic and its proof-theoretic profile in Section 1.2, I turn to the task of reviewing the singularist side of the debate in Section 1.3.

## 1.2 $\text{PL}^-$ and $\text{PL}$

In this section, I describe the language of a Plural Logic without non-logical plural predicates (which I call  $\text{PL}^-$ ) and an extension thereof including non-logical plural predicates (which I call  $\text{PL}$ ). I also describe deductive calculi for each of them.

### 1.2.1 The language of $\text{PL}^-$ : $\mathcal{L}_{\text{PL}^-}$

$\mathcal{L}_{\text{PL}^-}$  is an extension of the language of First-Order Logic with identity; it has plural terms and quantifiers and a logical predicate of plural membership to be read ‘is/are among’.<sup>14</sup>

#### Logical vocabulary

Singular variables	$x, y, \dots$
Plural variables	$xx, yy, \dots$
Connectives	$\neg, \rightarrow$
Universal quantifier	$\forall$
A two-place predicate to be read ‘is/are among’	$<$
A singular identity predicate	$=$

#### Non-logical vocabulary

---

<sup>14</sup>For the sake of simplicity, I omit functional terms from the description of  $\mathcal{L}_{\text{PL}^-}$  and  $\mathcal{L}_{\text{PL}}$  (as well as from the higher-level plural languages presented in Chapter 3).

Singular constants	$c, d, \dots$
Plural constants	$cc, dd, \dots$
Singular $n$ -adic predicates	$P_n, Q_n, \dots$

### Formation rules

I use  $t, u\dots$  and  $tt, uu\dots$  as meta-variables for singular and plural terms, respectively.

Singular variables and constants are singular terms.

Plural variables and constants are plural terms.

For  $t$  and  $u$ ,  $t = u$  is a formula.

For  $t, u, tt$  and  $uu$ ,  $t < u$ ,  $t < tt$ ,  $tt < t$  and  $tt < uu$  are formulae.

For  $t_1, \dots, t_n$  and  $P_n$ ,  $P_n(t_1, \dots, t_n)$  is a formula.

If  $\phi$  and  $\psi$  are formulas, so are  $\neg\phi$  and  $(\phi \rightarrow \psi)$ .

If  $\phi$  is a formula, so are  $\forall x\phi$  and  $\forall xx\phi$ .

Nothing else is a term or a formula of  $\mathcal{L}_{\text{PL}^-}$ .

### Defined expressions

Plural Identity:  $tt = uu := \forall x(x < tt \leftrightarrow x < uu)$

Mixed Identity:  $t = tt := \forall x(x < tt \rightarrow t = x)$

Existential quantification:  $\exists x\phi := \neg\forall x\neg\phi$  and  $\exists xx\phi := \neg\forall xx\neg\phi$

## 1.2.2 The language of PL: $\mathcal{L}_{\text{PL}}$

$\mathcal{L}_{\text{PL}}$  is  $\mathcal{L}_{\text{PL}^-}$  plus non-logical plural predicates, that is, predicates that take plurals as arguments. All predicates have a fixed number of positions, each of which can take both singular and plural terms as arguments.

We simply substitute our old rule of formation of singular predications with the following (where  $t_1, \dots, t_n$  are meta-variables standing for singular and plural terms):

For  $t_1, \dots, t_n$  and  $P_n$ ,  $P_n(t_1, \dots, t_n)$  is a formula

We can draw a semantic distinction between distributive and collective predicates. A distributive predicate is one for which the following is analytic: it holds of some objects if, and only if, it holds of each of them. For example, ‘are calico cats’, ‘are 15 years old’ and ‘run a marathon’ are of this sort. By contrast, a collective predicate is not distributive, that is, it is not analytically true that it holds of some things if, and only if, it holds of each of them. For example, ‘surround a castle’, ‘are siblings’ and ‘are 12 in number’ are all collective. We take the analogous formal notions to be defined in terms of models: a predicate is distributive

if, and only if, in any given model it holds of some objects if, and only if, it holds of each of them. Moreover, we take a collective predicate to be such that in any given model, it may or may not be the case that it holds of some objects if, and only if, it holds of each of them (thus, a predicate is collective when it is not specified that it is distributive).

Even though this is an important distinction for many applications, I will refrain from marking it in the object language and simply take all predicates to be collective by default (note that this does not forbid them from being true of some things and also of each of them in a given model). Whenever the distinction is required, we can simply make the necessary stipulation in the semantics. Moreover, the analogous axiom scheme would be as follows (supposing that the position occupied by  $uu$  is the only distributive one of the predicate  $\phi$  and representing a plurality of arguments with  $\vec{x}$ ):

$$(\text{Dis}) \quad \forall xx(\phi(\vec{x}, xx) \leftrightarrow \forall x(x < xx \rightarrow \phi(\vec{x}, x)))$$

Let me make a parenthetical remark before moving on to the proof theory. Admitting plural predicates allows us to cover multigradedness as a special case of plural predication. In this framework, a multigrade predicate is one which can hold of plural terms denoting different numbers of objects (even a single object). For instance, ‘wrote a book’ is one such predicate: ‘Frege wrote a book’, ‘Oliver and Smiley wrote a book’, ‘Burgess, Hazen and Lewis wrote a book’ are all true.<sup>15</sup>

### 1.2.3 Proof theory

In what follows I present a proof theory for  $\mathcal{L}_{\text{PL}}$  in axiomatic form. We start with the standard deductive system for First-Order Logic with identity:

---

<sup>15</sup>The fact that our predicates have a fixed number of positions seems to preclude the possibility of varigradedness. MacBride (2005) has described varigrade predicates as predicates which have a variable adicity. He gives ‘believe’ as an example of a predicate which needs to be analysed as being varigrade. Consider these occurrences of the relation: ‘a believes P’, ‘a believes that b Rs’ and ‘a believes that b Ls c and not-Ls d’. The claim is that the first occurrence is dyadic, the second one is triadic and the third one is pentadic. However, I am not convinced that ‘believe’ is varigrade. The relation denoted by ‘believe’ can be easily analysed in a way according to which it does not come out as varigrade. In particular, it can be analysed as denoting a relation holding between agents and propositions. What appears to occur in MacBride’s examples is that different levels of propositional analysis are involved in each case. For instance, the first sentence is one where the content of the belief is represented in a merely propositional language, whereas the other two are represented in a first-order language. Moreover, the difference between the last two is one that concerns the form of the belief involved. The fact that this alternative explanation is available casts doubt on MacBride’s suggestion. I will proceed on the assumption that there are no such things as varigrade predicates.

$$\phi \rightarrow (\psi \rightarrow \phi)$$

$$(\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi))$$

$$(\neg\phi \rightarrow \neg\psi) \rightarrow (\psi \rightarrow \phi)$$

(SUI) *Singular Universal Instantiation*

$$\forall x\phi(x) \rightarrow \phi(t), \text{ where } t \text{ is free for } x \text{ in } \phi.^{16}$$

(SUG) *Singular Universal Generalisation*

From  $\phi \rightarrow \psi(x)$  infer  $\phi \rightarrow \forall x\psi(x)$ , provided  $x$  does not occur free in  $\phi$  or in any premise of the deduction.

(SRI) *Singular Reflexivity of Identity*

$$\forall x(x = x)$$

(Ind) *Indiscernibility of Identicals*

$$\forall x\forall y(x = y \rightarrow (\phi(x) \rightarrow \phi(y))), \text{ where } y \text{ is free for } x \text{ in } \phi(x).$$

(MP) *Modus Ponens*

From  $\phi$  and  $\phi \rightarrow \psi$ , infer  $\psi$

To this, we add an axiom of reflexivity of membership for singulars:

(SRM) *Singular Reflexivity of Membership*<sup>17</sup>

$$\forall x(x < x)$$

And axioms and rules for the plural fragment:

(PUI) *Plural Universal Instantiation*

$$\forall xx\phi(xx) \rightarrow \phi(tt), \text{ where } tt \text{ is free for } xx \text{ in } \phi.$$

(PUG) *Plural Universal Generalisation*

From  $\phi \rightarrow \psi(xx)$  infer  $\phi \rightarrow \forall xx\psi(xx)$ , provided  $xx$  does not occur free in  $\phi$  or in any premise of the deduction.

---

<sup>16</sup>We say that a term  $t$  is free for  $x$  in  $\phi$  if  $x$  does not occur free in  $\phi$  within the scope of  $\forall y$ , where  $y$  is any variable occurring in  $t$ .

<sup>17</sup>The reason why the relation of plural membership is reflexive is that there are no such things as singletons, therefore it is natural to say that everything is among itself in the sense that it is itself. For instance, I am one of my mother and I and I am also one of myself (or the plurality which consists of only me). Another reason for this will become clear when we turn to higher-level plurals, in Chapter 3.

(PC) *Plural Comprehension*

$\exists x\phi(x) \rightarrow \exists xx\forall x(x < xx \leftrightarrow \phi(x))$ , where  $\phi$  is a formula of  $\mathcal{L}_{\text{PL}}$  that contains  $x$  and possibly other variables free but contains no occurrence of  $xx$ .

This accords with the usual view according to which pluralities can consist of a single individual.<sup>18</sup>

(NE) *Non-Emptiness*

$\forall xx\exists x(x < xx)$

This axiom ensures that every plurality consists of at least one individual.

(PRM) *Plural Reflexivity of Membership*

$\forall xx(xx < xx)$

In order to obtain a calculus for PL, we need only add an axiom-schema of plural extensionality:

(P-Ext) *Plural Extensionality*

$\forall xx\forall yy[\forall x(x < xx \leftrightarrow x < yy) \rightarrow (\phi(xx) \leftrightarrow \phi(yy))]$ , where  $yy$  is free for  $xx$  in  $\phi(xx)$ .

Given our definition of plural identity, this axiom can also be seen as a plural version of the Indiscernibility of Identicals.

### 1.3 Singularism: an overview

In this section, I describe three different singularist semantics for Plural Logic. Given that my aim is not to show that pluralism is to be preferred over singularism, I shall not stop to consider objections to singularism in much detail. I shall only point to some of the alleged weaknesses of these theories for the sake of delineating what motivations the pluralist has to reject them.

Singularist theories need to appeal to collectivizing objects – that is, objects that have members, constituents or parts – as the semantic values of plural terms in order to deliver correct truth conditions for plural expressions. The singularist views I consider in this chapter differ on the kind of collectivizing object they make use of: set-theoretic singularism

<sup>18</sup>I will discuss this in section 2.3.3.

assigns plural terms sets of objects from the first-order domain, property singularism assigns them properties of the objects from the first-order domain and mereological singularism, mereological sums thereof.<sup>19</sup>

### 1.3.1 Set-theoretic singularism

Set-theoretic singularism is the view that plural terms denote sets of objects from the first-order domain and interprets plural quantification as singular quantification over sets. In other words, it is like standard model theory for plurals.<sup>20</sup> For example, according to this view, ‘the member states of the European Union’ denotes a set that has as members each of the states of the European Union.

The main problem with this form of singularism, as Florio (2014a) notes, is its inability to convey absolutely general discourse. As originally pointed out by Boolos (1984), when our discourse is intended to be about domains such as that of absolutely everything (and, a fortiori, that of all sets), set-theoretic singularism does not deliver the right interpretations – that is, when built on a standard background set theory.

Note that set-theoretic singularism imposes two different (although related) restrictions onto our models. On the one hand, it forbids the assignment of non-set-sized collections as semantic values of the object language expressions. On the other, it forbids the existence of non-set-sized domains. In the terminology of Cartwright (1994), it adheres to the All-in-one Principle, the view that the objects in a domain of discourse always make up a set or some set-like object.

Each of these limitations has a different unwanted consequence for the purpose of giving semantics for Plural Logic. Firstly, excluding the possibility of non-set-sized denotations is a problem if one is concerned with singling out the intended interpretations of plural expressions. For example, consider the sentence

**(1.1)** *Some things are the sets which are not members of themselves.*

For the analysis of this sentence to capture its intuitive meaning, its quantified term would need to be assigned the set of all sets as semantic value, but this is not available here, on pain

---

<sup>19</sup>For the sake of brevity, in my presentation of the singularist proposals I ignore the question of the semantic value of predicates (both of singular and plural predicates). This is sufficient for our purposes since the main objections pluralists have raised against singularism concern the analysis of plural terms. I will tackle the analysis of predicates in Chapter 2.

<sup>20</sup>It has been advocated in Landman (1989a, 1989b) and Schwarzschild (1996).

of contradiction.

Secondly, the All-in-one Principle presents a problem for the characterization of plural logical truth and consequence. Intuitively, in order to define these notions, one needs to consider all possible models, including the non-set-sized ones, but in this framework those are not available.<sup>21</sup> Hence set-theoretic singularism does not capture an adequate notion of plural logical consequence.

To recap, one should be discontent with this option if one takes semantics as a tool that, among other jobs, is supposed to capture the intuitive meanings of the object language expressions. Moreover, even if one does not share this view of semantics, they may want to rule out set-theoretic singularism on the grounds that the limitations it imposes onto the size of domains are problematic for the very end of describing an adequate relation of logical consequence.

### 1.3.2 Property singularism

Property singularism is the view that plural terms denote properties.<sup>22</sup> For example, according to property singularism, the term ‘Venus and Serena’ picks out a property instantiated only by Venus and Serena.

Prima facie, property singularism appears to overcome the problem set-theoretic singularism suffers from regarding absolutely general discourse, since a sentence such as (1.1) is true in a model according to this semantics if, and only if, there is a property which is instantiated by all the non-self-membered sets. This is available as long as there is at least

---

<sup>21</sup>Note that Kreisel’s squeezing argument (Kreisel (1967)) is not available for Plural Logic. Kreisel’s result shows that the first-order notion of logical consequence is extensionally correct even if non-set-sized models are ignored. Unfortunately, this move is not available in the case of Plural Logic, since Kreisel’s proof relies crucially on a completeness result and Plural Logic under a suitable semantics is not complete (we will see this in more detail in the next chapter). Roughly speaking, Kreisel’s proof is as follows. Take an intuitive notion of validity, call it ‘I-validity’. First, it seems that if a sentence  $q$  is proof-theoretically derivable from a set of sentences  $P$ , then the argument from  $P$  to  $q$  is I-valid. In other words, whatever is derivable according to our calculus should be intuitively valid. Next, we also have that if an argument from  $P$  to  $q$  is I-valid, then the argument is model-theoretically valid. Now, if the language in question is that of First-Order Logic under the usual standard model-theory, then given its completeness result, we know that if the argument from  $P$  to  $q$  is model-theoretically valid, then it is proof-theoretically derivable. This closes the circle, and we can conclude that what is I-valid in First-Order Logic is model-theoretically valid and vice-versa. Thus we can safely omit non-set-sized models in our model-theoretic approach.

<sup>22</sup>This view has been defended in Florio (2010) and Florio (2014a).

a property which holds of all non-self-membered sets – which even under a sparse view of properties is plausible – and that domains can be non-set-sized (possibly construed as properties as well).

Unfortunately, there is a paradox analogous to Russell’s lurking in the vicinity; it arises when the plural discourse one intends to provide semantics for is itself about properties.

Consider the sentence

**(1.2)** *Some things are the properties which do not instantiate themselves.*

Not only does this seem to be an ordinary sentence about properties, but it seems true. In order to capture its intended meaning, property singularism would have to assign the quantified plural term the property of being a property which does not instantiate itself. But is there such a property? Suppose there is one. Then, as one would expect, the question is whether this property instantiates itself. If it does, then it must be a property which does not instantiate itself. But if it does not, then it should instantiate itself. A natural way to block this paradox is to accept a picture of properties as typed. On this picture, there is a first type of properties which can only be instantiated by objects. Properties that can be instantiated by type 1 properties are then said to be of type 2. And so on. It is then natural to maintain that there is no property of being a property which does not instantiate itself, but for each type  $k$  there is a property of type  $k$  of being a property of type  $k - 1$  that does not instantiate itself.

Unfortunately, the price to pay for this solution is that we no longer seem to be able to capture the intended meaning of statements concerned with absolutely everything, since now all quantifiers are restricted to one particular type of entities.

Nevertheless, there is a certain view of properties which suggests that this sort of criticism is not pertinent. There is an intuitive sense in which properties are not like the basic objects which instantiate them. In fact, there is an intuitive sense in which properties are not objects at all: they are just the ways objects are (where ways are not objects themselves). The view that higher-order expressions do not denote objects in the same way that names do, often dubbed ‘higher-orderism’, is quite widely held in the literature on higher-order logic. Accordingly, the semantic values of plural terms would not be counted among the things one is supposed to talk about when one talks about everything.<sup>23</sup>

Be that as it may, object-language talk about properties seems to bring about another difficulty for property singularism. Consider the following sentences:

---

<sup>23</sup>See Williamson (2003), Rayo and Yablo (2001), Krämer (2014) for some developments of this view.



(1.3) *My neighbour's dogs chase each other.*

(1.4) *The property of being one of my neighbour's dogs chases each other.*

Unless we bring properties into the first-order domain of quantification, property singularism makes the former true if, and only if, it verifies the latter too. If we had the property of being one of my neighbour's dogs among our first-order objects, then we could assign the subject of (1.3) a property from the second-order domain of objects and that of (1.4) its first-order counterpart; thus distinguishing their truth-conditions. But given the view that properties are not first-order objects, this move does not appear to be available. Another way out of this entanglement would be to argue that object-language talk of properties is not legitimate. However, this move would seem to disallow the analysis of higher-order property talk (e.g. 'The property of being a mammal is instantiated', 'The property of being a mammal is a biological kind', etc). This move is thus not desirable. I conclude that, under a property singularist reading, plural language is unjustifiably limited in the sorts of discourse it can regiment.

Finally, another difficulty faced by property singularism is posited by distributive predicates. These are predicates which admit both singular and plural terms as arguments. For instance, 'passed the exam':

(1.5) *Harry passed the exam.*

(1.6) *Harry, Ron and Hermione passed the exam.*

According to property singularism, (1.5) is true if, and only if, a first-order property holds of an individual, and (1.6) is true if, and only if, a second-order property holds of a first-order property.

This is problematic because it disallows the valid inference from (1.6) to (1.5).

Florio (2014a) argues that this difficulty can be overcome by allowing properties to be flexible in the sense that they can apply to entities of any lower type.<sup>24</sup> Although this liberalization is clearly technically available, it is less clear that it is justified. It appeals to an unorthodox conception of properties – the intuitive distinction between properties of different and strict types is lost under this liberalization of the notion. Besides this, we need properties to be extensional to function as the values of plural terms (in the sense that co-extensional

---

<sup>24</sup>Note that he discusses this issue in a slightly different context – that of the so-called 'equivocity objection' as presented in Oliver and Smiley (2016, Section 4.3 and p. 250). Incidentally, I believe the reasoning behind this objection is not compelling, as I will argue in n148.

properties are identical). This is yet another restriction on an otherwise potentially fruitful notion of property. And there are other *prima facie* differences between properties and plurals which add to the implausibility of property singularism. For instance, whereas properties may be uninstantiated, pluralities may not be empty. As argued in Rayo and Yablo (2001, pp. 86-7), all of these divergences suggest that it would be beneficial for semantics to keep properties and plurals separate, ultimately hindering the motivations for adopting property singularism.

To put it differently, in order to properly serve as semantic values for plurals, properties must be understood in a way which brings them very close to pluralities. Since pluralism, as we will see, consists in taking plural terms at face value, this suggests that property singularism introduces an unnecessary detour and that, ultimately, pluralism is to be preferred.

### 1.3.3 Mereological singularism

Mereological singularism takes mereological sums as semantic values of plural terms, rather than sets or properties.<sup>25</sup> *Prima facie*, mereological singularism seems to fare better than set-theoretic and property singularism when it comes to capturing absolutely general discourse. However, a bit of digging shall reveal that the mereological approach suffers from a similar, albeit less obvious, limitation.

First, there is nothing spurious about the mereological sum which has all sets as parts; thus, for example, ‘the non-self-membered sets’ would receive a correct interpretation in this framework. Moreover, there is no paradox analogous to Russell’s concerning sums – there is nothing problematic about the mereological sum of all mereological sums either. That’s because the sum analogous to Russell’s set would be the sum which has as parts all sums which are not parts of themselves, but there is no such sum, since there are no such objects – all sums are parts of themselves. But what about the sum which has as *proper parts* all the sums which are not proper parts of themselves? Let’s call it *s*. No sum is a proper part of itself, so *prima facie* *s* would not be vacuous. And indeed its existence would give rise to a paradox. If *s* exists, then *s* is not a proper part of itself (no sum is!), thus it should be, since it fulfills said condition. However, this is hardly a problem for mereology, since there does not seem to be anything counter-intuitive about the fact that *s* does not exist.

Thus the problems derived from capturing absolutely general discourse do not seem to arise in this framework, for provided one takes domains to be sums, nothing stops one from

---

<sup>25</sup>It has been proposed, among others, by Link (1983, 1998).

taking the sum of everything as a legitimate domain.

Nevertheless, mereological singularism has been criticised on other grounds. Nicolas (2007) provides a very comprehensive overview of the objections which can be raised against using sums as semantic values of plural terms. He shows that some of them can be easily overcome, but not all. In particular, some of them are solved by extending the mereological framework along the lines of Link (1983). Before presenting this elaboration of mereological singularism, let me explain the main objection that ordinary mereological singularism faces.

According to mereological singularism, from

**(1.7)** *Harry, Ron and Hermione met at Hogwarts.*

follows that Harry, Ron and Hermione's molecules met at Hogwarts. This is because even though 'Harry, Ron and Hermione' and 'the molecules of Harry, Ron and Hermione' represent different decompositions of the same sum, they are assigned that sum as their common referent, thus sanctioning the inference just described.<sup>26</sup>

This difficulty arises from the fact that mereological sums, unlike sets, do not have a unique decomposition into parts; hence the relation of parthood does not select a unique collection of objects of which the predicate in question holds. This problem is faced by many predications; think of analogous examples with predicates such as 'are three', 'are friends' or 'admire each other'.

However, this can be amended by distinguishing certain objects of the domain as salient individuals; and this is precisely what Link's variant of mereological singularism does. We shall call Link's account 'two-fold mereology', since it makes use of two different (although interrelated) structures of atoms and sums. Link's theory has two different parthood relations, one of individual part operating on a domain of atomic individuals, and one of material part operating on a domain of matter. The domain of individuals is closed under an operation of individual sum corresponding to the relation of individual part. Analogously, the domain of matter is closed under an operation of material sum corresponding to the relation of material part. The two structures are connected via a function of material constitution, which assigns each individual sum its material sum.

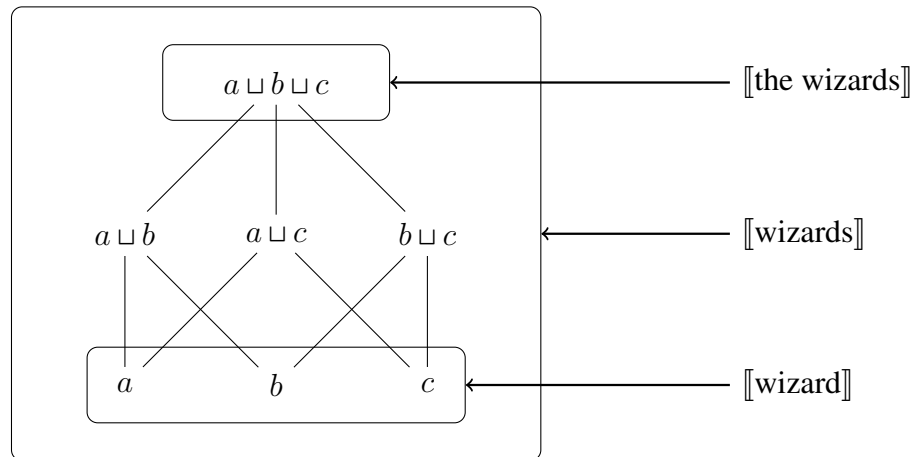
Individual sums are formed from atomic individuals via a relation of individual sum formation, which is commutative, associative and idempotent:

**(1.8) Definition (Individual sum formation)**  $a \sqcup b$  is the smallest sum that has the individual atoms  $a$  and  $b$  as parts.

---

<sup>26</sup>This has been noted, for example, by Oliver and Smiley (2001, p. 293).

Leaving aside material sums (which are irrelevant for our purposes), we are left with a domain of quantification which includes atomic individuals and individual sums thereof and which can be partially ordered via a relation of individual part, hence forming a complete atomic join-semilattice.<sup>27</sup> It is complete because it is closed under sum formation and it is atomic because all the atomic parts of the sums belong to it. Here is an example of such a structure:



According to mereological semantics, a singular common noun denotes a set of atomic individuals and a plural noun formed from it denotes the closure of that set under sum formation. For example, if ‘wizard’ denotes  $a$ ,  $b$  and  $c$ , ‘wizards’ denotes  $a$ ,  $b$ ,  $c$ ,  $a \sqcup b$ ,  $a \sqcup c$ ,  $b \sqcup c$  and  $a \sqcup b \sqcup c$ . Finally, plural definite descriptions denote the maximal element of the lattice formed from the corresponding atoms. In this case, ‘the wizards’ would denote  $a \sqcup b \sqcup c$ . This last sort of term is what we are interested in here.

Thus, for example, even though the matter of Harry, Ron and Hermione is the same as the matter of their molecules, neither the molecules nor the matter are individual atoms of the sum of Harry, Ron and Hermione (which are only each of the three Hogwarts’ students). In other words, individual sums have a unique decomposition into atomic individuals. What the salient atomic individuals are in each case depends on context, although in some cases it can be read off the syntax of terms. Thus under Link’s framework the problem described above is circumvented because predicates are taken as holding of the atomic individual parts of the sum. However, Link’s framework appears to suffer from other problems.<sup>28</sup>

In the first place, an obstacle arises when one makes two-fold mereology the subject matter of the plural object language. In the scenario where one is using plural language to regiment two-fold mereology itself, the relevant individual sums (invoked in the semantics

<sup>27</sup>I follow the reconstruction of Link’s structure in Landman (1989a, pp. 561-5).

<sup>28</sup>I mainly follow Nicolas (2007) here.

of plurals) would be individual and material sums. In other words, the salient objects in the theory of two-fold mereology are individual and material sums. Now let us focus on the domain of individuals and the relation of individual part and let us suppose that  $a$  and  $b$  are individual sums and  $c$  is their individual sum. Then,  $a \sqcup b = a \sqcup b \sqcup c$ , since they have the same individual parts. But whereas it is true that  $a$  and  $b$  are two individual sums, it is not true that  $a$ ,  $b$  and  $c$  are two individual sums. Unfortunately, two-fold mereological singularism cannot distinguish between these two scenarios and it will assign the same truth-conditions to the sentences describing each of them. Thus in this framework some statements involving talk of individual sums do not receive a correct interpretation.

One may complain that this is not the sort of discourse we find in ordinary contexts. However, this reply has little force, since there is no principled reason why the object language should not have mereology as its subject matter. Plural Logic should be as universally applicable as possible and hence it should be able to regiment discourse about anything, including mereological sums.

Nevertheless, one may attempt to bypass this difficulty without abandoning the whole framework. For instance, by taking into account in the semantics the internal syntactic structure of terms in the form of lists (which are indeed the terms causing this kind of trouble), one could establish that the individuals relevant towards truth-conditions are those picked out by each of the terms of the list.

However, discourse about mereology cannot always be amended in this way. Consider expressions of the form ‘ $x$  is one of the  $N$ s’. Two-fold mereological singularism dictates that this sentence is true if, and only if,  $x$  is an atomic individual part of the individual sum of the  $N$ s. But now consider this other expression: ‘ $x$  is one of the individual sums’. Analogously to the previous case, this sentence is true if, and only if,  $x$  is an atomic individual part of the individual sum of the individual sums. But this implies that individual parts which are not atomic are not individual sums, which is absurd. Thus it seems that even under the guise of a two-fold theory, mereological semantics is not entirely satisfactory, as it prevents Plural Logic from properly regimenting mereological talk.<sup>29</sup>

Furthermore, once Link’s modification of mereological singularist semantics is brought into the picture, part of the initial rationale for taking up the mereological route appears to be lost. An alleged advantage of the mereological approach was its simplicity. However, once Link’s distinction between two parthood relations is in place, part of that simplicity is lost. This would be justified if the picture painted by Link was indeed needed. Nevertheless,

---

<sup>29</sup>See Nicolas (2007, p. 14) for more details.

when providing semantics for plurals, the material sums and the relation of material part play no role at all. All that matters in order to provide truth conditions for plural statements is the individual decomposition of sums. But reference to a sum of individual atoms is exactly analogous to plural reference to some objects.<sup>30</sup> As we have seen, the structure needed for this is that of a complete atomic join-semilattice. So, from this point of view, there is a strict equivalence between the domain of Link's two-fold mereology without material sums and that of a Plural Logic with only plural variables (the relevant atoms would be the first-order objects and the relevant relation would be that of plural membership).<sup>31,32</sup>

One may wonder what role material sums play in Link's framework then. The answer is that they are there mainly to provide semantic values for mass terms and to connect them with those of plural terms, as indicated above. Mereological singularism was motivated in the first place by the observation that mass terms behave in some contexts like plural terms, a fact which led Link to connect the semantics of the two. For example, both seem to be cumulative in the following sense: the sum of this water and that water is water; similarly, the plurality of these children and those other children is a plurality of children. However, Nicolas (2008) has shown that in fact the meaning of mass terms is not adequately accounted for with mereological semantics. One of the problems he presents is the problem of minimal parts. According to mereological semantics, 'This is water' is true if, and only if, the sum denoted by 'this' is a part of the sum of everything that is water. However, an atom of hydrogen is not water, even though it is part of a molecule of water. Similarly, a leg of a chair is not furniture, even though it is part of some furniture.

Hence it appears that after all the mereological approach does not have the advantage of

---

<sup>30</sup>At this point, one may wonder: how do pluralist semantics manage to overcome the problems of mereological semantics, if they are structurally equivalent? Should not pluralist semantics be faced with similar problems when being used to provide semantics for the language of plurals?

If one takes pluralist semantics seriously, then it appears not. This is indeed where the advantage of using plurals in formal semantics resides. The difference between the two accounts has to do with the alleged ontological innocence of pluralist semantics. Link's semantics has it that every plural expression denotes an object, a certain individual sum. Therefore, it seems it should be possible to talk about these objects themselves in the object language. But as we have seen, Link's semantics does not work well in that case. By contrast, under a pluralist interpretation, a plural expression simply denotes several objects at once, so this difficulty does not seem to arise: there are no new objects about which to regiment discourse and thus no new predicates to be added to the object language (such as 'individual sum'). We will look into pluralism in detail in the next chapter.

<sup>31</sup>See Oliver and Smiley (2016, sec. 12.7) for a description of the algebra of plurals.

<sup>32</sup>In the next chapter (sec. 2.3.3), I show that Plural Logic can be developed with a single type of variables (i.e plural ones).

being easily extendible to an account of mass terms. Hence, part of the rationale to adopt it is lost.

## **1.4 Recap**

In this chapter, I have presented the main alternatives to pluralism and I have pointed to some of their problems. The main difficulties they face have to do with their failure to provide intuitively correct interpretations for certain expressions. Firstly, set-theoretic singularism cannot account for the meaning of terms which denote collections which are too big to form sets, on pain of contradiction. Property singularism cannot easily account for talk of properties. Finally, mereological singularism, even in its two-fold version, cannot account for certain statements concerning mereological sums. Thus, it seems that all of these approaches fail to account for some ordinary uses of plural language; accordingly, they limit the ability of Plural Logic to regiment ordinary discourse to certain subject matters.

# Chapter 2

## E Pluribus Plures

### 2.1 Introduction

The first half of this chapter (Section 2.2) is devoted to addressing the main objections raised against pluralism, the view that plurals can be safely incorporated into a formal language and interpreted at face value. Again, my main aim is not to convert the reader to pluralism, but to show which are the assumptions underlying the pluralist view. This will be crucial for the rest of this dissertation, where I will argue that the pluralist should be inclined to endorse higher-level extensions of PL.

Moreover, in the second half (Section 2.3), I will discuss some issues concerning ordinary plurals that shall prove relevant in our subsequent discussion of Higher-Level Plural Logic. As we will see, some of the arguments and positions discussed in Chapters 3-7 will hinge on decisions made with respect to the plural fragment. Instead of spreading their discussion throughout this dissertation, I will tackle them here and refer back to this chapter when necessary.

### 2.2 Pluralism: a defence

While we have seen that there is more than one singularist approach to the semantics of plurals, there appears to be only one suitable pluralist approach: the one according to which plural terms should be taken at face value, that is, as denoting the objects they intuitively denote. In other words, as referring plurally to objects from the first-order domain. As we shall see, this ultimately amounts to allowing for multivalued interpretation functions in the



semantics.

There are two main decisions to make when it comes to giving semantics to Plural Logic with non-logical predicates. One concerns the semantic value of plural terms. The other one, the semantic value of plural predicates. After giving the formal account of my preferred form of pluralist semantics for PL, I will discuss the former issue in Section 2.2.2 and I will turn to the latter in 2.2.3.

Tackling plural terms and predicates separately is not just narratively convenient. PL without non-logical predicates is worth exploring on its own, given its significant expressive power. As shown by Boolos,<sup>33</sup> PL without non-logical predicates can interpret Monadic Second-Order Logic (hereafter, ‘M2OL’).<sup>34</sup> For example, with it we can express a second-order sentence like ‘There is a property which all objects have’ (i.e.  $\exists X \forall x Xx$ ) as ‘Some objects are such that every object is one of them’ (i.e.  $\exists xx \forall x (x < xx)$ ). More generally, that PL without non-logical predicates interprets M2OL is shown by the existence of the following translation scheme, where  $\phi^*$  and  $\psi^*$  are the translations in PL without non-logical predicates of the M2OL formulae  $\phi$  and  $\psi$ :

$$\begin{aligned} (t = u)^* &= t = u \\ (Pt)^* &= t < pp \\ (\neg\phi)^* &= \neg\phi^* \\ (\phi \rightarrow \psi)^* &= \phi^* \rightarrow \psi^* \\ (\exists x\phi)^* &= \exists xx\phi^* \end{aligned}$$

The translation for formulae involving second-order quantification is not as straightforward, given the fact that whereas predicates can be assigned the empty property or set, pluralities are never empty. In order to deal with this divergence, Boolos proposes a disjunctive translation, where one of the disjuncts addresses the empty case explicitly. Let  $\phi$  be a M2OL formula with  $X$  free. And suppose  $\phi^{**}$  is the translation of the result of replacing each occurrence of  $Xx$  in  $\phi$  with  $x \neq x$ . The resulting translation clause is:

$$(\exists X\phi)^* = \exists xx\phi^* \vee \phi^{**}$$

Moreover, it is easy to construct a reverse translation scheme (from PL without non-logical predicates to M2OL). These translations map theorems of PL without non-logical

<sup>33</sup>In Boolos (1984, 1985).

<sup>34</sup>The reason why it cannot interpret full Second-Order Logic will become clear in section 2.2.3.2.

predicates to theorems of M2OL and vice-versa; thus showing that these two languages are equi-interpretable.

The topic of the relation between PL and M2OL is one of the most debated in connection with plurals. Although addressing this issue in detail falls outside the scope of this thesis, I agree with Rayo and Yablo (2001) that conflating plurals and predicates in all contexts would be a mistake. In fact, this follows from my criticism of property singularism in the previous chapter. Nevertheless, I believe this identification can be fruitful in certain cases. In particular, I will help myself to it in Chapter 7 in order to examine the potential of PL and a higher-level extension thereof in a strengthening of neo-Fregeanism.

### 2.2.1 Pluralist semantics for PL

Before describing a pluralist model for PL, let us have a look at the notion of plural reference and informally clarify how it is being understood here by pointing to some of its most salient features.

Firstly, pluralities are extensional in the following sense:

**(2.1) Principle (Extensionality)** *Some objects are the same as some other objects if each of the former is identical to one of the latter and vice versa.*

Moreover, they are unrestricted:

**(2.2) Principle (Unrestricted Composition)** *Any objects whatsoever are some objects.*

Let me clarify this principle. *Prima facie*, it is rather unilluminating due to its being so blatantly true. The idea behind it is that the existence of some objects hinges exclusively on the existence of each of them. Therefore, it takes nothing else for a plurality to exist than for each of some objects to exist. No conditions at all are imposed on some objects for them to be some objects.

Finally, they are rigid:

**(2.3) Principle (Rigidity)** *If an object is among some objects, then it is so necessarily.*

In other words, it is not the case that an object could have not been among some objects, for, had that been the case, those objects would have been some other objects. Pluralist semantics aims to capture these characteristics. We are now ready to have a look at its formal development.

A pluralist interpretation of PL is one which assigns each plural term one or more than one object from the first-order domain at once.

We take a model  $M$  for  $\mathcal{L}_{PL}$  to be a multivalued function which assigns a single or multiple objects to ‘ $\forall$ ’ (this encodes the domain of quantification). Moreover, it assigns a single object to each singular constant, a single<sup>35</sup> or multiple objects to each plural constant and a property or a relation to each predicate. One can thus think of one such model as a plurality of ordered pairs.

In this description of pluralism I adhere to the view that predicates denote properties (or relations) which are not to be counted among the values of the first-order terms. This is the view we call ‘mixed pluralism’. Below I will discuss this and an alternative view on the semantics of plural predicates, but let us take mixed pluralism for granted for the time being.

We also have multivalued functions  $s$  which assign a single value to each singular variable and a single or multiple values to each plural variable.

## Domain

The domain is a plurality  $dd$  (i.e. the value or values of ‘ $\forall$ ’)

## Interpretation of terms and predicates

For each constant  $c$ ,  $M_s(c)$  is an individual among  $dd$  (and  $M_s(c) = M(c)$ ).

For each constant  $cc$ ,  $M_s(cc)$  is an individual or some individuals among  $dd$  (and  $M_s(cc) = M(cc)$ ).

For each variable  $x$ ,  $M_s(x)$  is an individual among  $dd$  (and  $M_s(x) = s(x)$ ).

For each variable  $xx$ ,  $M_s(xx)$  is an individual or some individuals among  $dd$  (and  $M_s(xx) = s(xx)$ ).

For each  $n$ -place predicate  $P_n$ ,  $M_s(P_n)$  is a property, if  $n = 1$ , or an  $n$ -place relation, if  $n > 1$  (and  $M_s(P_n) = M(P_n)$ ).

---

<sup>35</sup>It is usually accepted that plural terms can have a single value. For the time being, I take this for granted, but I come back to this topic in Section 2.3.3 below.

**Satisfaction** Here I use  $a$ ,  $b$  and  $a_k$  ( $1 \leq k \leq n$ ) as metavariables for singular terms and  $t$  and  $u$  as metavariables for both singular and plural terms.

$M_s(a = b) = 1$ <sup>36</sup> if, and only if,  $M_s(a) = M_s(b)$ .

$M_s(t < u) = 1$  if, and only if,  $M_s(t) = M_s(u)$  or  $M_s(t)$  is an individual and  $M_s(u)$  are at least two individuals and the former is a member of the latter.<sup>37</sup>

$M_s(P_n(a_1, \dots, a_n)) = 1$  if, and only if,  $M_s(P_n)$  holds of  $(M_s(a_1), \dots, M_s(a_n))$ .

$M_s(\forall x\phi) = 1$  if, and only if, for every assignment  $s'$  which is an  $x$ -variant of  $s$   $M_{s'}(\phi) = 1$ .<sup>38</sup>

$M_s(\forall xx\phi) = 1$  if, and only if, for every assignment  $s'$  which is an  $xx$ -variant of  $s$   $M_{s'}(\phi) = 1$ .

$M_s(\neg\phi) = 1$  if, and only if,  $M_s(\phi) = 0$ . And  $M_s(\phi \rightarrow \psi) = 1$  if, and only if,  $M_s(\psi) = 1$  or  $M_s(\phi) = 0$ .

### Truth, Logical truth and Logical consequence

$\phi$  is true in  $M$  if, and only if, for all  $s$ ,  $M_s(\phi) = 1$ .

$\phi$  is a logical truth if, and only if, for all models  $M$   $\phi$  is true in  $M$ .

$\Gamma$  logically implies  $\phi$  if, and only if, for any model  $M$ , if each of  $\Gamma$  is true in  $M$ , then  $\phi$  is also true in  $M$ .

#### 2.2.1.1 Metatheory

It can be shown in the usual way that PL under a pluralist semantics is sound.<sup>39</sup> Moreover, it can be shown that PL is not compact.<sup>40</sup> Consider the formula:

<sup>36</sup>Expressions of the form ' $M_s(\phi) = 1$ ' are short for ' $\phi$  is true in  $M$  relative to the assignment  $s$ '. Analogously for ' $M_s(\phi) = 0$ '.

<sup>37</sup>Given that there are pluralities of a single object, we say these formulae are true whenever an object is among a certain plurality of objects, where the latter may consist of a single object (in which case the relation is equivalent to singular identity). The sentence is not true when the relation holds between two pluralities one of which is a proper sub-plurality of the latter, since we want to distinguish plural membership from plural inclusion (which can be defined in terms of membership in the usual manner).

<sup>38</sup>A  $\phi$ -variant of  $s$  is an assignment that only differs from  $s$  at most in what it assigns to  $\phi$ .

<sup>39</sup>See Oliver and Smiley (2016, ch. 12) for details.

<sup>40</sup>I follow Yi (2006, p. 262-4) and Oliver and Smiley (2016, ch.13).

$$\exists xx(\forall x(x < xx \rightarrow \exists y(y < xx \wedge A(x, y))))$$

Let us call it  $\gamma$ . And consider the set of formulae  $\Gamma = \{A(a_n, a_{n+1}) : n \text{ is a natural number}\}$ . It is easy to check that  $\Gamma \models \gamma$ , but there is no finite subset  $\Gamma^* \subset \Gamma$  such that  $\Gamma^* \models \gamma$ . Take any such finite subset. There will be  $a_k$  and  $a_{k+1}$ , such that  $A(a_k, a_{k+1})$  and such that there is no  $a_{k+2}$  for which  $A(a_{k+1}, a_{k+2})$ , thus falsifying  $\gamma$ .

As a consequence, PL is not strongly axiomatisable. In particular, there is no adequate axiomatic calculus that captures the logical relation between the sentences in  $\Gamma$  and  $\gamma$ . If there were one, given that any syntactic derivation is finite, there would be a finite subset of  $\Gamma$ ,  $\Gamma^*$ , from which to carry out the derivation, against the failure of compactness.

That PL is also not weakly axiomatisable follows from the fact that it can serve to finitely axiomatize a version of Peano Arithmetic which is categorical and therefore semantically complete. Thus, by Gödel's theorem, it follows that the logical truths of the underlying logic are not effectively enumerable.

## 2.2.2 The semantic value of plural terms

In this section, I turn to an evaluation of the pluralist account of plural terms by mainly addressing the objections which have been raised against it.

### 2.2.2.1 Rationales for pluralism

What reasons are there for endorsing pluralism in the first place? Firstly, as mentioned above, under a pluralist understanding PL *prima facie* overcomes the problem of absolutely general discourse: an absolutely general domain can be assigned to its quantifiers since there is nothing troubling about the plurality of everything.<sup>41</sup> Moreover, pluralism provides interpretations for plural sentences that match their intuitive meaning more generally. Famously, under this framework we can interpret a sentence like (2.4) without appealing to anything other than critics and their admiring one another:

**(2.4)** *Some critics admire only one another.*

---

<sup>41</sup>It must be noted that this is not unanimously accepted. For example, Linnebo and Rayo (2012) have argued that PL resolve the problem of absolute generality, in the sense described here, only at the expense of a huge proliferation of its ideological commitments (in the Quinean sense of the term), thus ultimately not much is achieved. I return to this topic in Chapter 7, where I will try to undermine this view.

As proved by Kaplan<sup>42</sup> this sentence cannot be correctly regimented in First-Order Logic. This follows from the fact that its second-order formalization,

$$\exists X(\exists x Xx \wedge \forall u \forall v (Xu \wedge A(u, v) \rightarrow Xv \wedge u \neq v))$$

with ‘ $u = 0 \vee u = v + 1$ ’ instead of ‘ $A(u, v)$ ’, is true in all and only the non-standard models of arithmetic. To see that it is false in the standard model, consider any suitable set of natural numbers. If its initial element is 0, then take  $u = v = 0$ . If its initial member is  $a \neq 0$ , then take  $u = a$ . And to see that it is true in the non-standard models, consider one such model. Since any such model has a subset which is closed under immediate predecessor, the formula is satisfied. This can never be the case for a first-order sentence, hence the non-firstorderizability result.

Thus (2.4) requires a second-order regimentation. However, it would be highly counter-intuitive if the truth value of (2.4) were to depend on the existence of some set-like entities or some properties (as a second-order reading is typically seen to demand). Fortunately, we can instead use a PL regimentation:  $\exists xx[\forall u \forall v (u < xx \wedge Auv \rightarrow v < xx \wedge u \neq v)]$ . Which, under a pluralist reading, presupposing a domain of critics, makes the sentence true simply in virtue of the existence of some suitably related objects.

Finally, pluralism makes PL an attractive tool to carry out nominalist projects, due to the plausibility of the claim that it makes PL ontologically innocent. As mentioned in the Introduction, different sorts of nominalist philosophers should welcome pluralism as good news.

However, pluralism has received a good number of objections. In what follows, I survey the ones which I take to be most pressing and I show how the pluralist may respond to them.

### 2.2.2.2 Objections and replies

**Objection 1: Against using ordinary plurals in the metalanguage** Recall that pluralist semantics employs ordinary plural terms and quantifiers in the metalanguage.

I will refer to any semantics which re-uses a certain object language expression (or rather its metalinguistic counterpart) in the metalinguistic statement of the truth-conditions of a sentence involving that expression as a ‘homophonic semantics’. Homophonic semantics cannot be taken to be problematic in themselves unless one is willing to deem the whole

---

<sup>42</sup>And reported in Quine (1950, sec. 38).

model-theoretic enterprise as misguided – homophony is ubiquitous in model-theoretic semantics; for instance, it is used to account for the meaning of first-order quantification and of most logical connectives. It is often agreed that model-theoretic semantics have two main aims: (1) to explain how the semantic value of a complex expression arises from the semantic values of the simple expressions composing it, and (2) to specify what should be sufficient for a speaker to know in order to understand a certain language. These aims are perfectly compatible with the homophonic strategy. There is nothing wrong with it in itself.

Nevertheless, any semantics which employs a metalanguage which is not clear enough to the relevant speakers will not be of much use to them. It is precisely in this sense that pluralist semantics has been objected to. Pluralist semantics makes use of metalinguistic plurals in order to fully capture the right features of the object language plural expressions (crucially, ontological innocence). However, the critics of pluralism take plural terms to belong to the fragment of ordinary language which is in need of clarification and, as a result, they believe that the meaning of their formal counterparts cannot be accounted for homophonically.

For example, Shapiro has raised this concern:

It might be noted, however, that plurals in general seem to be rather complex, and there is no consensus among linguists concerning how they are to be understood. (Shapiro, 1991, pp. 225-226.)

Here Shapiro<sup>43</sup> is referring to pluralist semantics when used to interpret M2OL. But the complaint is as pertinent in the case where the semantics is used to express the truth-conditions of plural formulae. According to Shapiro, given that there is no agreement among linguists upon the exact meaning of plural expressions, it is doubtful whether they are adequate for formal semantics.

I believe that the advocate of pluralism has a reply available. Firstly, there are various senses in which one may appeal to *clarity* in this discussion. In particular, there appear to be two main candidates. On the one hand, there is clarity among experts, philosophers of language or formal semanticists. Those seek to explain the meaning of the expressions in question in a systematic and global way, that is, in connection with other fragments of natural language. This is the sense invoked by Shapiro. On the other, there is the sense in which an expression is clear among native speakers of the language in question – this must be cashed out in terms of an agreement between a large enough majority of speakers with respect to the

---

<sup>43</sup>He has also raised this worry in Shapiro (1993, pp. 471-2).

correct usage of the expression (i.e. with respect to grammaticality, inferential profile, etc). We call these two senses, ‘semanticist-clarity’ and ‘speaker-clarity’.

Here is how the pluralist can reply to the sceptic. In the first place, semanticist-clarity does not seem to be a relevant criterion in this context. In this sense, plurals would certainly not be clear enough to figure in semantic theorizing, since linguists disagree as to the correct semantic account of plurals – there is mostly disagreement between adherents of set-theoretic and mereological singularism. However, if this gives us enough reason to cast doubt on pluralist semantics, so does it to cast doubt on other well-established formal semantic theories, since homophony is ubiquitous among model-theoretic approaches. For instance, one finds it in the model-theoretic semantic account of the logical connectives and, just as in the plural case, one can hardly say that linguists have reached an agreement in the debate around coordination. The same occurs with the expression ‘there is’, used to account for the meaning of singular first-order existential quantification. Formal semanticists still discuss its semantic contribution. Hence it seems that this sense of ‘clarity’ or ‘intelligibility’ is not sufficiently discriminatory; it makes of the model-theoretic approach to semantics a doomed enterprise.

By contrast, speaker-clarity seems to be a relevant criterion and in fact it appears to be available when it comes to plurals. Plurals appear to be speaker-clear enough. There is an overwhelming agreement upon the meaning of ordinary plurals: any competent speaker (of ordinary English, in this case) has a perfect command of plurals and no systematic misunderstandings arise from their being used in ordinary talk. This should be enough for the purposes of developing model-theoretic semantics.

Importantly, speaker-clarity is discriminatory; it does not deem all ordinary language idioms fit for purpose. Although a clear-cut distinction between suitable and unsuitable expressions may not be available, there are fragments of natural language which are quite indisputably not speaker-clear enough to figure in semantic theorizing. For example, the ordinary indicative conditional appears to provide one such case. The meaning of ‘if A, then B’ has been the focus of an important debate in the philosophy of logic. The reason why its meaning is subject to such disagreement among experts appears to be that ordinary speakers have conflicting intuitions about it – for instance, people disagree as to the import of the so-called paradoxes of material implication. The reason why none of the different semantic theories proposed to analyze the notion fully captures the pre-theoretic conception of the indicative conditional seems to be that there is no such unique conception – there are multiple clashing ones. At least, this is a plausible diagnosis of the situation. Thus, the most we can aim for is a non-homophonic treatment which captures part of the understanding of the notion. A homophonic treatment would not do the job, for it would not pick out a single



understanding. By contrast, this does not seem to be the case with respect to plurals. Plurals do not give rise to disagreements of the sort that affect indicative conditionals.

Admittedly, the notion of plural reference can be finessed, made more precise in various ways – e.g. one can ask whether reference to one thing is a case of reference to various things – but all those precisifications can be captured within a pluralist understanding. In other words, the core idea of pluralist semantics – that plurals denote some things at once – does not appear to be disputed by ordinary speakers. It seems nothing else can be required of ordinary plurals to be legitimately added to our semantics. Therefore, the use of ordinary plurals in semantic theorizing is not problematic in itself.

**Objection 2: The metalanguage is singular in disguise** The next complaint is typically raised in relation with the syntax of ordinary plurals. Thus it can be turned into an objection about pluralist semantics for PL as well. It is the claim that ordinary plural language should be understood as being singular language in disguise (and thus that pluralist semantics is singularist semantics in disguise). At the basis of this complaint is a certain singularist intuition. As put by Resnik:<sup>44</sup>

I find that I cannot process many sentences containing plural quantifications without understanding them in terms of collections. [...] How else are we to understand the phrase ‘one of them’ other than as referring to some collection and as saying that the referent of ‘one’ belongs to it? (Resnik, 1988, p. 77)

Even though this gives rise to a standoff of contrary intuitions with respect to the meaning of ordinary plurals, it does not suffice to undermine pluralism (for one may analogously reply by expressing their having the opposite intuitions). However, there are two ways of making this objection more compelling. Firstly, it can be argued that plural language can always be paraphrased away in favour of singular language – i.e. that all ordinary sentences involving plurals can be rephrased as sentences making use exclusively of singular expressions – by providing translation schemes and showing that they suffice to eliminate all plural expressions. Secondly, it can be argued that plural language can be analysed away – i.e. that all ordinary statements involving plurals can be given semantics that show that no plural reference or quantification are really present.

However, to begin with, the fact that something can be paraphrased or analysed in a certain way does not mean that it should. Thus from the fact that there are singularist para-

---

<sup>44</sup>See also Parsons (1990, p. 326) for the expression of a similar concern.

phrases or analyses of plural language does not follow that pluralism must be abandoned. Be that as it may, both moves appear to be resistible.

On the one hand, that ordinary plural language cannot be systematically paraphrased away has been argued at length in Boolos (1984), Oliver and Smiley (2001), Oliver and Smiley (2016, chs. 3, 4), Yi (2005) and Florio (2014a), and I will not delve further into it. Nowadays that plurals are not eliminable by paraphrase seems to be a widely accepted claim. On the other hand, whether ordinary plural language can be analysed away is precisely the question I tackled in Chapter 1, where I gave some arguments to the effect that the main candidate singularist analyses of PL are not satisfactory (although we focused on semantics for a formal plural language, most of the considerations put forward applied to ordinary plurals – this is precisely because of the role pluralists assign to natural language in semantic theorizing). Provided one accepts certain assumptions (importantly, that absolutely general discourse is possible), if our arguments are correct, one should be convinced by now that, at least, plurals are not easily analysable away.

**Objection 3: Lack of an ordinary universal plural quantifier** Another objection to the pluralist understanding of PL has been raised by Parsons and reported by Boolos.<sup>45</sup> The latter writes:

Charles Parsons has pointed out to me that although second-order existential quantifiers can be rendered in the same manner we have described, it is curious that there appears to be no nonartificial way to translate second-order universal quantifiers. (Boolos, 1984, p. 68)

Boolos acknowledges this much: there does not seem to be any naturally sounding ordinary reading of the universal plural quantifier. However, he sees this fact as a reason to take it as a derived notion, defined in terms of the existential quantifier (i.e. ‘It is not the case that there are some objects such that they do not...’).<sup>46</sup>

In any case, this objection can be countered as follows. In the first place, one can push the line that, in fact, the universal plural quantifier does have naturally sounding renderings. Here are two such proposals: ‘any objects are such that...’ and ‘for any given objects, it is the case that they...’. It looks to me as though they are good English and they capture the intended meaning of the plural universal quantifier. Moreover, one can argue that whether

<sup>45</sup>See also (Resnik, 1988, p. 86) for another expression of this objection.

<sup>46</sup>Others have followed him on this, for example, Rayo (2006).

this quantifier has a natural English reading or not is not of much relevance. After all, why should the contingent fact that the vernacular lacks an expression have consequences for the legitimacy of a formal language? I am inclined to take the presence of ordinary renderings as lending support to the legitimacy of that formalism. However, I do not think the lack thereof can serve to cast doubt on the intelligibility of a formal device. For one thing, one would need to carry out a formidable task in checking all ordinary languages, and even if there was confirmation that no ordinary language contains such-and-such expressions, this would still fall short of a decisive reason for rejecting the corresponding formalism. The reasons why natural languages lack certain expressive devices are diverse and need not be a sign of the unintelligibility of the formalism in question. By contrast, a single occurrence of a certain expression in a natural language provides strong support for the claim that that expression is intelligible. In this case, good news appears to be the only possible news.

**Objection 4: Weaker systems of PL** Hazen has objected to pluralist semantics on completely different grounds. His objection is put in terms of pluralist semantics for M2OL, but an analogous objection would apply to pluralist semantics for PL. The idea is expressed in the following passage, where by ‘Platonistic semantics’ he means what we mean by ‘singularist semantics’ and by ‘impredicative semantics’ he means a semantics which assigns second-order quantifiers all the pluralities which can be formed with objects from the first-order domain:

On the pluralistic reading, full, impredicative M2OL carries no ontological commitments beyond those of the first-order theories it is added to, but, since there is no way of making sense of the relevant restrictions to the comprehension principle on the pluralist reading, predicative or ramified second-order logics have to be interpreted Platonistically and do carry commitment to a domain of attributes. On the other hand, the predicative logics are mathematically weaker than impredicative second-order logic, and are in an obvious sense subsystems or parts of it. The pluralist is then in the anomalous position of holding that someone (the predicativist) who accepts part of second-order logic is ontologically committed to more than someone who accepts all of it! (Hazen, 1993, p. 135)

Let me rephrase his argument in terms more suitable for our purposes. Pluralist semantics for PL is usually assumed to give PL the expressive power of M2OL under standard semantics, that is, it is supposed to make the plural quantifiers range over all the pluralities

we can form with objects from the first-order domain. One reason why one might think that pluralist semantics enforces this kind of reading of the plural quantifiers is that, according to it, there is no specific domain for the plural quantifiers. Hence, in particular, there can be no specific restricted domain for them (i.e. a domain which would be analogous to a subset of the power-set of the first-order domain). In other words, since PL under a pluralist reading is ontologically innocent, only a full reading of the plural quantifiers is available. However, Hazen's argument goes, restricted semantics (such as semantics where the background principle of comprehension for the values of the plural variables is predicative) need to be expressed by appealing to a restricted domain and thus a specific plural domain (distinct from the first-order domain). Thus, whereas when describing full semantics we can make do with a single domain of basic objects, when we attempt to describe weaker semantics we must make use of a new domain of entities in order to capture said restriction. Hence, according to a Quinean-style criterion of ontological innocence, it turns out that PL under restricted semantics is ontologically more costly than PL under full semantics, since it is committed to an additional domain of objects. But this is absurd, since PL under full semantics is expressively stronger than PL under weaker semantics and expressively stronger theories should be ontologically at least as expensive as weaker theories.

If this is the right reading of Hazen's argument, then I believe there are two ways to challenge it.

Firstly, one may follow Florio and Linnebo (2015), who have proposed a way of describing restricted semantics for PL that does not make use of any resources not already present in the description of the full pluralist semantics. Thus they reject the idea that the only way to capture restricted interpretations is to make use of an additional domain. They propose that we represent the range of the plural quantifiers by means of a plural collective property  $D$ . The pluralities (of objects from the first-order domain) which instantiate  $D$  will be those over which plural quantifiers range. Florio and Linnebo argue that this semantics is *as innocent as* the full one. One of their key observations is that when developing pluralist semantics for an absolutely unrestricted plural language we need to help ourselves to a strictly more expressive metalanguage – in particular, we need to help ourselves either to plural collective predication or to superplural reference.<sup>47</sup> Thus ascending one (expressive) level up seems unavoidable in any case.

However, although Florio and Linnebo's proposal suffices to show that Henkin-style semantics can be legitimately developed in a pluralist fashion, it is not clear that this is all we

---

<sup>47</sup>See Rayo (2006, pp. 35-8) for details.

need to tackle Hazen's worry.

An important motivation for adopting Henkin-style semantics for second-order languages arises from a certain view on the ontology of the values of the second-order terms. For instance, one typically endorses a predicativist view because of a prior endorsement of some form of constructivism with respect to whatever it is that is picked out by second-order expressions. The same would hold for plurals.

Being a predicativist about plurals would have to be motivated by a certain conception of what it takes for a plurality to exist. However, it seems that the very notion of plural reference precludes such a conception: as captured by the principle of Unrestricted Composition above, some objects ('a plurality') exist as long as each of them exists, it takes nothing else for them to exist. Thus, although Florio and Linnebo's proposal delivers as promised, it is not clear that its adoption can be motivated in a philosophically coherent way. At least, one of the main rationales for adopting a restricted semantics does not appear to be available when it comes to plural quantification.

Nevertheless, if I am right about this, then this provides us with a new way to respond to Hazen's objection. We can respond that there are no understandings of PL weaker than the full one. Pluralism delivers semantic determinacy, since although we can make sense of a restricted understanding thereof (in the sense of not helping ourselves to any extra machinery), the very nature of plural quantification prevents one from having the right rationale for adopting such an interpretation.<sup>48</sup>

**Objection 5: Plural quantification is always ontologically committal** Finally, one may reject pluralism on the basis that plural quantification, as opposed to plural reference, is what brings about ontological commitment. Even if one accepts that plural terms are not ontologically committing, the objection goes, once quantification into plural position is brought into the picture, a special domain of quantification is needed in order to make semantic sense of it.

There seem to be two different ways of motivating this thought. Here is the first one. Whereas we can understand talk of specific pluralities without commitment to anything other than each of the individuals of the plurality, this is not possible when we consider plural quantification. In order to understand the notion of a plural quantifier we need to understand what picking out an arbitrary plurality consists of and in order to do so we need to make use

---

<sup>48</sup>This is something which some plural logicians have argued for. See, for instance, Hossack (2000, pp. 440-1).

of combinatorial and set-theoretic resources. Essentially, understanding what an arbitrary sub-plurality is requires a prior understanding of what a subset is.<sup>49</sup> However, in this form the objection appears to have little force and can be avoided by disentangling the question of what one is required to have previously understood in order to understand plural quantifiers from the question of what objects plural quantifiers commit us to. Whether these two questions can be conflated in this way is not clear and appears to be in need of justification.

Be that as it may, there is another way of putting the complaint that plural quantification is always ontologically committal. It arises from a certain view about quantification in general: the view that quantification is always ontologically committing. The idea is to push the Quinean thought that the only legitimate sort of quantification is singular first-order quantification and thus that any other forms of quantification are singular first-order in disguise. In other words, no form of quantification can be interpreted without the aid of a domain of objects for its quantifiers to range over. In particular, plural quantification is singular quantification in disguise.

However, this take on the import of quantification is not compulsory. There is at least an alternative conception thereof which would block this complaint. It has been endorsed in Prior (1971) and further developed by Simons (1997). I will call it, following MacBride (2006, pp. 444-7), ‘neutralism’. Neutralism is not a particular semantic theory, but rather a proposal to construe semantics in a certain way. As MacBride puts it, neutralism is ‘an invitation to think about quantification in a manner to which we are ill accustomed’. Let me briefly make a case for its plausibility.

Neutralism can be described as the view that we should remain neutral as to the meaning of quantification in general and deem the meaning of each sort of quantifier dependent on the meaning of the semantic category of the expressions it binds.

Let’s say that a quantifier is singularly nominal if, and only if, it is into singular name position (i.e. it is a singular first-order quantifier). A consequence of neutralism, given that there are semantic categories other than that of singular names and that quantification into them is possible (at least, in a syntactic sense), is that non-singularly-nominal quantification is legitimate. Another consequence of neutralism is that quantification does not by itself commit us to the existence of any objects. It is only quantification into the position of ontologically committing expressions that carries commitment to a domain of objects. In other words, the commitments of quantifiers are derivative of the commitments of non-quantified

---

<sup>49</sup>A similar argument, although not intended to show that PL is ontologically committing, can be found in Linnebo (2003).

expressions. This view is partly Quinean and partly anti-Quinean in spirit. It is Quinean in the sense that it is compatible with the claim that singular first-order quantification is associated with a domain of objects. However, it is anti-Quinean in that it rejects the claim that all quantifiers function like the singular first-order ones.

But what reasons are there to adhere to neutralism? One rationale for neutralism comes from the intuition that non-quantified statements are semantically prior to their quantified counterparts; that the meaning of the latter is somehow derived from the meaning of the former. As put by Prior:

Why, then, should he suppose that the quantified forms  $\exists\phi\phi(a)$ ,  $\exists\phi\exists x\phi(x)$ , etc., commit us to the existence of sorts of entities to which we are not committed by the forms  $\phi(a)$ ,  $\psi(a)$ ,  $\exists x\phi(x)$  from which they follow? [...] The alleged emergence of these new ontological commitments has an almost magical air about it. (Prior, 1971, p. 43)

Furthermore, Prior and Simons argue that one of the alleged reasons for taking singularly nominal quantification as the only legitimate form of quantification is wrong-headed. Their diagnosis is that an important reason why that view appears plausible is that all the naturally sounding readings of quantified statements in ordinary English are singularly nominal, i.e. the quantified positions are read as standing for singular names. For example, ‘ $\exists X X(a)$ ’ is read ‘there is a property which  $a$  instantiates’ or ‘there is something that  $a$  is’. Similarly with quantification into sentential position: ‘ $\forall p(p \vee \neg p)$ ’ is read ‘all propositions are such that either they hold or they don’t’. But this cannot be a good reason to treat logical quantifiers as singularly nominal. As I argued when discussing the lack of a natural reading of the plural universal quantifier, natural language renderings of formulae can serve to lend support to the legitimacy of a certain formalism, but a lack thereof cannot be taken as withdrawing such support. As put by Simons,

Probably Quine’s view arose because in English we naturally nominalize when using the idiom some etc., but this vernacular point should cut no ice logically. (Simons, 1997, p. 263)

Moreover, there are actually instances in ordinary English of what seem to be non-singularly-nominal quantifiers. In particular, quantifiers into adverbial position: ‘somehow’ and ‘however’. Prior imagined analogous quantifiers for the sentential case: ‘everywhenever’

and ‘somewhether’. His point is that it is plausible that the fact that English does not contain these expressions is merely accidental.<sup>50</sup>

An obvious, although not compulsory, development of neutralism is provided by substitutional semantics. Substitutional semantics is compatible with neutralism in the sense that, according to it, the truth-conditions of quantified statements depend directly on the truth-conditions of their instances. Hence what must exist for them to be true is just what must exist for their instances to be true. This makes the meaning of quantified expressions depend on the meaning of non-quantified expressions simply in the sense that the former are shorthand for the latter. However, substitutional semantics has often been dismissed on the basis that it cannot account for the expressive power of some theories. Roughly speaking, substitutional semantics would be suitable if it were the case that for us to grasp the meaning of a quantified statement it were required that we had an antecedent ability to refer to every object over which the quantifiers range. However, this does not appear to be the case, as one may lack the means to denote a certain object in a first-order domain of objects, while being perfectly able to grasp the statement ‘ $\forall x(x = x)$ ’.<sup>51</sup> I will not explore the substitutional development of PL for these very reasons.

In any case, neutralism and the way in which Prior and Simons motivate it should add plausibility to pluralism. In fact, neutralism has also been associated with the view that the best and only way to provide formal semantics for quantification is to re-use the object language quantifiers in the metalanguage.<sup>52</sup> This is because if we make use of a metalinguistic counterpart of the object language quantifiers in the semantics (possibly extending English with some new expressions), then we will avoid reducing the object language quantification to singularly-nominal quantification in the semantics. As we have seen, pluralism exploits this strategy. Plural quantification is non-singularly-nominal, because despite being nominal, it is not singular: even though it demands the existence of a domain of objects, it does not demand the existence of a domain of objects over which to range singularly. Therefore, I conclude that the plausibility of neutralism undermines the objection that all quantification

---

<sup>50</sup>See Rayo and Yablo (2001) for a proposal as to how to read second-order quantifiers in ordinary English.

<sup>51</sup>More precisely, substitutional semantics is problematic in the scenario where there are objects for which we have no names, properties for which we have no corresponding predicate, etc. Such theoretically possible situations, in fact, can be seen to obtain once we turn our attention to mathematics. For example, according to ZFC there are uncountably many sets, but our languages can only produce countably many expressions and hence countably many names for sets. Therefore, there are sets which are not definable. Despite this, substitutional semantics have indeed been developed for PL. See Urbaniak (2013), for a modal substitutional account of PL which allegedly overcomes the issues just outlined.

<sup>52</sup>See van Cleve (1994), p. 588, for an expression of this view.



is ontologically committing and lends support to pluralism.

\*\*\*

I hope to have provided a comprehensive enough survey of the main objections to pluralism and the strategies one may follow in order to resist them. Naturally, these strategies rely on some assumptions for which I have not argued. In the conclusions of this chapter, I will stop to compile them and elucidate them. As I said before, they will prove relevant in the remainder of this thesis.

### **2.2.3 The semantic value of plural predicates**

Finally, we turn to the semantic value of non-logical singular and plural predicates. There are two main candidate semantic theories to extend pluralist semantics in order to cover the whole of PL. I call them ‘mixed pluralism’ and ‘superpluralism’. They differ on the sort of semantic value they assign to predicates, however they coincide in that the semantic values of predicates cannot be first-order objects. As shown by Florio (2014b), an untyped view of predicate reference according to which predicates denote first-order objects goes against a plural version of Cantor’s theorem. According to it, there are more pluralities than objects (provided there are at least two objects). Thus, if properties are objects, then we obtain that there are more pluralities than properties, which makes some interpretations of plural predicates unavailable. Thus, following Florio’s result, both mixed pluralism and superpluralism appeal to entities distinct to the values of first-order terms as values of predicates.

On the one hand, according to mixed pluralism, predicates denote predicable entities – entities that can be predicated of objects: possibly properties or concepts. In the statement of the pluralist semantics above I have talked of properties and relations rather than concepts, and I shall continue to do so in the upcoming discussion, but I do not intend to take sides on the kind of predicable entity mixed pluralism should make use of. On the other hand, superpluralism has it that predicates stand for higher-level pluralities – i.e. pluralities of pluralities.

Let’s briefly consider them one at a time.

### 2.2.3.1 Mixed pluralism

According to mixed pluralism, whereas terms pick out objects, predicates pick out predicable entities. Among the latter, there must be some which have plural argument positions, that is, which admit plural terms as arguments.

**Objection: Violation of Absolute Generality** The main problem with this approach is that we appear to lose the capacity to quantify over everything there is – recall that, as we saw in the first chapter, this is one of the main motivations for pluralism. However, as is well-known, this is a potential problem for any typed approach, since in these frameworks we lack a form of quantification that ranges over both the values of terms and those of predicates. Hence, in light of Florio’s result, it seems that this is a limitation any adequate extension of pluralist semantics will suffer from. Perhaps, one just needs to bite the bullet at this point.

Moreover, there is room to bypass this difficulty in this case. One would need to argue that whatever it is that expressions falling in higher types mean, they do not pick out additional objects. In the case of mixed pluralism, one would need to argue for a nominalist view of predicable entities. For example, one could argue that predicable entities are not objects of any sort, but rather are ways objects are (in the monadic case) or ways objects stand to each other (in the polyadic case) – where, importantly, ways are not objects.<sup>53</sup>

### 2.2.3.2 Superpluralism

There is another way to extend pluralist semantics: to resort to higher-level plural reference in the metalanguage. According to this view, whereas plural terms and singular predicates pick out pluralities, plural predicates pick out second-level pluralities, where second-level pluralities are, roughly speaking, pluralities of pluralities. Importantly, pluralities of pluralities are not supposed to be new objects, just as basic pluralities themselves are not supposed to be new objects.

Higher-level plurals will be the subject matter of the rest of this dissertation, thus I will only give a very brief presentation of the topic here, for the sake of discussing this semantics.

To get a more intuitive grasp of what higher-level plurals are, here are some alleged ordinary English examples: ‘those students and their professors’, ‘the specialists competing

<sup>53</sup>We mentioned this view in Chapter 1 (Section 1.3.2).

for the same jobs',<sup>54</sup> 'my friends, organised in two groups'. If they are to count as higher-level plurals, these expressions should be understood as denoting objects that are internally grouped in a certain way, but do so without picking out groups, sets or collections. Their only referents are first-order objects.

It is worth pointing out that there is no need to employ a full second-level plural language in order to give semantics for PL. In particular, we do not need quantification into second-level plural position for this purpose, but only second-level plural reference (since PL does not have quantification into predicate position).

Importantly, second-level pluralities are not predicable entities. That is, they are not the kind of entity that can be instantiated. Second-level plurals are not predicates; they are referring expressions.

According to this semantics, a predicate holds of a single object if, and only if, that object is among the objects denoted by the predicate; analogously, a predicate holds of some objects if, and only if, those objects are among the second-level plurality denoted by the predicate. Hence, we need as well as metalinguistic second-level plural terms, a meta-linguistic expression capturing the relation of membership between objects and pluralities and between pluralities and second-level pluralities. I shall use the expression 'is/are among' to indicate that an object belongs to a plurality and also to indicate that a plurality belongs to a second-level plurality. For example, take the plural term 'Venus and Serena' and the second-level plural 'the Philosophy students and their professors'. We say that Serena is among Venus and Serena; and we say that the Philosophy students are among the Philosophy students and their professors.

Given that nowadays Higher-Level Plural Logic is seen with scepticism, it is to be expected that superpluralism would be welcomed with the same incredulous stare. The legitimacy or lack thereof of Higher-Level Plural Logic will be the focus of the remainder of my thesis, thus I shall not stop to consider it here. For the sake of our arguments, let us suppose that higher-level plural reference is a legitimate device, which can be employed in semantic theorizing. Interestingly, the specific use to which we intend to put them here – i.e. as semantic values of predicates – faces challenges of its own. In what follows I consider the ones I take to be most pressing.

---

<sup>54</sup>This term should be interpreted as picking out the specialists who are competing for job A, those who are competing for job B, etc.

**Objection 1: Empty predicates** Firstly, there is the problem of vacuous predicates – predicates which hold of nothing. As we will see, just as there is no empty plurality, there is no such thing as the empty second-level plurality. Thus we appear to lack what would be the obvious intended semantic value for empty predicates.<sup>55</sup>

There are two ways we could go about approaching this issue. One would be to allow for terms of any levels to denote not only individuals/pluralities of any level, but also a dummy object we can call ‘nothing’. However, Florio (2010) points out that this strategy goes counter the view, usually held by the pluralist, that model-theoretic semantics should aim not only to capture adequate notions of logical truth and consequence, but also to assign intuitively correct interpretations to the expressions of the object language. He proposes, instead, that we allow for failures to refer in our semantics and that we take empty predicates as failing to denote. This seems indeed to be more in consonance with speaker’s intuitions. However, as Florio himself shows, this prevents some truth-conditional clauses from mirroring the logical structure of the sentences they interpret. In order to show this, he asks us to consider this passage: ‘We expected John and Mary to be two people who care for each other. But they are not what we expected them to be. That’s not surprising. There are no people who care for each other!’

Given that the last sentence says that the predicate ‘care for each other’ is empty, according to this view, the predicate fails to denote. But in that case we cannot interpret the first two sentences as meaning something like: (i) there is a second-level plurality of those who care for each other and we expected John and Mary to be among that second-level plurality, but (ii) John and Mary are not among the second-level plurality we expected them to be among. Hence, if we allow for failures of reference we need to further complicate the semantics.

Therefore, in this context, there appears to be a trade-off between capturing the logical structure of the object language in the semantics and capturing the intuitive semantic values of its expressions. Something has to give. As far as I can see, this is one of the limitations of the superpluralist approach to the semantics of predicates.

**Objection 2: Polyadic predicates** In the second place, things get murky when we consider not only monadic but also polyadic predicates. As we mentioned, Boolos’ pluralist account of M2OL is not readily extended to cover full 2OL. Accordingly, a superpluralist account of full PL is not readily available, given the presence of polyadicity. Which plurality

---

<sup>55</sup>I mentioned this objection in passing when considering property singularist semantics for plural language (Section 1.3.2).

corresponds to the predicate ‘is older than’ in (2.5)?

(2.5) *Harry is older than Hermione.*

And which second-level plurality corresponds to ‘teamed up against’ in (2.6)?

(2.6) *Harry and Hermione teamed up against Ron.*

Both relations are dyadic; the former takes singular terms in both positions and the latter takes a plural term in one position and a singular one in the other one. With regards to the former, we cannot just assign it the plurality of pluralities of people such that one of them is older than the other one, since this strategy would not allow us to distinguish, for example between (2.5) and ‘Hermione is older than Harry’. Turning to the latter, we encounter the same problem. We cannot assign it the plurality of all pluralities such that some of the objects in them teamed up against another object in them, since that would not be fine-grained enough to distinguish between (2.6) and ‘Harry and Ron teamed up against Hermione’.

A natural suggestion is that we assign them pluralities of ordered pairs. In the case of (2.5), the plurality of all pairs of people such that its first member is older than its second one. In the case case of (2.6), the plurality of all pairs such that its first member is a plurality of people and its second member is the person they teamed up against.<sup>56</sup> However, the need for pluralities of *pairs* demands that we go strictly beyond the resources offered by plural reference to basic individuals, thus casting doubt on the prospects of superpluralism.

\*\*\*

I conclude that at least at first sight superpluralism faces more important objections than mixed pluralism. Even though a lot more can be said about this topic, I will leave it aside in the rest of this thesis, since it is not our main concern. Our focus, from now on, is on referring expressions, not on predicates.

## 2.2.4 Taking stock

To recap, we have so far (in the previous chapter and in this section) considered four candidate semantics for PL without non-logical predicates: three forms of singularism (i.e. set-theoretic, property and mereological singularism) and the pluralist approach. The objections

---

<sup>56</sup>Note that the ordered pair needed in the case of (2.6) is not an ordinary one: it requires that one of its members is a plurality. However, this is unproblematic, since we can code these unorthodox ordered pairs using techniques already available to us, as shown in Linnebo and Rayo (2012, pp. 38-41). I am grateful to Øystein Linnebo for pointing this out to me.

faced by the singularist views have to do to a large extent with their inability to handle absolutely general and topic neutral discourse, discourse about everything and anything. Pluralism does not seem to face these difficulties, since it does not introduce any new entities liable to be members of the first-order domain of quantification. Moreover, we have surveyed the main objections raised against pluralism and showed how one may respond to them. Many of the rebuttals we have seen rely on a view of semantics as having among its aims that of correctly capturing speaker's intuitions. More generally, throughout our survey it has become clear that the pluralist and the singularist have different attitudes towards the import of ordinary language and the role it plays in semantic theorizing. I will return to this topic at the end of this chapter. Finally, when moving on to the discussion of semantics for full PL, we considered two accounts of the meaning of predicates: mixed pluralism and superpluralism. We concluded that the former is less objectionable and thus to be preferred, although not much that will be said in what follows will hinge on this.

## 2.3 Some remarks on ordinary plurals

To finish this chapter, I discuss three different issues having to do with the ordinary plural idiom. These are topics which will come up in our discussion of higher-level plurals, thus I shall occasionally refer back to this section throughout the rest of this thesis.

### 2.3.1 Order and repetition

The first issue we need to tackle arises most clearly in relation with plural terms in list form. These sort of terms may consist of a list of singular terms – e.g. ‘Harry, Ron and Hermione’, but also of a list of plural terms – e.g. ‘The historians and the philosophers’, or of a mixed list – e.g. ‘Harry and his friends’.<sup>57</sup>

I shall take lists as referring expressions – as expressions whose semantic contribution is to supply some objects.<sup>58</sup> In particular I shall interpret lists as functional expressions, where the conjunctive ‘and’ takes some referring expressions as inputs and delivers a new referring expression. Lists can have a variable length, thus the function sign ‘and’ is multigrade, i.e.

---

<sup>57</sup>As I will argue in Chapters 3 and 4, lists of plurals and mixed lists admit also of a higher-level plural interpretation. However, we focus here on their plural reading, that is, as simply denoting some objects.

<sup>58</sup>See Oliver and Smiley (2016, ch. 10) for a series of arguments to the effect that the understanding of lists as referring expressions is adequate.

takes a variable number of arguments.

These terms will prove especially important for us because the process of list formation admits syntactic iteration (understood as list nesting), thus giving rise, allegedly, to higher-level plurals.

Two important questions arise regarding lists: (i) what is the role of lists in conveying order?; and (ii) how does repetition of items in a list affect truth-conditions? Let us tackle them one at a time.

**Order** Sometimes sentences involving plurals in the form of lists are such that the order in which the lists are laid out has an effect on their truth-conditions. For example, the following appear to have different truth-conditions, even though their plural subject denotes exactly the same objects:

(2.7) *John and Mary arrived at school in that order.*

(2.8) *Mary and John arrived at school in that order.*

Since the notion of plural reference, as it stands, cannot account for such differences, some authors (in particular, Hewitt (2012)<sup>59</sup> and Ben-Yami (2013)) have proposed elaborations of the notion of plural reference which are capable of making distinctions with respect to order. Their accounts are similar in that they make plural terms be inherently ordered – in the sense that they denote objects in an order.

For example, Ben-Yami proposes that his notion of articulated reference<sup>60</sup> includes the feature of order of articulation. In other words, the order of articulation is part of the articulation of a certain term. The way in which he captures the idea that order does not always matter for truth-conditions is by arguing that, even though terms in list-form are always ordered, not all predicates are sensitive to their order:

Although articulation due to a structured phrase always articulates the reference in a certain order, this order is usually irrelevant to the meaning of what is said; but phrases such as ‘in that order’ or ‘respectively’ can make use of this order and affect the meaning of what is said. (Ben-Yami, 2013, p. 91)

---

<sup>59</sup>It should be noted that Hewitt has since then changed his views about this issue, as communicated in private conversation.

<sup>60</sup>Which I shall discuss in Chapter 4.

However, Florio and Nicolas (2015) have argued, in my view convincingly, that views which make order inherent to the reference of the terms cannot account for a wide enough range of cases. Their main limitation arises in cases where orders other than the one captured via the syntax of the term are relevant:

(2.9) *Harry, Ron, and Hermione arrived in the order they were called.*

In this sentence, two orders are relevant towards truth-conditions: the order in which Harry, Ron and Hermione arrived and the order in which they were called. (2.9) is true just in case the two orders are the same. Neither of them needs to coincide with the order in which they are listed in the subject. A proposal like Ben-Yami's cannot account for this, since it links each plural term with the order associated with its syntactic form – i.e. lists refer to some objects in the order in which they are listed.

Florio and Nicolas offer an alternative view which covers these cases too. I shall follow their lead on this. They argue that plural terms simply contribute according to the usual view on plurals, that is, by supplying some objects in a non-ordered fashion. Order arises only in the presence of certain special expressions – ‘in that order’ or ‘respectively’. What these modifiers do is supply some external indexings and it is by appeal to these indexings in the semantics that order is seen as playing a role in determining truth-conditions. An indexing is a function  $f$  from a plurality of indices  $ii$ , related by a salient order  $<$ , to a plurality of objects. The function  $f$  is multivalued and surjective. An indexing matching is defined as follows. Let  $f$  be an indexing of  $xx$  based on  $(ii, <_1)$ , and let  $g$  be an indexing of  $yy$  based on  $(jj, <_2)$ . We say that  $f$  matches  $g$  with respect to identity when these two conditions are satisfied:

- (i) There is an order-preserving isomorphism  $H$  between  $(ii, <_1)$  and  $(jj, <_2)$ .
- (ii) For any  $i$  in  $ii$  and  $j$  in  $jj$ , if  $H(i, j)$ , then  $f(i) = g(j)$ .

For example, consider sentence (2.9) again. There are two salient indexings in it. One,  $f$ , tracks the order in which Harry, Ron and Hermione arrived. The other one,  $g$ , tracks the order in which they were called. Let us take as indices the first three positive integers (with their usual ordering). And suppose that  $f(1) = \text{Hermione}$ ,  $f(2) = \text{Ron}$  and  $f(3) = \text{Harry}$  and that  $g(1) = \text{Harry}$ ,  $g(2) = \text{Ron}$  and  $g(3) = \text{Hermione}$ . Now (2.9) is true if, and only if,  $f$  matches  $g$  with respect to identity. First, the relevant order-preserving isomorphism between the indices used by  $f$  and  $g$  is the trivial automorphism. Let's call it  $H$ . Is it the case that for any  $i$  and  $j$  such that  $H(i) = j$ ,  $f(i) = g(j)$ ? In this case,  $f(1) = g(1)$ ,  $f(2) = g(2)$  and  $f(3) = g(3)$  are false, thus (2.9) is false.



I shall follow Florio and Nicolas on this and take lists as being non-ordered. From now on, we shall leave the issue of order aside and rely on the availability of the devices just described when necessary.

**Repetition** Next we need to look at repetition of items within a list. In natural language, repetition within a list is often used for emphasis:

(2.10) *The government's priorities are education, education and education.*

Obviously, this case does not demand that we invoke multiple reference in its truth-conditions. Rather, this construction appears to introduce a conversational implicature – i.e. that the government's top priority is education. Nevertheless, there are cases that do seem to demand that we take repetition as playing a role in determining truth-conditions. Consider, for example:

(2.11) *The students' scores were 5, 1, 1 and 1.*

If we were to take '5, 1, 1 and 1' to denote the grades 5 and 1 simpliciter, we would get the wrong truth-conditions.<sup>61</sup> What mechanism should be employed in the semantics in order to account for this phenomenon? Florio and Nicolas argue that their account of order should be applied to repetition as well. Once again, the idea is that we need not distinguish between the semantic value of lists involving repetition and those co-extensive lists which do not (or which involve a different sort of repetition), since all of them make the same contribution to truth-conditions: they denote the same objects. Thus, the list '5, 1, 1 and 1' and the list '5 and 1' denote the same plurality of numbers, that of 1 and 5.

But by, once again, bringing in external indexings and given that indexings may be non-injective, we can account for the differences between (2.11) and (2.12):

(2.12) *The students' scores were 5 and 1.*

The semantic difference between these sentences would rest on the fact that, even though both '5, 1, 1 and 1' and '5 and 1' denote the numbers 1 and 5, the predicate 'The students'

---

<sup>61</sup>Moreover, there are terms that involve what we could call 'accidental repetition'. Consider the following sentence: 'The lecturer and the grader have different salaries.' And suppose that the lecturer and the grader happen to be the same person, who has two different jobs. In this case, we would not obtain the correct truth-conditions by saying that whoever is the lecturer (and the grader) has different salaries, since this seems false. Moreover, we cannot substitute 'the lecturer' with 'the grader' without a change in truth-conditions. Accidental repetition involves a form of intensionality, which I shall call 'restricted reference'. This topic shall be explored in detail in Chapters 5 and 6 and I shall leave it aside for the moment.

scores were' holds of a plurality relative to an indexing and each sentence involves a different indexing,  $f$  and  $g$ . Using again positive integers as indices, on the one hand,  $f(1) = 5$ ,  $f(2) = 1$ ,  $f(3) = 1$  and  $f(4) = 1$ , and, on the other,  $g(1) = 5$  and  $g(2) = 1$ .

Note that when it comes to repetition, the indexing approach, as it stands, results in over-representation. Let me explain this with an example. In the case of (2.11), we would have to conclude that the indexing  $f$  above and the indexing  $f'$ , where  $f'(1) = 1$ ,  $f'(2) = 5$ ,  $f'(3) = 1$  and  $f'(4) = 1$  determine different truth-conditions. This would be a case of over-representation, since order is irrelevant in (2.11) (at least, there is available a reading under which it is irrelevant). Nevertheless, this can be amended by taking the predicate as holding of a plurality relative to any indexing of a class of isomorphic indexings (with respect to the number of indices that are mapped to each object).

Despite the two phenomena being tractable in the same way, I believe there is an important difference between order and repetition. As opposed to order, whether syntactic repetition is semantically relevant or not does not in general depend on the predicate involved. In other words, lists are such that if they involve (or not) multiple syntactic occurrences of co-referential terms, that has an effect on truth-conditions by default. By contrast, the same does not obtain for order: lists are not such that the syntactic order of their components has an effect on truth-conditions by default. The latter needs to be triggered by a modifier in the predicate; the former needs no such triggering. In other words, lists are always assumed to convey multiplicity, but not order. For instance, the list 'Harry, Hermione and Ron' is substitutable by 'Hermione, Ron and Harry' in all contexts, except for those that involve modifiers such as 'in that order'. By contrast, the same list is in general not substitutable by 'Harry, Harry, Hermione, Ron, Hermione and Harry'. In fact, I cannot think of an example in which this would be allowed, but we can leave the door open to there being some predicates which would – they would be predicates involving a modifier to the effect that syntactic repetition must be ignored. Our point still holds.

We see this in example (2.11) above. There is no modifier indicating that multiple objects are indeed contributing to truth-conditions and, indeed, no such modifier is needed.

Moreover, using indices is not necessary when it comes to repetition. Recall that the main motivation for adopting the indexing approach comes from cases in which there are orderings relevant to truth-conditions distinct from the one captured by the syntactic order of the list. We do not seem to find analogous cases when it comes to repetition. *Prima facie*, an analogous case would be:

**(2.13)** *Nadal has won the Men's US Open one more time than the Men's Wimbledon.*

In which the predicate points to two co-extensional pluralities each involving a different number of occurrences of Nadal. However, this example is not really analogous to the one in (2.9), in the sense that it is a case in which a certain object, Nadal, carries out a number of different actions, and not a case in which more than one object (e.g. Nadal, Nadal and Nadal) carry out different actions collectively (in (2.9) some objects collectively carried out an action in a certain order). Thus, there is no need to compare different indexings at all. We can interpret (2.13) as involving a complex singular predicate which, as such, may be satisfied by Nadal alone. I have not been able to come up with any example of a collective plural sentence involving competing repetitions. My working hypothesis is that, unlike cases involving competing orderings, these cases are not at all common.

Overall, I conclude that while order demands something like the indexing approach, repetition does not. Judging from the workings of ordinary English, we would be entitled to take plural terms as possibly denoting the same object multiple times and thus to liberalize the notion of plural reference accordingly. Nevertheless, given that we are already equipped with the indexing device and in order not to further complicate our formalism, we will also make use of it whenever repetition is relevant.

### 2.3.2 Pseudo-singularity

A term is pseudo-singular if, and only if, it is syntactically singular and semantically plural. Oliver and Smiley (2016) have given, among others, the following examples of pseudo-singulars: ‘the couple’, ‘this pair’, ‘the deck (of cards)’, ‘that class’, ‘the plurality’ and ‘the committee’.

Pseudo-singularity plays an important role in the debate around plurals. For instance, our use of the term ‘plurality’ is only justified if its pseudo-singular reading is indeed defensible. Moreover, the notion is important in the debate on higher-level plurals too. Recall that, roughly speaking, a higher-level plural is the plural of a plural. Ordinary languages such as English do not admit the syntactic iteration of the plural suffix, a fact from which many have concluded that higher-level plurals are not found in this language. However, if it were the case that some singular-looking expressions were really plural, given that we could syntactically pluralize them, the resulting expressions would appear to be higher-level plurals: ‘the couples’, ‘these pairs’, ‘the decks’, ‘those classes’, ‘the pluralities’, ‘the committees’. This would be a strong argument for the legitimacy of higher-level plural reference.

In this section I shall give two reasons to think that, indeed, these singular-looking ex-

pressions are plurals in disguise.

**Plural Override** First of all, pseudo-singulars display an unusual grammatical behaviour. In the words of Max Black:

It is a peculiarity of English and many other languages to admit collective expressions such as ‘the Hungarian Quartet’ and ‘the Cabinet’ which are allowed to behave, at least part of the time, as if they were singular names or descriptions. [...] This ambivalent role of such expressions is betrayed in the absence of firm rules for their grammatical ‘number’ – thus [...] an expression like ‘the Cabinet’ can be indifferently followed by a verb in the singular or the corresponding plural. (Black, 1971, pp. 631-2)

Pseudo-singulars admit what is called ‘plural override’: while they are usually accompanied with singular determiners, they admit a plural verb and plural anaphoric reference back to them. For example, whereas one would usually say ‘this committee’ or ‘this couple’, the following plural predications are correct:

(2.14) *The committee are unanimously in favour of the decision.*

(2.15) *The couple who live on the third floor get along very well.*

And here are some examples of plural anaphora linked to syntactically singular terms:

(2.16) *The committee is meeting this morning. They will most likely veto our proposal.*

(2.17) *That couple lives in my building. They got married last month.*

Note that plural override is most common with animate collective nouns. By contrast, inanimate collectives usually trigger a singular agreement:

(2.18) *The crockery was lying on the table.*

Moreover, some contexts favour plural override. For example, contexts in which terms are modified by a numeral:

(2.19) *Three crew have arrived late this morning.*

Or when they are modified by determiners that select a plural count noun, such as ‘many’ or ‘all’:

(2.20) *Many crew have arrived late this morning.*

(2.21) *All the team have done their best.*

While this can be found in British English, it is not so common in American English.<sup>62</sup> However, it can also be found in other languages, mostly in its anaphoric form (e.g. in Catalan, Spanish, French or Latvian.)<sup>63</sup> Hence, it turns out that, even according to surface form, there is something odd about those singular terms. They do not display the features of most syntactically singular terms. This gives us a *prima facie* reason to think that they are not as they appear.

**The one-many identity puzzle** There is another argument in favour of pseudo-singularity. It is triggered by the puzzle posited by one-many identity statements.

Whitehead and Russell were a pair of logicians, and a suit is thirteen cards. But how can the two men be one pair? How can thirteen cards be one suit? How can the same thing (things) be one and many? (Oliver & Smiley, 2016, pp. 306)

The problem concerns a well-known metaphysical question: how can a relation of identity hold between one and many things? Oliver and Smiley's suggestion is that the problem is only apparent; it derives from a linguistic confusion: the apparently singular terms involved in one-many identity claims are actually plurals in disguise - they are pseudo-singulars. For example, 'a pair of logicians' picks out two people; the same goes for 'a suit', which refers to thirteen cards. Hence, despite appearances, one-many identity statements express plural identity claims, which are unproblematic – usually defined in terms of singular identity as captured by my formal definition of plural identity in Chapter 1: the *x*s are identical with the *y*s if, and only if, each of the *x*s is identical with one of the *y*s and vice versa. The fact that we can explain away the one-many identity puzzle by appealing to pseudo-singular reference provides substantial evidence for the legitimacy of the latter.

Hereafter, I proceed on the basis that pseudo-singularity is a legitimate linguistic phenomenon.

---

<sup>62</sup>See (Oliver & Smiley, 2016, p. 305).

<sup>63</sup>This has been reported by native speakers of these languages.

### 2.3.3 Empty and singular pluralities

Finally, we need to decide whether there are such things as pluralities which are empty or which consist of a single object.

Of course, plural terms may denote a single object or nothing. For instance, ‘Harry’s friends’ whenever Harry has a single friend or no friends at all. The question is whether we would still say that those terms refer to some objects (or to a plurality) in those cases. In other words, should Plural Comprehension be formulated so as to imply the existence of pluralities which are empty or singular? Of course, we already answered this when describing our formalism. It follows from Plural Comprehension that there are indeed singular pluralities, and it follows from Non-Emptiness, that there are no vacuous ones. Here I want to provide reasons for these decisions.

On the one hand, there is a reason why vacuous reference should not be considered a species of plural reference. It follows from the pluralist understanding of plural reference as ontologically innocent. Since ‘a plurality’ is understood as short for ‘some objects’, in the absence of objects there can be no plurality. Contrast it with the notion of set. A set is understood as being an entity in its own right, that is, distinct from its members (individually and collectively); it is precisely this being something over and above some objects that enables us to speak of the empty set. Since sets ‘add something’ to their members, we can think of this additional thing separately from its members.<sup>64</sup> This explains the naturalness of the notion of empty set. By contrast, to repeat, this is not the case when it comes to plural reference. In this case, if the objects are gone, so is the plurality. This explains why we find it unnatural to talk about an empty plurality and suggests that, indeed, there is none. Empty reference is not plural reference.<sup>65</sup>

Note that this is not to say that plural terms cannot fail to denote. Everything just said is compatible with endorsing a free version of PL, where terms may lack referents altogether. Although my formalism is not that of a free logic, it can be modified to allow for failures of reference. Oliver and Smiley (2016, ch. 13) and Rayo (2006) have proposed two different versions of a free PL. Rayo admits that terms may fail to denote while dismissing the possibility that a predicate may hold of a term which fails to refer. By contrast, Oliver and Smiley admit both. They use the term ‘zilch’ as a term which is empty by logical necessity. This term can be taken as primitive or else as defined in a plural language. For instance, we can

<sup>64</sup>This corresponds to the lasso-conception of sets as described in Lewis (1991, pp. 42-45).

<sup>65</sup>Even though this view is widely accepted in the literature, there are some dissonant voices. See, for example, Burgess (2004).

say that ‘zilch’ is short for ‘the non-self-identical things’. Oliver and Smiley offer a number of arguments and examples to show that it is possible for a predicate to be true of what a certain term denotes, even if that term fails to denote anything (e.g. ‘are non-existent’). Similarly, they argue that functions may be co-partial, that is, may assign something to zilch (e.g. ‘Everything but’ in ‘Everything but the present King of France’). Importantly, zilch is not supposed to be a dummy object – some sort of null thing drawn from an extraneous domain. Rather, ‘zilch’ fails to denote of necessity, thus when we say that a term denotes zilch, what we mean is that it fails to denote anything.<sup>66</sup>

In any case, saying that plural terms may fail to denote is very different from saying that there are empty pluralities. To see this, think about a free set-theoretic object language. In this language, a term would be able to denote the empty set but might also fail to denote altogether. Plural language does not have something analogous to the former, since there is no such thing as the empty plurality.

On the other hand, there are a number of reasons why singular reference should be considered a species of plural reference and, thus, plural reference should be understood as being non-strict (i.e. as being either strictly plural – involving at least two referents – or singular). To begin with, the view that the basic semantic notion is that of plural reference broadly understood, of which singular reference is a special case, is coherent with the intuition, expressed for example in Simons (1982) and McKay (2006, pp. 27-32), that the fact that singular reference has been traditionally privileged and, indeed, taken as the only sort of reference is an accidental and arbitrary development, resulting from an unjustified bias against the plural. Why should reference to one object be considered more salient than reference to two, three or any other number of objects?

Secondly, taking the singular to be a species of the plural receives support from the observation that there is nothing special about singular predicates – predicates which take singular terms as arguments. In fact, they are just a limiting case of a variety of distributive predicates, the other end of which being populated by collective predicates. Predicates which distribute down to sub-pluralities of objects of a certain plurality are just as common as those that distribute down to individuals.

Moreover, treating the singular as a species of the plural adds simplicity to the resulting logic since it opens the door to dispensing with two types of variables, thus making for a simpler picture (i.e. one that involves less expressive devices). Supposing that plural vari-

---

<sup>66</sup>Since nothing that we will say hinges on going for a free or a non-free language, we will stick to a non-free language for simplicity.

ables are understood non-strictly, as possibly referring to a single object, we can define the notions of singularity and of singular existence as follows:

**(2.22) Definition (Singularity)**  $S(uu) := \forall xx(xx < uu \rightarrow uu < xx)$ .

**(2.23) Definition (Singular existence)**  $\exists x\phi(x) := \exists xx(\phi(xx) \wedge S(xx))$ .

Thus we could reduce our primitive logical symbols according to this definition (although from a practical point of view it is preferable to retain the notational distinction between singular and strict plural variables). In any case, the important observation is that we could paraphrase every expression involving singular reference with a non-strict plural expression, showing that the only necessary device is that of non-strict plural reference.

A fourth reason to take the singular as a limiting case of the plural is the asymmetry between cases where plural terms accidentally denote a single object and cases where singular terms accidentally denote more than one object. For example, consider the sentence

**(2.24)** *The new neighbours moved in today.*

And suppose I happen to have only one new neighbour. It seems that in this case, (2.24) would not be false, but simply infelicitous. That there are more than one neighbour is a mere conversational implicature, rather than part of the truth-conditions of the statement. This is supported by the fact that one can cancel this implicature: ‘The new neighbours moved in today. That is, the only new neighbour did.’ Moreover, one can also reinforce the implicature without sounding redundant: ‘The new neighbours moved in today. And, yes, there is more than one of them.’

The converse does not seem to hold. If I happen to have more than one new neighbour and I utter:

**(2.25)** *The new neighbour moved in today.*

then it seems I have done something wrong that I had not done in the previous case. At the very least, the implicature that there is a single neighbour living upstairs is not readily cancellable: ‘The new neighbour moved in today. Well, there is more than one of them’. The second clause looks like it is attempting to correct a mistake, rather than merely cancelling an implicature. Moreover, the sentence cannot be reinforced without redundancy: ‘The new neighbour moved in today. And, yes, there is only one new neighbour.’ This shows that plural reference can be used to convey general information about grammatical number, not ruling out the singular case. This suggests, once again, that singular reference is a special case of plural reference.



I conclude that, at the very least, there is a sense in which plural reference has singular reference as a limiting case and this will be the sense relevant to the rest of this dissertation. Plural reference, from now on, is non-strict.

## 2.4 Conclusions

This and the previous chapter are the only ones exclusively devoted to plurals. From now on, I turn my attention to higher-level plurals. These two chapters will serve as the basis of what follows in two different ways. On the one hand, the decisions made in the last section in connection with order and repetition, pseudo-singularity and the relation between vacuous, singular and plural reference, will be important at various points in Chapters 3-7. On the other, my arguments in Chapter 1 and in the first part of this chapter should have convinced the reader who shares the pluralist's assumptions that pluralist semantics are adequate for PL; and thus that an investigation of an extension of PL from a pluralist perspective is, at the very least, not doomed from its very beginning.

But what are these assumptions? Both the objections we raised against the singularist proposals and our defence of pluralism were based on certain assumptions concerning the role and aims of semantic theorizing and the importance of natural language for legitimizing formal languages. Even though it is not my intention to convince the reader that these assumptions should be accepted, it is important to bear them in mind, since the rest of this thesis will start out from the acceptance of pluralism and thus will be most relevant to those who share some of these tenets.

One of the main ideas guiding the pluralist is that

**(2.26) Principle (Aim of semantics)** *Formal semantics should aim to capture speakers' intuitions about the meaning of the object language expressions.*

This principle is made precise by two different desiderata. One is that our semantic theory should assign the intuitively correct semantic values to expressions. As expressed by Rayo:

[T]here is more to model-theory than a characterization of logical consequence. Conspicuously, model-theory might be thought to deliver a generalized notion of reference [...]. (Rayo, 2006, p. 244)

The other one is that truth-conditional clauses should mirror the surface structure of the corresponding object language expressions. As we saw when surveying superpluralist semantics for plural predicates, these two requirements are not always compatible. However, whenever compatible, I will assume both to be desirable from the point of view of the pluralist.

Note that if we are to take seriously speakers' intuitions about their own language, then we must be able to assign quantifiers the absolutely general domain. Thus a consequence of (i) is the following:<sup>67</sup>

**(2.27) Principle (Absolute Generality in Semantics)** *Formal semantics should capture absolutely general interpretations of the object language.*

The requirement of generality plays a central role in the debate around plurals and, as we will see, also in that around their higher-level counterparts.<sup>68</sup> This is especially interesting for us since it seems to make many of the arguments laid out compelling for the formal semanticist as well. From the viewpoint of linguistics, the requirement that ordinary expressions apparently denoting everything be taken at face value in the semantics seems important. Admittedly, the formal semanticist is not concerned with capturing the speaker's intuitions in the semantics, in general. However, cases involving absolutely general discourse appear to be more pressing than others. For example, think about a term denoting a plurality too big to form a set, such as 'the ordinals'. The formal semanticist who adheres to set-theoretic singularism does not have available the obvious choice of set, on pain of contradiction. Nevertheless, she still needs to provide an alternative semantic value for 'the ordinals'. The worry is two-fold. On the one hand, that for whatever set we assign to this term, there will always be an ordinal which is not in it. On the other, that the singularist will need to provide a way to find restricted denotations for general terms in a principled manner, and this does not seem an easy task.

Leaving the role of semantics aside, the baggage of the pluralist contains a second important guiding principle, which I have also mentioned in passing:

**(2.28) Principle (Justification from natural language)** *The presence of an expression in ordinary language supports the claim that the corresponding class of formal expressions (interpreted at face value) is legitimate.*

<sup>67</sup>In turn, this relies on certain assumptions, for instance on the assumption that the all-inclusive domain is not indefinitely extensible, in the sense that there is a definite plurality which consists of everything. See Rayo and Uzquiano (2006, pp. 4-6) for an overview of this topic.

<sup>68</sup>That Absolute Generality should be adopted is something I shall not argue for. For a compilation of arguments for and against it, see Rayo and Uzquiano (2006).

For example, this is the way in which Boolos in his seminal papers appears to make use of the fact that English contains plural reference. Of course, this idea is connected with principle (2.26), according to which formal semantics should aim to capture speakers' intuitions concerning the meaning of their own language. The reason why the pluralist takes the presence of an expression in natural language as speaking in favour of adopting its formal counterpart is that she takes natural language (in general) as providing enough clarity for formal purposes. This optimistic view towards the precision of ordinary idioms is the same which leads her to aim to match speaker's intuitions in her semantic theorising. Since ordinary language is, in general, reliable enough for formal purposes, semantics for formal languages are most accurate when approximating ordinary speaker understanding.

Finally, although it has not played any role in this chapter, there is another principle guiding some pluralists. It is the endorsement of certain metaphysical views, in particular, of certain forms of nominalism. Although this has indeed been an important element in the debate around plurals, I have chosen to give it a secondary role in this thesis. None of the crucial arguments given from now on will rely on the prior acceptance of any form of nominalism, although readers of a nominalist inclination shall have an additional motivation for engaging with the project of developing and defending Higher-Level Plural Logic.

## **Part II: Superplurals**



# Chapter 3

## Superplurals: An Elucidation

### 3.1 Introduction

Some advocates of PL believe that it can be extended by adding higher-level plural (hereafter, ‘HLP’) terms and quantifiers to it. Importantly, second-level plurals do not stand in predicate position with respect to plurals, but rather stand to them as the latter stand to singulars (analogously for higher levels). They are the result of iterating the step from the singular to the plural; they are pluralized plurals. Higher-Level Plural Logic (hereafter, ‘HLPL’) is supposed to be a lot more expressive than PL while inheriting some of its key characteristics – most crucially, its ontological innocence. It is because of this alleged gain in expressive power free of any additional ontological costs that HLP reference and quantification have attracted attention in the recent literature on plurals.

In order to follow more easily the upcoming discussion, it may be useful for the reader to know which English terms have a good claim to being higher-level plurals. Of course, English does not admit iteration of the application of the plural suffix (typically, ‘-s’ or ‘-es’), thus it does not have terms like ‘objectses’ or ‘thingses’. However, English has other terms which plausibly fall within the category of HLP terms. These terms are lists of plurals (e.g. ‘the students and their professors’), definite descriptions with pseudo-singular head nouns (e.g. ‘the couples living in the building’), plural definite descriptions built from collective predicates (i.e. ‘the specialists competing for the same jobs’) and plurals accompanied by certain appositive phrases (i.e. ‘the kids, organised in two groups’). I will discuss each of these forms in detail in the next chapter (as well as argue that they indeed should be counted as HLP), but when I speak of HLP terms from now on, it will be useful for the English-speaking reader to think of them as being of one of these sorts.

Interestingly, friends of plurals are divided when it comes to the legitimacy of the notion of HLP reference. Some, like Rayo (2006) and Oliver and Smiley (2016) have embraced it and developed formal systems which incorporate it. They claim that HLPL gives us the expressive power of a type theory and of a form of set theory, respectively, without committing us to anything other than some individuals. By contrast, others, for instance Lewis (1991), McKay (2006) and Uzquiano (2004), have argued against its availability.

Moreover, not all authors who admit higher-level plurals see them as good news for pluralism. Some authors have appealed to the fact that PL is extendible to HLPL to argue against the alleged advantages of PL. Most notably, Linnebo (2003) and Linnebo and Rayo (2012). The view that HLP quantification is bad news for pluralism turns on the observation that if we accept the legitimacy of additional levels of plurals, first-level plurals no longer appear to be as expressively powerful as they might have seemed. Once an infinite hierarchy of higher-level plurals is accepted, the very same objections raised against type theory with respect to its capacity to capture absolutely general discourse can be turned against HLPL. Briefly, since we do not have quantification across all levels, it seems that we are unable to formalize discourse about all levels within HLPL.<sup>69</sup> Moreover, once HLPL is in place, PL seems to lose some of its applications. For example, while one might have thought that plural talk sufficed to speak of all sets, and thus to replace proper-class talk, the availability of HLP talk suggests that we can go beyond proper-class talk. But in set theory there is no talk of classes of proper-classes, since proper-classes are assumed not to be members of other classes. Hence plurals would turn out to be unsuitable to regiment proper-class talk in the first place.

My view is an optimistic one. I believe that some philosophical endeavours would benefit from making use of an HLP formalism. However, arguing for this claim is not at the center of this dissertation. I will only turn to it in the last chapter, where I will defend HLPL from some of the objections just mentioned and I will advocate for its usefulness in a neo-logicist project. Instead, my main aim is to argue that, for better or for worse, the acceptance of first-level plural logic inevitably leads one to the acceptance of its higher-level counterparts. To sum up, my stance is, primarily, opposed to those pluralists who dislike higher-level plurals or think they can safely disregard them and, secondarily, it is opposed to those who think HLPL has nothing new to offer or that it speaks against PL itself.

As mentioned in the Introduction, the notion of HLP reference has been received with significantly less enthusiasm than its plural counterpart by the philosophical community. Not

---

<sup>69</sup>We return to this objection in Chapter 7.

only has its presence in natural language been doubted, but its very intelligibility has been questioned: supposing English had expressions like ‘objectses’ or ‘objectseses’, what could they possibly mean (if anything)?

An interesting exception is that of Russell (1903). Even though Russell does not elaborate much on the issue, he briefly considers the idea of HLP reference when looking into the notion of class-as-many as applied to higher types – i.e. to classes of classes. According to a plausible interpretation of Russell’s work, a class-as-many is simply a plurality of objects. Interestingly, in *Appendix A*, Russell appears to be sympathetic towards a higher-level notion of class-as-many. He writes:

Thus a class of classes will be many many’s; its constituents will each be only many, and cannot therefore in any sense, one might suppose, be single constituents. (Russell, 1903, Section 489)

In the same page, he raises a complaint (for the sake of his own argument):

The main objection to this view, if only single terms can be subjects, is that, if *u* is a symbol standing essentially for many terms, we cannot make *u* a logical subject without risk of error. We can no longer speak, one might suppose, of a class of classes; for what should be the terms of such a class are not single terms, but are each many terms. We cannot assert a predicate of many, one would suppose, except in the sense of asserting it of each of the many; but what is required here is the assertion of a predicate concerning the many as many [...]. (*ibid.*)

In this passage, Russell seems to be saying that higher-level pluralities appear problematic because we seem incapable of expressing the idea that a plurality is one of some pluralities, which he sees as a consequence of the fact that we appear, more generally, incapable of predicating something collectively of some objects. However, he ultimately takes the argument laid out in the above quotation as a *reductio* of the alleged impossibility of collective predication:

The logical doctrine which is thus forced upon us is this: The subject of a proposition may not be a single term, but essentially many terms; this is the case with all propositions asserting numbers other than 0 or 1. (Russell, 1903, Section 490)



Here he is referring to collective predicates such as ‘are two in number’.<sup>70</sup> Despite my sympathy towards Russell’s views, I believe more work – that is, besides defending the notion of collective predication – needs to be done if HLP reference and quantification are to become respected apparatuses in logic and philosophy. In this and the next chapters I undertake this task.

More specifically, in this chapter, I present and survey the debate around higher-level plurals. In Section 2, I give a precise presentation of a formal HLP language, its proof theory and the semantics I favour. In Section 3, I describe the usual sceptical response towards higher-level plurals. Finally, in Section 4, I offer an elucidation of the notion of HLP reference with the aim of assuaging some of the concerns with respect to it. I will turn to a pluralist defence (i.e. a defence based on the prior acceptance of pluralism) of HLPL in the next chapter.

## 3.2 Higher-Level Plural Logic

As far as I am aware, only two HLP formal languages have been developed to date. They can be found in Rayo (2006) and in Oliver and Smiley (2016). The main difference between them is that the former has infinitely many types of terms, whereas the latter makes use of a single type of term that can denote individuals, first-level pluralities, second-level pluralities and so on.

As one would expect, the question of which setting is to be preferred, the typed or the untyped one, does not have a straightforward answer. Even though I believe there is nothing fundamentally defective with either path, I shall opt for the typed approach. One reason to do so is that the typed approach allows us to make explicit in the syntax the idea, held by defenders of HLPL, that all we need in order to get more expressive power are new expressive resources – there is no need for ontological proliferation. In particular, all we need are new ways of referring to our basic objects: second-level plurally, third-level plurally, etc. In an untyped framework, we could still argue that there is no ontological proliferation by using a typed metalanguage to express the semantics of the HLP object language. Alternatively, we could appeal (in an untyped way) to first-level pluralities, second-level pluralities, etc.

---

<sup>70</sup>Russell’s attitude towards higher-level plurals appeared to change after *The Principles of Mathematics*. See Klement (2014) for an argument that his scepticism towards higher-level plurals is one of the reasons why he eventually abandoned his view on classes as being classes-as-many, which was ultimately to be replaced with the no-classes theory of the *Principia*.

as semantic values of the terms employed in the object language and argue that our singular metalanguage is shorthand for an artificial HLP extension of English (for example, that ‘a second-level plurality’ is short for ‘objectses’). However, in either case, the claim of ontological innocence would not be explicitly captured by the surface form of the object language, thus possibly losing plausibility.

An important characteristic of my hierarchy of types is that it is cumulative. And, in fact, it is cumulative in two ways. Firstly, it is cumulative with respect to plural membership. A reason for adopting a membership-cumulative hierarchy is given by the fact that we need to regiment not only expressions such as ‘the students and their lecturers’, but also mixed expressions such as ‘Carla and her students’ or ‘Russell and Whitehead, Hilbert and Bernays, and Frege’. If these terms are to be counted among the HLP terms,<sup>71</sup> then we need to allow that not only do they have components of an immediately lower level, but also of any lower level. For example, in my hierarchy, as we will see, ‘Carla and her students’ is of level 2, ‘her students’ is of level 1, and ‘Carla’ is of level 0. This suggests that our types are cumulative in the sense that expressions of the form  $x < y$  are well-formed whenever  $y$  belongs to a level higher than that of  $x$ , but not necessarily the level immediately above that of  $x$ .<sup>72</sup>

Moreover, I think the hierarchy of plurals should be further liberalized by letting terms of level  $k$  denote individuals and pluralities of up to level  $k$ . This is another way in which the hierarchy of HLP expressions is cumulative. I will refer to it as ‘reference-cumulativity’. This is motivated by our claim (to be defended in Section 3.4) that plural reference of level  $k$  is a species of reference of level  $k + 1$ . In other words, that terms of level  $k + 1$  may function as terms of level  $k$  (e.g. terms that may denote pluralities may also denote a single thing, terms that may denote second-level pluralities may also denote a single plurality or a single thing, etc). Accordingly, our view as to the meaning of HLP expressions differs from the view endorsed in Williamson (2003) with respect to higher-order expressions in the sense that for us, the meanings of terms that belong to different levels need not be fundamentally different; some of them are species of others. They are thus different in that each level opens up the possibility to refer in a new way, but it does not turn its back on the modes of referring introduced at lower levels. Thus, expressed in a somewhat misleading (but idiomatic) way,

---

<sup>71</sup>To see that these terms are plausibly understood as HLP, note that there are predicates which can function as applying collectively to mixed lists, even though the lists do not admit a mere plural reading. For example, ‘Carla and her students admire each other’, understood as expressing that Carla admires her students and they admire Carla in turn (but they do not admire each other), would be one such case. We will see more examples throughout this dissertation.

<sup>72</sup>We will liberalize further the conditions on well-formed formulae involving  $<$ . This is only the first step in said liberalization, as it were.

our typed picture is as follows: terms are typed, but types only put an upper bound as to the sorts of things (i.e. individuals, first-level pluralities, second-level pluralities, etc) the terms in them can denote.

Having introduced reference-cumulativity, any term of the form ‘ $k$ th-level plurality’ becomes ambiguous between a non-cumulative and a cumulative reading. According to the former, it denotes pluralities which are only of one sort (the ones that are introduced at that level). According to the latter, it can denote pluralities of any level up to  $k$ . In order to disambiguate between these two understandings, I will often talk of ‘strict  $k$ th-level pluralities’ (i.e. non-reference-cumulative) and ‘non-strict  $k$ th-level pluralities’ (i.e. reference-cumulative) (analogously for ‘ $k$ th-level plural reference/term’).

On a related note, whenever I speak of HLP terms or reference I mean it in a general sense, unless otherwise indicated: as including terms and reference of all levels, including the basic plural level and the singular level. The same holds when I talk of higher-level pluralities: these include pluralities of all levels, including the basic level as well as single objects, unless otherwise indicated.<sup>73</sup>

One may wonder whether predicates are typed. My favoured answer is that they are not. Neither logical nor non-logical predicates are syntactically typed: they can take terms of any level in any of their positions. This goes hand in hand with the fact that I treat predicates as genuinely distinct from HLP terms (that is, as I said, I will not pursue the superpluralist route to semantics for predication presented in Chapter 2). Thus the reasoning behind syntactically typing terms does not transfer to the case of predicates. However, as I explain below (Sec. 3.2.4), they are semantically typed in the sense that they denote higher-order entities over which first-order singular or plural quantifiers cannot range.

In what follows I present the languages  $\mathcal{L}_{\text{HLP}^-}$  and  $\mathcal{L}_{\text{HLP}}$ . Analogously to the plural case, the former does not contain non-logical predicates other than the first-order ones,<sup>74</sup> but

---

<sup>73</sup>Having membership and reference-cumulativity is another of the reasons why the relation of plural membership is reflexive. According to my account  $<$  must be interpreted as being analogous to set-membership, except that we say that any given higher-level plurality is a member of itself. Thus it is reflexive. Since we can refer to, for example, an individual with a first-level plural term, it seems one may want to express the idea that the plurality denoted by that term has a single individual member. In other words, that the plurality consists of a limiting case of plurality and it is an individual (recall, an individual and its plurality are one and the same thing). But since we can also refer to that individual with a singular term, and we want it to be the case that the membership relation is retained regardless of which terms we use to refer to individuals and pluralities, we must allow that every individual (more generally, every higher-level plurality) is a member of itself.

<sup>74</sup>In this case, predicates will thus be syntactically typed in the sense that they admit only singular terms as arguments. But this will be abandoned when moving on to  $\mathcal{L}_{\text{HLP}}$ .

the latter does. Note that in the description of these languages I abandon the notation used in Chapters 1 and 2 for singular and plural terms (i.e.  $u$ ,  $uu$ ) and I use superscripts to mark the level to which a certain term belongs instead (i.e.  $u^k$ , for  $0 \leq k$ ).

### 3.2.1 The language of HLPL<sup>-</sup>: $\mathcal{L}_{\text{HLPL}^-}$

$\mathcal{L}_{\text{HLPL}^-}$  extends  $\mathcal{L}_{\text{PL}^-}$  by having HLP terms, and quantifiers and by allowing  $<$  to hold of HLP terms.

#### Logical vocabulary

Variables of level $0 \leq k$	$x^k, y^k, \dots$
Connectives	$\neg, \rightarrow$
Universal quantifier	$\forall$
A two-place predicate to be read ‘is/are among’	$<$
A singular identity predicate	$=$

#### Non-logical vocabulary

Constants of level $0 \leq k$	$c^k, d^k, \dots$
Singular n-adic predicates	$P_n, Q_n, \dots$

#### Formation rules

I use  $t^k, u^k, v^k, \dots$  as meta-variables for terms of level  $k$ .

Variables of level  $k$  and constants of level  $k$  are terms of level  $k$ .

For  $t^0$  and  $u^0$ ,  $t^0 = u^0$  is a formula.

For  $t^k$  and  $u^j$ ,  $t^k < u^j$  is a formula.

For  $t_1^0, \dots, t_n^0$ ,  $P_n(t_1^0, \dots, t_n^0)$  is a formula.

If  $\phi$  and  $\psi$  are formulas, so are  $\neg\phi$  and  $(\phi \rightarrow \psi)$ .

If  $\phi$  is a formula, so is  $\forall x^k \phi$ .

Nothing else is a term or a formula of  $\mathcal{L}_{\text{HLPL}^-}$ .

#### Defined expressions

Higher-Level Plural Identity  $t^k = t^l := \forall x^m (x^m < t^k \leftrightarrow x^m < t^l)$ , for  $m = \max(l, k) - 1$  and either  $k \neq 0$  or  $l \neq 0$ .

Existential quantification  $\exists x^k \phi := \neg \forall x^k \neg \phi$

### 3.2.2 The language of HLPL: $\mathcal{L}_{\text{HLPL}}$

In order to obtain  $\mathcal{L}_{\text{HLPL}}$  we need only add non-logical predicates that take HLP terms as arguments to  $\mathcal{L}_{\text{HLPL}^-}$ . All predicates have a fixed number of argument positions, and as I explained above, each of them can take any sort of terms as arguments. We thus remove the formation rule for singular predication and add the following formation rule, for  $0 \leq m_i$  and  $1 \leq i \leq n$ :

For  $t^{m_1}, \dots, t^{m_n}$  and  $P_n$ ,  $P_n(t^{m_1}, \dots, t^{m_n})$  is a formula.

Just as in the case of PL, I assume that all argument positions are collective and stipulate to the contrary when necessary by adding the axiom (for predicate  $P$  whose only distributive position is the one occupied by  $x^n$ ; and where  $\vec{x}$  represents the rest of arguments):

$$\text{(HLP-Dis)} \quad \forall x^n (P(\vec{x}, x^n) \leftrightarrow \forall x^{n-1} (x^{n-1} < x^n \rightarrow P(\vec{x}, x^{n-1})))$$

### 3.2.3 Proof theory

In describing the calculus of  $\mathcal{L}_{\text{HLPL}^-}$  and  $\mathcal{L}_{\text{HLPL}}$  I largely follow the treatment of cumulative type theory in Linnebo and Rayo (2012).

Let us start with  $\mathcal{L}_{\text{HLPL}^-}$ . As expected, we start with the standard deductive system for First-Order Logic with identity (minus the axioms for the quantifiers, which are redundant in the present framework):

$$\phi \rightarrow (\psi \rightarrow \phi)$$

$$(\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi))$$

$$(\neg\phi \rightarrow \neg\psi) \rightarrow (\psi \rightarrow \phi)$$

(SRI) *Singular Reflexivity of Identity*

$$\forall x^0 (x^0 = x^0)$$

(Ind) *Indiscernibility of Identicals*

$$\forall x^0 \forall y^0 (x^0 = y^0 \rightarrow (\phi(x^0) \rightarrow \phi(y^0))), \text{ where } y^0 \text{ is free for } x^0 \text{ in } \phi(x^0).$$

(MP) *Modus Ponens*

From  $\phi$  and  $\phi \rightarrow \psi$ , infer  $\psi$ .

Next we add the axioms and rules for HLP quantifiers. All of the following hold for  $0 \leq k$ .

(HLP-UI) *Higher-Level Plural Universal Instantiation*

$$\forall x^k \phi(x^k) \rightarrow \phi(t^k), \text{ where } t^k \text{ is free for } x^k \text{ in } \phi.$$

(HLP-UG) *Higher-Level Plural Universal Generalisation*

From  $\phi \rightarrow \psi(x^k)$  infer  $\phi \rightarrow \forall x^k \psi(x^k)$ , provided  $x^k$  does not occur free in  $\phi$  or in any premise of the deduction.

The rest of axioms are analogous to those of PL, except for (LR), which is introduced to capture the cumulativity informally described above:

(HLP-C) *Higher-Level Plural Comprehension*

$\exists x^k \phi(x^k) \rightarrow \exists x^{k+1} \forall x^k (x^k < x^{k+1} \leftrightarrow \phi(x^k))$ , where  $\phi$  is a formula of  $\mathcal{L}_{\text{HLPL}}$  that contains  $x^k$  and possibly other variables free but contains no occurrence of  $x^{k+1}$ .

(HLP-NE) *Higher-Level Plural Non-Emptiness*

$$\forall x^k \exists x^{k-1} (x^{k-1} < x^k)$$

(LR) *Level-Raising*

$$\forall x^k \exists x^l (x^k = x^l), \text{ for } 0 \leq k \leq l$$

This axiom schema captures the reference-cumulativity of the HLP hierarchy.

(HLP-RM) *Higher-Level Plural Reflexivity of Membership*

$$\forall x^k (x^k < x^k)$$

Finally, in order to obtain a proof theory for  $\mathcal{L}_{\text{HLPL}}$ , we need only add the following axiom schema:

(HLP-Ext) *Higher-Level Plural Extensionality*

$\forall x^k \forall x^l (\forall x^m (x^m < x^k \leftrightarrow x^m < x^l) \rightarrow (\phi(x^k) \leftrightarrow \phi(x^l)))$ , for  $m = \max(k, l) - 1$  and either  $k \neq 0$  or  $l \neq 0$ , where  $x^l$  is free for  $x^k$  in  $\phi(x^k)$ .

Given how we have defined HLP identity, this can be seen as a HLP version of Indiscernibility of Identicals.

### 3.2.4 Semantics for $\mathcal{L}_{\text{HLPL}^-}$ and $\mathcal{L}_{\text{HLPL}}$

A model  $M$  for  $\mathcal{L}_{\text{HLPL}^-}$ , like before, assigns the universal quantifier some individuals (encoding the domain), singular terms single individuals and plural terms either single individuals or more than one individual at once. Furthermore,  $\mathcal{L}_{\text{HLPL}^-}$  models assign HLP terms of level higher than 1 single individuals, more than one individual or pluralities of higher levels. Given that in English there is no obvious way to express HLP reference (in an artificially extended English we could talk of reference to many *individualses*, some *objectses* or more than *one things*), I cannot properly describe the assignment to terms of level higher than 1, but I need to speak of higher-level pluralities as if they were single objects (we could alternatively speak of ‘some pluralities’, but not much would be gained, for we would still need to talk of pluralities in the plural, which admits of no obvious paraphrase into proper English).

Note that PL has sometimes been defended on the basis that it is possible to give a mathematically acceptable notion of plural semantic value. As we have seen, we need only take the interpretation function to be multivalued. At first sight, it appears the same move is not available in the HLP case. We would need to have something like a multi-multivalued interpretation function. But any attempt to make precise this notion seems to deliver something that in no way resembles a function or a relation, as we know them. For instance, one option would be to have ordered pairs which take pluralities as one of their coordinates, but this forces us to abandon the usual conception of ordered pairs as having objects as coordinates. Moreover, this only gives us the interpretation of second-level plurals, but it is not sufficient for the third-level. This may seem a point where the merits of PL and HLPL come apart, thus justifying the stance we are arguing against in this thesis: the endorsement of PL and the rejection of HLPL.

However, I believe this reasoning should be rejected, since the claim that multivalued interpretation functions are mathematically acceptable relies on the prior legitimacy of plural reference. I believe the reason why one finds that an interpretation function can be multivalued, is that one finds that the idea of an expression having *many values* makes sense in the first place. In other words, one thinks the expression ‘the values of’ is meaningful and should be taken at face value. However, my diagnosis is that this does not come from a certain prior conception of what is mathematically acceptable, but rather from a conception as to what is linguistically acceptable – originating on what appears acceptable according to one’s ordinary linguistic abilities. The latter justifies the former and not vice versa.

Against this claim one could think that our ability to grasp multivalued functions arises from our ability to grasp relations. This idea goes hand in hand with, more generally, thinking

of functions as relations: just as a (univalued) function is a restricted sort of relation (i.e. a relation in which for any first coordinate there is a unique second coordinate), a multivalued function is a relation where this restriction has been dropped. However, as suggestive as this correspondence is, there is a leap from the notion of relation to that of a function and, in particular, to that of a multivalued function. Relations crucially involve the notion of *holding of*; by contrast, functions crucially involve the notion of *the value of*. In other words, an account of relations makes no use of the notion of a value assignment; despite it involving the directionality necessary for the distinction between the notion of an input and that of an output, it does not involve the further idea of picking out a certain object relative to another one. That relations and functions are different kinds of animals is supported by the fact that expressions picking them out have different grammatical forms: when combined with terms a relation symbol gives rise to a sentence, but a function symbol gives rise to a term. This has been argued by Oliver and Smiley (2013, pp. 145-6):

Making a relation out of a function requires introducing *holds of*, and making a function out of a relation requires introducing *the value of*. Granted, in the first case no extra conceptual apparatus or notation is needed, since the relation  $f(x) \equiv y$ <sup>75</sup> holds of  $x$  and  $y$  if, and only if,  $y$  is the value of  $f(x)$  [...]. In the converse case, however, supplying the missing ingredient *the value of* does require additional conceptual apparatus, namely definite descriptions. (Oliver & Smiley, 2013, p. 146)

Thus, although we can model a function as a relation, the former crucially involves some additional linguistic apparatus. In particular, when looking at multivalued functions the missing ingredient is *the values of*, thus plural description. So our point stands that linguistic command of plurally referring expressions is presupposed in the acceptance of multivalued functions.

Hence, HLP reference cannot be expected to be justified on the basis that its associated interpretation function is mathematically acceptable, since we should expect the latter to depend on whether the informal notion of HLP reference can be justified in the first place – on whether it makes sense to say that an expression has ‘many valueses’. In other words, the fact that Boolos’ construction of an interpretation as a plurality of pairs appears to be unavailable in the HLP case is not surprising. A plurality of pairs (at least under the usual conception of pairs) can only encode singular and plural assignments. In order to encode

---

<sup>75</sup>Oliver and Smiley use a symbol of identity which allows terms to fail to denote, since they work in the framework of a free logic.



HLP assignments we would need interpretations to be higher-level pluralities of pairs. In other words, we would need to be able to higher-level plurally quantify over pairs to capture the truth-conditions of HLPL. Since we do not appear to have HLP quantification in ordinary language, this gives us the impression that the Boolosian approach to semantics is out of reach in the HLP case.

I submit that if English (or other natural languages) had these linguistic forms, multi-multivalued interpretation functions would be as mathematically acceptable as univalued and multivalued ones. Since, as I will explain in the next chapter, I believe that natural languages do indeed have these forms, I hold that multi-multivalued functions are legitimate. However, in order to express my semantics in proper English, I shall say, in a slightly misleading way, that terms denote higher-level pluralities (in the non-strict sense).

\*\*\*

In what follows I describe a semantics in which a model  $M$  is a function from object language expressions to individuals. Moreover, functions  $s, s'$ ... assign values to the variables in an analogous manner. As I said, in order to express our semantics in proper English, I will talk of  $M$  and  $s$  as assigning *higher-level pluralities* (or rather, ‘pluralities of level  $k$ ’, for precision) to terms. This is indeed what makes my semantics homophonic.<sup>76</sup> I call this semantics ‘higher-levelist semantics’; the view according to which this semantics is suitable for  $\mathcal{L}_{\text{HLPL}}$  is, in turn, dubbed ‘higher-levelism’, as I mentioned before.

**Higher-levelist semantics for  $\mathcal{L}_{\text{HLPL}}$**  A higher-levelist model  $M$  and an assignment  $s$  for  $\mathcal{L}_{\text{HLPL}}$  are extensions of the pluralist models and assignments for  $\mathcal{L}_{\text{PL}}$  we presented in the previous chapter. We shall adhere to an extended version of Mixed Pluralism (i.e. the view that while terms denote single or multiple first-order objects, predicates denote higher-order entities: properties or relations). To that end I assume the existence of higher-order entities to serve as semantic values of predicates. It is important that, in light of Florio’s results against untyped pluralism (also mentioned in Chapter 2), properties and relations are not taken to be first-order objects. In a Fregean fashion, I shall consider them to be higher-order entities over which the first-order quantifiers (i.e. the HLP quantifiers of any level) cannot range.

The metalanguage I use is cumulative in the same sense as the object language. Thus, when I talk of an  $n$ th-level plurality in the metalanguage I am using this term in a non-strict

---

<sup>76</sup>The two formal developments of HLPL that can be found in the literature today, Rayo (2006) and Oliver and Smiley (2016), follow this approach.

way: an  $n$ th-level plurality may consist of an individual, some individuals or a plurality of up to level  $n$  (speaking in the strict sense). Also where  $n = 1$ , an  $n$ th-level plurality is a plurality of individuals and, where  $n = 0$ , it is just an individual.

### Domain

The domain is a plurality  $dd$  (i.e. the value or values of ‘ $\forall$ ’).

### Interpretation of terms and predicates

For each constant  $c^k$ ,  $M_s(c^k)$  is a plurality of level  $k$  of objects among  $dd$  (and  $M_s(c^k) = M(c^k)$ ).

For each variable  $x^k$ ,  $M_s(x^k)$  is a plurality of level  $k$  of objects among  $dd$  (and  $M_s(x^k) = s(x^k)$ ).

For each predicate  $P_n$ ,  $M_s(P_n)$  is a property or relation of adicity  $n$  (and  $M_s(P_n) = M(P_n)$ ).

### Satisfaction

$M_s(t^0 = u^0) = 1$  if, and only if,  $M_s(t^0) = M_s(u^0)$ .

$M_s(t^k < u^j) = 1$  if, and only if,  $M_s(t^k) = M_s(u^j)$  or  $M_s(t^k)$  is a member of  $M_s(u^j)$ .

$M_s(P_k(t_1, \dots, t_k)) = 1$  if, and only if,  $M_s(P_k)$  holds of  $(M_s(t_1), \dots, M_s(t_k))$ .

$M_s(\forall x^k \phi) = 1$  if, and only if, for every assignment  $s'$  which is an  $x^k$ -variant of  $M$ ,  $M_{s'}(\phi) = 1$ .

$M_s(\neg \phi) = 1$  if, and only if,  $M_s(\phi) = 0$ . And  $M_s(\phi \rightarrow \psi) = 1$  if, and only if,  $M_s(\psi) = 1$  or  $M_s(\phi) = 0$ .

Truth, logical truth and logical consequence are defined as before.

### 3.2.5 The hierarchy of higher-level pluralities

Having described the language and its semantics, one may wonder how many higher-level pluralities there are. We must keep in mind that, strictly speaking, there are no higher-level pluralities at all, but only individual objects. The question of how many higher-level

pluralities there are must be understood as shorthand for the question of how many ways there are of higher-level plurally refer to individual objects.

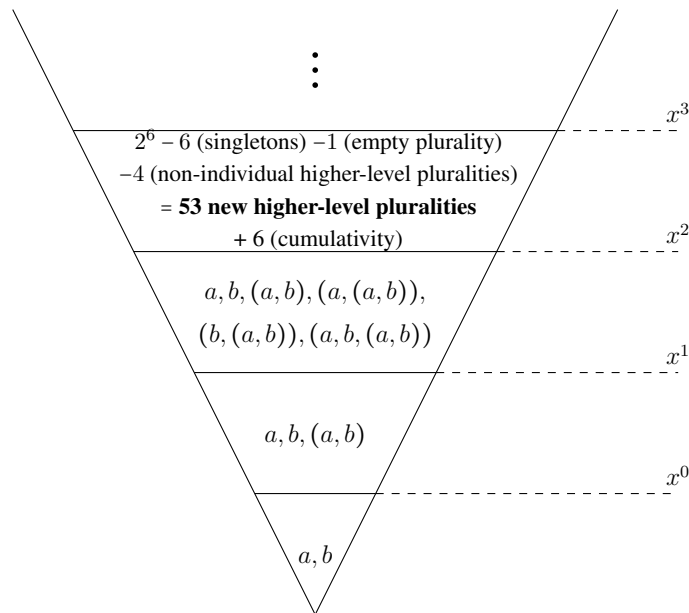
In order to find out how many higher-level pluralities there are, we need to answer two different questions. Firstly, how many higher-level pluralities there are in a given level and, secondly, how many levels there are.

Let's start with the former question. How wide is each of the levels? First of all, in order to count the higher-level pluralities at any given level, we must keep in mind that there is no such thing as an empty plurality and that there are no such things as singleton higher-level pluralities – i.e. the higher-level pluralities that have a single member are identical to that member. This springs from the putative ontological innocence of HLPL. Just as at the basic plural level, a plurality consisting of one individual is identical with that individual, a higher-level plurality having as member a single higher-level plurality is identified with the latter. In other words, given ontological innocence, just as it is impossible to make sense of an ontological distinction between an object and the plurality of that object, it is likewise impossible to distinguish between a plurality and the plurality of that plurality.

Hence, assuming that our domain of quantification consists of  $\kappa$  objects (where  $\kappa$  may be infinite), there are  $2^\kappa - (1 + \kappa)$  first-level pluralities (understood strictly) of those – that is all the pluralities we can form from  $\kappa$  individuals minus the empty higher-level plurality and the singletons of the individuals. But since our hierarchy is cumulative, we say that first-level plurals can denote  $2^\kappa - (1 + \kappa)$  first-level pluralities (understood strictly) plus everything that has appeared below, namely, the  $\kappa$  individuals. Thus the first plural level consists of  $2^\kappa - 1$  higher-level pluralities and is thus analogous to the power set of the basic domain minus the empty set. More generally, if level  $n$  contains  $\kappa$  individuals and  $\mu$  non-individual higher-level pluralities, then level  $n + 1$  contains  $\kappa$  individuals plus all non-individual higher-level pluralities generated from the higher-level pluralities below. How many new higher-level pluralities are generated from  $\kappa + \mu$  higher-level pluralities? A total of  $2^{\kappa+\mu} - (1 + \kappa + 2\mu)$ , since we need to subtract the empty set and all the singletons of higher-level pluralities and the non-individual higher-level pluralities which appeared below (which are generated again). If, due to cumulativity, we add everything that appeared below, then we have that each level  $n + 1$  contains  $2^{\kappa+\mu} - (1 + \mu)$  higher-level pluralities, if level  $n$  contains  $\kappa$  individuals and  $\mu$  non-individual higher-level pluralities.

This is what the hierarchy looks like for the case where we start off with two individuals  $a$  and  $b$ . I make use of the following informal notation to represent higher-level pluralities:  $(a, b)$  is the first-level plurality of  $a$  and  $b$ ,  $(a, (a, b))$  is the second-level plurality that consists

of the individual  $a$  and the first-level plurality of  $a$  and  $b$ ,  $(a, b, (a, b))$  is the second-level plurality that consists of  $a$ ,  $b$  and the first-level plurality of  $a$  and  $b$ , etc.



The second question concerns the height of the hierarchy of levels. Linnebo and Rayo (2012) have argued that the theory of types should be liberalized to allow infinitary types. Their argument is based on three main assumptions, including the idea that absolutely general discourse is available. Put in slogan form:

**Semantic Optimism:** Given an arbitrary language, it should be possible to articulate a generalized semantic theory for that language.<sup>77</sup>

**Union:** For  $\gamma$  a limit ordinal, suppose that one is prepared to countenance languages of order  $\beta$ , for  $\beta < \gamma$ . Then one should also countenance languages of order  $\gamma$  (i.e. languages containing variables of type  $\beta$ , for every  $\beta < \gamma$ ), on the grounds that they would be made up entirely of vocabulary that had been previously deemed legitimate.

**Absolute Generality:** One's first-order quantifiers can meaningfully be taken to range over absolutely all objects.

The argument goes as follows. Given Absolute Generality, one should expect our language to have models with domains of any possible size, including the domain of everything.

<sup>77</sup>Where a generalized semantic theory for a language is a theory of all possible interpretations the language might take.

Hence, given Semantic Optimism, our model theory should countenance interpretations of any possible size. From this follows that a generalized semantics for a language of order  $\beta$  can only be given in a language of order  $\beta + 1$ .<sup>78</sup> This motivates the ascent from a language of order  $n$  to any language of finite order higher than  $n$ . Finally, given that for the limit ordinal  $\omega$ , we have motivated the ascent to  $n$ th-order languages, for  $n < \omega$ , Union finally takes us to the acceptance of  $\omega$ -order languages. Once one has accepted  $\omega$ -order languages, then, by running an analogous argument, she is brought to accept even higher-order languages.

As it stands,  $\mathcal{L}_{\text{HLPL}}$  does not go beyond finite levels. Even though the language could be easily modified so as to allow for infinite levels and none of what I will argue in the rest of this thesis hinges on this issue, there is a reason why the HLP logician may be wary of moving on to accept infinite levels. A transfinite extension of the hierarchy of levels would rely on an extremely high idealization of natural language, since natural language does not contain anything even remotely close to transfinite levels of HLP reference and quantification. Given the sort of considerations the pluralist takes as relevant to legitimize a formal language, this level of idealization may be best avoided by us. Although in general it does not seem problematic to move well beyond natural language when developing an artificial language, given our endorsement of a homophonic approach, it is more pressing for us that we do not depart substantially from natural language. For this reason and for simplicity, I will proceed on the basis that infinite levels are out of the picture.<sup>79</sup>

---

<sup>78</sup>This has been proved in Rayo and Uzquiano (1999) and Rayo (2006).

<sup>79</sup>Note that given this limitation, some potential applications of HLPL turn out to be unavailable. In particular, the nominalist about sets cannot use the HLP hierarchy to eschew sets altogether. Although this may be seen as an important loss, it receives independent support from the following considerations. Firstly, it is coherent with the view that there is no such thing as the empty plurality or singleton pluralities. This introduces an asymmetry between the hierarchy of pluralities and that of sets. Secondly, nominalist applications of plurals appear to be in tension with one of the pluralist's main guiding principles: that we should take speaker's linguistic intuitions at face value as much as possible. Nominalist projects are always revisionist with respect to the language regimenting the sort of objects they intend to eschew. Of course, this may not always be a problem – in particular, it would not be a problem if the language or theory affected by it were not widely accepted and taken at face value by its speakers or practitioners. But the case of set theory is clearly one in which nominalism incurs a substantial semantic revisionism. When Oliver and Smiley (2016, ch. 15) suggest that set theory rests on a mistake, they may be treating set theory unjustly. Whereas plural language is taken at face value, set-theoretic language is not; but does one have a better claim to be taken literally than the other? Even if our hierarchy of terms was indeed adequate to translate set theory, an independent argument would be needed to show that this form of nominalism is compatible with the views motivating the adoption of higher-levelism in the first place.

### 3.3 Against Higher-Level Plural Logic

The fact that the notion of HLP reference has been looked at with apprehension should not come as a surprise; were it legitimate, it could help us settle some big debates in the philosophy of logic, the philosophy of mathematics and even in metaphysics. Quite a lot appears to be at stake, so, not surprisingly, many are wary of the optimism of some. In this section, I survey the different forms that the scepticism towards higher-level plurals has taken.<sup>80</sup>

**Unintelligibility** The main objection raised against HLP reference is that it is inherently unintelligible – that it is not possible for us to make sense and thus utilise such alleged linguistic device.

One of the advocates of this sceptical line is Simons, who uses the term ‘manifold’ to mean what we mean by ‘plurality’:<sup>81</sup>

We might look upon different plural expressions as relating to the objects in a different way. Consider the case of the chairs against the wall. It might be that we should wish to look upon the role of different referring expressions like this: ‘these chairs’, ‘these pairs of chairs’, ‘these pairs of pairs of chairs’. [...] It is one thing, however, to draw diagrams like this showing how we may group and subgroup individuals into larger or smaller groups:<sup>82</sup> it is quite another to think that we have made any semantic sense of these diagrams in terms of higher-order manifolds. (Simons, 1982, pp. 192-193)

Lewis was also sceptical on this matter. He briefly tackles the issue in his *Parts of Classes* when discussing Boolos’ approach to plurals:

[Boolos’ view of the relation between second-order and plural quantification] hints that the third, fourth, and higher orders cannot be far behind but what

<sup>80</sup>Bear in mind that in the quotations offered to illustrate these, authors use different terminology to refer to higher-level plurals. These are some of the names that have been used: ‘perplurals’ (Hazen, 1997; McKay, 2006), ‘pluplurals’ (Rosen & Dorr, 2002; Simons, 2016), ‘plurally plurals’ (Hossack, 2000; McKay, 2006; Rumfitt, 2005; Uzquiano, 2004), ‘hyperplurals’ (Cotnoir, 2013), ‘superplurals’ (Oliver & Smiley, 2016; Rayo, 2006).

<sup>81</sup>Since then, he seems to have changed his views. See Simons (2016) for an optimistic approach to higher-level plurals.

<sup>82</sup>Simons represents the alleged corresponding higher-order manifolds by means of tree-style diagrams.

might plurally plural quantification be? (Infinite blocks of plural quantifiers? – That will be only a skimpy third order, and no start at all on the fourth.) (Lewis, 1991, pp. 70-71)

This is meant as a criticism to Boolos' conception of M2OL. The unavailability of higher-level plurals could be seen as a problem for Boolos since one would expect the equi-interpretability of PL<sup>-</sup> and M2OL to extend to higher orders. Like Lewis, Rumfitt (2005) sees the unintelligibility of higher-level plurals as an obstacle for a Boolosian approach to plurals. He writes:

We can all understand what is meant by saying that a speaker is quantifying plurally over some objects. But what is meant by saying that he is quantifying plurally plurally over objects? The only sense I can make of this is as saying that one is quantifying plurally over objects with many members. But again, that brings in ontological commitments not incurred by quantification over the first-order domain. (Rumfitt, 2005, p. 13)

The idea behind these expressions of bafflement seems to be that since a plurality is always a plurality of things, pluralities would need to be things themselves (i.e. collectivizing entities) in order to be collectable into other pluralities. But this would be at odds with their alleged ontological innocence.<sup>83</sup>

**Lack of higher-level plurals in natural language** The allegation of unintelligibility has received support from the apparent lack of higher-level plurals in ordinary language. We find this line of criticism expressed, for example, in Uzquiano (2004, p. 439) and Lewis (1991, pp. 70-71). In the words of Uzquiano:

If English contained quantifier phrases, which, one could argue, behaved like plurally plural quantifiers, then perhaps that would be some evidence, albeit inconclusive, for the coherence of plurally plural reference. In the absence of such evidence, however, advocates of plurally plural quantification must first make it plausible that plurally plural quantification could intelligibly be introduced into the language. (Uzquiano, 2004, p. 439)

---

<sup>83</sup>See Linnebo (2017, Sec. 2.4).

In fact, even some advocates of HLPL explicitly claim that, at least probably, there are no higher-level plurals in natural language. For instance, this view is held by Hazen (1997), Linnebo (2003) and Rayo (2006).<sup>84</sup> Some, like Linnebo (2003), explain their absence (in this case, in English) as the result of syntactic efficiency.<sup>85</sup>

The reason why English contains no separate devices for higher plural quantification is that a language that does will be cumbersome and unpractical, and that ordinary English, just as it is, offers better ways of expressing essentially the same content [...]. Instead of having a stock of separate syntactical devices to handle second-order plural quantification and maybe even more devices to handle plural quantification of higher orders, English allows essentially the same content to be expressed by singularizing the first-order pluralities. (Linnebo, 2003, p. 87)

**Dispensability** Some authors have also argued that although there may be HLP terms in ordinary language, they can and/or must be understood in a non-HLP way, in other words, they are shorthand for mere plurals. For example, McKay thinks that we can and must always turn an apparent HLP term into a plural term denoting complex objects:

The language of perplurals can be understood if we build in singularizing assumptions. Given those singularizing assumptions, perplurals are expressible by ordinary plurals applied to the ‘higher-level’ objects that the singularizing process introduces. If we do not make the singularizing assumptions, then there is no evident way to understand perplurals [...]. (McKay, 2006, p. 138)

\*\*\*

What can be said against these claims, if anything? Firstly, the higher-levelist may push the view that the legitimacy of a formal language should not hinge on the contingent fact that natural languages happen to display a linguistic phenomenon or other, but only on the fact that we do actually have a good grasp of that linguistic phenomenon. She may add that one shall get a clear enough understanding of HLPL by simply mastering its use.<sup>86</sup> This

<sup>84</sup>Rayo only claims that English, in particular, does not contain such devices.

<sup>85</sup>Although he seems to have slightly changed his mind about this issue, as can be seen from the more optimistic take in Linnebo (2017).

<sup>86</sup>See Rayo (2006, p. 227), for an endorsement of this approach.



would be akin to the view endorsed by Williamson in the following fragment with respect to higher-order logic:

Perhaps no reading in a natural language of quantification into predicate position is wholly satisfactory. If so, that does not show that something is wrong with quantification into predicate position, for it may reflect an expressive inadequacy in natural languages. We may have to learn second-order languages by the direct method, not by translating them into a language with which we are already familiar. After all, that may well be how we come to understand other symbols in contemporary logic, such as  $\supset$  and  $\diamond$  [...]. At some point, we learn to understand the symbols directly; why not use the same method for  $\forall F$ ? We must learn to use higher-order languages as our home language. (Williamson, 2003, p. 459)

Little can be said against this stance. By the lights of the higher-levelist, who claims to have a clear grasp of the notions involved, the homophonic approach is entirely unproblematic. The discussion at this point mirrors the debate around higher-order logic and its semantics. Given that it is not clear that natural languages contain higher-order expressions, the legitimacy of the homophonic move in that area has been questioned too. The discussion differs from the one to be had concerning the use of homophonic semantics for the logical connectives, for first-order quantification or for plural quantification. In all of those cases, there appears to be a widespread agreement among speakers as to the meaning of the meta-linguistic expressions, which can be found in ordinary English and many other natural languages.

But is there any way to make progress in the debate regarding higher-level plurals? What can the higher-levelist do to counter the accusation of the unintelligibility of her metalanguage (and the related complaints as to the lack of ordinary HLP expressions and their dispensability)? I think there are four different strategies she can follow to try and counter the sceptical view. I believe they are jointly sufficient to shift the burden of proof towards the sceptic and I shall carry them out in what follows.

In the first place, the higher-levelist may be able to help her opponent reach an appropriate understanding by elucidating the notion further. In other words, the fact that the notion should ultimately be taken as primitive does not mean that one cannot attempt to clarify it to some extent. At the very least this could be used to support the view that there is nothing inherently incoherent about it. Secondly, she can show that even though there may not be

HLP terms and quantifiers in English, they may be found in other languages. Thirdly, she can show that what appear to be ordinary HLP expressions are, in fact, not eliminable in favour of merely singular or plural expressions. Finally, she can offer a non-homophonic semantics in terms which the sceptic claims to grasp correctly to clarify the meaning of the HLP expressions. Although this semantics would not be strictly speaking adequate for the higher-levelist, it would be the second best account available and it could serve as a bridge between the two parties.

In the next section, I turn to the first strategy by offering an elucidation of the notion of HLP reference. In the next chapter, I turn to the presence and the ineliminability of HLP expressions in natural languages. In Chapters 5 and 6, I develop the non-homophonic approach.

### 3.4 Elucidating higher-level plural reference

Naturally, the import of an elucidation is modest. The fact that we can elucidate a notion in a certain way does not show that such an elucidation is correct. My aim, here, is simply to give a sense of how I am thinking of this notion and to show that, at least *prima facie*, there is nothing incoherent about this way of conceiving of it. Whether this elucidation is defensible I will tackle in the next chapter, where I will provide reasons for optimism.

#### 3.4.1 The Iteration Principle

There is a substantial agreement in the literature that higher-level plurals, if legitimate, should be understood as whatever would result from ‘pluralizing the plural’. For example, we find the idea expressed in the following quotes:<sup>87</sup>

[...] a perplural (noun, pronoun, verb form...) is related to plurals as plurals are to singulars. As a semi-serious example, pretend our plural endings on nouns are iterable: then we could assert the existence of infinitely many cats by saying something like:

There are some catses such that for each cats among them there are some cats among them including at least one more cat. (Hazen, 1997, p. 247)

---

<sup>87</sup>See also (Uzquiano, 2004, p. 438).

A natural question that arises is whether the step from the singular to the plural can be iterated. Are there terms that stand to ordinary plural terms the way ordinary plural terms stand to singular terms? Let's call such terms 'superplural'. A superplural term would thus, loosely speaking, refer to several 'pluralities' at once, much as an ordinary plural term refers to several objects at once. (Linnebo & Nicolas, 2008, p. 186)

This idea is captured by the following principle:

**(3.1) Principle (Iteration Principle)** *Second-level plural terms are the result of an iteration of the step from the singular to the plural.*

The principle, as presented in the literature, concerns exclusively second-level plurals, but it may be generalized:

**(3.2) Principle (Generalized Iteration Principle)**  *$n + 1$ th-level plural terms are the result of iterating the step from the  $n - 1$ th-level plural to the  $n$ th-level plural (for  $n \geq 1$ ).*

The idea behind this principle is important since it is what marks the idiosyncrasy of HLP reference, what distinguishes it from other modes of reference, such as predicate reference. But not only that. Importantly, this principle also indicates that HLP reference inherits some of the features of plural reference, to which we turn next.

### 3.4.2 Inherited features

In the previous chapter, we mentioned some important characteristics of plural reference: Extensionality, Unrestricted Composition and Rigidity. Judging from the Iteration Principle, these must be transferred to HLP terms. In what follows, I express the resulting principles as they concern second-level plurals, rather than in full generality, for simplicity (here, when I speak of first-level or second-level pluralities, I mean it in the strict sense).

First of all, just as some objects are the same as some other objects if, and only if, each of the former is identical to one of the latter and vice versa, the following holds:

**(3.3) Principle (HLP-Extensionality)** *A second-level plurality  $a^2$  and a second-level plurality  $b^2$  are identical if, and only if, they have the same individuals/first-level pluralities as members.*

The idea is that just as we have extensionality at the plural level, going up the hierarchy of levels does not introduce any failures thereof.

Next, just as any objects whatsoever are some objects (i.e. a plurality), the following is also the case:

**(3.4) Principle (HLP-Unrestricted Composition)** *Any individuals/first-level pluralities form a second-level plurality.*

It takes nothing else for a plurality to exist than for each of some objects to exist, thus any things whatsoever form a plurality. The same holds of second-level pluralities. Take whatever things you like and then some other things: they form a second-level plurality.

Finally, just as it is the case that if an object is one of some objects, it is so necessarily, the following also holds:

**(3.5) Principle (HLP-Rigidity)** *If an individual or a first-level plurality are among a second-level plurality, then they are so necessarily.*

They could not fail to be among it without the second-level plurality changing its identity.

### 3.4.3 Elucidating the Iteration Principle

The scepticism towards HLP devices can be expressed as a form of scepticism towards the Iteration Principle, which, in the absence of further analysis, carries most of the weight in describing the intended interpretation of HLP reference and quantification.

When trying to clarify the significance of the Iteration Principle, one needs to get clear about two things: first, what the step from the singular to the plural is and, second, what the process of iteration consists of. A naïve interpretation of the principle would go as follows. First, describe the step from the singular to the plural as the fact that a singular term refers to one thing, whereas a plural term refers to many. Now, if the step of iteration is taken as a reasoning by analogy, then one obtains that a plural term refers to many things, whereas a second-level plural term refers to many *manys*. But, obviously, this does not help in answering the question driving the sceptic's incredulity: what are many *manys*?

Scepticism towards the Iteration Principle has been expressed even by advocates of higher-level plurals. In particular, we find it in Rayo (2006):

I would like to insist that thinking of super-plural quantification as an iterated form of plural quantification over pluralities would be a serious mistake. Plural quantification over pluralities can only make sense if pluralities are taken

to be ‘items’ of some kind or other. And a plurality is not an ‘item’: apparently singular quantification over pluralities is a syntactic abbreviation for plural quantification over individuals. (Rayo, 2006, p. 227)

However, I believe the friend of higher-level plurals should not dismiss the Iteration Principle, for, as I said above, it is what serves to intuitively distinguish higher-level plurals from other kinds of idioms.

Be that as it may, there are two ways to assuage the sceptic’s bafflement. Firstly, one may note that the sceptic appears to be taking too seriously the contingent fact that English seems to lack higher-level plurals. Many *manys* are simply many *manys*, as a speaker of an alien language would have it. Moreover, perhaps, we need not even appeal to alien languages. In fact, in the next chapter, I will show that some perfectly earthly languages display the HLP phenomenon.

Secondly, it is possible to give a rendering of the principle which does not lead to this cul-de-sac – not even in English, that is.

I would like to suggest a rendering of the principle which is in consonance with the idea that the singular is a species of the plural, as argued in Chapter 2. Recall the claim that referring to one object is a limiting case of referring to many objects. Just as we may refer to two, three or four objects, we may refer to only one of them.

What does the idea that the singular is a special case of the plural tell us about the Iteration Principle? Prima facie, it appears to undermine it, since we no longer seem to be able to describe a step from the singular to the plural that marks a difference in their semantics, since they are semantically on a par. However, it can also be seen as providing a clarification of it. This precisification of the principle takes the step from the singular to the plural to be the fact that the singular is a limiting case of the plural. By iteration, we obtain the analogous claim that the plural is a limiting case of the second-level plural. And so on. In other words, just as there is a way of being plural which consists in being singular, there is a way of being second-level plural which consists in being plural. This captures the idea that each step up the hierarchy opens up the door to more expressive forms of reference, of which the lower levels are special cases. According to this, the Iteration Principle can be reformulated as:

**(3.6) Principle (Revisited Iteration Principle)** *Just as singular reference is a limiting case of plural reference, plural reference is a limiting case of second-level plural reference.*

And in its generalized version:

**(3.7) Principle (Revisited Generalized Iteration Principle)** *Just as  $n-1$ th-level plural reference is a limiting case of  $n$ th-level plural reference, the latter is a limiting case of  $n+1$ th-level plural reference.*

Since the relation of being a limiting case is transitive, it follows that plural reference of level  $n$  is a limiting case of plural reference of level  $m$  for  $n < m$  – in accordance with the reference-cumulativity of our hierarchy. In particular, singular reference is a limiting case not only of first-level plural reference, but also of plural reference of any level.

As we have hinted to at various points, one can in proper English approximate what HLP reference consists of: it is reference to some things organised in groups. Described in this way, we can explain the fact that singular reference is a case of, for instance, second-level plural reference, by saying that referring to one object is a way among many of referring to  $n$  objects and referring to  $n$  objects is a way among many of referring to  $n$  objects in an organised way (namely organised in a single group).

In this explanation we have mentioned two parameters: the number of objects a term picks out and the organisation or grouping under which it does. Singular terms denote one object, plural terms denote one or more than one object, second-level plural terms denote one or more than one object organised in simple groups (i.e. groups that are not in turn organised in groups), and so on. Now, if we characterize second-level plural reference as reference to some objects organised in  $n$  simple groups, we can make the analogy between the step from the singular to the plural and the next steps more conspicuous. We can now say that the way in which first-level plural reference is a species of second-level plural reference is that it denotes some objects organised in  $n$  simple groups, where  $n = 1$ , just as singular reference denotes  $n$  objects, where  $n = 1$ .

However, this way of expressing what the iteration consists of brings to the fore a certain disanalogy: the step from the singular to the first-level plural appears to be an odd one out of all the steps. The rest of steps turn on the complexity of the organisation of the referents of the term, rather than on their number. While this is true, this disparity arises only because of the non-homophonic nature of our elucidation. But this should not come as a surprise. Being non-homophonic, our clarification is forced to appeal to something like *the internal organisation of the referents in groups*. However, the elucidation provides only an approximation to the notion of HLP reference.<sup>88</sup>

---

<sup>88</sup>In Chapters 5 and 6, I will elaborate on the strategy of appealing to non-HLP resources to describe the significance of HLP expressions. My strategy there will be to expand this elucidation into a proper semantic analysis of HLP language. Again, it will have to be taken as a mere approximation.

Strictly speaking, the Iteration Principle can only be fully captured in English by previously accepting that expressions like ‘many manys’ or ‘some objectses’ could be indeed intelligibly added to English. Once this is accepted, we can express the principle as saying that just as a first-level plural term denotes some objects, a second-level plural term denotes some *objectses*. And so on – having accepted the legitimacy of ‘objectses’, nothing deters us from accepting that of ‘objectseseses’, ‘objectseseseseses’, etc. In the next chapter, I will try to justify this move.

### 3.5 Recap

In this chapter, I have introduced the notion of HLP reference and the debate revolving around it. Moreover, I have offered an elucidation of the notion in an attempt to show that, *prima facie*, there is nothing inherently defective about it. Those who claim that the very notion of HLP reference is incoherent appear to be taking too seriously the contingent idiosyncrasy of the English language.

# Chapter 4

## Superplurals: a Defence

### 4.1 Introduction

In this chapter I turn to a defence of higher-levelism, the view that HLP reference can be incorporated into a formal language and taken at face value. In order to do that, on the one hand, in Section 4.2, I show that there are natural languages which display the HLP phenomenon in a clear manner and, on the other, in Section 4.3, that there are English expressions which appear to be HLP and which, in fact, are ineliminable. They are ineliminable in two different senses. First, they cannot be paraphrased away in the sense of being eliminated from the object language by rephrasing expressions containing them as merely plural expressions. Second, they cannot be semantically analyzed away, that is, they cannot be given an adequate semantics which renders them not HLP. I will conclude that not only other languages contain HLP terms, but so does ordinary English – and with it many languages which, like English, do not seem to contain them at first sight.

Most of the arguments put forward in this chapter will be based on the assumptions of the pluralist outlined at the end of Chapter 2. Thus, in this chapter I will show that the viewpoint according to which PL is legitimate but its higher-level extension is not is likely to suffer from an internal tension. The kind of considerations that give support to the legitimacy of HLPL are the same that are usually taken as giving support to the legitimacy of PL in the first place, so the advocate of PL should be willing to endorse HLPL too.



## 4.2 Higher-level plurals in natural language

As we have seen, some detractors of HLP notions claim that we cannot make sense of them partly on the basis that there are no such expressions in natural language. However, as I will try to show in what follows, not only is there evidence of their presence in other languages, but they appear to be present in ordinary English as well.

### 4.2.1 In English

As I have mentioned before, the examples of alleged English terms which have a good prima facie claim to being HLP are lists of plurals, descriptions built from pseudo-singular terms, plural definite descriptions involving collective predicates and plurals accompanied by certain appositive phrases. Let me stop to consider each of these at a time.

**Lists of plurals** Firstly, we find lists of plural noun phrases. E.g. ‘the old chairs and the new chairs’, ‘the Gordons, the Stewarts and the MacLeods’, ‘these and those’. Moreover, given that we are assuming that lists of referring expressions themselves are referring expressions, nested lists of singulars must be taken as being lists of plurals (e.g. ‘Whitehead and Russell, and Hilbert and Bernays’).<sup>89,90</sup>

One reason why it is thought that lists of plurals are HLP terms is that in their nested form they are the result of iterating the syntactic process of list formation, which is one process by which one may form a plural term. That is, they cohere with a syntactic version of the Iteration Principle.

Note that lists of pseudo-singulars are a species of this form of term. An example would be ‘this pair of shoes, that pair of shoes and this other pair of shoes’. Given that ‘the pair’ is a pseudo-singular, this is simply a list of plural terms.

**Definite descriptions with a pseudo-singular head noun** Secondly, there are higher-level plurals in the form of definite descriptions which have a pseudo-singular as their head noun phrase. For instance, ‘the most expensive pairs of shoes in this shop’, ‘my favorite teams’ and ‘those couples over there’.

<sup>89</sup>See Oliver and Smiley (2005, p. 1062), and Moltmann (2016, p. 25) for more examples.

<sup>90</sup>I shall use a comma followed by a conjunction in order to indicate where one nested list ends and the other one begins. Moreover, in order to avoid confusion, I shall never use the Oxford comma.

The existence of these suggests that Hazen's semi-serious example (i.e. 'catses') may not be so misguided after all: just as 'catses' would be a pluralization of 'cats' via an iterated application of the plural suffix, 'pairs' would be a pluralization of 'pair' by the same method. The fact that some semantically plural terms happen to be syntactically singular allows for the process of plural suffix application to be applied to (semantically) plural terms.

Moreover, pseudo-singulars allow us to easily form plurals of third level and above. Of course, we cannot iterate syntactic pluralization any further without abandoning the confines of good English, but we can conjoin terms like 'these pairs of shoes' to form lists: 'these pairs of shoes and those pairs of shoes'. These terms are similar to the nested lists we considered above, but enjoy more naturalness thanks to the pseudo-singularization involved. This would show that, even though they are rare, ordinary English contains not only second-level plurals, but also third-level ones.

**Plural definite descriptions built from collective predicates** Next, we have plural definite descriptions which are restricted by collective predicates. This kind of restriction facilitates a reading of the resulting phrases according to which they do not simply denote some objects fulfilling a certain condition, but, roughly speaking, all the groups of objects which do so. For example, 'the numbers whose product is larger than 25', 'the specialists competing for the same jobs' and 'the authors of multi-volume classics in logic' fall within this category.

Oliver and Smiley (2016, ch. 8) call this kind of terms 'plurally exhaustive descriptions'. They are interpreted as picking out all the objects that jointly satisfy the predicate in question. For example, 'the numbers whose product is larger than 25' denotes all the numbers which jointly satisfy the property of having a product larger than 25; thus it denotes numbers organised in groups. The examples above have also what Oliver and Smiley call a 'plurally unique description interpretation', according to which the expressions would pick out the only joint satisfiers of the relevant predicate in one occasion. Under this reading, the terms would refer merely plurally. For instance, consider the description 'the children who are the same age'. Depending on the context, one might correctly interpret it as picking out simply some children, the only children who are the same age. But this reading is not always available (for example, it would not be available in a case where there are various 3 years old and various 4 years old).

**Plurals accompanied by certain appositive phrases** Finally, we find plural terms followed by an appositive phrase explicitly describing a certain internal structure of the plural-

ity being denoted. For example, ‘the kids, organised in two groups’ or ‘the shoes, divided by colours’. The appositive marks a certain sub-division of the plurality which, allegedly, allows us to speak of them higher-level plurally.

## 4.2.2 In other languages

Whether these English terms are HLP or not is doubtful, because their surface form does not cohere with what would be the paradigmatic realisation of the Iteration Principle – namely, that of the iteration of the application of the plural suffix (that of Hazen’s ‘semi-serious example’: cat, cats, *catses*). I will go back to this issue and conclude that these terms should, indeed, be taken as HLP. But before doing that, in this section, I will show that even if English does not contain paradigmatically HLP terms, other languages do.

As far as I am aware, little has been written on the presence of HLP devices in natural languages other than English. In the philosophical literature, Linnebo (2017) is an exception. He has pointed out that Icelandic contains a group of expressions that appear to be HLP. As we will see, the linguistic literature on number suggests that Icelandic is not the only language that contains higher-level plurals. There are at least five other such languages: Finnish, Estonian, Breton, Khamtanga and Classical Arabic. In fact, some of the phenomena present in those languages are even more interesting for us, since they include instances of more general forms of HLP reference. This idea will become clear in a moment.<sup>91</sup>

**Icelandic** As pointed out by Linnebo (2017), the alleged higher-level plurals found in Icelandic are numeral phrases, that is, noun phrases whose determiner is numerical. The key observation is that Icelandic’s first four cardinal numbers have both a singular and a plural form and they can be combined with nouns in both of these forms:

Value	Singular	Plural
1	einn	einir
2	tveir	tvennir
3	þrír	þrennir
4	fjórir	fernir

<sup>91</sup>As is to be expected, none of the authors reporting these forms refer to them as ‘higher-level plurals’. This is because formal semanticists interpret basic plurals singularly to begin with. However, they do interpret them as being the plural of the plural (Corbett (2000) talks of ‘semantically composing plural on plural’). Moreover, both the reports of native speakers and the morphology of the expressions support the hypothesis that they are indeed ordinary higher-level plurals.

When the plural form is combined with a common noun, we obtain expressions which reportedly pick out groups of the objects being referred to by the head noun – as many as the original singular number. Moreover, they do not pick out any kind of group, but a specific kind: pairs of objects. For example, whereas ‘*einn skór*’ translates as ‘one shoe’, ‘*einir skór*’ means ‘one pair of shoes’. Analogously, ‘*tveir skór*’ means ‘two shoes’ (not necessarily forming a pair) and ‘*tvennir skór*’ means ‘two pairs of shoes’. And so forth. As we shall see the limitation to pairs is distinctive of the Icelandic case.

According to Linnebo, the plural numerical determiners allow us to talk about pairs of shoes as second-level pluralities rather than as first-level pluralities of individual objects. What is distinctive about these expressions, compared to the analogous English ones, is that they involve no mention of anything else other than shoes – i.e. pairs. That is, while in the English case in order to show that these phrases are HLP, we need to argue further that expressions like ‘the pair’ are pseudo-singular, in the case of Icelandic we can skip this step. They have a more explicit HLP form.

**Finnish and Estonian** Like Icelandic, Finnish contains plural numeral phrases. However, unlike Icelandic, Finnish contains plural forms of all cardinals, not only the first four. According to Hurford (2003), Estonian is ‘to a large extent’ similar to Finnish in this respect. I will focus on Finnish for simplicity.

These are some examples:

Value	Singular	Plural
1	yksi	yhdet
2	kaksi	kahdet
3	kolme	kolmet
10	kymmenen	kymmenet
50	viisikymmentä	viidetkymmenet
100	sata	sadat
1000	tuhat	tuhannet
10000	satatuhatta	sadattuhannet
1000000	miljoona	miljoonat

When the plural numerals of Finnish precede plural nouns, the resulting noun phrases usually mean something like ‘*n* groups of ...’, where *n* corresponds to the original singular numeral determiner. The most uncontroversial cases involve things that come naturally in

certain groupings, such as pairs – i.e. hands, socks, eyes. However, things which do not typically come in any specific kind of groupings can also be denoted by these phrases. For example, ‘neljät kupit’ means ‘four groups of cups’ and ‘kahdet paperit’ means ‘two sets of papers’.

Just as it occurred with Icelandic, there is no way to denote groups of objects in the plural simpliciter, but a specific number of groups must always be specified. This is not the case with the rest of languages we shall consider.

Let me make a parenthetical remark and reply to an objection that could be raised against the claim that the pluralized numeral expressions in Icelandic and Finnish are really HLP. In both cases, we find that ‘one’ counts with a pluralized form. For example, in the Icelandic case, phrases formed from the plural of ‘einn’ also receive the interpretation ‘one pair of...’. This may seem significant because expressions of this form (assuming that ‘pair’ is a pseudo-singular term) are not expected to be HLP. If ‘pair’ is considered a semantically plural term, then ‘one pair of shoes’ should be seen as referring to the shoes themselves.

However, there is a plausible explanation for this that is compatible with the claim that the other forms do give rise to HLP reference. It goes as follows. Pluralization of numeral phrases allows us to think of objects as organised in groups. In the cases where the numeral is higher than 1 this gives rise to HLP reference via pairs, that is, the resulting expression denotes some objects under their aspect of being organised, for example, in pairs (where with regard to shoes, for example, this has certain implications: there must be a right shoe and a left shoe, they must be of the same size, etc). In other words, as well as being HLP, these referring expressions are terms which denote some objects under a certain aspect thereof. This is what in Chapter 5 I will call ‘restricted reference’ or ‘reference under aspects’. With regard to the first numeral, pluralization gives rise to a plural expression which denotes some objects under a certain aspect thereof. In this case, the aspect of the shoes of forming a pair. As we shall see in Chapter 5, it can be argued that even though I can talk of two shoes as being a pair, this does not mean that I am talking of a complex object, the pair. Importantly, restricted reference is often present when pseudo-singularization is present, although it can arise via other mechanisms.<sup>92</sup>

Let’s move on. Interestingly for the higher-levelist, Finnish appears to have plural forms of other determiners as well:

---

<sup>92</sup>I will tackle the connection between restricted reference and pseudo-singularization in Chapter 5, Section 5.3.3.1.

English	Singular	Plural
pair, couple	pari	parit
a few	muutama	muutamit
many	moni	monet
several	usea	useat
few, not many	harva	harvat

Hurford argues that, despite appearances, only the first one, ‘parit’, is to be interpreted as being HLP:

The plural forms of the last four quantifiers given above, ‘muutamit’, ‘monet’, ‘useat’ and ‘harvat’, are not interpreted in the same way as the plural forms of the numerals [...]. Many Finns have been taught that the difference between singular ‘moni’ and plural ‘monet’ corresponds to a difference between English ‘many a boy’ and ‘many boys’, though many an English speaker would find it difficult to say exactly what the semantic difference between these English expressions actually is. (Hurford, 2003, p. 32)

Note that Hurford refers to these expressions as ‘quantifiers’. Even though philosophers do not typically refer to ‘many’ as a quantifier, we do consider something of the form ‘many shoes’ one. Thus, from these determiners we can form quantifiers (in our sense) by attaching them to common nouns. Hence, if ‘pair’ as a determiner has a pluralized form, it is to be expected that the quantifier we form from it is a pluralized plural quantifier – that is, an HLP quantifier. The resulting form would mean ‘some pairs’. The presence of an HLP quantifier in a natural language is an important step forward for the advocate of higher-level semantics for HLPL, which makes use not only of primitive HLP reference, but also of quantification.

To sum up, judging from the reports considered here, Icelandic and Finnish have in common the fact that their only HLP terms are numeral phrases (phrases which always specify the number of groups involved). Finnish, by contrast with Icelandic has the HLP reading available for all numerals and allows for interpretations which do not involve pairs, but a generalized notion of group. Finally, Finnish, unlike Icelandic, seems to count with an HLP quantifier: ‘some pairs’.

**Breton** Another language containing apparent higher-level plurals is the Celtic language Breton. An important difference with respect to Icelandic and Finnish is that the HLP terms

present in Breton are not numeral phrases (thus specifying the number of groups of objects being denoted), but phrases denoting various things as being divided into any number of groups.

According to Corbett (2000), whom I follow in this and the next subsection, Breton has two different sorts of HLP terms. Firstly, HLP terms formed from pluralizing a dual noun. A dual term is a term that denotes exactly two objects. In Breton, a dual noun is a noun prefixed with the dual ‘daou’. This prefix is obligatory with the nouns which have it, which typically are nouns denoting parts of the body or clothing that come in pairs. When we pluralize the dual we obtain an HLP referring expression picking out pairs of objects. For example,

Breton	lagad	daoulagad	daoulagadoù
English	eye	(two) eyes	pairs of eyes

Secondly, Breton has a more liberal form of HLP, since it allows composition of plural suffixes:

Breton	bugel	bugale	bugaleoù
English	child	children	groups of children

In this example, the first plural suffix (‘e’) is irregular and the second one (‘où’) is regular. Even though this is excellent news for the higher-levelist, it must be noted that this phenomenon is not very common. Corbett (2000, pp. 36-7) reports that the composed form of the plural is not available for all nouns and that it is not always understood as an HLP.<sup>93</sup>

**Khamtanga** Khamtanga, a Cushitic language, displays a phenomenon analogous to the liberal HLP form found in Breton:<sup>94</sup>

<sup>93</sup>For example, the double plural form does not receive an HLP interpretation when applied to the singular ‘shoe’ (see Stump (1989, pp. 270-1)):

Breton	botez	botoù	boteier
English	shoe	pair of shoes	indeterminate number of shoes

<sup>94</sup>While the following examples were reported in Appleyard (1987, p. 252), they were originally recorded a whole century before, in Renisch (1884). Appleyard reports that even though in his study he found similar forms, they had evolved into mere alternative first-level plural forms. Hence Corbett concludes that it would not be surprising if the distinction above had been lost. Nevertheless, this is irrelevant for the higher-levelist; it suffices for them that Khamtanga displayed the HLP in the past.

Khamtanga	ieferā	iefír	iefírt
English	child	children	crowds of children

Khamtanga	lálā	lal	lálāle
English	bee	bees	swarms of bees

**Classical Arabic** Finally, Ojeda (2015) reports the presence of HLP terms in Classical Arabic. The plural in Classical Arabic can be pluralized only when the basic plural is a broken one. A broken plural is a form of the plural where the singular stem is altered, rather than a plural suffix being added to it. Normally, pluralization of broken plurals does not lead to an HLP interpretation, but it does in some cases. It is in the cases where broken plurals “‘assimilate’ to singulars’, to put it in Ojeda’s words. To illustrate what this intermediate stage of assimilation consists of, let us look at a couple of examples:

Arabic	raḥṭun	ʔarḥuṭun	ʔarāḥiṭu
English	tribe	association of tribes	associations of tribes

Arabic	baladun	bilādun	buldānum
English	village	land	lands

In both of these cases, the intermediate form does not have the English translation one would expect (i.e. ‘tribes’ and ‘villages’, respectively), but is translated into an apparently singular term. This syntactically singular term is then pluralized giving rise to the final form, which, in its surface, is merely a plural.

Ojeda observes:

[...] the meanings of the primary plurals are not entirely predictable from the meaning of Plurality<sup>95</sup>, and the meanings of the corresponding stems. It may thus be appropriate to regard the first pluralization as a derivational process which produces a new singular out of an old one. [...] It seems more likely, however, that the new singular arises diachronically rather than synchronically – i.e. as the historical reanalysis of a plural as a singular. (Ojeda, 2015, p. 320)

One may be thereby tempted to conclude that the intermediate form is singular and thus its pluralization results in a mere plural, but that would be a bit too quick. Ojeda adds:

<sup>95</sup>Plurality is a semantic rule specifying the denotation of the plural inflection.



Typically, a basic singular will denote the individuation of a kind – the set of proper individuals which instantiate a kind. A derived singular, on the other hand, will tend to denote a coarser partition of a kind instead. (Ojeda, 2015, p. 320)

Thus even though the exact meaning of the derived singular form is unpredictable, what we can indeed predict is that it will denote a partition of the union of the singletons denoted by the singular form, to put it in set-theoretic terms. Thus this form functions, denotationally, as a pluralization of the original form. But what is then the sense in which their *exact* meaning is unpredictable? My guess is that these terms are restricted referring expressions, in the sense mentioned above and to be further developed in the next chapter. The derived singulars are pseudo-singular terms which, as such, have multiple referents (are semantically plural), but pick them out under a certain aspect thereof – e.g. the aspect of forming an association or of being organised as a land. This goes hand in hand with Ojeda’s remark:

An association of tribes is more than a group of tribes; [...] and a land is more than a bundle of villages. Derived plurals may therefore *specialize* in meaning and refer only to particular groups, collections or bundles. But this does not detract from the fact that the final, specialized, meaning required an intermediary coarsening of the initial meaning. (Ojeda, 2015, p. 321)

Thus it is plausible that there is an intermediate stage in which the derived singular (i.e. ‘association of tribes’) and the derived plural (i.e. ‘associations of tribes’) must be interpreted as a first-level plural and a second-level plural, respectively. It is only in a final semantic shift that *something is added* to those forms, namely, an aspect under which the terms denote the objects in question. My claim is that it is only in order to convey this aspect that singularization is needed. This may all sound slightly vague at this point, but, as I said, it will become clearer when we turn to the topic of restricted reference in the next chapter.

Finally, Ojeda also notes that in Classical Arabic the plural of the plural can in turn be pluralized. This would indicate that not only second-level plurals, but also plurals of higher levels can be found in natural language. He gives as example the noun ‘sect’, which has up to a triple plural form.<sup>96</sup>

\*\*\*

---

<sup>96</sup>However, he adds ‘some disapprove of such formations’ (Ojeda, 2015, p. 322).

I conclude that there is strong evidence of the presence of terms (and even a quantifier) in various languages which both display an HLP form and are reported by native speakers to be understood as one would expect HLP terms to be understood. The most uncontroversial examples concern objects which come usually grouped in a certain way, the pair-grouping being the most common one.

I am not oblivious of the fact that some of the reports are said to be controversial and in general indicate a tendency of these forms to disappear. Moreover, from a broader perspective, this phenomenon appears to be uncommon and few languages are reported to display it. In fact, Corbett warns us by beginning the section on composed numbers (the semantic category among which we find HLP terms) as follows:

These are a rare phenomenon. Occasionally we find one number built as it were on another. Recall that we are making semantic distinctions here; using the form of one number as the base for building another is not unusual; it is also common to add number morphology ‘to itself’ as it were [...]. But to base one number semantically on another is rare. (Corbett, 2000, p. 36)

Nevertheless, this is irrelevant for us. In order to make a case for the legitimacy of HLP reference, a few scarce reports are more than enough. I think we must conclude that HLP reference is indeed present in natural language. If the kind of evidence we have encountered were not enough to settle this issue, it is not at all clear what could be. I submit that the claim that there is no such thing as speaker understanding and command of the HLP phenomenon is incorrect and is likely to have arisen due to the Anglophone-centric framework in which the debate around HLP reference takes place.

In my view, this fact justifies to a large extent the use of HLP expressions taken at face value in formal languages – or, in other words, the use of higher-level semantics to account for the meaning of HLPL. This conclusion should be especially persuasive for the advocate of plurals, who often offers similar rebuttals to the complaint that the plural idiom is not suitable for semantic theorizing.

### **4.3 Indispensability**

Some theorists have argued that even if there appear to be higher-level plurals in natural languages, this does not justify us in taking them at face value, given that they are not fun-

damental parts of ordinary speech – they can always be eliminated.

That higher-level plurals are dispensable may be argued in two different ways. Firstly, it might be argued that we can always paraphrase away apparent higher-level plurals in favour of mere plural expressions. That is, that we can eliminate them at the level of the object language. Secondly, it might be argued that even if those expressions are not eliminable in this sense, they can be interpreted in ways that show that they are not really HLP. That is, that we can analyse them in a non-homophonic fashion thereby eliminating them from the metalanguage used in their semantics.<sup>97</sup>

It is important to note that even if the sceptic is right that higher-level plurals are not indispensable parts of language, this does not mean that they cannot be made sense of and be put to use. The fact that we can make do without them does not imply that we should. Thus although, if they worked, these objections would weaken the position of the higher-levelist, they would not be conclusive. Nevertheless, I shall try to show that in fact HLP terms are not eliminable in any of the two ways just described.

### 4.3.1 Against paraphrasing away

Some authors have suggested that HLP reference is always paraphrasable away in favour of mere plural reference. This discussion mirrors the analogous debate around plural reference. In this sort of debates, typically the sceptic describes how some of the expressions could be paraphrased away and its opponent responds with some counter-examples, expressions which cannot be eliminated in any of the ways suggested by the sceptic.

One of the earliest authors to endorse this eliminative view is Max Black. In Black (1971), he advocates a nominalist reading of set theory according to which, set-talk is elliptical for plural-talk. Consequently, the set-theorist needs not commit herself to the existence of sets, but only of the members of the alleged sets. *Prima facie*, it may seem that Black endorses an extension of this view to higher-order sets:

---

<sup>97</sup>This distinction corresponds, roughly, to what Florio (2014a) calls Regimentation Singularism and Semantic Singularism in the debate around plurals. The former is the view that plural ordinary expressions are to be regimented as singular when formalised. The latter is the view that they are not, but that their semantics is to be expressed in singular terms. Although we do not talk of regimentation in a formal language, but rather of object language paraphrases, the distinction is effectively the same, since when we say that an ordinary expression *X* is to be paraphrased as *Y*, this has as a consequence that a regimentation of *X* would be a formal counterpart of *Y* (and vice versa).

Set talk (the use of plural referring expressions) is especially convenient when we cannot or need not identify the corresponding memberships [...]. But these and other considerations that give talk about sets its rationale can also lead naturally to forming lists of sets (second-order lists). [...] there will naturally emerge lists in which plural referring expressions follow one another ('the finance committee, the membership committee and the rules committee'). (Black, 1971, pp. 632-3)

However, immediately after this, he writes:

I do not think this interesting and undoubtedly useful extension of primitive set talk offers any serious obstacles for our program [...]. We need only be sure that we have at our disposal adequate devices for connecting such 'second-order' discourse with the lower level discourse already discussed [...]. In short, we need to know how we could, if necessary convert the more abstract talk about 'sets of sets' into assertions about sets simpliciter (sets composed of persons or other things that are not sets). (Black, 1971, p. 633)

Judging from this paragraph, it seems that he advocates higher-level plurals only as a superficial form – that is, as long as they are reducible to simple plurals (since, for him, 'assertions about sets simpliciter' are plural assertions). Moreover, he thinks they must be paraphrasable away in favour of plural expressions which do not pick out sets or other complex objects.

Another advocate of the paraphrasability thesis is McKay (2006). Unlike Black, however, he thinks that HLP terms can always be paraphrased away by appealing to singularizers, where singularizers are what we called 'collectivizing entities', 'objects that have 'members' (or other constituents)' (McKay, 2006, p. 138). Thus he thinks that HLP expressions are shorthand for plural expressions picking out several sets or other set-like entities at once. This is also Linnebo's view, as expressed in the quotation at the beginning of this chapter: 'English allows essentially the same content to be expressed by singularizing the first-order pluralities.' (Linnebo, 2003, p. 87)

In what follows I shall argue that whereas the paraphrasability objection would have some force if paraphrases of the kind advocated by Black were always available, that is not the case. And it is when we need to recur to the latter strategy, the one McKay and Linnebo have endorsed, that the objection loses its appeal.

### 4.3.1.1 Strategy I: Flattening

The strategy of flattening consists in eliminating HLP terms in favour of plural terms denoting the same sort of objects the original terms were supposed to denote (i.e. first-order objects).

This strategy allows one to paraphrase statements involving distributive predicates. For instance, (4.1) can be paraphrased as the conjunctive statement (4.2) thereby eliminating any apparent HLP reference:

(4.1) *The separatists and the unionists appeal to democracy in their arguments.*

(4.2) *The separatists appeal to democracy in their arguments and the unionists appeal to democracy in their arguments*

Moreover, a similar move is available when it comes to collective predicates built from dyadic relations. For example, in (4.3), the collective monadic predicate ‘... blame each other for the current state of affairs’ can be replaced with the dyadic predicate ‘... blame ... for the current state of affairs’. Just as before, one needs to make use of a conjunctive paraphrase:

(4.3) *The separatists and the unionists blame each other for the current state of affairs.*

(4.4) *The separatists blame the unionists for the current state of affairs and the unionists blame the separatists for the current state of affairs.*

However, as we shall see shortly, things are usually not so simple and in many cases one needs to follow the strategy suggested by McKay and Linnebo and bring collectivizing objects into the picture.

### 4.3.1.2 Strategy II: Reification

This is the strategy I call ‘reification’. It consists in eliminating HLP terms in favour of plural terms denoting objects having the relevant basic individuals as members or parts.

Consider the following sentence:

(4.5) *These people, those people and these other people played against each other in a 3-way game.*

As argued in Linnebo and Nicolas (2008), (4.5) cannot be paraphrased in terms of two-way games, that is, as something like ‘These people played against those people, and the

latter played against these other people...’. A sentence like this would not quite capture the meaning of the original sentence, which involves some sort of game in which each of three teams plays against the other two simultaneously. Thus the foe of HLP terms appears to be forced to abandon Flattening and to paraphrase it along the lines of:

**(4.6)** *This group of people, that group of people and this other group of people played against each other in a 3-way game.*

This paraphrase eliminates the list of plurals in favour of a list of singular terms.

However, there are various problems with this strategy. Firstly, if this strategy were adopted, the pluralist would need to explain how it is that (4.7) should not be paraphrased as (4.8) as well:

**(4.7)** *These people played a game.*

**(4.8)** *This group of people played a game.*

In other words, Reification seems to motivate a similar move with respect to ordinary plurals. Analysing them away in favour of singularizers would result in a more homogeneous analysis. But this would be in clear tension with the view that we are assuming in this thesis, according to which plurals can and should be understood as plurally referring expressions. Of course, the foe of plurals will be more than happy with both of these moves, but our arguments are not targeted at her.

Moreover, the acceptance of this paraphrase leads to the acceptance of an object-language entailment relation that is *prima facie* counter-intuitive. A sentence that is apparently about people entails a sentence that is about groups. In other words, by asserting something about people I appear to be forced into asserting the existence of some collectivizing objects, groups. This seems wrong.<sup>98</sup>

Another example is given by a pluralized version of the Geach-Kaplan sentence (which we presented in Chapter 2):

**(4.9)** *Some teams hate only one another.*

This is HLP, since ‘team’ is a pseudo-singular term (at least in some contexts) and, thus, ‘some teams’ would be an HLP quantifier.

---

<sup>98</sup>Even though this is not uncontroversial, the claim that paraphrasing a sentence as another one that explicitly invokes more ontology is unacceptable is a widely held assumption by pluralists. See, for example, Boolos (1984). Nevertheless, it is not crucial for our arguments.

Firstly, there is no paraphrase available for (4.9) via Flattening. This is because of the result, presented in Chapter 2, that the plural version of the Geach-Kaplan sentence does not have a first-order paraphrase (provided the domain of quantification is held fixed). If we were to find a plural paraphrase for our HLP Geach-Kaplan sentence, this paraphrase could be turned into a singular paraphrase of the original sentence by simply substituting ‘team’ with a genuine singular term. But drawing from the result of non-firstorderizability, we know that this is impossible, hence no such plural paraphrase exists.

An alternative paraphrase is (4.10), which is obtained via Reification:

**(4.10)** *There is a set such that if a team belongs to it, then, if another team belongs to it, then the former hates the latter, and any team which is hated by a team that belongs to it belongs to it.*<sup>99</sup>

(4.10) is analogous to the paraphrase of the original Geach-Kaplan sentence proposed by Quine (1950, p. 239). This paraphrase appeals crucially to the existence of a certain set and, again, it appears that the former sentence does not entail the latter from the point of view of the ordinary speaker.

A more pressing difficulty with Reification has to do with the cardinalities of certain denotations. The problem is that for terms with non-set-sized denotations there is not even a candidate set to be invoked in paraphrases via Reification. As before, this worry arises in the discussion around the eliminability of plurals and transfers to the present discussion. For example, consider:

**(4.11)** *The cardinals, the ordinals and the transitive sets overlap.*

It is easy to check that we cannot apply Flattening to (4.11). For example, as observed by Linnebo and Nicolas (2008, pp. 193-4), who consider a similar example, an obvious choice of paraphrase via Flattening would be:

**(4.12)** *The cardinals overlap the ordinals, the ordinals overlap the transitive sets, and the transitive sets overlap the cardinals.*

But this sentence is not equivalent to (4.11). To see this, think about the scenario in which all of the conjuncts in (4.12) are true and yet it is not the case that something is a cardinal, an ordinal and a transitive set.

The obvious alternative is to use Reification as follows:

---

<sup>99</sup>See Oliver and Smiley (2016), p. 40.

(4.13) *The set of cardinals, the set of ordinals and the set of transitive sets overlap.*

But, because of the set-theoretic paradoxes, we know that there are no such sets, thus preventing (4.13) from receiving its intended interpretation. One may alternatively paraphrase it as:

(4.14) *The class of all cardinals, the class of all ordinals and the class of all transitive sets overlap.*

However, similar problems arise with classes. Consider:

(4.15) *The proper classes and the non-proper classes are disjoint.*

This sentence cannot be paraphrased via Flattening unless one is willing to accept that being disjoint is a defined notion. If classes are our collectivizing entities of choice, the obvious alternative is:

(4.16) *The class of proper classes and the class of non-proper classes are disjoint.*

But there are no such classes by definition – proper classes cannot be members of other classes. Moreover, if one were to admit that there are other collectivizing entities which have classes as members (i.e. higher-order classes), analogous problems would re-arise one level up.<sup>100</sup>

I shall come back to cardinality problems in the next section.

Finally, a more general objection can be raised against any attempt to paraphrase HLP terms away. As McKay (2006, p. 48) himself acknowledges, any such attempt will likely involve a variety of different strategies to tackle different HLP statements. The fact that there does not seem to be a unique or systematic way in which to construct the paraphrases not only suggests there to be implausible irregularities, but leaves the door open to finding expressions which resist paraphrase and suggests that HLP expressions should be taken as basic.

### 4.3.2 Against analyzing away

Even if they cannot be in general eliminated from the language, some theorists believe that HLP expressions can be given a semantics which shows that they are not really HLP. In

---

<sup>100</sup>Note that using an informal notion of collection in the object language will not help, because we will need a formal one in the semantics, if we are to assuage worries about paradoxes regarding the informal collections. Hence the problem will re-arise at the metalinguistic level.



the philosophical literature, Linnebo and Nicolas (2008) and Ben-Yami (2013) have made proposals along these lines. In linguistics, almost all authors who have worked on this topic are of this opinion. In particular, there are two main views on the semantic contribution of the expressions listed above. One current of thought has it that HLP terms are really mere plural terms under a cover-reading. This view has been advocated in Gillon (1987, 1992) and Schwarzschild (1996).<sup>101</sup> The other line of thought stems from Link (1984) and has been further developed and modified by Landman (1989a, 1989b). According to Link and Landman, HLP terms denote pluralities of groups.<sup>102</sup>

Nothing of what I argued in the previous section shows that one cannot give adequate truth-conditions for apparent HLP statements which make use of predicates and names that are intuitively about a different subject matter from the object language statements or that make different ontological demands from them. In other words, paraphrasing away and analyzing away come apart in general. In the previous section, I argued that the former is not in general available, but this does not rule out the latter being a suitable option. The problems we just presented with respect to paraphrasing away would only re-arise if the semantics sanctioned the wrong object-language entailments, but one thing may not lead to the other – be it because the object language lacks some of the expressive resources of the metalanguage or because there are models which block the entailments.

For example, consider again sentence (4.9):

*Some teams hate only one another.*

One may well think that this sentence is true in all models in which there is a set such that if a team belongs to it, then, if another team belongs to it, then the former hates the latter, and any team which is hated by a team that belongs to it belongs to it. That is, one could give the truth-conditions for this sentence in set-theoretic terms. However, this need not lead to the further claim that the Geach-Kaplan sentence implies an object-language sentence about sets (namely the object-language counterpart of the metalinguistic statement of its truth-conditions). The object language may lack the expressive resources of the metalanguage or there may be a model under which the inference fails.

However, one may also think that the existence of paraphrases and of certain semantic

---

<sup>101</sup>Although neither Gillon nor Schwarzschild endorse their semantics in the context of the HLP debate, Linnebo and Nicolas (2008) note its relevance for the present topic.

<sup>102</sup>In fact, as I showed in Chapter 1, all of the proposals in linguistics make use of individual sums or sets rather than pluralities. However, for the purpose at hand, one can think of those proposals as building on a pluralist understanding of basic plurals. The objections raised for the original proposals are as relevant when they are considered as extensions of pluralist semantics.

analyses are not independent. For instance, under the usual view that mutual paraphrases share their meaning and the assumption that model-theoretic semantics must capture meaning in some strong sense – in the sense that model-theoretic semantics not only serve to capture the right notion of logical consequence, but also to capture the meanings of individual expressions – one would expect counterparts of the truth-conditional clauses to be available as paraphrases in the object language.

Be that as it may, in this section I shall argue that analyzing HLP terms away is not as easy a task as it may seem. Let us look at the proposals listed above one at a time.

#### 4.3.2.1 Multigradedness

The first account I am going to consider is in Linnebo and Nicolas (2008). The authors take the locus of HLP readings of sentences to fall on the predicate. According to them, HLP predicates are multigrade predicates which take a variable number of plural terms as subjects. Thus, they are predicates that take lists of plurals with a variable number of items as subjects. Even though Linnebo and Nicolas do favour HLP expressive devices and do not seem to intend it in this sense, their proposal can be taken as a reductive analysis of the HLP idiom, since it reduces it to another linguistic phenomenon: multigradedness.

Although their proposal does not involve a semantic analysis strictly speaking, it is of interest to us, since it suggests that a semantic analysis based on the notion of multigradedness could be developed, which would have HLP predicates as a special case.

I believe such account would face some difficulties. One of its problems is that it would fall short of covering all HLP predicates. For example, it would not deem a predicate like ‘being arch-rivals’ in

(4.17) *Barcelona’s supporters and Real Madrid’s supporters are arch-rivals.*

a HLP predicate, since it is not multigrade.<sup>103</sup> The same would hold of many other predicates, including predicates which make specific reference to numbers such as ‘play against each other in a 3-way game’ in (4.5).

These predicates are of a fixed grade, but are similar in all other respects to multigrade predicates that take a variable number of plural terms as arguments. For example, compare the predicates just considered with ‘are rivals’ and ‘play against each other’, respectively. It

---

<sup>103</sup>I am assuming an interpretation of ‘are arch-rivals’ according to which only two parties can be arch-rivals. Alternatively, one could appeal to a predicate along the lines of ‘are each other’s biggest rivals’.

would be an unwelcome consequence of one's account if these predicates were to receive a different semantic treatment than their multigrade counterparts.

Moreover, the multigradedness analysis would apply to some intuitively non-HLP predicates, which suggests, again, that it is materially inadequate. If multigradedness is a sufficient condition for being a higher-level plural, then the predicate in

(4.18) *My cousins and their kids are noisy.*

should be HLP. But it needs not be. In fact, a more natural reading of this sentence is one according to which either all of those people together are noisy or each of them is. In either case a mere plural reading of the subject is adequate.

Finally, Linnebo and Nicolas' notion would not cover non-list HLP terms.<sup>104</sup> For example, consider

(4.19) *The people forming the three circles play against each other in a three-way game.*

If (4.5) gets classified as involving an HLP predicate, so should (4.19). But, according to Linnebo and Nicolas' account, it does not. The problem is that it is not obvious how their account should be extended in order to cover these cases. Overall, I conclude that the multigradedness account, despite being adequate in certain cases, should be ruled out, due to its material inadequacy.

### 4.3.2.2 Articulated reference

In the philosophical literature, we find another proposal as to how to understand HLP terms. Like the multigradedness one, it does not consist of a proper semantic theory for HLP expressions. Instead, it provides a quasi-syntactic condition to identify them and only a brief suggestion as to how their semantics works. It is Ben-Yami (2013)'s theory of articulated reference.

According to Ben-Yami, the notion of HLP reference should be rejected and an alternative account of the meaning of the alleged ordinary HLP terms should be adopted. The alternative notion is that of articulated reference:

**(4.20) Definition (Articulated reference)** *A term refers in an articulated way if, and only if, it refers to some objects by virtue of containing other referring expressions that refer to some of those objects (possibly to one of them).*

<sup>104</sup>It must be noted that the authors do not claim to have given an exhaustive account.

Prima facie, this definition intends to shed light on the semantic workings of a certain kind of term by appealing to a syntactic fact (i.e. an expression ‘containing’ another expression). Articulated terms refer to some objects by virtue of some of their components referring to a proper part of them.

Under this view, only lists of terms get classified as articulated terms. This has two unwelcome consequences. Firstly, other types of terms, which seem equivalent to lists do not fall, prima facie, under the articulated type of referring expressions. This would even be a problem by Ben-Yami’s lights, since he acknowledges that those terms should be understood in the same way:

[A]rticulation of reference might result not only from lists of referring expressions, as in all cases we discussed so far, but also from the use of a general term that applies to several particulars together and not one by one. (Ben-Yami, 2013, p. 97)

But he seems to think his notion is readily extendible to cover those too:

The application of the concept of articulated reference to definite descriptions such as ‘the joint authors etc.’ is an *extension* of that concept [i.e. articulated reference] as introduced above. [...] because ‘joint authors etc.’ applies to particulars two by two (in this case), the definite description behaves under predication the way the articulated noun phrase ‘the authors of Principia Mathematica and the authors of Grundlagen der Mathematik’ does. In this sense the articulation involved here can be seen as an extension of articulation as first defined. (ibid., p. 98)

But the fact that Ben-Yami needs to explain this extension in terms of the predicate ‘being a joint author’ holding of several individuals collectively, strongly suggests that a unifying semantic account is in order, one that supersedes his articulated reference account. Taking articulated reference to hinge on a syntactic feature of expressions is unsatisfactory.

Another problem with this account is that, according to the definition of articulated reference, not only terms such as lists of plurals are deemed articulated, but also terms that are typically taken to be mere plurals such as lists of singulars. For example, ‘Serena and Venus’ refers to Serena and Venus in virtue of ‘Serena’ referring to Serena and ‘Venus’ referring to Venus, thereby fitting the definition of ‘articulated term’. In fact, this appears to be one of

the main reasons why Ben-Yami believes that his notion of articulated reference should not be taken as an elucidation of HLP reference, but rather as showing that there is no such thing as HLP reference.

Nevertheless, this appears to be a problem for his account, because it draws a semantic distinction where there is none. Under his view, a list of singular terms like ‘Serena and Venus’ is as much an articulated term as a list of plural terms. There is some initial plausibility to this claim, since both types of terms share the feature of being in list-form, which may indicate an underlying semantic commonality. But there is a problem with rendering mere lists of singulars articulated. On the one hand, given the substitutability of co-referring terms in transparent contexts, mere lists of singulars cannot be taken to be articulated, unless their non-list equivalents are also taken to be so. For example, a list like ‘Serena and Venus’ is substitutable *salva veritate* in all transparent contexts by the definite description ‘the Williams sisters’, hence if the former is articulated, so is the latter. But this would render all plural terms articulated (or at least, those which can be substituted *salva veritate* by a list of singular terms – namely, those with a finite denotation). On the other, if semantically equivalent terms can be such that one has articulated reference but not the other, then it is no longer clear what the semantic import of articulated reference is.

I conclude that the articulated reference account is unsatisfactory due to its material inadequacy.

### 4.3.2.3 Cover-based semantics

In the third place, we find cover-based semantics. This has been advocated in Gillon (1987, 1992) and Schwarzschild (1996). Cover-based semantics was proposed as an extension of set-theoretic semantics for plurals. Cover readings are intended to account for interpretations of predicates that fall between the collective and the distributive ones, considered limiting cases thereof under this framework. Under a cover reading, the relevant predicate needs not apply collectively to all the referents of the subject nor distribute down to the individual level, but may stay in between, distributing down to some sub-pluralities of the plurality picked out by the subject.

The proposal is that the interpretation of any plural statement depends on a prior and contextually-determined choice of cover. A cover is defined as follows:

**(4.21) Definition (Cover)** *Given a set  $x$ ,  $C$  is a cover of  $x$  if, and only if,*

- (i)  *$C$  is a set of subsets of  $x$ .*

- (ii) Every member of  $x$  belongs to some member of  $C$ .
- (iii)  $\emptyset$  is not in  $C$ .

Cover-based semantics makes use of a distributive operator  $D$ , relativised to a contextual choice of cover. This operator attaches to a plural predicate and forces it to apply to the elements of the relevant cover. Suppose  $F$  is a plural predicate,  $x$  is a set and  $D_{Cov}$  is a distributive operator relativised to the cover picked out by the free variable over sets of sets  $Cov$ . This is the semantic rule for the  $D_{Cov}$ -operator:

**(4.22) Definition ( $D_{Cov}$ -operator)**  $D_{Cov}F(x)$  if, and only if,  $Cov$  is a cover of  $x$  and  $\forall y(y \in Cov \rightarrow F(y))$ .

Under this proposal, the old distributive reading of predication is the limiting case where  $Cov$  is the set of singletons of members of  $x$  and the old collective reading is the limiting case where  $Cov = \{x\}$ .

For instance, under this semantics,

**(4.23)** *Russell and Whitehead, and Hilbert and Bernays wrote multi-volume logic books.*

is true with respect to the cover  $\{\{\text{Russell, Whitehead}\}, \{\text{Hilbert, Bernays}\}\}$ .

This analysis also allows for overlap of the members of a cover. For instance, consider the sentence

**(4.24)** *Hammerstein, Rodgers and Hart wrote musicals.*

(4.24) is true because Hammerstein and Rodgers wrote musicals together and so did Rodgers and Hart, but neither of them wrote musicals on his own nor did they write musicals jointly in some other combination. Hence the sentence is true only with respect to the cover  $\{\{\text{Hammerstein, Rodgers}\}, \{\text{Rodgers, Hart}\}\}$ .

An interesting aspect of Gillon and Schwarzschild's work is that they show that even mere plurals demand a cover-reading sometimes. To see this, consider:

**(4.25)** *The shoes cost 75 euros.*

Assuming that 'the shoes' denotes more than one pair of shoes, this is usually interpreted as meaning that each relevant pair of shoes costs 75 euro. In that case the predicate would have to be interpreted as applying to an intermediate cover, which has as members the set of each pair.

As presented so far, cover-based semantics can only account for distributive predications over second-level plural subjects (denoting objects organised in simple groups). In fact, this was the purpose for which this analysis was originally designed. However, in the previous section we made a case for the claim that English contains predicates which collectively apply to more than one plurality at once. This linguistic phenomenon was displayed, for instance, by sentences (4.5), (4.9) and (4.11). Importantly, the predicates in (4.5), (4.9) and (4.11) cannot be taken as distributing over the different plural components of their subjects. However, they cannot be taken as mere basic collective predicates either: the HLP feature of the subjects must be taken into account in order to grasp the right meaning of the sentences. For example, with respect to sentence (4.5), the predicate ‘played against each other in a three-way game’ cannot be interpreted as applying collectively to a mere plurality of objects, for under this interpretation we would be losing some of the information that this sentence gives us: namely, that these people, those people and these other people formed each a team and played against each other. Keeping in mind that it is a mere manner of speaking, we can say that the predicate applies collectively to three pluralities.

If I am right that English contains this sort of predication, then the covers-based account is unsatisfactory as it stands, since it focusses exclusively on readings intermediate between the distributive and the collective by placing the locus of the different readings entirely on the presence of a certain cover and deeming all predicates of the same kind – as applying distributively to the elements of the relevant cover. Cover-based semantics disallows cases like the one just described, where a predicate applies to many members of a cover jointly.

Nevertheless, cover-based semantics can be modified so as to account for collective predications over second-level plurals simply by not forcing cover readings to be accompanied by the distributive operator. In this context, this move amounts to acknowledging predication of higher-order sets. For example, we could introduce a collective operator defined as follows:

**(4.26) Definition (C-operator)**  $C_{Cov}F(x)$  if, and only if,  $Cov$  is a cover of  $x$  and  $F(Cov)$ .

Moreover, once collective readings of predications of second-level plurals are brought into the picture, it is a small further step to allow distributivity and collectivity over plurals of higher levels. We simply liberalize the notion of cover by allowing covers to be subsets of the power-set of the power-set of  $x$ , of the power-set of the power-set of the power-set of  $x$ , and so on. Now a sentence of the form  $P(a_3)$ , where  $P$  is collective and  $a_3$  is a third-level plural is analyzed as having the form  $C_{Cov}P(a_3)$  where  $Cov$  is a set of sets of sets of the objects denoted by  $a_3$ .

Note that the cover-based approach seems incompatible with the view that the terms

enumerated above are really HLP, since saying that a predicate applies to a term under a cover is the same as saying that the predicate holds of a certain set (i.e. the cover). Linnebo and Nicolas (2008) appear to disagree with this. They seem to take cover semantics as giving a genuine HLP account of terms such as lists of plurals:

But the semantics of such sentences makes covert appeal to a superplural term and to universal quantification over the pluralities of the superplurality (i.e. the cover) that this term denotes. (Linnebo & Nicolas, 2008, p. 192)

However, in this case no appeal to higher-level plurals seems to be taking place, but rather to singular reference to sets. If cover-based semantics is adequate, it suggests that HLP terms are not really HLP, but plurals picking out sets.

In any case, cover-based semantics has a couple of important limitations in its application to HLP expressions.

**Cardinality restrictions** Most importantly, this semantics has the limitation that it only delivers correct truth-conditions when the denotation of the terms is set-sized. For example, cover-based semantics does not account for the correct meaning of (4.11):

*The cardinals, the ordinals and the transitive sets overlap.*

This is because there is no cover available that would capture the intended denotation of its subject.<sup>105</sup> Once again, this is especially problematic for those advocating pluralist semantics for plural terms (who would take cover semantics as a complement to their semantics for the HLP fragment), since they would, by contrast, get the right truth conditions for (4.27):

(4.27) *The ordinals do not form a set.*

If one's semantics gets it right with respect to (4.27), but goes astray when moving onto (4.11), something seems to have gone wrong.

One may reply that what really occurs in (4.11) is that 'the ordinals, the cardinals and the transitive sets' just picks out a plurality – the plurality of all ordinals, cardinals and transitive sets. However, (4.11) would then turn out to be truth-conditionally equivalent to (4.28) (given that, under their usual set-theoretic definitions, all cardinals are ordinals, which in turn are transitive sets):

---

<sup>105</sup>One could define class-sized covers, but we could raise analogous problems for those by bringing talk of classes into the object language.



(4.28) *The transitive sets overlap.*

But (4.28) does not have the same truth-conditions as (4.11) (if it is meaningful at all).

**The availability of covers** Another problem faced by cover-based semantics arises from the fact that, according to it, HLP readings are available for all terms, regardless of their syntax. Thus it all comes down to the interpretation of the predicate. But how are we to decide when a predicate should be interpreted as applying to a certain cover? First note that there are predicates that have more than one reading, for instance ‘play against each other’ may hold collectively of some objects (‘The kids play against each other’), but also of some groups (‘The kids and their parents play against each other’). Thus it cannot be inherent to the predicate whether it takes a certain kind of cover or other as subject. However, neither can it be the case that all covers are equally available in any given context: some terms are able to trigger an available reading of the predicate rather than another. Cover-based semantics, as it stands, provides no means to capture this fact.

This has been pointed out by Lasersohn (2006), who has argued that cover-semantics can make true sentences that are intuitively false:

However, covers-based analyses face a challenge in dealing with examples like ‘The T.A.s earned exactly \$20,000.’: Suppose John, Mary and Bill are the T.A.s, and each of them earned \$10,000. In this case, the predicate ‘earned exactly \$20,000’ holds of each cell of the cover {{John, Mary}, {John, Bill}}, but [that] sentence [...] is not intuitively true in this situation. (Lasersohn, 2006, p. 643)

The advocate of cover-based semantics needs to complement her account with a story as to when a certain cover-reading is available in a given case. I conclude that, as it stands, it is unsatisfactory.

#### 4.3.2.4 Group-based semantics

Finally, group-based semantics for HLP terms are presented as an extension of mereological semantics for plurals. However, everything that I say in what follows applies equally well to a group-based extension of pluralist semantics.

Recall that according to mereological semantics, plural terms denote a sum of all the individuals falling within the extension of the corresponding singular term. How can this be extended to higher levels? First of all, note that sum formation cannot be exploited to construct the reference of higher-level plurals – since the sum of two sums results in a sum of the same type. For example, consider the following sentences, which have apparent higher-level plurals as subjects:<sup>106</sup>

(4.29) *The cards below 7 and the cards from 7 up are separated.*

(4.30) *The cards below 10 and the cards from 10 up are separated.*

If their subjects denoted sums of sums, then they would be truth-conditionally equivalent – since the sum of the sum of cards below 7 and the sum of cards from 7 up is just the sum of all cards, and the same holds for the subject of the second sentence. This is why Link (1984) and Landman (1989a, 1989b) propose to use groups instead of sums at this point.

Groups are new atomic individuals added to the original domain of individual atoms and sums.<sup>107</sup> Groups are related to sums via two functions. First, an injective and multivalued function, group-formation, takes us from sums to groups.<sup>108</sup> Second, a non-injective function of membership-specification which brings us back to sums.

Going back to sentences (4.29) and (4.30), a group-based interpretation would assign ‘the cards below 7 and the cards from 7 up’ a sum of two groups as reference. Given that ‘the cards below 10 and the cards from 10 up’ would be assigned a sum of two other groups, the two sums are distinct (they do not have the same parts, given that groups are atomic) and that explains the difference in truth-conditions. Moreover, the process of group formation can be iterated by allowing groups to be formed from sums of groups via group-formation, thus giving rise to groups of groups.

Link and Landman’s proposal can easily be seen to be in tension with the idea that those terms are genuinely second-level plural. Taken at face value, the account renders them mere plurals (in their analysis, sum-denoting expressions) which denote a special sort of individual object: groups.

<sup>106</sup>These examples come from Landman (1989a, p. 595).

<sup>107</sup>I am leaving aside the domain of material atoms and sums, since it plays no role here.

<sup>108</sup>The fact that group-formation is multivalued has to do with the application of group semantics to tackle non-extensional occurrences of plural terms – where we need to allow for two different groups to be formed from exactly the same individual sum. We can ignore this issue for the moment. I will come back to it in the next chapter.

Before going on, it is worth mentioning Moltmann (2016)'s account of HLP. Her proposal is similar to the group-based one, but, instead of groups, she appeals of integrated wholes. Integrated wholes are very similar to groups, but there is a restriction onto the formation of integrated wholes, which are hence not available for all pluralities. According to her, integrated wholes are pluralities which fulfill a certain integrity condition (they are either the maximal plurality falling under a property or else standing in a particular relation). However, her account faces the same problems the groups account faces and which I describe in what follows.

**Cardinality restrictions** Admittedly, we do not know much about what groups are. At least, in the context of this debate no clear account of groups is being advanced. But it is plausible that groups must be taken to be set-theoretic in nature. Let me argue for this.

Firstly, given some plausible assumptions, there is a paradox analogous to Russell's affecting groups. Let us call the atomic individuals making up the sum from which a certain group is formed 'the members' of the group. Under this view, a non-self-membered group is such that it is not an atomic individual making up the plurality from which it is formed. It is obvious that there are some non-self-membered groups. Now consider the sum of all the non-self-membered groups. If we can form a group from any given sum, paradox ensues in a familiar way. Let us call the group of all non-self-membered groups  $G$ . Is  $G$  a member of  $G$ ? Well, if it is, then it is a non-self-membered group, which means it is not. And if it is not, that shows it is non-self-membered, in which case it must be a member of it.<sup>109</sup>

How can one block this paradox? It seems one can do one of two things: either restrict sum formation or restrict group formation. However, under the usual conception that any objects whatsoever form a plurality,<sup>110</sup> one is forced to accept that any objects form a sum (since any sum, as defined by Link, corresponds to a plurality and vice versa). So we cannot restrict sum formation. Hence, we are left with the option of restricting group formation: not all pluralities can be subsumed into groups. In light of this, we need to regiment groups somehow. The natural thing to do is to take groups to be ZFC sets, which leads to group-based semantics facing the very same problem cover-based semantics encountered: the problem of accounting for non-set-sized denotations.

---

<sup>109</sup>Note that Moltmann's account suffers from the same problem, since there does not seem to be any reason why the integrated whole of all the non-self-membered integrated wholes should not exist, given that they are the maximal plurality falling under the property of being a non-self-membered integrated whole.

<sup>110</sup>Note that there are dissenting voices on this issue. See Linnebo (2010) and Linnebo (2016). I will come back to this discussion in Chapter 7.

**Anaphora and mixed predications** Moreover, the group-based approach does not fare well with respect to anaphoric reference. Consider the following sentence, involving an anaphoric use of ‘they’:

(4.31) *These players and these other players are arch-rivals and they are training hard to prepare for the derby.*

In this sentence, the pronoun ‘they’ is linked to ‘these players and these other players’, however whereas in the first conjunct the subject appears to function as an HLP term, the subject of the second conjunct appears to function as a mere plural. This is because whereas ‘being arch-rivals’ takes into account the way in which the players are referred to (as belonging to two different groups), ‘training hard to prepare for the derby’ does not take that information into account.

The reason why anaphora is a problem for group-based semantics is that according to their usual understanding, the subjects of each conjunct would pick out exactly the same, in this case, a plurality of groups, hence leading to the conclusion that the second conjunct is true if, and only if, each of some groups of players is training hard to prepare for the derby. But this is wrong: ‘training’ is an ordinary singular predicate – some people are training if, and only if, each of them is (at least, there is one such interpretation of ‘training’). Thus it should distribute down to the individual level. Moreover, once again, this is a particularly problematic move for the advocate of PL, who interprets statements like the second conjunct as consisting of a plural subject and a simple distributive predicate.

Perhaps the group-based approach could be modified so as to account correctly for anaphoric cases. However, there does not seem to be any easy way to do this. Since group-based semantics places the whole locus of ambiguity on the term, the required modification will have to assign distinct semantic values to the original term and the anaphoric one. Even though they would be systematically connected, a story as to how this mechanism works needs to be provided.<sup>111</sup>

Furthermore, this modification would not straightforwardly account for cases of mixed

---

<sup>111</sup>Moreover, things look less promising with respect to quantified statements involving anaphora – if these are available in one of the languages surveyed above. Consider the following sentence, expressed in an extension of English with HLP quantification:

(4.32) *Some playerses are arch-rivals and they are training hard to prepare for the derby.*

In this case, shifting the value of the subject of ‘are arch-rivals’ or that of ‘are training hard to prepare for the derby’ is not an option. If this kind of sentences were available, then this would posit a further problem for a group-based analysis.

predication:

**(4.33)** *These players and these other players are arch-rivals and are training hard to prepare for the derby.*

In this example, since no pronoun is provided as the subject of the second sentence, we cannot amend it directly by assigning two different semantic values to the two subjects present. Instead, we would need to posit an implicit anaphora and proceed as before. Moreover, note that there is a class of related cases which cannot even be amended in this way:

**(4.34)** *These players and these other players, who are arch-rivals, are training hard to prepare for the derby.*

In this case, the positing of an implicit anaphoric pronoun is even more far-fetched, for it requires that we analyze this sentence as a conjunctive sentence like (4.31). Furthermore, this move is not always available. For example, consider the following, involving a restrictive appositive phrase:

**(4.35)** *The players who are arch-rivals are training hard to prepare for the derby.*

There is a reading of this sentence according to which the subject, which is a definite description restricted by a collective clause, denotes a sum of groups (i.e. the teams which are arch-rivals). However, the predicate is a basic distributive predicate, requiring that the subject simply denotes the relevant players. This sentence, unlike the previous one, cannot be paraphrased as a conjunction, thus demanding further complications of the framework. Even though this may be corrigible, the fact that the group-based account requires these complications does not speak in its favour. Moreover, this is again especially challenging for the pluralist, since, as we argued, one of the pluralist's guiding principles is that, other things being equal, semantics should take the logical form of the object language expressions at face value, but all of these amendments consist in a certain modification of the form of the original sentence. Our diagnosis is that what is at fault is the fact that the ambiguity is placed entirely on the term. As we will see, the non-homophonic proposal we will put forward in Chapter 6 will survive this objection by partially delocalizing the ambiguity.

\*\*\*

I conclude that all of the current proposals as to how to analyse away HLP reference suffer from significant defects. Leaving aside the multigradedness analysis and the articulated reference account, which turned out to be materially inadequate, the other two accounts

impose unwanted limitations on the sizes of the denotations of terms. Sentences involving apparent HLP terms demand that we appeal to some notion in the semantics that captures the fact that HLP reference is somehow articulated, to put it in Ben-Yami's words. However, so far we have not found a mechanism that does the job properly, since all of the proposals to analyze away the superplural appeal to collectivizing objects, which gives rise to intuitively incorrect truth-conditions – going against one of the main guiding principles of the pluralist.

Moreover, on the one hand, the cover-based analysis suffers from problems due to the free availability of covers for higher-level plural terms of any level. And, on the other, the group-based analysis struggles with anaphoric reference.

The fact that we can neither paraphrase nor analyse away expressions which have a good prima facie claim to be higher-level plurals provides strong support to the thesis that those terms are indeed HLP. From now on, I proceed on this assumption.

## 4.4 Conclusions

The higher-levelist employs a homophonic semantics for her formal language. This is not problematic in itself, but it is problematic in cases where the meaning of the metalinguistic expressions is unclear. And in this particular case, there is a great disparity of opinions as to what the significance of apparent HLP expressions is. This has given rise to a widespread sceptical reaction towards higher-level plurals. Nevertheless, I believe that in this and the previous chapter the burden of proof has been considerably shifted towards the sceptic. I have done three main things to that end:

- (i) Firstly, I have provided a tentative elucidation of HLP reference which, although not constituting a reductive analysis thereof, should shed some light on the notion.
- (ii) Next, I have shown that there is substantive linguistic evidence of the presence of HLP in a number of ordinary languages.
- (iii) Finally, I have argued that the alleged instances of HLP reference in English are in general ineliminable, both in the sense that they cannot be paraphrased away and in the sense that they cannot be analysed away.

Most of the considerations and arguments I have provided should be especially pressing for the advocate of pluralism. Firstly, the linguistic evidence provided to carry out task

(ii) should move anyone who thinks that the presence of a certain expression in natural language supports the claim of its legitimacy as an element of a formal language (as the pluralist typically does). Secondly, the arguments used to support the indispensability of ordinary HLP terms (task (iii)) have been based, in part, on the principle that our theorising about language should mirror as closely as possible the speaker's own understanding of it, in particular, that it should allow for absolutely general interpretations.

An additional argument for the legitimacy of HLP reference can be given and I shall devote the next part of this thesis to develop it. I shall focus on the English natural language expressions enumerated in Section 4.2.1 (which from now on I take to be genuine HLP terms), and develop a non-homophonic semantic analysis for them employing only previously well-understood devices (i.e. singular and plural reference) – well-understood by the English speaker, that is. This should allow the higher-levelist to engage with the sceptic. My proposal will be an alternative to the accounts presented in the last section, which overcomes the problems faced by them. The resulting theory should thus be one which the sceptic can access without difficulty and therefore use as a ladder to arrive where the higher-levelist is.

There is a worry that any non-homophonic semantics will undermine the claim of any putative HLP expression to have a genuinely HLP way of signifying. While I share this worry, it is nonetheless possible to offer a non-homophonic semantics which is superior to the ones we have seen so far and thus delivers a closer approximation to a face value interpretation. This is what I aim to develop next.

## **Part III: Aspects**





# Chapter 5

## Reference and Aspects

### 5.1 Introduction

In this part of the thesis, I present my own proposal as to how to analyze the HLP fragment of English in a non-homophonic fashion. As I argued in the last two chapters, there is nothing defective with the higher-levelist approach to HLP reference and quantification. That approach is useful in that it gives those who have a prior grasp of these notions a compositional account of the meaning of complex expressions built from them. However, taking HLP expressions as primitive shall not help much those who do not have such previous understanding. My aim in this and the next chapter is to help those by sketching a non-homophonic account of higher-level plurals which employs only notions that are previously well-understood by English speakers – i.e. singular and plural reference and quantification. This analysis should serve as an alternative to the, admittedly, not very illuminating description of HLP reference as being reference to pluralities of pluralities or to some *objectses*.

When I speak of ordinary higher-level plurals in this and the next chapter, English speakers can think of any of the English expressions falling within any of the categories considered in Chapter 4: lists of plurals, plural definite descriptions with a pseudo-singular head noun, plural definite descriptions built from collective predicates and plurals accompanied by certain appositive phrases. All of my examples of HLP sentences will involve these types of term.

The aim in these chapters is the usual when developing formal semantics for ordinary language: I shall try to account for as much data as possible in the simplest way possible and, other things being equal, I shall aim for my account to cohere with accounts of other

fragments of English. Moreover, the resulting analysis should overcome the problems which undermined the theories presented at the end of the last chapter. Recall that the main problem of the alternative accounts was that they could not handle cases where the denotation of terms was non-set-sized. Moreover, the covers-based account did not discriminate between terms which trigger HLP readings and those which do not and the groups-based account could not account for some uses of anaphoras and mixed predications linked to HLP terms. One of my objectives will be to circumvent these difficulties.

The proposal I will put forward in what follows is, in slogan form, that HLP terms denote some objects *under their aspect of being organised in a certain way*. Moreover, I shall argue that this mode of reference is a species of the broader mode of reference which I call ‘restricted reference’ – i.e. reference to some objects *under a certain aspect thereof*.

According to this view, higher-level plurals refer to some objects under what I call a ‘cluster’. Clusters are plural properties (or perhaps relations, as we will see) which hold of some objects in so far as they are organised in various groups. Two higher-level plurals may be co-referential and yet contribute differently to the meaning of the sentences in which they occur; the different contributions being explained by means of the different clusters restricting the terms in question. This is in consonance with the idea that ontology is not all there is to the significance of HLP terms.

Before going into the details of my proposal, which shall be laid out in the next chapter, in this chapter I explore the notion of restricted reference in general – as it concerns singular as well as plural terms. More specifically, in Section 5.2, I begin by tackling restricted plurals and presenting a group-based solution to the puzzles of substitution concerning plurals. As we shall see, once again, groups, understood à la Link/Landman, fail to deliver correct results. In search of a solution, in Section 5.3, I take a step back and briefly survey the literature on the topic regarding singular reference. Having identified my favoured solution with respect to singulars, I move on to show how it can be expanded to apply to plurals. Finally, in Section 5.4, I come back to HLP reference and I argue that it should be analysed as a species of restricted plural reference. I finish by ruling out the possibility of taking HLP readings as being merely pragmatic, rather than semantically substantial.

## 5.2 Restricted plural reference: groups to the rescue?

As noted by Landman (1989b), co-referential plural terms are not always substitutable *salva veritate*,<sup>112</sup> even in seemingly ordinary sentential contexts.

Landman starts by considering noun phrases formed from pseudo-singular terms such as ‘committee’, ‘team’, ‘deck’, or ‘party’. He observes that two different committees, let’s say, the energy committee and the climate change committee, may have the exact same members and yet the terms denoting them not be substitutable in certain contexts. For example, (5.1) may be true, while (5.2) being false, even if the energy committee and the climate change committee have exactly the same members:

(5.1) *The energy committee paid an official visit to South Africa.*

(5.2) *The climate change committee paid an official visit to South Africa.*

A scenario which would block this substitution is one in which the members of the energy committee went to South Africa in their capacity as members of the energy committee and not of the climate change committee. The failure of substitution not only has to do with the lexical noun ‘committee’, but also with the sentential context where it appears: ‘paid an official visit to South Africa’. Informally speaking, we can say that this predicate<sup>113</sup> is sensitive not only to which people carried out the action, but also to their aspect of being members of a certain committee. By contrast, for example, the predicate ‘are 10 people’ would not be sensitive to such an aspect.

The failures we are interested in here may also concern rigid designators. For example, consider the demonstrative noun phrases ‘this committee’ and ‘that committee’. Demonstrative noun phrases are typically taken to be rigid; they denote the same object(s) in all possible worlds. Yet, we observe the same lack of substitutivity as before (assuming that the former refers to the energy committee and the latter to the climate change committee). We cannot infer (5.4) from (5.3):

(5.3) *This committee paid an official visit to South Africa.*

(5.4) *That committee paid an official visit to South Africa.*

To put it somewhat metaphorically, the reason why substitution fails to preserve truth-conditions is that committees have a life of their own, beyond their members.

<sup>112</sup>Hereafter I shall use ‘substitutable’ to mean ‘substitutable *salva veritate*’.

<sup>113</sup>In this and the next chapter I will often use ‘predicate’ in its logical sense, i.e. a sentence with an empty term-position, rather than in its narrower grammatical sense.

Interestingly, one of the reasons Landman proposes to expand Link's approach to plurals with groups is to tackle this phenomenon. According to Landman, groups may not only serve to give structure to plural terms (as observed by Link (1984)) and thus account for HLP interpretations, but also to account for certain failures of substitutivity. In the previous chapter we tackled the former application of group-based semantics. Here we are interested in the latter (although, as we will see shortly, these two applications are interrelated).

In what follows, I discuss the group-based solution as added to a pluralist semantics, rather than to a mereological one. But almost everything that I say applies to the mereological case as well.

Under this framework, we would solve the puzzles of substitution above as follows. We would assign an expression such as 'the energy committee' a group, instead of some individuals, as its semantic value. Given that a group is an individual in its own right, there is no reason to think that we can substitute 'the energy committee' by 'the climate change committee'. They are no longer co-referential, even though they are both related via membership-specification to the same individuals.

Unfortunately, groups, once again, fail to deliver the right results.

**Problem I: Ontological proliferation** Firstly, the group-based approach demands an enormous proliferation in the ontology of our semantics. As pointed out by Landman, the increase in ontology is of much bigger proportions than it may have seemed at first. This is due to the fact that the phenomenon exhibited by the examples above is much more pervasive than it seems; it does not only affect special noun phrases, like those involving committees or teams, but it affects a huge variety of apparently ordinary plural noun phrases too.

To see this, imagine a situation in which some people work as lecturers and also as graders. These are two different jobs for which they get paid separately. And suppose that those people decide to go on strike as lecturers, but not as graders – that is, they refuse to lecture, but not to mark assignments. In such a context, it would seem that the sentence

(5.5) *The lecturers are on strike.*

is true, whereas

(5.6) *The graders are on strike.*

is false.

Thus the solution based on groups ultimately demands that we introduce additional entities for every plural term occurring in such a context, hence becoming ontologically very costly.<sup>114</sup> One may not have any problem with ontological proliferation, but if one does (and some pluralists will in fact be so inclined), then the group-based proposal should be unattractive to them.

**Problem II: Cardinality restrictions** Secondly, as noted in Chapter 4, groups impose restrictions on the size of the denotations of terms. This problem re-emerges in this context.

One may think that we are able to use groups for the restricted aim of tackling failures of substitutivity, since the problem of non-set-sized denotations does not arise in the fragment of language affected from them. However, there are indeed uses of terms with non-set-sized denotations which violate substitutivity *salva veritate*. For example, consider:

(5.7) *The cardinals are used to measure size.*

(5.8) *The ordinals which are not bijective with smaller ordinals are used to measure size.*

Although ‘the cardinals’ and ‘the ordinals which are not bijective with smaller ordinals’ are co-referential, their substitution appears not to be truth-preserving. Intuitively, this is because the predicate ‘are used to measure size’ is sensitive to the way in which we refer to those sets – i.e. as playing the role of cardinals or the role of ordinals. A similar situation takes place in the following pair of examples:

(5.9) *The cardinals are defined as the ordinals which are not bijective with smaller ordinals.*

(5.10) *The ordinals which are not bijective with smaller ordinals are defined as the ordinals which are not bijective with smaller ordinals.*

The former appears to be true, but not the latter.

It could be argued that these substitutions do not introduce changes in truth value, but rather that their results are less felicitous than the original expressions. Reports by native speakers appear to vary on this issue. We will see many more examples in the course of this chapter, which hopefully will convince the reader that these failures of substitutivity do occur. Moreover, the sceptic may still get on board with our account to the extent that it makes these readings available and gives an adequate account of them. In other words, even if one finds these readings are not always most readily heard, one may still concede that at least they are available in some contexts. If that is so, then the sceptic can see our account as applicable in the (for her, rare) occasions where those readings are indeed available.

<sup>114</sup>As we will see shortly, these failures also affect singulars.

**Problem III: Anaphora and mixed predications** In the third place, the problems having to do with anaphora and mixed predication identified in Chapter 4 transfer to the present context as well. We have already seen that group-based semantics cannot capture the meaning of sentences like (4.31):

*These players and these other players are arch-rivals and they are training hard to prepare for the derby.*

This is because, unless the account is modified in a suitable way, ‘they’ would be interpreted as picking out two groups, hence not delivering the right results with respect to the meaning of the second conjunct, which consists of a basic distributive predication. Moreover, we saw how this sort of difficulty becomes increasingly challenging when we consider mixed predication and sentences involving appositive clauses.

Not surprisingly, when groups are used to account for intensional uses of plurals analogous limitations arise. For instance, suppose that in the following sentence we are talking about the same lecturers as before, who are also graders:

(5.11) *The lecturers stayed at home today. They are on strike.*

In (5.11) the argument position of ‘stayed at home today’ and the argument position of ‘are on strike’ are not both equally transparent. Whereas the former predicate is insensitive to the fact that its subject denotes some people as having a certain job, the latter is not. The lecturers did not stay at home in their capacity as lecturers. They stayed at home simpliciter. However, they went on strike only in their capacity as lecturers. In particular, they did not go on strike in their capacity as graders. Thus whereas ‘the lecturers’ is open to co-referential substitution, ‘they’ is not. Given that the group-based solution, as it stands, forces the anaphoric ‘they’ to denote the plurality denoted by ‘the lecturers’ thus opening the door to co-referential substitution, we have yet another reason to reject it.

One way to avoid this would be to force transparent predicates, such as ‘stayed at home today’, to allow for substitutivity between group-denoting terms with the same membership-specification. However, this solution suggests that plural reference is unnecessary after all. Why have plural and group reference plus contexts which allow for substitution between groups with the same membership-specification when you can simply have group reference and contexts which allow such substitution? This would erase the distinction between terms which denote pluralities of objects and terms which denote the groups thereof, thus losing the advantages of a pluralist approach to plurals. Recall that groups would be added to the pluralist account to deal only with certain phenomena, one of which being opacity. This is because, as we have seen, pluralist semantics are more successful than group-based

semantics in various ways. I submit that if we are to employ groups to deal with failures of substitution, then we must only invoke them in opaque contexts, not across the board. I proceed on the assumption that, if the group-based approach were to carry any advantage, then it would have to be conceived solely as complementing the pluralist approach in the way just described: a term in a transparent context denotes some individuals; the same term in an opaque context denotes a group associated with them.

\*\*\*

In order to look for an alternative solution to the puzzles of substitution concerning plurals, I am going to turn to the literature on singular failures of substitution, which, as one would expect, is much more extensive than the parallel discussion concerning plurals. So let me take a step back and briefly turn to singular reference in search for a unified solution to the puzzles of substitution before I return to my proposal.

### 5.3 A Step Back: Restricted Singular Reference

As is well-known, attitude contexts pose a problem for the following triad of principles:

**(5.12) Principle (Compositionality)** *The meaning of a complex expression is determined by the meaning of its constituents and the expression's logical form. In particular, the truth value of a sentence  $S$  is determined, given  $S$ 's logical form, by the semantic values of  $S$ 's constituents.*

**(5.13) Principle (Semantic innocence)** *The semantic value of an expression is independent of the sentential context where it appears – in particular, it is independent of whether it appears in an attitude context or otherwise.*

**(5.14) Principle (Substitutivity)** *If sentence  $S$  is the result of replacing some expression in sentence  $T$  with a co-referential expression, then  $S$  and  $T$  have the same truth value.*

Prima facie, the first two imply the last one. If the truth value of a sentence is determined by the semantic values of its constituents (Compositionality) and the semantic value of a term is always the same (i.e. its reference never shifts) (Semantic Innocence), then substitution of terms which pick out the same object (or objects) in a sentence should preserve truth value (Substitutivity).



However, the usual examples from the literature on psychological attitude contexts (hereafter, ‘attitude contexts’) show that these principles are not always satisfied. If they were all true, the inference from (5.15)-(5.16) to (5.17) would be truth-preserving:

(5.15) *Lois believes that Clark Kent can’t fly.*

(5.16) *Clark Kent is Superman.*

(5.17) *Lois believes that Superman can’t fly.*

Something must go. There are various ways to go about tackling this problem. The three most famous approaches are the Fregean (Frege, 1892/1952), the Russellian (Russell, 1905, 1918) and the Kripkean (Kripke, 1980) ones. Unfortunately, none of them are exempt from problems. Without intending to survey this vast topic, let me go through them in the briefest way possible and point to some of the main difficulties they face.

Frege took the culprit of the problem to be Semantic Innocence – it is not in general true that expressions have an invariant semantic value in all contexts. In particular, in attitude contexts sentences do not have truth values as their semantic values, but they have their usual senses (i.e. propositions) instead. The sense expressed by a sentence is in turn determined by the senses of its components. Thus, the semantic value of a name in an attitude context also shifts: it is no longer an object but its usual sense – in this case, the way in which it picks out its usual referent. Substitutivity fails in attitude contexts, because co-referential terms may not have the same sense.

Frege’s solution has a problem: as it stands, it cannot handle some cases of anaphoric reference.

(5.18) *Lois believes that Superman can fly. And he indeed can.*

According to the Fregean, in this context, we cannot identify the meaning of ‘Superman’ with that of ‘he’ – the former is a way of thinking about Superman, the latter is Superman himself – which goes against the usual understanding of anaphora. More generally, the meaning-shift favoured by Frege greatly complicates the semantics. One way in which it introduces complications concerns embedded attitude contexts (i.e. ‘Lois believes that Clark Kent believes that Superman can fly’), where we need to posit further meaning shifts.<sup>115</sup> This suggests that, other things being equal, Semantic Innocence should be retained.

The Russellian approach takes names to be abbreviations of definite descriptions and applies a quantificational analysis to the statements where they appear (i.e. ‘The *P* *Q*s’ is

<sup>115</sup>See Pietroski (1996, pp. 343-6) for a discussion of this issue.

shorthand for ‘At least one thing *Ps*, at most one thing *Ps* and whatever *Ps Qs*’). This dissolves the problem of Substitutivity by simply paraphrasing away the apparent opaque argument position. The main problem with Russell’s approach to names is that there does not seem to be a fact of the matter as to which description is associated with each name; not only when looking at different people’s ways of describing an object, but even with respect to a single person. Kripke criticized Russell’s view on these grounds (among others) and advocated, instead, a Millian view of names, according to which names are very much unlike descriptions. They are rigid designators and their semantic value is the unique object they pick out in all possible worlds. The way in which the Kripkean appears to solve the puzzle of attitude contexts is by arguing that, in fact, inferences like the one above are truth-preserving, that is, that positions in attitude contexts are transparent, despite some people’s intuitions. However, although it is plausible that some positions in attitude contexts are transparent, it is highly implausible that all of them are. For instance, even though there is a sense in which Abraham Lincoln believed that slavery should be abolished in Donald Trump’s country, there is another sense in which he clearly did not. At the very least, we must allow statements involving attitude contexts to have a reading like the latter, where the positions inside the that-clause are not transparent. The Kripkean approach seems to be unable to account for the difference between these two readings and appears, thus, to be unsatisfactory.

Besides the fact that all of the classical approaches face difficulties in dealing with attitude contexts, Saul (1997) has noted that the problem of substitution affects not only sentences involving attitude contexts, but seemingly ordinary sentences too. For example, consider:

(5.19) *Clark went into the phone booth and Superman came out.*

Via substitution of co-referential expressions, we get the seemingly false:

(5.20) *Clark went into the phone booth and Clark came out.*

The following statements demonstrate this phenomenon too:

(5.21) *Superman always gets more dates than Clark Kent does.*

(5.22) *Chris hit Clark Kent, but he never hit Superman.*

Substitutivity appears to fail in cases which do not involve attitude contexts at all.

In fact, this is in consonance with Landman’s observation that substitutivity fails for seemingly ordinary plurals, like ‘the graders’, in seemingly ordinary contexts, like ‘are on

strike'. His observation is, in fact, readily applicable to singulars: think of the singular versions of (5.5) and (5.6), involving a lecturer and a grader who happen to be the same person.

Given that the problem appears to be a lot more pervasive than initially thought, any solution that intends to classify contexts simply into opaque and transparent becomes implausible. It seems that there are no such things as opaque contexts simpliciter. Rather, opaqueness is a relative matter: predicates are opaque relative to certain kinds of terms. For example, the context in (5.19) appears to be opaque for terms which make salient the way in which their referent is dressed, among others; it is sensitive to aspects which relate to ways of dressing.

In light of these considerations, some authors have proposed an alternative strategy to tackle this sort of puzzles of substitution. It is what I shall call 'the Logophor View'. The Logophor View has been advocated, in various forms, in Forbes (1990, 1997) and Pietroski (1996). These authors take the culprit of the problem of substitution to be the principle of Substitutivity itself, while holding fixed the other two principles: Compositionality and Semantic Innocence.<sup>116</sup>

The main idea of the Logophor View is that substitution failures both in attitude and non-attitude contexts are explained by the mechanism at play in the following example by Quine (1953):

**(5.23)** *Giorgione is so-called because of his size.*

Here Substitutivity fails because even though 'Giorgione' is directly referential, when replaced with 'Barbarelli', which denotes the same person, the substitution alters the reference of the logophor 'so', giving rise to a change in truth-conditions. A logophor is like an anaphor, except that it refers to its anchoring expression rather than to the latter's reference – in (5.23), 'so' refers to 'Giorgione', rather than to Giorgione.

The Logophor View posits a hidden logophor in all of the examples where Substitutivity fails. Semantic Innocence is retained, for all expressions are assigned the same semantic value in all contexts (although some, such as 'so' in 'so-called', get re-interpreted after composition), and Compositionality is retained, since it is still clearly the case that the semantic value of a sentence like (5.23) is determined by those of its components and its logical form.

---

<sup>116</sup>This approach to the puzzles falls within the Davidsonian tradition (Davidson, 1968) of accusing the Fregeans of unjustifiably discarding Semantic Innocence while agreeing with them that senses play an important role in explicating these puzzles.

For example, the failure in (5.19) is explained by making explicit the logophoric phrases at work:

(5.24) *Clark, so-attired, went into the phone booth and Superman, so-attired, came out.*

The same occurs in attitude contexts. Once its hidden logophoric phrase is made explicit, (5.15) becomes:

(5.25) *Lois believes that Clark Kent can't fly, so-labelled.*

Where this should be understood as saying that the situation of Clark Kent not being able to fly is such that Lois believes her so-labelled way of thinking of it.

### 5.3.1 Revisiting the Logophor View

However, the Logophor View faces some difficulties. In what follows, I point to some of them and propose ways to amend it so as to bypass them. This will lead us to my favoured approach to the puzzles of substitution.

**‘So’ is neither logophoric nor anaphoric** Firstly, it does not seem to be the case that ‘so’ is logophoric in general. It is plausible in Giorgione-like cases,<sup>117</sup> but it does not seem to work in other cases. For example, in ordinary sentences like (5.24) ‘so’ cannot work as a logophor. At least, unlike in Giorgione-like cases, there is no obvious way to eliminate the so-phrase that delivers a grammatical sentence.

In the case of (5.23), by taking ‘so-called’ to be short for ‘called *t*’, where *t* is a name of the relevant term, we obtain:

---

<sup>117</sup>Although there is even reason to doubt this. It has to do with translation. Consider the following sentence:

(5.26) *Germany is so-called because of the Gaul's reference to those who came from the East of the Rhine as 'Germani'.*

Now consider the translation of this sentence into Catalan:

(5.27) *Alemanya s'anomena així perquè els Gals anomenaven 'Germani' a aquells que provenien de l'est del Rin.*

Under the understanding that ‘so’ is a logophor, the latter sentence is false, since in Catalan, the name for Germany (i.e. ‘Alemanya’) has a different stem and thus a different etymology, but ‘així’, being the translation of ‘so’, would seem to pick out the term ‘Alemanya’. However, translation is supposed to be truth-preserving. Thus, even in Giorgione-like cases it seems that ‘so’ cannot simply stand for the name in question, but a more complex mechanism must be in play.

(5.28) *Giorgione is called 'Giorgione' because of his size.*

I will turn to ordinary contexts shortly. But first note that this paraphrase seems to deliver correct results in attitude contexts as well. For example, if we do the pertinent transformations, we obtain from (5.25):

(5.29) *Lois believes that Clark Kent can't fly, labelled 'Clark Kent'.*

However, even in attitude contexts, it is not clear that 'so' is acting as a logophor, since (5.29) does not quite capture the meaning of (5.25). It seems that the reason why Substitutivity fails in the attitude context is that Clark Kent is thought of by Lois as Clark Kent (which is conveyed by the fact that in the attitude context he is labelled 'Clark Kent'), but Substitutivity does not fail because he is so-labelled. In other words, he could be so-labelled (in Lois' thought, that is) and yet Lois could be thinking of him as Superman, which would make the sentence false (for instance, imagine a situation in which Lois thinks that Superman, the mysterious superhero, is called 'Clark Kent').

Be that as it may, the Logophor View appears to be more clearly unsuccessful in ordinary cases. Following the same method of paraphrase as above, in the case of (5.24), we obtain an ungrammatical sentence:

(5.30) *Clark, attired 'Clark', went into the phone booth and Superman, attired 'Superman', came out.*

A more complicated mechanism is called for. First of all, one may suggest that, in this case, we replace the so-phrase with an as-phrase. For example, take 'so-attired' as short for 'attired as such'. This, at least, seems to lead to grammaticality:

(5.31) *Clark, attired as such, went into the phone booth and Superman, attired as such, came out.*

Now the question is what role does 'such' play. Firstly, if 'such' were logophoric, then it would be substitutable by a name of the relevant term. Thus we would obtain:

(5.32) *Clark, attired as 'Clark', went into the phone booth and Superman, attired as 'Superman', came out.*

But this paraphrase is clearly incorrect. Secondly, one may think we should take 'such' as being anaphoric instead. However, this would not work either, for if 'such' denoted the object being referred to by the initial term, then we could use any other co-referring term to replace it. But, by doing so, we could obtain the seemingly false:

(5.33) *Clark, attired as Superman, went into the phone booth and Superman, attired as Superman, came out.*

Finally, one may think that ‘such’ is simply a place-holder for the relevant term. It does neither denote the term nor its denotation, but it simply replaces it. According to this, in the case of (5.24), we would obtain

(5.34) *Clark, attired as Clark, went into the phone booth and Superman, attired as Superman, came out.*

Prima facie, this paraphrase is more plausible. However, I believe it is unsuccessful as well. To see this observe that Clark is in some occasions dressed as an ordinary human being and, in others, as a mysterious superhero. Thus, saying that Clark is attired as Clark does not pin down the relevant attire.

In my view, the correct diagnosis of these entanglements is that ‘such’ does neither denote the object referred to by the subject nor the term itself. Moreover, it is not a placeholder for the term. Rather, ‘such’ denotes an aspect of the object being referred to by the subject, where aspects are to be thought of as properties, as I will explain in detail shortly.

In general, aspects can be pointed to by the very same term occurring in an opaque context. In fact, what seems to explain the puzzles of substitution is that, unless the contrary is indicated, usually objects are taken to be referred to under the aspect or aspects conveyed by the very same term used to refer to them. This appears to be the default interpretation.<sup>118</sup>

Once a few further modifications of the Logophor View are carried out, we will see that this proposal appears to work in all the cases we have seen so far.

**Modifying the form of as-phrases** Another modification of the Logophor View seems needed: we must generalize the way in which terms become restricted. The Logophor View proposes that we use as-phrases of the form ‘ $\phi$ ed as  $x$ ’, where ‘ $\phi$ ed’ is a past participle and  $x$  is a referring expression – e.g. ‘attired as Superman’. However, it appears we should drop the participle and more generally speak of objects as  $x$ , under their  $x$  aspect. The reason for

<sup>118</sup>Whereas it is clear which aspect or aspects of a certain object a definite description points to (namely, those picked out by the predicates involved in the description), this is not the case with proper names. It seems that they must point to collections of aspects, but for instance, it is not at all clear which aspects ‘Clark Kent’ points to. For the time being I assume there is some fact of the matter as to which collection of aspects any proper name denotes. We can think about it as the characteristics of Clark Kent that people in general attribute to him when they think of him as Clark Kent. I leave this aside, since in fact, proper names are not going to be very relevant in the remainder of my arguments.

this is that in many cases, for example in (5.21), it is not clear what  $\phi$  should be. To see this try to substitute  $\phi$  in

(5.35) *Superman,  $\phi$ -ed as Superman, always gets more dates than Clark Kent,  $\phi$ -ed as Clark Kent, does.*

There is no clear candidate verb.

To go back to (5.24), I believe the situation is not well captured by saying that Clark Kent went into the phone booth attired as Clark Kent. Rather we should say that he went into the phone booth as Clark Kent. His having a Clark Kent aspect does not only determine his attire but a number of other features – his attitude, his strength, his intentions – many of which seem to play a role in determining the truth-conditions of (5.24).

**The source of opaqueness** A further reason to modify the Logophor View turns on the observation that there is no fact of the matter as to whether a predicate is accompanied by an implicit as-phrase or not, that is, whether a predicate is opaque or not. As we said above, opaqueness is a relative matter. This is because whether a certain predication should be interpreted as opaque depends, in part, on the choice of argument term.

In other words, the choice of argument not only determines what complement the as-phrase has but also, in part, whether there is an implicit as-phrase present at all. This is because an argument term may make salient a certain restriction thus triggering an opaque reading of the predicate.

Let me illustrate this with an example. Consider the sentence:

(5.36) *Theresa May is in favour of the new law proposed in parliament.*

This has an unrestricted reading according to which, Theresa May, regardless of which aspect we consider her under, likes a certain law. By contrast, the following does not seem to facilitate such an unrestricted reading:

(5.37) *The British PM is in favour of the new law proposed in parliament.*

Here we seem to be compelled to interpret the sentence as saying that Theresa May, in her role of being the British PM, likes a certain law. But this could be false while the former being true. For instance, imagine a scenario in which she personally approves of that law but, as PM, she votes it down.

My diagnosis of what occurred in the latter case is that the subject term makes a certain aspect of Theresa May salient and thus triggers an opaque reading of the predicate. In other

words, the predicate ‘is in favour of the new law proposed in parliament’ has two readings, a transparent one (which is insensitive to the role the subject plays in politics) and an opaque one (which is sensitive to it). Hence, it seems that the choice of term in some cases decides what the final reading of the sentence will be.

Nevertheless, it cannot be inherent to referring expressions that they are always restrictively predicated of either. That depends, in part, on what is predicated of them; on whether the predicate is sensitive to some aspects or others. For instance, ‘The lecturers’ in ‘The lecturers are on strike’ appears to demand a restricted reading. By contrast, the same term in ‘The lecturers are in their thirties’ does not. Although the term ‘the lecturers’ denotes some people under their aspect of working as lecturers, only the former predicate is sensitive to this restriction. Hence, the source of opacity cannot be solely in the argument term either. My final proposal will have it that both terms and predicates are partly responsible for the emergence of opaque contexts.

**Non-default restrictions** Moreover, there are cases in which the aspect made salient by the term is not the default one; it is not the one denoted by the term itself (or, rather, pointed to).

For example, consider:

(5.38) *The lecturer is 35 years old.*

This sentence does not seem to demand a restricted reading. The argument position occupied by ‘the lecturer’ is transparent. However, consider

(5.39) *The kid is 35 years old.*

In most contexts, we would expect this to be false. However, in some, it may be true and demand, for instance, that the term is interpreted as restricted by the aspect of the kid of playing a certain role in a theatre play. Making its meaning more conspicuous with an as-phrase, we obtain:

(5.40) *The kid, as a character of the play, is 35 years old.*

These sort of cases suggest two things. One, that the aspect restricting a given term may sometimes be provided by context, rather than by the term itself. And two, that opacity may sometimes arise from the need for unlikely interpretations. The predicate ‘is 35 years old’ appears to be mostly insensitive to aspects of the objects it applies to, but in this case it



must be interpreted as being sensitive to the aspect of the kid of playing a certain role in a play.

In other words, whereas the Logophor View only allows the argument to determine the content of the hidden as-phrase directly, it seems it can also affect it indirectly, when a different complement needs to be posited in order to deliver the correct truth-conditions.

\*\*\*

Now we can see how the modified version of the Logophor View would account for all the cases we have seen so far. First, it appears to deliver the right results in ordinary cases. For instance, we say that Clark Kent went into the phone booth under his Clark Kent aspect (where this may be construed as a collection of aspects), but he did not go into it under his Superman aspect (again, this may be seen as a collection of aspects). This explains the failure of Substitutivity. Moreover, cases of attitude context are also resolved in this way, because the term inside the attitude context is taken to denote an object under a certain aspect thereof: whereas Lois thinks that Clark Kent, under his Clark Kent aspect, can't fly, she does not think the same of Clark Kent under his Superman aspect. Moreover, cases of translation like that of n117, can be resolved in this way as well, since we can now analyse (5.26) as appealing to the aspect of Germany of being called 'Germany' in English. And, since, as we have seen, there is room for non-default interpretations (i.e. for the aspect at play not being the one denoted by the term employed as subject), we can analyse (5.27) as appealing to the same aspect, rather than the aspect of Germany of being called 'Alemanya' in Catalan. This explains why (5.27) may be true and indeed a good translation of (5.26) – that is, in contexts which facilitate such non-default interpretation.

All of these ideas will be made more precise shortly.

### 5.3.2 My proposal: taking aspects seriously

From now on, I will refer to terms which make salient an aspect of their referent as 'restricted terms'. Moreover, I will call the corresponding mode of reference, 'restricted reference'. Restricted reference is reference under aspects, reference that makes salient a certain aspect of the object picked out.<sup>119</sup>

---

<sup>119</sup>Prima facie, not all referring expressions are restricted. For example, it is plausible that demonstratives are unrestricted (except for when restriction is provided by context). The question of whether there are in-

As we have seen, there is no such thing as opaque predicates simpliciter. Whether an application of a predicate to a term is interpreted as opaque depends, in part, on whether and how the term is restricted and, in part, on the meaning of the predicate in question, in particular, on whether it is sensitive to the restriction of the term. In other words, on the one hand, if the argument term is restricted but the predicate is insensitive to such restriction, then the resulting predication is transparent. On the other, if the predicate is sensitive to certain restrictions, but the term is not restricted in any of those ways, then the resulting predication is transparent too. The term being restricted and the predicate being sensitive to the restriction are, in general, necessary and sufficient conditions for a restricted predication to take place.<sup>120</sup>

My view is similar to the Logophor View in that it makes terms retain their meaning in all contexts (i.e. it respects Semantic Innocence). However, it differs from the Logophor View, most importantly, on the one hand, in that I take as-phrases as pointing to aspects and, on the other, in that I take terms as having some power to bring about an opaque reading of the predicates attached to them. This is because I do not take predicates as always being (or not) accompanied by a hidden as-phrase. Instead, as-phrases are only implicitly present in cases in which terms are suitably restricted.

We can finally state a new, more refined, rule of substitutivity *salva veritate*:

**(5.41) Principle (New Substitutivity)** *In context  $P(t)$  we can substitute  $t$  by  $u$  salva veritate iff  $t$  and  $u$  are co-referential and either they are not restricted by any aspect to which  $P$  is sensitive or they are restricted by an aspect to which  $P$  is sensitive and they are restricted by the same aspect.*

**Aspects** Let me say more about what aspects are. I believe we should think of them as being intensional properties – in the sense that co-extensional properties need not be identical.

Firstly, the reason why we take aspects to be properties is that, as pointed out by Asher (2006), in languages such as Spanish and French, as-phrases take predicates or generic nouns as complements rather than full noun phrases. In the following sentences the relevant nouns are ‘político’ and ‘avocate’ (the corresponding indefinite noun phrases would be ‘un político’ and ‘une avocate’):

---

deed unrestricted terms will have to be left for future research. But even if this question had to be answered negatively, this would not suppose a problem for my proposal.

<sup>120</sup>Except for cases where context makes it clear that the term in question must be interpreted as being restricted. That is, cases like that of (5.39).

(5.42) *Mariano, como político, es corrupto.*

(5.43) *Lila, en tant qu'avocate, a un bon salaire.*

Secondly, aspects must be intensional in the sense that two different aspects may apply exactly to the same objects. For example, the aspect (or, rather, collection of aspects) of being Superman and the aspect (again, rather, collection of aspects) of being Clark Kent are aspects of one and only one object and yet they are distinct.<sup>121</sup>

In this sense, the notion of restricted reference is Fregean: there is something irreducibly intensional about aspects. However, there is an important difference between Fregean senses and aspects. Senses are in a many-one relation with referents. Two different senses may be ways of thinking about the same object, but terms which express one and the same sense always have the same reference. Thus the sense of a term can be said to determine the term's referent. Aspects are like senses in that terms that make salient different aspects may refer to the same thing, but they are unlike senses in that two terms may be restricted by the same aspect and yet not refer to the same thing. Aspects are not in a many-one relation with referents. For instance, we may refer to the same person under her two different aspects of being a father and a painter, but we may also refer to two different people under their common aspect of being teachers. Senses fix the reference of a term, whereas aspects fix (part of) the semantic contribution of a term.

**Types** To finish the presentation of my proposal, I make precise the ideas presented so far following, in part, the type-theoretic proposal in Asher (2006).

In my account, terms will be assigned types, where types should be seen as saying something about the metaphysical nature of the objects denoted by the terms that fall under them. For example, terms may be of type *physical object*, *event*, *informational object*, *job holder*, *artifact*, etc. If a term is of a certain type, then we also say that its referent is of that type. Under this framework, an aspect is a characteristic which is had only by objects of a certain type and thus determine the latter. For example, the type *physical object* has as corresponding aspects that of being heavy, being located at a certain place, etc. Thus when we refer to

---

<sup>121</sup>In fact, even aspects which necessarily apply to the same objects may be distinguished. Thus aspects may have to be thought of as being hyperintensional. This seems necessary to account for the truth value of some statements. For example, a certain geometrical figure, under its equiangular triangle aspect, might be thought of by Ann as an example of a figure with three equal angles. But it is possible that Ann does not think it has three equal angles under its equilateral triangle aspect. However, I leave this aside for the time being and concentrate on intensional aspects.

an object under an aspect thereof we are simultaneously determining its type.<sup>122</sup> Moreover, we can only refer to an object under a certain aspect thereof if the object instantiates, in fact, that aspect.

Predicates will also be assigned a type. Their type will indicate which type of aspects they are sensitive to (i.e. for which sorts of terms the predicate gives rise to an opaque context).

Moreover, I will make use of a complex type: what Asher calls the ‘dot type’. This type is formed from two or more simple types. We say that they are proper ‘sub-types’ of the complex type (an improper sub-type of a type is itself). For example, *informational object•artifact* is a complex type formed from the simple types at each side of the dot. The complex type allows us to capture the idea that a term may be restricted by various aspects at once.

Let me give an example in which the complex dot type will prove useful. In this type-driven framework, a predication succeeds if, and only if, the predicate and the term are of compatible types. More precisely, as I will explain shortly, the predication succeeds if, and only if, the term has the type of the predicate as one of its proper or improper sub-types.

For example, consider the following:

(5.44) *Lunch took forever.*

Here we are considering a lunch as being of the simple *event* type, that is, under its aspect of being a lunch kind of event.

However, in some cases we may have to recur to a complex type. This occurs, for instance, in some cases of anaphora:

(5.45) *Lunch took forever, but it was delicious.*

If we were to take the meaning of ‘lunch’ as picking out an *event* type of object, we would not get the right truth-conditions with respect to the second predication, since the second predicate demands that the argument be of *physical object* type (in fact, this move would be analogous to the one proposed in the group-based framework for plural terms).

---

<sup>122</sup>Note that there is no clear-cut distinction between aspects and types. Aspects are properties and their corresponding types are the sort of entity which typically has those properties. However, being a certain sort of entity or another can itself be seen as a property. Thus any proper taxonomy of types and aspects is bound to be relative. As the reader can imagine, providing such a taxonomy will not be part of this work. I shall need only a small and well-delineated fragment thereof, which I sketch below and describe in more detail in the next chapter.

Following Asher, our preferred solution is to establish that the term be of the complex type *event•physical object*. Now each of the predicates involved is seen to apply to the lunch under different types: the lunch as an event and the lunch as a physical object. Each predicate exploits one of the simple types from which the complex type of the term is composed.

Moreover, in some cases, adjustments to the type of terms can occur so as to ensure success. These are cases like that of (5.39). Even though the term ‘the kid’ is not initially classified as restricted by the aspect of playing a certain role in a theatre play, we can expand the type of the term to include the type of that aspect.

Importantly, in all cases in which terms are of a complex type, we will always leave a proper sub-type unspecified. This captures the idea that the aspects picked out by a certain restricted term are only a proper part of the whole typology of an object. That is, restricted reference to an object tells us that that object has some specific aspects, but also that it has other (unspecified) aspects. We indicate the unspecified type with a ‘?’. For example, in

(5.46) *This book, as a paddle, is useless.*

the as-phrase is making explicit the restriction of the subject to the aspect of being a paddle and thus forces it to be of complex type *?•physical object*.

When a predicate applies to an object under its unspecified type we say that it is transparent for that term, since it does not take into account any of its specific proper sub-types. Thus, under this framework, an argument term is occupying a transparent position if, and only if, we can substitute it by any other co-referring term which has as a proper or improper sub-type the unspecified type. This demands that we start off by assigning all terms a type which includes as proper or improper sub-type, the unspecified type.

This finalizes the exposition of my type-driven account of opacity. This theory, as is customary, focuses on singular predication. Let me explain next how we can extend it to plurals, before turning to how all of this can be applied to the semantics of higher-level plurals.

### 5.3.3 Extending the account to plurals

Given that I characterized aspects as properties, all it takes for the present framework to extend to plurals is that one accepts the legitimacy of plural monadic properties and relations as playing the role of aspects. But, given that I am assuming the pluralist standpoint and thus

taking for granted the legitimacy of plural predicates (and thus of plural properties), this is readily available here.

Importantly, the type-driven approach, as applied to the plural fragment of the language, appears to overcome the problems of the group-based account. Firstly, the problem of ontological proliferation may be addressed by noticing, first, that aspects are not the referents of restricted terms, but rather part of their meaning-fixing mechanism. Moreover, one may hold a nominalist view about properties and relations and argue that aspects, unlike groups, have a good claim to not being new objects in our domain of quantification. Although this is controversial, the door is open for one to develop this line of defence. I will go back to this issue in Chapter 6, when looking into the ontological innocence of Cluster Semantics, so let me put it aside for the time being.

Secondly, taking as-phrases as invoking properties or relations does not impose any restrictions on the sizes of the main subjects. For example, we can speak without problem of the ordinals which are not bijective with smaller ordinals under their aspect of being cardinals, since the property of being a cardinal need not have a set-sized extension.<sup>123</sup>

Thirdly, this approach is successful in dealing with anaphoric reference into opaque contexts. Recall sentence (5.11):

*The lecturers stayed at home today. They are on strike.*

According to the present account, ‘are on strike’ is sensitive to *job holder* aspects, hence it holds of the referents of ‘the lecturers’ under their aspect of being lecturers. On the other hand, one can also affirm that the lecturers stayed at home, without taking ‘stayed at home’ as a predicate that applies to objects of *job holder* type. This predicate is not sensitive to one particular aspect of the objects being picked out and this is captured in our framework by making it hold of objects of unspecified type – i.e. ‘the lecturers’ and thus ‘they’ above should be understood as picking out some objects, namely the lecturers, under their complex type ?•*job holder*.

To sum up, the type-driven account is successful in accounting for the failures of Substitutivity of singular terms as observed in both simple and attitude-ascription sentences. Moreover, it can be easily extended to account for the analogous failures concerning plural terms while *prima facie* overcoming the problems faced by the group-based account considered in the beginning of this chapter. The fact that the type-driven approach provides a

---

<sup>123</sup>I am assuming that properties and relations should not be set-theoretically analysed (i.e. identified with their extensions, understood as sets).

unified solution for a number of different cases can only be seen as an additional reason to adopt it.

### 5.3.3.1 Pseudo-singularity and restricted reference

Additionally, the notion of restricted plural reference sheds light on an otherwise puzzling phenomenon related to the notion of pseudo-singular reference.<sup>124</sup>

One of the claims defended in Chapter 2 was that pseudo-singular reference is a genuine linguistic phenomenon present in natural language. Even though we found good reasons to accept this conclusion, a certain difficulty derives from its acceptance. According to the view that pseudo-singulars are disguised semantically plural terms, Substitutivity between co-referential pseudo-singulars and the corresponding plurals is to be expected. However, in some cases, such substitutions are not truth preserving. For example, given that ‘couple’ is one of the terms we took to be pseudo-singular, sentence (5.47) would seem to be paraphrasable as (5.48) (supposing that ‘this couple’ refers to the same people as ‘those people’):

(5.47) *This couple is great.*

(5.48) *Those people are great.*

However, this seems to be a clear case where Substitutivity (understood in its old, simple, version) fails. For example, think of the scenario where those people are great as a couple, but are not great in general (whatever that means!). In that case, the former would be true and the latter false.

Now, equipped with the notion of restricted reference and the principle of New Substitutivity we can predict and explain this failure. In (5.47) we are denoting some objects considered under their aspect of being in a romantic relationship (and ‘is great’ is sensitive to this), whereas in the latter, we are considering them unrestrictedly. Thus New Substitutivity predicts that the substitution is not truth-preserving. This receives support from the fact that (5.47) is paraphrasable as

(5.49) *Those people, as a couple, are great.*

The predicate ‘are great’ when combined with the restricted term ‘those people, as a couple’ can be seen to become sensitive to the fact that the objects of which it is predicated

---

<sup>124</sup>The connection between the notion of pseudo-singularity and that of restricted reference was briefly mentioned in Chapter 4, Section 4.2.2.

form a couple. In other words, being great is a property that people may have simpliciter, but also under a certain aspect – e.g. as colleagues, as artists, as parents.

I conclude that restricted reference and predication can explain some of the failures of Substitutivity concerning pseudo-singulars. Yet another reason to adopt this proposal.

## 5.4 HLP reference as a species of restricted reference

Let me finally return to HLP reference. How does the theory of restricted reference relate to the HLP idiom? My claim is that ordinary HLP terms should be seen as a species of restricted terms, as a species of terms which make salient a certain aspect of their referents.

Singular terms can be restricted in many different ways. Plural terms can be restricted in any of the ways singular terms can, but also in ways not available to singular terms. The reason for this is that, since aspects correspond to properties or relations and all singular properties can hold of more than one object at once (that is, in a distributive way), every singular aspect can play the role of a plural aspect as well. For example, just as we can consider a book as an informational object, we can consider many books as informational objects. However, the existence of collective properties – properties which hold of more than one object at once while not necessarily holding of each of them – and of relations brings about a proliferation of restricted plural terms, since there are more aspects under which they may be restricted. For instance, helping ourselves to collective properties or relations we can refer to some people under their aspect of being co-workers, under their aspect of forming a circle or under their aspect of being 12 in number. The corresponding simple types would be *mutual relationship*, *arrangement* and *number*, respectively.<sup>125</sup> These are clearly not available for singular terms.

My proposal is that HLP terms be seen as plural terms restricted by one such collective aspect. In particular, under what I shall call ‘a cluster’. A cluster is the property of some objects of being organised or arranged in a certain way.

The way in which this gets implemented in the type-driven approach is by making particular clusters be the aspects and the different sorts of clusters be the types. For instance, a cluster which consists of the property of organising six objects in three groups of two objects each is an aspect of six objects and, roughly speaking, it is of the type *cluster which divides*

---

<sup>125</sup>For a (non-exhaustive) taxonomy of collective types, one may look at the list of kinds of collective predicates in Oliver and Smiley (2016, pp. 116-120).



*objects in simple groups* (giving rise to a second-level plural term). As we will see in the next chapter, there will be a cluster-type for each level of the HLP hierarchy.

When one refers to some objects higher-level plurally, one refers to them under their aspect of being clustered in a particular way. Thus two HLP terms may have the same referents but denote them under two different clusters and hence contribute differently to the truth-conditions of the sentence where they occur.

In the previous section, I sketched a general theory of restricted reference. One of the advantages of that account is that even though it is compatible with the pervasiveness of the failures of Substitutivity, it can make distinctions between transparent and opaque contexts. Recall that, according to the type-driven approach, it is only relative to types of terms that sentential contexts can be said to be transparent or opaque. So it is not possible to classify contexts as transparent or opaque simpliciter. Nonetheless, it is possible to specify which combinations of context and argument give rise to opacity. Doing this across the board is a huge task that I neither intend to nor could carry out. However, I shall try to contribute to it by focussing on HLP expressions and giving a systematic account of which contexts are opaque for them. Before doing this in the next chapter, let me justify my claim that HLP terms should be analysed as a species of restricted terms.

### 5.4.1 Rationale

**Clusters as collective properties or relations** If one agrees that plural terms may be restricted, then one must buy into the legitimacy of plural properties and relations as playing the role of aspects. If clusters are plural properties or relations, then it would be arbitrary to allow for restriction of plural terms to other aspects, but not to clusters. Thus in order to show that HLP expressions are restricted expressions, we need to first justify the claim that clusters are either plural properties or relations.

Prima facie, one may think that clusters are plural properties and, in particular, collective properties. This is because there seem to be collective predicates that pick out clusters: ‘are grouped evenly’, ‘are organised in colour-groups’, ‘are classified by type’, ‘form teams’, ‘are divided into two’, etc. In fact, clusters seem to belong to the class of collective properties Oliver and Smiley call ‘arrangement and derangement’ properties (these are properties denoted by predicates such as: ‘assemble’, ‘are concatenated’, ‘congregate’, ‘are dense’, ‘are evenly distributed’, ‘form a circle’, ‘gather’, ‘are in sequence’ or ‘are separated’).<sup>126</sup>

<sup>126</sup>See Oliver and Smiley (2016, pp. 119-120).

However, the predicates listed above do not seem to be capable of making fine-grained enough distinctions between different clusters; rather they distinguish between kinds of clusters (i.e. the clusters which consist of even groups, the clusters which consist of groups of objects of the same colour, etc). We need something which allows us to make finer distinctions.

Relations appear to be more suitable in this respect. Although the distinction between collective properties and relations is not easy to pin down, there is a characteristic of certain relations which collective properties certainly cannot have. It is called ‘differential application’ and it is possessed by non-symmetric relations in particular. We say that a relation has differential application if, and only if, there is more than one way in which it can apply to a given plurality of objects.<sup>127</sup> For example, the relation ‘is taller than’ can be applied to Venus and Serena in two different ways, the results of which describe two different scenarios; thus it is non-symmetric. Clusters appear to admit differential application as well. That is, for any given cluster, there are different ways in which it may apply to some things.

Recall that a cluster is instantiated by some objects when those are organised in certain groups.<sup>128</sup> For example, think of the cluster giving rise to two groups of three objects each. If there was no repetition of referents, this cluster would be instantiated by a plurality of six objects. But there is more than one way in which those objects can instantiate it. For example, consider the plurality of the Holy Family and the Three Wise Men. We must distinguish between the case where the Holy Family form one group and the Three Wise Men another one and the case where Joseph, Mary and Balthazar form one group and Jesus, Melchior and Gaspar another one. Yet, both are cases where one and the same plurality instantiates one and the same cluster.

Differential application seems to be an indicator that clusters are relations and, indeed non-symmetric relations. Thus, in what follows, I take clusters to be relations rather than collective properties. However, not much hinges on this decision; what is really important for us is that given that we had already accepted that plural terms could be restricted by collective properties or relations, we must allow that they be restricted by clusters.

**Analogous failures of Substitutivity** Moreover, HLP terms display failures of Substitutivity analogous to those displayed by the restricted terms considered so far.<sup>129</sup> Consider the

<sup>127</sup>See Fine (2000) and MacBride (2007) for a discussion of this notion.

<sup>128</sup>Hereafter ‘group’ is not used in its technical sense in linguistics, but informally, unless otherwise indicated.

<sup>129</sup>See Ben-Yami (2013) for a similar diagnosis of the failures of Substitutivity and for more examples.

following:

(5.50) *Serena, Angelique, Karolina and Simona played against each other in the tournament.*

(5.51) *Serena and Angelique, and Karolina and Simona played against each other in the tournament.*

Both of these sentences have more than one reading, but there are at least some available readings thereof according to which substitution of their subjects fails to preserve truth. Namely, the interpretation of (5.50) according to which all of the four players played against each other and the interpretation of (5.51) according to which each pair played a different game – that is, Serena played against Angelique and Karolina played against Simona. Under those readings, these sentences have different truth-conditions.<sup>130</sup> Even though the two subjects denote the same players, the subject of (5.51) does so under a certain cluster, whereas the subject of (5.50), being a mere plural term, does not convey any such aspect. Thus it is plausible that the reason why Substitutivity fails is that one is restricted by a cluster whereas the other one is not.<sup>131</sup>

Let us look at another pair of statements:

(5.52) *The students and their lecturers met in separate rooms.*

(5.53) *The students, their art lecturers and their science lecturers met in separate rooms.*

Suppose that ‘the lecturers’ in (5.52) denotes the art and the science lecturers and no-one else. This pair of sentences is distinct from the first one in that in this case both subjects are of the same syntactic type (i.e. simple lists of plurals), and thus of the same cluster-type. However, they denote the same objects under different clusters. Roughly speaking, the subject of (5.52) denotes its referents as belonging to two groups, whereas the subject of (5.53) conveys the idea that they belong to three different groups. The difference in the cluster carrying out the restriction in each case is the source of the difference in truth-conditions.

I conclude that failures of Substitutivity involving HLP terms have an analogous origin to those involving other restricted terms and, thus, should be treated on a par.

<sup>130</sup>This would also be the case if we interpreted (5.50) as saying that they played doubles.

<sup>131</sup>In fact, for the sake of uniformity in my analysis, I shall say that basic plural terms, like the simple list in (5.50), are restricted by another cluster – i.e. the vacuous cluster. We will see this in the next chapter.

### 5.4.2 Pragmatics?

There is a question we must consider before going on: why are the failures of Substitutivity involving HLP terms a semantic rather than a merely pragmatic phenomenon?

This question has been raised both in the narrower debate around HLP terms and in the debate around the puzzles of substitution of co-referential terms more generally. Given that we just argued that failures of Substitutivity involving HLP expressions are a species of failures of Substitutivity more generally, we should once again take a step back and consider the broader debate on the semantics/pragmatics divide.

Salmon (1986), Saul (1997) and Soames (2002) have advocated a Millian view of names and attributed the failures of Substitutivity to pragmatics. The idea is that even though it may seem as though substitution of co-referential terms sometimes leads to a change in truth-conditions, what is really going on is that there is a change in the conversational implicatures of the relevant sentence.

(5.54) *Lois has always believed that Clark Kent can fly.*

The claim is that the reason why one tends to find this sentence odd is that it has the pragmatic implicature that Lois would put her belief as expressed in the that-clause. In support of this, we see that the following sentence, including a cancellation clause of the implicature just described, seems true:

(5.55) *Lois has always believed that Clark Kent can fly, but she wouldn't put it that way.*

However, this argument is not entirely convincing, for adding cancellation clauses to apparently opaque statements does not always result in felicitous statements. For example, consider:

(5.56) *Superman always gets more dates than Superman does, that is, he gets more dates when appearing as Superman than when appearing as Clark.*

(5.57) *Chris hit Clark Kent, but he never hit Clark Kent, that is, he hit Clark Kent when appearing as Superman, but not when appearing as Clark Kent.*

These sentences appear to be infelicitous and the corresponding sentences without the cancellation clause appear to be (necessarily) false.

Hence there is at least one reason to reject the view that the failures of Substitutivity are in general reducible to pragmatic considerations. Moreover, I believe that a stronger case can

be made in the specific case of HLP terms. So let me take a step forward and come back to the debate around HLP expressions. In linguistics, some authors<sup>132</sup> have suggested that HLP readings of predicates, understood as being readings intermediate between the distributive and the collective reading, do not have a role in semantics, but only in pragmatics. The main argument for this conclusion is based on the principle of Mereological Generalization (which is usually stated in terms of groups and sums, but which I state in plural terminology):

**(5.58) Principle ((Plural) Mereological Generalization)** *Whenever  $P$  is true of a higher-level plurality, then  $P$  is true of the individuals which constitute that higher-level plurality.*

The claim is that even though we normally express certain propositions as involving HLP reference, that is only because using mere plural reference would result in a misleading way of expressing the situation at hand, but not in a false statement. For example,

**(5.59)** *The young animals and the old animals were herded separately.*

is true if, and only if,

**(5.60)** *The animals were herded separately.*

is true (supposing that ‘the animals’ denotes all and only the relevant young and old animals).

This is supposed to show two things. Firstly, that ‘were herded separately’ does not demand an HLP argument and, secondly, that ‘the young animals and the old animals’ is substitutable *salva veritate* with ‘the animals’. Moreover, this motivates the adoption of the dual of Mereological Generalization, Upwards Closure (which I also bring to the plural framework):

**(5.61) Principle ((Plural) Upwards Closure)** *Whenever  $P$  is true of some individuals, then  $P$  is true of any higher-level plurality constituted by those individuals.*

Here the idea is that the inference from (5.60) to

**(5.62)** *The pigs and the cows were herded separately.*

is truth-preserving (supposing that the pigs and the cows are all and only the relevant animals).

As a consequence, (5.62) is true in the situation where the animals were herded separately by age, even though it is a misleading way of describing the situation. In favour of these claims, (5.62) can be shown to be true by adding a cancellation clause to it:

<sup>132</sup>See Link (1998), Schwarzschild (1993), Schwarzschild (1996, ch. 5).

**(5.63)** *The pigs and the cows were herded separately, but not grouped in this way.*

If these principles obtained, this would show that substituting different co-referential HLP terms in a certain context does not make any difference with respect to truth-conditions and thus that they contribute the same semantic value to the propositions where they appear. However, there is a problem with this argument. Even though the series of statements just presented does seem to be truth-preserving,<sup>133</sup> the application of the principles of (Plural) Mereological Generalization and (Plural) Upwards Closure is not always truth-preserving. On the one hand, (Plural) Mereological Generalization can be seen to fail in the following case. Suppose the following is true:

**(5.64)** *The short and the tall kids are equally loud.*

If (Plural) Mereological Generalization were true, the following would have to be true (supposing that the subjects are co-referential):

**(5.65)** *The kids are equally loud.*

However, this is either false or its being true involves a shift in the interpretation of the predicate. (5.65) strongly suggests the interpretation according to which each kid is as loud as each other kid in the plurality. This is because, unlike ‘were herded separately’ which only applies to groups, ‘are equally loud’ can naturally describe two different kinds of situation: those in which groups of things are collectively as loud as each other and those in which individuals are as loud as each other. By contrast, ‘were herded separately’ does not have a natural reading analogous to the latter, that is, as holding collectively of some individuals. Hence, (Plural) Mereological Generalization appears to fail in this case.

On the other hand, (Plural) Upwards Closure does not hold either, since predicates may hold of basic pluralities in a collective way. For instance, (5.66) is true, but (5.67) is not:

**(5.66)** *2, 3, 4 and 6 are co-prime.*<sup>134</sup>

**(5.67)** *2 and 4, and 3 and 6 are co-prime.*

Given that ‘are co-prime’ is a basic collective predicate, (5.67) demands an interpretation according to which the predicate distributes down to the two pairs: 2 and 4, on the one hand, and 3 and 6, on the other. But it is neither true that 2 and 4 are co-prime nor that 3 and 6 are.

I hope to have shown that neither (Plural) Mereological Generalization nor (Plural) Upwards Closure obtain. Neither everything that holds of an HLP term holds of a mere plural nor vice-versa. I conclude that co-referential HLP terms involving different clusters make different contributions to the meaning of the sentences where they appear.

<sup>133</sup>Although reports by native speakers appear to cast doubt into this as well.

<sup>134</sup>Two or more integers are co-prime if, and only if, the only positive integer that divides all of them is 1.

## **5.5 Recap**

In this chapter, I have argued in favour of a variant of the Logophor View in order to tackle the puzzles concerning substitution of co-referential terms. Moreover, I have suggested a precisification of this view in the form of a type-driven approach. I finished the chapter by giving reasons to see HLP reference as a species of restricted reference – in particular HLP reference is present when reference is restricted by a cluster, an aspect of some objects according to which they are organised in various groups. In the next chapter, I present the details of a non-homophonic semantics for HLP expressions based on the ideas laid out so far.

# Chapter 6

## Cluster Semantics for Higher-Level Plurals

### 6.1 Introduction

In this chapter I describe a semantics for ordinary HLP expressions based on the ideas introduced in the previous chapter. I start, in Section 6.2, by looking at HLP terms and predicates from a static perspective, that is, assuming their interpretation is fixed. Having done this, in Section 6.3, I turn to the compositional viewpoint. Since I hold that HLP terms and predicates are just a species among many of restricted terms and aspect-sensitive predicates, in this part I hope to contribute to the general task of explaining which predicates are opaque for which terms (recall that in our framework there is no such thing as an opaque predicate simpliciter – what counts as an opaque predicate is a relative matter). Finally, in Section 6.4, I show that this semantic approach overcomes the problems faced by its rivals.

### 6.2 Higher-level plural terms and predicates

#### 6.2.1 Cluster types and their aspects

First of all, let me establish some terminology. I will call terms restricted by clusters ‘cluster-restricted terms’. When we refer to some objects via a cluster-restricted term, the specific cluster doing the restricting is the aspect under which we consider the objects. The corresponding type depends on the sort of cluster involved. The idea is that clusters can be more



or less complex (i.e. involve more or less nesting of groups). We classify clusters according to their complexity: clusters of level 1, clusters of level 2, etc. There is a cluster-type for each level of complexity. Hence terms are of type  $cluster_k$  (which is short for ‘cluster of level  $k$ ’) if, and only if, they are restricted by a certain  $cluster_k$ .

I shall give a more detailed account of what clusters there are below. For the moment, as I argued in the previous chapter, we should think of clusters as being relations which hold between some objects if, and only if, the objects are internally clustered in some way. In other words, if, and only if, as a result of instantiating the cluster they are seen as belonging to different groups, where groups may have groups as well as basic individuals as members.<sup>135</sup>

We thus classify clusters as follows:

- (1) A  $cluster_1$  does not organise objects in any groups.<sup>136</sup>
- (2) A  $cluster_2$  organises objects in simple groups.
- (3) A  $cluster_3$  organises objects in groups of groups.

And so on.

Let us look at some examples to clarify these ideas. In order to represent clusters I use the same informal notation that I used in Chapter 3 to represent higher-level pluralities.

We can represent the cluster restricting ‘Russell and Whitehead, and Hilbert and Bernays’ as  $((u^0, v^0), (w^0, t^0))$ , where each letter is a singular variable (as indicated by the superscripted ‘0’) representing a singular argument position. The term denotes four people who instantiate the cluster consisting of four objects organised in two groups of two objects each. The way in which the plurality of Russell, Whitehead, Hilbert and Bernays instantiates this cluster according to the term above, can be represented as any of the following:  $((r^0, w^0), (h^0, b^0))$ ,  $((w^0, r^0), (b^0, h^0))$ ,  $((h^0, b^0), (r^0, w^0))$ ,  $((b^0, h^0), (w^0, r^0))$ ,  $((r^0, w^0), (b^0, h^0))$ ,  $((w^0, r^0), (h^0, b^0))$ ,  $((h^0, b^0), (w^0, r^0))$  and  $((b^0, h^0), (r^0, w^0))$ , where  $r^0$ ,  $w^0$ ,  $h^0$  and  $b^0$  are singular constants denoting each of the logicians. For the sake of simplicity, I shall usually use the representation corresponding to the literal syntax of the natural language expression.

We can represent more complex clusters as follows. For example, the cluster made salient in ‘the historians and Alba and Bruna, and the philosophers and Carla’ is a  $cluster_3$  and can

<sup>135</sup>Recall that, unless otherwise indicated, I do not use ‘group’ in its technical sense as used in linguistics, but in an informal sense.

<sup>136</sup> $cluster_1$  do not really group the objects they restrict at all, except in a vacuous way. The reasons why we have added them to the hierarchy of clusters will become clear shortly.

be represented as  $((u^1, (u^0, v^0)), (v^1, w^0))$ . The resulting plurality can be represented as  $((h^1, (a^0, b^0)), (p^1, c^0))$ .

Each cluster has a certain number of places. When we say that a plurality instantiates a cluster we mean that all of the individuals of that plurality occupy one of its places (where more than one individual may occupy one place collectively, when the place is plural). Hence, there are many ways in which a cluster can be instantiated by one and the same plurality – one for each way of assigning its objects to the places of the cluster. We call each of these ways ‘an application’. As we have seen, for each plurality that instantiates a cluster, there are some applications that give rise to the same restricted pluralities, given that there is an isomorphism between them that preserves group-membership, as it were. I will come back to these issues in Section 6.2.6.

To sum up, a cluster-restricted term denotes some objects under their aspect of instantiating a certain cluster via a certain application. In order to express this more succinctly, I will say, as a convenient shorthand, that a cluster-restricted term denotes a clustered plurality (i.e. some objects under a cluster relative to an application). I shall represent clustered pluralities as ordered triples of the form  $\langle p^1, C, app \rangle$ , where  $p^1$  denotes the objects being referred to,  $C$  denotes the cluster doing the restriction and  $app$  denotes the application of the cluster to the objects (i.e. a function from the places of  $C$  to  $p^1$ ).

## 6.2.2 The hierarchy of cluster-restricted terms

We can describe a hierarchy of cluster-restricted terms according as to the complexity of the cluster that restricts them.

As explained in the last chapter, we use the symbol  $\bullet$  to represent the complex dot-type, a type which allows us to capture the idea that an object or some objects are restricted by more than one aspect simultaneously. Recall that we allow dot-types with more than two simple sub-types.

In particular, I take all cluster-restricted terms to be of complex type  $cluster_1 \bullet cluster_2 \bullet \dots \bullet cluster_n$ , for some  $n$ .<sup>137</sup>

This is how we analyse HLP reference as cluster-restricted reference:

---

<sup>137</sup>In a broader context, we would also include the unspecified type  $?$  as a sub-type of these complex types. However, given that we are only interested in failures of substitution due to cluster restrictions, we can ignore this here.

- (1) A first-level plural is a plural of type  $cluster_1$ . E.g. ‘the Williams sisters’.
- (2) A second-level plural is a plural of type  $cluster_1 \bullet cluster_2$ . E.g. ‘Russell and Whitehead, and Hilbert and Bernays’.
- (3) A third-level plural is a plural of type  $cluster_1 \bullet cluster_2 \bullet cluster_3$ . E.g. ‘these students and their lecturers, and those other students and their lecturers’.

And so on.<sup>138</sup>

Whether there are examples of type  $cluster_1 \bullet \dots \bullet cluster_k$ , for  $3 < k$ , in ordinary English is an open question. Of course, one can form them, for example by nesting lists and using pseudo-singulars, but they may not be easily found in everyday talk.

The motivation behind assigning higher-level plurals a complex type is that they make salient various *layers of cluster information*, as it were. To make this idea more precise we need to define the notion of sub-cluster first.

A cluster  $C_1$  is a sub-cluster of a cluster  $C_2$  if, and only if, it results from merging all the groups in  $C_2$  of a certain level of complexity (or of more than one level). For example, the cluster  $((a^0, b^0), (c^0, d^0), ((e^0, f^0), (g^0, h^0)))$  is of level 3 and has as sub-clusters of level 2  $((a^0, b^0, c^0, d^0), (e^0, f^0, g^0, h^0))$  and  $((a^0, b^0), (c^0, d^0), (e^0, f^0), (g^0, h^0))$ , depending on which groups are merged.<sup>139</sup>

Equipped with the notion of a sub-cluster, we say that if a cluster-restricted term is of level  $n$ , then it is also restricted by all the proper sub-clusters (of level  $k$  for  $1 \leq k < n$ ) of the relevant cluster $_n$  and by the relevant cluster $_n$  itself.<sup>140</sup> The reason why we need this, as will become clear in section 6.3, is that any given predicate will ignore some of these clusters whenever it is not sensitive to certain types of cluster. This is, precisely, what will make a predicate transparent for some terms.

I shall speak of each sub-plurality of the referents of a cluster-restricted term that is classified into one group by the relevant cluster as a plural member of that clustered plurality (as usual, I use singularizing jargon only as a convenient shorthand). For example, the

<sup>138</sup>Note that this appears to be at odds with the reference-cumulativity of the HLP hierarchy, according to which plural reference of level  $n$  is a species of plural reference of any higher level. However, one should simply take the analysis of HLP reference just outlined as corresponding to the strict notion of HLP reference. When moving on to giving semantics for HLPL, we would say that a term of level  $n$  denotes clustered pluralities of level  $k$  for  $k \leq n$ .

<sup>139</sup>As we will see, the fact that each cluster may have more than one sub-cluster of a given level will give rise to certain ambiguities.

<sup>140</sup>Possibly by more than one sub-cluster of a given type as we just saw with our example.

clustered plurality picked out by ‘Russell and Whitehead, and Hilbert and Bernays’, has as plural members only the plurality denoted by ‘Russell and Whitehead’ and that one denoted by ‘Hilbert and Bernays’. This is exactly analogous to our old notion of plural member of a higher-level plurality. I shall also say that a plural member is among a certain plurality, in consonance with our old terminology. And, as before, a plural member may consist of a single individual, but may not be empty.

In what follows, I consider two ways of refining HLP reference: ordered HLP reference and HLP reference with repetition. We tackled both of these topics with respect to the plural level, back in Chapter 2. Let me briefly summarize the conclusions I reached there and show how they can be implemented in the cluster-based approach to HLP expressions.

### 6.2.3 Order

Recall our favoured account of order in connection with plurals in the form of lists. The basic idea, as proposed in Florio and Nicolas (2015), was that plural terms simply contribute some objects (according to the usual view on plural reference) and that when order is seen to play a role in determining truth conditions, we help ourselves to an external indexing – a function  $f$  from a plurality of indices, related by a salient order  $<$ , to the referents of the term.

Given that indexings can be multivalued, all of what we said with respect to plurals can be applied to second-level plural terms. For example,

**(6.1)** *The children and their parents had lunch in this order.*

Suppose this sentence is true if, and only if, the children had lunch before their parents.<sup>141</sup>

The present account captures this idea as follows. Two indexings are present:  $f$  tracks the order in which the children and their parents had lunch and  $g$  tracks the order in which they are listed. Supposing that the sentence is indeed true and that the children are  $a$  and  $b$  and their parents are  $c$  and  $d$  (and taking, once again, positive integers as indices), we have

---

<sup>141</sup>The presence of the modifier ‘in this order’ restricts the interpretations to those that take the syntax of the list at face value. In particular, we must rule out the interpretation according to which the children and their parents considered as a simple plurality had lunch in that order, since in this case, ‘in this order’ would be meaningless. Moreover, we must also rule out the interpretation according to which ‘had lunch in that order’ is distributive, since the terms ‘the children’ and ‘their parents’ also lack any internal syntactic structure for ‘in this order’ to track.

that  $f(1) = a, b$ ,  $f(2) = c, d$ ,  $g(1) = a, b$  and  $g(2) = c, d$ . The sentence is deemed true since  $f(1) = g(1)$  and  $f(2) = g(2)$ .

Furthermore, this apparatus is suitable for terms of higher level as well, given that clusters of higher complexity appear to be immune to facts about order. In other words, indexings assign either an individual or a simple plurality to each index, but never a clustered plurality itself. For example, consider:

**(6.2)** *Annie and her parents, Bonnie and her parents, and Connie and her parents arrived in the order they were called.*

First of all, this statement has more than one possible interpretation, but let us suppose that it is true just in case the three pluralities denoted by ‘Annie and her parents’, ‘Bonnie and her parents’ and ‘Connie and her parents’, respectively, arrived in the same order as they were called. In this case, the cluster instantiated by each of these pluralities is irrelevant towards truth conditions. Hence, the term is ultimately interpreted as a second-level plural – i.e. some of the sub-clusters taking part in the restriction are ignored by the predicate. Moreover, this is also the case with the alternative reading of the sentence in which the predicate distributes down to the second-level plural components of the list. As far as I know, all of the statements involving order are such that they ignore clusters of level higher than 2. I submit that all sentences involving order are such that their predicates will ignore all types except  $cluster_1$ ,  $cluster_1 \bullet cluster_2$  and  $cluster_1 \bullet cluster_2 \bullet cluster_3$  (the latter, under a distributive reading of the predicate). Thus the account of order favoured regarding plurals can be readily extended to HLP reference.

Nevertheless, in order to simplify our framework and given that we are already assuming the existence of clusters, understood as relations, and of applications, understood as functions from cluster positions to objects, we can identify applications with indexings and cluster positions with indices. We need only assume an order holding between the cluster positions.

Let me illustrate this by informally describing the truth conditions of (6.2). According to this account, (6.2) is true if, and only if,

- (i) there are some objects: Annie, Annie’s parents, Bonnie, Bonnie’s parents, Connie and Connie’s parents;
- (ii) s.t. they instantiate the cluster  $C = (1,2,3)$ <sup>142</sup> relative to applications  $f$  (tracking the order in which they arrived) and  $g$  (tracking the order in which they were called);
- (iii) and  $f(1) = g(1)$ , ...,  $f(3) = g(3)$ .

<sup>142</sup>I use positive integers to name each of the plural cluster positions in order to capture the relevant ordering.

### 6.2.4 Repetition

When discussing plurals in Chapter 2 we argued that repetition seems to be an inherent aspect of plural reference and thus that the latter could be liberalized to allow for multiple reference to the same object. However, given that the indexing approach can be applied to repetition and given that we had already made use of it to account for order in plural expressions, we concluded that we should exploit it to account for repetition in plural expressions too, for the sake of simplicity.

As explained in Chapter 2, there are two sorts of repetition affecting plural terms. Both of them transfer to higher-level plurals. Firstly, just as repetition is sometimes made explicit in the syntax of plural terms, the same occurs with regard to higher-level plurals:

(6.3) *The winners of the three rounds were the kids, the kids and the adults.*

Here, we cannot substitute ‘the kids, the kids and the adults’ with the co-referential ‘the kids and the adults’ without altering the truth conditions of the sentence.

Secondly, just as it occurred with plurals, there are HLP terms that involve accidental repetition. The following sentence is exactly analogous to the example we gave concerning plurals:

(6.4) *The lecturers and the graders have different salaries.*

First of all, suppose that one of the lecturers is also a grader, so that there is an overlap between the plurality of lecturers and that of the graders. This is a case where one object belongs to more than one of the pluralities picked out by an HLP term. Now suppose that not only one of the lecturers happens to be a grader as well, but all of them are also the relevant graders. In this case the plurality of the lecturers and that of the graders are one and the same.

Thus there are two ways in which HLP terms may have repetition among its referents. On the one hand, (i) we must allow for individual objects to occur multiple times as the referents of a single term and, on the other, (ii) we must open the door to multiple occurrences of the members of a higher-level plurality. As follows from the description of the HLP hierarchy in Chapter 3, the existence of some higher-level pluralities involving cases of (i) follows from HLP Comprehension and thus does not require an external indexing. These are cases in which one and the same object appears as a member of different members of a higher-level plurality. For instance, according to the HLP Comprehension axioms, for any two objects  $a$  and  $b$ , there is the higher-level plurality  $(a, (a, b))$ . This involves repetition, but only occurring in different members of the higher-level plurality. The same occurs with

$((a, b), ((a, b), (c, d)))$ , which exists by HLP Comprehension if  $a, b, c$  and  $d$  exist. However, pluralities which have multiple occurrences of one and the same member do not exist by HLP Comprehension and can only be obtained via the use of an external indexing.

Let us turn to these. First, the indexing approach has no difficulties handling cases of non-accidental repetition, such as that of (6.3). Given that indexings may be multivalued and non-injective, we can simply assign the same objects to two different indices (or more), thus obtaining the desired truth conditions by making predicates hold of pluralities relative to indexings, rather than pluralities simpliciter. Once again, for the sake of simplicity and economy, we shall take indexings as applications and indices as cluster positions. Let me clarify this by informally stating the truth conditions of (6.3). That sentence is true if, and only if,

- (i) there are some objects, i.e. some kids and some adults;
- (ii) s.t. they instantiate the cluster  $C = (a^1, b^1, c^1)$ <sup>143</sup> relative to applications  $f$  (tracking who were the winners of the different rounds) and  $g$  (tracking the cluster restriction made salient by the HLP term);
- (iii) and there is an isomorphism between  $f$  and  $g$  with respect to the number of times that they assign each plurality to a position.<sup>144</sup>

Unfortunately, things are not so straightforward in cases of accidental repetition. For example, consider again sentence (6.4). It is not the case that (6.4) is true if, and only if, its predicate holds of the plurality of lecturers (who are also graders) relative to a certain indexing, since the corresponding truth-conditions would also make true the following:

**(6.5)** *The lecturers and the lecturers have different salaries.*

Even though the two components of the list in (6.4) are co-referential, we cannot substitute one with the other without altering the truth conditions of the sentence. We must be able to discriminate between these two sentences. As the reader will suspect, this is nothing else than a case in which each of the components of the list is itself a restricted term in the general sense described in the previous chapter. In the next subsection, I shall show that, fortunately, we need not abandon Florio and Nicolas' indexing approach, but we can simply supplement it with the tools presented in the previous chapter in order to cover these cases.

<sup>143</sup>In this case, the cluster positions are not assumed to be ordered.

<sup>144</sup>The assignments need not be identical, since, at least under a reading of the sentence, order is irrelevant. It is enough that the two applications assign each plurality the same number of times.

### 6.2.5 Restricted components

We need an account of HLP reference that allows for accidental repetition and that forbids substitution of co-referential components within the same list (at least, in some cases). As we have just seen, the indexing approach to repetition is not sufficiently fine-grained to make suitable distinctions.

My diagnosis is that the reason why we cannot substitute one term with the other in (6.4) is that they denote some objects under different *job-holder* aspects, which is in turn taken into account by the predicate ‘have different salaries’. Having a certain salary or another is a property that does not hold of people as such (unless we assume that everyone who has a salary has a single job), but it holds of people qua job holders, under their aspect of having a certain job.<sup>145</sup>

Given that we have already adopted a type-driven approach to HLP reference, one that focusses on cluster-types, we may as well employ the very same mechanisms involved there to explain what goes wrong in substitutions like the one giving rise to (6.5). We need only acknowledge that the items of an HLP term in list form can themselves be restricted by aspects.

In order to account for this in the semantics, we shall say that the relevant application is such that its output values may be restricted by certain aspects: in this case, the property of being lecturers and the property of being graders.

Once again, let me give an example to clarify things. The truth conditions of (6.4) would be, informally, the following. That sentence is true if, and only if,

- (i) there are some objects,  $a^1$ ;
- (ii) s.t. they are restricted by the cluster  $C = (u^1, v^1)$  relative to the application  $f$  (where  $f(u^1) = a^1$  and  $f(v^1) = a^1$ );
- (iii) and s.t.  $f(u^1)$  are restricted to their lecturer aspect,  $L$  and  $f(v^1)$  are restricted to their grader aspect,  $G$ ;
- (iv) and the predicate ‘have different salaries’ holds of  $(L(f(u^1)), G(f(v^1)))$ , where this represents the clustered plurality which has as members two occurrences of the same

---

<sup>145</sup>Note that the same would apply to plural terms displaying the same phenomenon. For example, it applies to the analogous example we presented in Chapter 2 (n61):

*The lecturer and the grader have different salaries.*



plurality of people considered under their aspect of being lecturers and under their aspect of being graders, respectively.

Thus, I conclude that order and repetition, including accidental repetition, can be accounted for by mechanisms we already had in place: clusters and applications (and some additional apparatuses, such as an external ordering of the argument positions of clusters).

### 6.2.6 A background theory of clusters

Having had a look at various examples and at the mechanisms needed to account for a number of linguistic phenomena, we are now ready to sketch a background theory of clusters. As we argued in the last chapter, clusters can be taken to be relations of various adicities.

I have mentioned at various points that there are clusters of level 1. That is, clusters that do not really group the objects they restrict at all, except in a vacuous way. A plural term restricted by a cluster<sub>1</sub> is simply a basic plural term, and thus semantically contributes some objects. Note that repetition and order both support the existence of clusters of level 1, since equipped with the latter we can substantially simplify our semantics by getting rid of the notion of an indexing and using the application present in any instantiation of a cluster instead.

Let us say that a group is of complexity of level  $n$  if, and only if, it is the result of applying the operation of group formation  $n$  times. For example, a group of groups is of complexity 2. Then we say that an  $n$ th-level cluster may organise objects into groups of all levels of complexity lower than  $n$ , not only of the immediately lower level. This corresponds to the membership-cumulativity enjoyed by the HLP hierarchy. However, certain restrictions are in place:

- (i) Any cluster <sub>$n$</sub>  organises objects in at least one group of complexity  $n - 1$ .
- (ii) Any cluster <sub>$n$</sub>  organises objects in at least two groups.

Condition (i) disallows the occurrence of one and the same cluster at two different levels and condition (ii) forbids the existence of clusters analogous to singleton sets.

Finally, let us say something about applications. Clusters, as I showed in the last chapter, admit differential application – they have more than one way to apply to the same objects. Each of these ways, as explained above, is an application, a function from the argument

positions of a cluster to the referents of the term. Applications are possibly non-injective (allowing for repetition) and possibly multivalued (allowing for plural assignments).

Since not all the positions in a cluster admit differential application, we say that a cluster-restricted term denotes some objects under a certain cluster, relative to a plurality of applications which are isomorphic with respect to plural membership (rather than to a single application).

### 6.2.7 Distributivity and collectivity as two sides of the same coin

When describing PL and HLPL, we agreed to take all of the non-logical plural predicates, by default, as picking out collective properties, that is, properties such that if they hold of a higher-level plurality, that does not imply that they hold of each of the members of that plurality (or vice-versa). To see that this move is unproblematic, one can point to the fact that we can always paraphrase away distributive predicates in English by making use of a quantifier.<sup>146</sup> For example, instead of saying that Serena and Venus are tennis players, we can say that anyone who is one of Serena and Venus is a tennis player. As we saw, the same holds in HLP cases. Instead of saying ‘The kids and their parents are forming a circle’ (understood distributively) we say ‘Any things which are among the kids and their parents are forming a circle’. The reason we can paraphrase away distributive uses of predicates is that we can make use of the very same predicates and apply them to terms of a lower level. Note that in our paraphrases we neither dispense with the predicate nor with the plural or HLP term. Instead, we quantify restrictedly over the individuals or pluralities denoted by the term and apply the predicate to the quantified variable. All that is paraphrased away is the application of the predicate to the HLP term.

Now let us say that a predicate is of type  $cluster_k$  if, and only if, it holds collectively of pluralities restricted by  $clusters_k$ . This idea can be made more precise as follows: A predicate holds of  $cluster_k$  type terms if, and only if, the predicate is sensitive to the fact that the referents of the term instantiate a  $cluster_k$ , that is to say, if, and only if, it takes the aspect of the objects being so clustered into account, in the sense that this fact plays a role in determining the truth conditions of the resulting sentence.

We can now express the idea that distributivity and collectivity are two sides of the same coin as follows: A predicate of type  $cluster_k$  is collective for terms of type  $cluster_1 \bullet \dots \bullet cluster_k$  and distributive for terms of type  $cluster_1 \bullet \dots \bullet cluster_{k+1}$ . Put differently, a pred-

<sup>146</sup>This is pointed out in Florio (2010, p. 10).

icate can hold collectively of objects restricted by a cluster<sub>k</sub> if, and only if, it can hold distributively of objects restricted by a cluster<sub>k+1</sub>.<sup>147</sup>

This gives us a neat picture of the hierarchy of predicates that are sensitive to cluster restrictions. At the bottom level – the singular level – we find only collective predication: a predicate such as ‘is red’ holds of a single object collectively (this is a degenerate case of collectivity, but we include it for the sake of homogeneity). If we move one level up and apply a singular predicate to a plural term, we encounter the first sort of distributivity: some things are red if, and only if, each of them is red. At this level there are also predicates for which such a biconditional does not hold. These are the plural collective predicates. If we move one level up and apply those to a second-level plural term, then we encounter a second sort of distributivity. And so on.

## 6.3 Higher-level plural sentences

We are finally ready to move to the compositional viewpoint and investigate what the exact source of opaqueness for HLP terms is.

### 6.3.1 The emergence of higher-level plural interpretations

To begin with, we must depart from a given taxonomy of typed terms and predicates. This is what I have been describing in the previous sections. Let me recap.

I take the sorts of ordinary HLP terms listed in the introduction of the last chapter as inducing cluster-restricted interpretations. Those terms, recall, were of the following sorts: lists of plurals, plural definite descriptions with pseudo-singular head nouns, plural definite descriptions built from collective predicates and plurals accompanied by certain appositive phrases. Thus, I take all of these as being of a complex cluster-type, having as sub-types all the cluster-types up to a certain level. For example, the term ‘the dog lovers and their dogs, and the cat lovers and their cats’ is of complex type  $cluster_1 \bullet cluster_2 \bullet cluster_3$ , as can be read off its syntax (being a list of lists of plurals). Moreover, I take plural terms as being of simple type  $cluster_1$ .

As I explained above, the reason why we must assign HLP terms of level  $n > 1$  a complex cluster-type to begin with is that they are restricted not only to a cluster of level  $n$ , but also to

<sup>147</sup> An analogous claim has been defended in Landman (1989a, pp. 590-593).

all its sub-clusters of lower level. To go back to the example above, the term ‘the dog lovers and their dogs, and the cat lovers and their cats’ contains cluster information of various levels, the highest one being of level 3. This term tells us about its referents that they are organised in two groups (i.e. the dog lovers and their dogs, on the one hand, and the cat lovers and their cats, on the other) of two groups each (i.e. the dog lovers, their dogs, the cat lovers and their cats), each of which is a basic plurality of individuals. This is important, since these layers of information are the ones which will be available to be taken into account by different predicates, once composition has taken place.

Predicates will get classified according as to the sorts of individuals/pluralities they can hold of collectively: singular objects, pluralities restricted by clusters<sub>1</sub>, pluralities restricted by clusters<sub>2</sub>, etc. However, most predicates may hold collectively of terms of different cluster-types. The most typical cases of ambiguity are those like ‘admire each other’. This predicate can take as argument a simple plurality, in which case the admiring holds between any pair of objects in the plurality:

**(6.6)** *The historians admire each other.*

But it can also be seen to apply to a plurality restricted by a cluster<sub>2</sub>, in which case a natural interpretation is that the admiring holds between pluralities:

**(6.7)** *The historians and the philosophers admire each other.*

When appearing in a sentence, in order to distinguish between these two readings, we will need to disambiguate the predicate and interpret it as being sensitive to clusters of a single type (possibly a complex one). This is because, for instance in the latter case, even though the term is of type *cluster*<sub>1</sub> • *cluster*<sub>2</sub>, the predicate may be interpreted in such a way that it ignores part of the complexity of that restriction and simply applies to it under its cluster<sub>1</sub> aspect. In other words, (6.7) can be interpreted as meaning that each individual who is either a historian or a philosopher admires any other individual who is either a historian or a philosopher.<sup>148</sup>

---

<sup>148</sup>At this point, one may worry about the equivocity objection (Oliver & Smiley, 2016, Sec. 4.3 and p. 250). The idea behind the objection is that a predicate such as ‘carried the box upstairs’ (for instance, in ‘Anna’s parents carried the box upstairs’ and in ‘Anna carried the box upstairs’) is univocal and simply holds of different sorts of terms, rather than being ambiguous between a singular reading, a plural reading, a second-level plural reading, etc. However, I believe there is a problem with this objection. In cases like that of the interpretation of (6.7) I just described, we need to either make predicates ambiguous between various readings, against the claim that they are univocal or else make terms ambiguous between various readings. According to the latter strategy, in this case, ‘the historians and the philosophers’ would be a plural in some contexts and a second-level plural in others. However, this would demand that we abandon Semantic Innocence with respect

Moreover, there is an additional element to take into account when it comes to HLP predicates. Predicates can hold of a plurality in two different ways: collectively or distributively. If a predicate holds collectively of some individuals/pluralities, then it simply holds of them. Nothing else needs to be said. But if a predicate holds distributively of some individuals/pluralities, then it also holds of each of their members. Therefore two distinct parameters may be revised when it comes to assigning a semantic value to a predicate in a sentential context: its type – the sort of cluster information it is sensitive to – and the way in which it holds of its arguments (i.e. collectively or distributively).

Finally, it is possible that a term be retyped to be of a different complex type than the one it started from. For example, this is the case with a term such as ‘the kids’ when attached to a  $cluster_1 \bullet cluster_2$  predicate (such as ‘compete against each other’) and such that context makes it obvious that ‘the kids’, despite looking plural, must be interpreted as being a second-level plural (the context must provide the relevant cluster under which the objects in question are restricted, for instance context might make it obvious that we are talking about the kids in one room and the kids in another room). This allows for cases analogous to (5.39) in the previous chapter:

*The kid is 35 years old.*

where an unlikely, non-default, interpretation of the term was required.

Having said this, in what follows I present the various possible combinations of HLP terms and predicates and the interpretation of the sentences they give rise to.

In order to clarify ideas, I will give an example for each possible case. In order to describe the examples, I use the following notation. I use subscripted numbers and the symbol ‘•’ to represent the cluster-type of terms (for instance ‘[Your kids and his kids]<sub>1,2</sub>’ means that the term ‘Your kids and his kids’ is of type  $cluster_1 \bullet cluster_2$ ). Moreover, I shall use the superscript ‘*Dis*’, when a predicate is interpreted distributively. In its absence, the predicate is interpreted collectively. Singular terms and predicates shall be marked with a subscripted *s*. Moreover, when a predicate is ambiguous between various readings, I will use more than one subscript. For instance, ‘admire each other’ may be sensitive to clusters of level 1, but also of level 2. I shall capture this fact with the following notation: [admire each other]<sub>1,2</sub>.

---

to terms which, as we have seen, would posit problems for anaphora and mixed predications, among others. Thus, I submit that we must opt for the former strategy. Admittedly this forces us to introduce the possibility of equivocity between homonymous predicates, but I believe that this loss does not suppose a big problem, since there is some plausibility to the intuition that homonymous predicates may have indeed different meanings sometimes.

For each example, I first give a pseudo-interpretation of the sentence, prior to disambiguating the predicate and/or retyping the term, and then I list its possible final interpretations (when there is more than one possible final reading, I enumerate them with letters: a), b), c), etc).

\*\*\*

Consider a plural term of type  $cluster_1 \bullet \dots \bullet cluster_{n-1} \bullet cluster_n$  (it may be of simple type  $cluster_1$ ). And suppose it occupies the only position of a certain monadic predicate.<sup>149</sup>

- (i) If the predicate only holds of individuals, it ignores all cluster restrictions and it is interpreted distributively.

<p>[Your kids and his kids]<sub>1•2</sub> [are noisy.]<sub>s</sub>  a) [Your kids and his kids]<sub>1•2</sub> [are noisy.]<sub>s</sub><sup>Dis</sup></p>
--

According to the resulting interpretation, each of your kids and his kids is noisy. The fact that the subject term is restricted by a  $cluster_2$  is ignored by the predicate, which is a simple singular predicate, applying distributively to the term as restricted by a vacuous  $cluster_1$ .

- (ii) If the predicate is of type  $cluster_k$  (univocally),<sup>150</sup> then

- (ii.1) If  $n = k$ , there is a single possible interpretation.

<p>[Your kids and his kids]<sub>1•2</sub> [are equally diverse.]<sub>2</sub>  a) [Your kids and his kids]<sub>1•2</sub> [are equally diverse.]<sub>2</sub></p>
--

The resulting sentence should be read as saying that your kids are diverse to the same degree that his kids are. The predicate in this case does take into account all the cluster-information provided by the restriction of the subject.

- (ii.2) If  $n > k$ , then either the predicate ignores some sub-clusters but is applied collectively or is applied distributively (also possibly ignoring some sub-clusters).

<sup>149</sup>Expanding these rules to polyadic predicates is straightforward, but I will focus on the monadic case for simplicity.

<sup>150</sup>For example, the predicate ‘are equally diverse’ seems to be a predicate that holds of a single type of term (i.e. second-level plural), assuming that a single object cannot be equally diverse to another single object. Moreover, the predicate ‘voted against the European Constitution’, used in the next example, also appears to be of this sort if interpreted as meaning that a majority of people voted against it, that is as applying to pluralities collectively and not to individuals. In any case, it must be acknowledged that univocal predicates are rare. Usually, predicates are ambiguous between various readings.

[The French and the Dutch] <sub>1•2</sub> [voted against the European Constitution.] <sub>1</sub> a) [The French and the Dutch] <sub>1•2</sub> [voted against the European Constitution.] <sub>1</sub> b) [The French and the Dutch] <sub>1•2</sub> [voted against the European Constitution.] <sub>1</sub> <sup>Dis</sup>
--

The first interpretation says that all of those who were French or Dutch voted collectively against the European Constitution. The second one is similar, but since the predicate is interpreted distributively, it says that, on the one hand, the French voted collectively against the European Constitution and, on the other, so did the Dutch.

- (ii.3) If  $k > n$ , then the type of the term may be expanded to include the type of the predicate among its components.

[The kids] <sub>1</sub> [are equally diverse.] <sub>2</sub> a) [The kids] <sub>1•2</sub> [are equally diverse.] <sub>2</sub>
---

The resulting sentence says that the kids, restricted by a contextually-provided cluster<sub>2</sub> are equally diverse, i.e. that some of the kids are as diverse as some others.

- (iii) If the predicate can hold of terms of various types, then context will have to be used to determine the interpretation of the predicate.

[The kids and the adults] <sub>1•2</sub> [get easily annoyed by each other.] <sub>12</sub> a) [The kids and the adults] <sub>1•2</sub> [get easily annoyed by each other.] <sub>2</sub> b) [The kids and the adults] <sub>1•2</sub> [get easily annoyed by each other.] <sub>1</sub> c) [The kids and the adults] <sub>1•2</sub> [get easily annoyed by each other.] <sub>1</sub> <sup>Dis</sup>
---

The first disambiguation says that the kids get annoyed by the adults and vice-versa. The second one says that each one of the kids and the adults gets annoyed by every other kid and adult. Thus the predicate picks out a basic collective property which applies to all of the kids and adults without distinction. Finally, the third one says that each of the kids gets annoyed by every other kid and each of the adults gets annoyed by every other adult, but is silent on whether a kid gets annoyed by an adult or an adult gets annoyed by a kid. Thus the predicate is interpreted exactly as in the previous case, as a basic collective predicate, but it now distributes down to each of the plural terms making up the list ‘the kids and the adults’.

We can finally appreciate more clearly which predicates are opaque for which terms and describe a specific rule of substitutivity *salva veritate* for HLP expressions:

**(6.8) Principle (HLP Substitutivity)** *In context  $P(t)$ , where  $P$  is sensitive only to clusters<sub>k</sub>, we can substitute  $t$  by  $u$  *salva veritate* iff  $t$  and  $u$  are co-referential and they are restricted by*

*the exact same clusters of level up to  $k$  (or level  $k + 1$  if  $P$  is interpreted distributively).*<sup>151</sup>

In other words, if a monadic predicate is only sensitive to clusters <sub>$k$</sub> , then co-referential terms will be substitutable in its argument position as long as they are restricted by the exact same clusters of up to level  $k$ . Whether they are restricted to clusters of higher level (and to which ones exactly) does not matter, for the predicate will ignore those.<sup>152</sup>

For example, in the case of ‘Your kids and his kids are noisy’ (interpreted as above: [Your kids and his kids]<sub>12</sub> [are noisy.]<sub>s</sub><sup>Dis</sup>), one can substitute the subject by any other co-referential term: a basic plural, but also a plural of higher level, since the predicate, being a basic singular predicate, will ignore all cluster restrictions of level higher than 1.

### 6.3.2 Selection restrictions

As we have seen, predicates that are sensitive to a given cluster-type can be combined with terms of a number of different types, thereby giving rise to grammatical and possibly true sentences. For instance, they can at least be combined with terms of any cluster-type higher than their own. In the case where they are combined with a term of only one level higher, this may trigger a distributive interpretation. Whenever they are combined with terms of even higher level, they will ignore part of the typology of the term (i.e. some layer(s) of cluster information).

Moreover, given that expansion of the type of a term is usually available, even terms that do not carry enough cluster information may be seen to form true sentences when combined with those predicates. We saw this phenomenon with respect to:

(6.9) *The kids are equally diverse.*

In this sentence, ‘the kids’ could be retyped as a term of type  $cluster_1 \bullet cluster_2$ , even though its syntax does not make salient such restriction.

The question we want to address in this section is: which terms (if any) give rise to a defective sentence when combined with a given predicate? By saying ‘defective’ I intend to

---

<sup>151</sup>If  $P$  is ambiguous between various interpretations, then we can substitute  $t$  by  $u$  *salva veritate* if, and only if, they are co-referential and they are restricted by clusters of any level the predicate is sensitive to and they are restricted to the same clusters of up to that level (or a level higher if  $P$  is interpreted distributively).

<sup>152</sup>Whenever a predicate ignores certain layers of cluster information, since it is possible that a cluster has more than one sub-cluster of a given level (we saw this in Section 6.2.2.), this will give rise to ambiguity.



be neutral between various diagnoses – for instance, some may take defective sentences to be necessarily false; others may take them to lack a truth value.

Prima facie, this only occurs in cases where terms do not carry enough cluster information for the predicate to compute and such information is not available via context. However, my hypothesis is that there are no selection restrictions at all when it comes to cluster-restricted terms and predicates. In order to argue for this claim, let me consider two cases where one may expect to obtain an anomalous sentence.

Firstly, a case in which terms do not denote enough objects (in number) so as to be retypable as the predicate demands. Observe that, prima facie, for each cluster-type there is a minimum number of objects that terms must denote in order to be liable to restriction to clusters of that type. For example, we need at least three objects to be able to obtain a second-level plurality.<sup>153</sup> Two objects cannot be divided into simple groups at all, but only into individuals, which gives rise to a basic plurality. In general, if a higher-level plurality consists of  $n$  objects, then a term referring to them can be at most restricted by a cluster $_{n-1}$ .<sup>154</sup>

Nevertheless, recall that there is always the possibility of repetition of the referent of a certain term, thus possibly reaching the necessary threshold for the predicate to be applicable to it. This would need to be made obvious by the context, of course. For example, consider:

**(6.10)** *Venus, Serena and Rafa are equally well-coordinated.*

Prima facie, there is something wrong with (6.10). However, it is possible that the context determines that ‘Venus, Serena and Rafa’ denotes for example, Venus twice. For example, think of a case in which Venus and Serena form a team and Venus and Rafa form another team. The sentence could then be interpreted as saying that Venus and Serena, and Venus and Rafa are equally well-coordinated. This may look far-fetched, but it is available.

A related case is that in which predicates make explicit the number of groups that must be involved in the restriction of their term. For example, ‘compete against each other in a 3-way game’. This predicate demands a term of type  $cluster_1 \bullet cluster_2$ . In this case we know

<sup>153</sup> Assuming there are mixed higher-level pluralities, as I do.

<sup>154</sup> To see this, suppose the contrary is the case: consider a higher-level plurality consisting of  $n$  objects restricted to a cluster $_n$ . Now consider the cluster $_2$  restricting it. It must be such that, at least one of the groups in which it classifies the objects has two objects (otherwise, we would obtain a mere plurality). Thus we know that the cluster $_2$  restriction classifies the objects in at most  $n - 1$  groups. Now consider the next cluster, a cluster $_3$ . Again we know that it must be such that it puts the groups obtained via the cluster $_2$  in groups. Since it must at least classify two groups together, we know that it gives rise to at most  $n - 2$  groups. And so on. Thus, once we get to the cluster $_{n-1}$ , we know that it gives rise to at most 0 groups, which is absurd.

that the term in question needs to denote at least four objects. Three things are not enough, since they cannot be considered as forming three different groups. This follows from our view that for a cluster to count as a cluster<sub>2</sub>, only one of the groups it gives rise to needs to consist of more than one object. Thus, for example, the following sentence would seem to be defective:

**(6.11)** *Venus, Serena and Rafa are organised in four groups.*

However, once again, repetition may be at play here. Suppose this is uttered in a context which makes clear that ‘Venus, Serena and Rafa’ refers to Venus, Serena and Rafa multiple times. For example, suppose that Venus, Serena and Rafa are actors playing six different roles in total in a play. And suppose that the sentence is interpreted as saying that Venus, Serena and Rafa, as playing those characters (thus restricted reference comes into play doubly in this case!), are organised in four groups. Again, this is far-fetched, but it is nonetheless available.

I conclude that, given the possibility of repetition, there are no selection restrictions for predicates which are sensitive to cluster restrictions.

## 6.4 Overcoming the problems of other accounts

To close this chapter, let me show that the semantics sketched in this chapter overcomes the problems faced by the cover-based and the group-based accounts surveyed in Chapter 4.

On the one hand, cardinality restrictions are not a problem for Cluster Semantics, since we have taken clusters to be relations, which as such, can apply to any number of objects. Some may be tempted to think of relations, and thus of clusters, set-theoretically. Under this view, HLP terms would denote some objects under their aspect of being the urelemente on which a certain set is built. This move may indeed help those who are used to working in the usual model-theoretic framework. Under this view, we would lose Unrestricted Composition, since there would be fewer sets than pluralities – there would be no non-set-sized pluralities. However, that may not be much of a problem as long as one keeps in mind that thinking of clusters as sets is just a convenient way to approximate the notion of cluster.

On the other hand, anaphora and mixed predications are easily accounted for, since we now take anaphoras as tracking the complex types of the terms they are linked to and account for transparency and opaqueness by means of the interpretations of the predicates involved.

For example, once again recall (4.31):

*These players and these other players are arch-rivals and they are training hard to prepare for the derby.*

Here is how it gets interpreted in our framework (under a certain reading of ‘are arch-rivals’, which is ambiguous):

[These players and these other players]<sub>1•2</sub>[are arch-rivals]<sub>12</sub> and [they]<sub>1•2</sub>[are training hard to prepare for the derby.]<sub>s</sub>  
 a) [These players and these other players]<sub>1•2</sub>[are arch-rivals]<sub>2</sub> and [they]<sub>1•2</sub> [are training hard to prepare for the derby.]<sub>s</sub><sup>Dis</sup>

This interpretation takes ‘are arch-rivals’ as a second-level collective predicate, being sensitive to all the cluster-information provided by the subject and takes the second predicate as a basic singular predicate which ignores the second-level cluster restriction of the subject and applies distributively to the plurality denoted by it (as saying that each player is training hard).

I believe this way of dealing with anaphora and mixed predication is especially appealing due to its naturalness. Not only our semantics makes the right predictions in this sense, but it also gives an intuitively correct description of the mechanisms behind them.

Moreover, the problem faced by cover-based accounts, which did not provide any means to distinguish between terms that trigger HLP readings from those that do not is also solved by our approach, since we have established that certain kinds of terms make salient HLP readings and described mechanisms by which predicates may or may not take that into account. Moreover, we have also left some room for special cases in which context may trigger such interpretations.

Finally, let me return to the issue of ontological proliferation mentioned in the last chapter. As we saw, given that failures of Substitutivity are a lot more pervasive than one may have thought, a singularist account which simply shifts the reference of terms when appearing in opaque contexts (e.g. from sums to groups), has a very high ontological cost. Naturally, most formal semanticists will not be moved by this consideration. However, being mostly interested in moving the pluralist, the topic of ontological innocence is indeed of relevance for us, since many pluralists are motivated by nominalist inclinations.

So the question is whether there is a case to be made for the ontological innocence of restricted reference and, more specifically, for cluster-restricted semantics. Recall that, in

this framework, truth-conditions for restricted expressions involve existential claims about aspects – in this case, clusters. Thus, at least *prima facie*, HLP language does incur commitment to those.

However, it is plausible that aspects are not as ontologically substantive as first-order objects: they facilitate different ways of referring to first-order objects, but they are not the referents of our terms; they are not the values of any object-language variables. In the specific case of HLP reference, terms, while simply denoting objects (which are uncontroversially part of our ontological commitments), do so by bringing to the foreground a certain property instantiated by them, namely a cluster. The cluster, however, is best not accounted for as being part of the reference of the term in question (which, after all, is a referring expression, not a predicate), but rather as part of a meaning-fixing mechanism (together with all the other aspects (if there are any) playing a role in the restriction of the term). The idea is that just as in the examples involving some lecturers and some graders from the last chapter (sentences (5.5) and (5.6)) the only ontological commitment seems to be to some people, the same obtains for sentences involving co-referential HLP terms. What changes when we employ different HLP terms to refer to some objects is the way in which we refer to the objects, but not the objects we refer to.

In this and the last chapter, I gave a number of reasons for adopting this view of HLP terms independent of ontological considerations. Since our view is made easily compatible with the claim of ontological innocence, this supports the claim that HLP terms are indeed ontologically innocent under Cluster Semantics. The fact that an independently motivated theory allows us to make sense of the ontological innocence of HLP reference should be seen as progress by someone aiming to establish the stronger claim that HLP reference is indeed ontologically innocent. But can we give a principled argument for the ontological innocence of HLP language under Cluster Semantics?

We mentioned above the idea that clusters may not be as *ontologically substantive* as first-order objects. A reason why one might think that clusters present no substantial demands on reality is that they pertain to the speaker's way of thinking about some objects. Whether a certain restriction takes place seems to hinge in some cases merely on whether the speaker conceives of the objects as so restricted. For instance, consider a case in which there are eight chairs in a room and someone refers to them higher-level plurally as organised in two groups of four chairs each (e.g. 'these four chairs and those four chairs'). And suppose they do that simply on the basis that they are planning to move four of those chairs to another room. Is there, at the time of uttering that expression, a fact of the matter as to whether those eight objects are restricted by the corresponding cluster? Or is the restriction merely

capturing a subjective way of thinking about certain objects? It may seem the latter is the case. Nevertheless, a necessary condition for a term to be restricted by a certain aspect is that the referent(s) of the term do in fact instantiate that aspect.<sup>155</sup> Thus, truth conditions for cluster-restricted expressions demand not only that a cluster is made salient, but also that the cluster is instantiated as a matter of fact. Not everything that is objectively anchored in the world carries a corresponding ontological commitment – we should not conflate these two ideas. Moreover, that something belongs to the subjective sphere does not immediately show that it is not ontologically substantial. However, the fact that we cannot defend ontological innocence by appealing to the presence of a subjective mechanism is not good news for the nominalist. Ultimately, it seems that a defence of the ontological innocence of restricted reference and predication and, in particular, of cluster-restricted expressions, will have to go through the usual path of advocating a nominalist view of properties and relations in general.

Be all of this as it may, it is important to keep in mind the role that Cluster Semantics plays in our overall view. Cluster Semantics provides only a model of what I take to be the intended interpretation of HLP language, the higher-levelist interpretation, which can only be appropriately captured homophonically. As a model, it merely approximates the homophonic understanding; it misses part of it. Cluster Semantics is nothing more than the least inadequate alternative to the higher-levelist approach.

## 6.5 Conclusions

In this chapter, I have presented Cluster Semantics, a semantics for ordinary HLP language that does not employ metalinguistic HLP expressions. Even though there may well be adequate alternative non-homophonic semantic theories for HLP expressions, I believe Cluster Semantics would be a strong contender if we were to pick one. It has three main desirable features.

Firstly, it accounts for a good deal of data. Importantly, it overcomes the problems its main rivals faced. Secondly, it can be embedded in a bigger picture as a fragment of a semantic analysis for restricted reference, which I sketched in the previous chapter. This

---

<sup>155</sup>A plausible view of clusters is that any given objects instantiate all clusters they can. That is to say, they instantiate all clusters which involve the right number of objects: not too many and not too few. Since we have repetition available, the resulting view is that a plurality of  $k$  objects instantiates all clusters which have at least  $k$  positions. How this transfers to aspects in general will have to be the subject of future research – certainly, we do not want to say that any object(s) instantiate(s) any aspect whatsoever; however, it is also not clear what criterion we should use to delineate the plurality of aspects a given object(s) can potentially be restricted to.

adds plausibility to the view by dissolving worries of ad-hocness. Finally and more subtly, I believe this semantic theory is in accordance with the view often endorsed by the pluralists that one of the desiderata of our semantic theories is that they are coherent with the speakers' linguistic intuitions. For one thing, it deals better with absolutely general reference than its singularist rivals do. This is already an important gain in this respect. But, more generally, the idea that when we speak higher-level plurally of some objects we are simply speaking of them as considered under a certain cluster appears to be in consonance with our own linguistic intuitions. For instance, it seems very natural to say that, when one talks of Russell and Whitehead, and Hilbert and Bernays, one is simply talking of the four logicians, although one is talking about them as organised in two groups. In particular, one is talking about the same people one talks about when referring to them via a simple list. (Just as when one says 'Hesperus', one is talking about the same object one does when one says 'Phosphorus', but in another way.) At the very least, this seems to be a more natural thought than the idea that in one case we speak of some sets or some groups and, in another, of some people. Given that this thesis is directed at the advocates of plurals and they are likely to hold this as a desirable feature of semantic theories, they should be especially inclined to welcome Cluster Semantics as a suitable option.



## **Part IV: Numbers**





# Chapter 7

## Superplural Logicism

### 7.1 Introduction

In this chapter, I will look into the prospects of Superplural Logicism, a form of neo-logicism which employs third-level plural quantification.

In recent years, plurals have been at the centre of a small revival of neo-logicism. In particular, they have been employed in various modifications of the strand of neo-Fregeanism led by Wright and Hale.<sup>156</sup> As is well-known, Wright and Hale propose to employ 2OL augmented with Hume's Principle to derive the Peano Axioms. More precisely, they provide second-order definitions of the primitives of arithmetic relative to which the Peano Axioms are theorems of 2OL augmented with Hume's Principle. This result is known as 'Frege's Theorem'.

Hume's Principle can be stated as follows:

**(7.1) Principle (HP)**  $\forall F \forall G (N(F) = N(G) \leftrightarrow F1 - 1G)$

Where ' $F1 - 1G$ ' is shorthand for a second-order formula which states the existence of a bijection between the objects under  $F$  and the objects under  $G$ .

The principle is read as: any properties  $F$  and  $G$  are such that the number of the  $F$ s is the same as the number of the  $G$ s if, and only if, there is a bijection between the  $F$ s and the  $G$ s.

Given that this form of neo-logicism crucially employs 2OL, some of the criticisms which

---

<sup>156</sup>Wright (1983), Hale (1988), Hale and Wright (2001).

can be directed at it turn specifically on the alleged lack of logicity of 2OL, which has been accused of not being pure logic on different grounds.<sup>157</sup> The most relevant for us concerns its ontological commitments. One reason to think that 2OL is non-logical is that it appears to be ontologically committed to entities which do not fall within the first-order domain of quantification; in particular, it has been accused of being part of mathematics, since its most common interpretation makes second-order quantifiers range over sets – 2OL would thus seem to be ‘set theory in sheep’s clothing’, as Quine (1970, p. 68) famously put it.

It is at this point that Plural Logic may make a positive contribution to neo-Fregeanism. As one can imagine, the logicist proposals which make use of plural quantification turn on Boolos’ observation that M2OL and PL without non-logical predicates are equi-interpretable.<sup>158</sup> Since *prima facie* PL without non-logical predicates has a better claim to logicity than 2OL (due to its putative ontological innocence), substituting the latter with the former seems to be a promising way of strengthening neo-Fregeanism. I call any proposal which exploits this fact ‘plural logicism’.

However, things are not straightforward for a plural logicist programme. Since HP involves quantification over dyadic second-order variables, it demands that we go beyond M2OL in our interpretation. Unfortunately, as we have seen, PL without non-logical predicates cannot give us that much. Here is where Third-Level Plural Logic without non-logical predicates (hereafter, ‘3LPL\*’) enters the picture in our framework, since, as I will show, we can recover the expressive power of the relevant fragment of full 2OL with this fragment of HLPL.

None of the other plural logicist proposals currently available in the literature makes use of higher-level plurals. This is not surprising in light of the fact that HLPL is typically looked at with scepticism. However, given the defence of HLPL offered so far, I believe we should explore the prospects of bringing HLPL into the logicist arena. Of course, in order to show that 3LPL\* does indeed offer an advantage over its second-order rival much will be needed to be argued for; the defence offered until now is only to the effect that HLPL can and is best given a higher-levelist interpretation and thus has a good *prima facie* claim to ontological innocence. However, this falls short of showing that it has a better claim to logicity than 2OL. One of the aims of this chapter will be to address this issue.

Here is the plan. In Section 7.2, I describe one of the plural logicist proposals which has been advanced recently and point to some of the difficulties it faces. Next, in Section

---

<sup>157</sup>See MacBride (2003, pp. 135-142) for a survey of the debate around the logicity of 2OL.

<sup>158</sup>See the translation scheme in Chapter 2, Section 2.2.

7.3, I describe my own account. In Section 7.4, I argue that my proposal overcomes some of the problems faced by the other pluralist approach. Finally, in Section 7.5, I examine the prospects of the claim that 3LPL\* is purely logical.

## 7.2 Plural Logicism via Pair Abstraction

At least three different plural logicisms are found in the literature today. The first one is that of Boccuni (2011, 2013). Her proposal follows Frege's own logicism by making use of a restricted version of Basic Law V. The other two proposals, Hewitt (unpublished) and Pleitz (2017), by contrast, stick more closely to Wright and Hale's approach, by abandoning Basic Law V and proving a version of Frege's Theorem from principles analogous to HP. Since my proposal proceeds via a principle analogous to HP, I shall leave Boccuni's account out of this chapter, for the sake of focus.

Note that one of the advantages of employing an abstraction principle akin to HP in a logicist programme is that such abstraction principles introduce numbers, the basic components of arithmetic, by appealing to their paradigmatic application: counting. This desideratum is what Wright (2000) calls 'Frege's Constraint':

**(7.2) Principle (Frege's Constraint)** *A satisfactory foundation for a branch of mathematics should explain its basic concepts so that their applications are immediate.*

Thus, at least *prima facie*, my proposal will fulfil this requirement too.

As I said above, since plurals do not provide an interpretation of polyadic predicates, pluralist approaches are not straightforwardly equipped to deal with the polyadic variable present in HP. Hewitt's and Pleitz's way around this issue is to interpret polyadic relations as plural terms denoting ordered pairs, while taking the notion of ordered pair as primitive. However, they differ on how the existence of pairs is to be established: Hewitt makes use of an abstraction principle for pairs, whereas Pleitz adopts a form of postulationism. Since tackling the merits (or lack thereof) of a postulationist approach to pairs would take us too far afield, I shall also leave Pleitz's account out of my evaluation. In what follows I expound Hewitt's proposal and point to some of the difficulties it faces.

To start, let me address more generally the question: How are we to recover polyadicity in a pluralist framework?

An initial thought is to take ordered pairs for granted, as entities in the domain of the

plural quantifiers. However, this would be at odds with the purpose of the neo-Fregean programme by not only presupposing the existence of a plethora of entities without providing a further story as to why this is legitimate, but also by presupposing the existence of entities which are suspiciously set-theoretic in nature.

Alternatively, one could use a pairing function.<sup>159</sup> Clearly, any such function, if defined in arithmetic (for instance,  $f(a, b) = 2^a \cdot 3^b$ ), would put the cart before the horse for the purposes of neo-Fregeanism. But perhaps we could circumvent this by defining a pairing function such as  $f(a, b) = \{\{a\}, \{a, b\}\}$ . However, this method would also be incompatible with the epistemological and ontological strictures of neo-Fregeanism, since it would presuppose a set-theoretic and, as a consequence, a mathematical domain.

More generally, invoking a pairing function would make specific demands on the size of our background ontology: for there to be a pairing function, we need our domain to consist of either a single object or of infinitely many (this is because if the first-order domain of objects consists of  $n$  objects, then there are  $n^2$  pairs thereof). In the words of Shapiro and Weir:

This throws a monkey wrench into the works. The neo-logicist wants to use Hume's Principle and therewith Frege's Theorem to establish that the universe is infinite, by showing that the natural numbers exist (and are distinct). But on the present plan, she cannot even formulate Hume's Principle (via plural quantification with pairing) without first showing that the universe is either non-plural or infinite. (Shapiro & Weir, 2000, p. 173)

We need to look somewhere else to recover polyadicity. Hewitt (unpublished) proposes an alternative path: to introduce pairs via an abstraction principle:

**(7.3) Principle (Pair Abstraction)**  $\langle x_1, x_2 \rangle = \langle x_3, x_4 \rangle \leftrightarrow (x_1 = x_3 \wedge x_2 = x_4)$

This principle is typically taken as capturing a necessary and sufficient condition characterizing pairs. Kanamori (2003, p. 289) says that it is 'the instrumental property which is all that is required of the ordered pair'.

Pair Abstraction does not have the paradigmatic form of an abstraction principle, since its right-hand side does not make use of a dyadic relation, but rather a four-place one:  $E(a, b, c, d) := a = c \wedge b = d$ . Under the usual definition of equivalence relation,  $E$  does

<sup>159</sup>This has been proposed, for example, in Boolos (1985, p. 330, n4).

not count as one. However, it does count as one as long as we take the variables ‘two at a time’ (Shapiro, 2000, p. 337). According to this liberalization of the notion of an equivalence relation, a four-place relation like  $E$  counts as an equivalence relation if, and only if:

- (i)  $\forall x \forall y E(x, y, x, y)$
- (ii)  $\forall x \forall y \forall z \forall w (E(x, y, z, w) \rightarrow E(z, w, x, y))$
- (iii)  $\forall x \forall y \forall z \forall w \forall r \forall s (E(x, y, z, w) \wedge E(z, w, r, s) \rightarrow E(x, y, r, s))$

Once the notion of equivalence relation is thus liberalized, Pair Abstraction has a good claim to serve as an implicit definition of the ordered pair operator. And once this is in place, we can make use of term-forming operators for pairs in a plural formulation of HP. The resulting principle is what Hewitt calls ‘Boolos’ Principle’ (hereafter, ‘BP’):<sup>160</sup>

**(7.4) Principle (Boolos’ Principle)**  $\forall x^1 \forall y^1 [N(x^1) = N(y^1) \leftrightarrow \exists z^1 (\forall z < z^1 \exists x_1 \exists x_2 (z = \langle x_1, x_2 \rangle) \wedge \forall x < x^1 \exists! z < z^1 \exists! y < y^1 (z = \langle x, y \rangle) \wedge \forall y < y^1 \exists! z < z^1 \exists! x < x^1 (z = \langle x, y \rangle)))]$ <sup>161</sup>

where ‘ $N(x^1)$ ’ is a functional term to be read ‘the number of the  $x^1$ s’ (the function picked out by  $N$  takes more than one object at once as arguments and outputs a single value: a number).

In PL augmented with BP, Pair Abstraction and suitable definitions, Hewitt shows that we can interpret Peano Arithmetic (in Hewitt (unpublished, sec. 5)), thus proving a result analogous to Frege’s Theorem. BP, just as HP, appears to provide a characterization of the notion of cardinal number and one that indeed is linked to its basic application: counting, thus fulfilling Frege’s Constraint. Moreover, *prima facie*, the use of an abstraction principle for ordered pairs should be unproblematic by the lights of the neo-Fregean.

Nevertheless, this path is not exempt from problems. Let us start with the difficulties faced by the principle of Pair Abstraction. On the one hand, Pair Abstraction suffers from a problem analogous to the Caesar Problem: it does not settle whether Julius Caesar is identical with some pair; it does not tell us anything about the truth value of identity statements involving pairs of a form distinct from the left-hand side of Pair Abstraction.

<sup>160</sup>Hereafter I use the notation introduced in Chapter 3 for higher-level plural terms with a small modification: I use superscripts to indicate the level of the terms (1 indicates that the term is a basic plural, 2 indicates that the term is a second-level plural, etc). However, for the sake of simplicity, I drop the superscripted 0 from singular terms and represent them simply with single letters.

<sup>161</sup>Hereafter, I use restricted quantifiers, understood as abbreviations analogous to those used in set theory: ‘ $\forall x < x^1 \phi$ ’ is shorthand for ‘ $\forall x (x < x^1 \rightarrow \phi)$ ’.

But, most importantly, Pair Abstraction seems to suffer from additional problems of its own. Firstly, it has models which contain non-well-founded pairs. For instance, it has a model whose domain contains a single object  $p$ , which is the pair of itself and itself, that is  $p = \langle p, p \rangle$ .<sup>162</sup> This is problematic because it shows that Pair Abstraction is insufficient to capture the intuitive notion of ordered pair as involving well-foundedness.

As pointed out by Tennant (2008), well-founded pairs can be characterised as fulfilling the following two conditions:

- (i) The relation of being a coordinate of an ordered pair is such that its transitive closure is finite;
- (ii) and is such that it does not contain loops – i.e. no object is a coordinate of itself.

The latter condition fails in the model above. This can be seen as an especially problematic instance of the Caesar Problem, since one cannot appeal to the usual solution whereby one distinguishes categories of objects via the criteria of identity associated with them: two objects belong to the same category if, and only if, whether they are identical to a third object or not is decided by the same criterion of identity.<sup>163</sup> This solution is not available in the present case, since non-well-founded and well-founded ordered pairs share their criterion of identity.<sup>164</sup> Be that as it may, Hewitt (in press) proposes a rebuttal of this objection on different grounds.

Hewitt's response turns on the observation that one of the ideas behind the use of abstraction principles in a logicist programme is that if a speaker understands the expressions occurring in the right-hand side of the principle, she is then in a position to understand those occurring in the left-hand side. Under this view of abstraction principles, it appears impossible for Pair Abstraction to introduce an expression such as  $p$ , where  $p = \langle p, p \rangle$ , since in order to understand it, the speaker would need to previously have a grasp of the expressions occurring in the right-hand side, among which we find  $p$  itself.

In Hewitt's own words:

We can rule out models of [Pair Abstraction] containing non-well-founded pairs as deviant. [...] These only arise as possibilities when one considers the abstraction principle as a formal statement apart from considerations about language use and the practice of reference. (Hewitt, in press, p. 9)

<sup>162</sup>This has been pointed out by Pleitz (2017, p. 213), who in turn relies on Tennant (2008).

<sup>163</sup>See Hale and Wright (2001, ch. 14) for a discussion of this proposal.

<sup>164</sup>This has been pointed out in Hewitt (in press).

However, this answer appears to fall short of solving the problem of non-well-founded models. According to Hewitt, an ordered-pair-referring expression  $p$  cannot pick out a pair one of whose coordinates is denoted by the very same term  $p$ . This relies on the more general (and plausible) view that we cannot refer to an object in a way that demands that we are antecedently able to refer to that object in that way. However, to point this out is not sufficient to solve the problem of unintended models. Hewitt needs to tell us what we are to do in order to rule them out. His remarks suggest a predicative restriction of Pair Abstraction. However, note that modifying the language and making it two-sorted in the sense that terms in the left-hand side pick out pairs and terms in the right-hand side pick out non-pairs, will have the unwanted consequence that we will not be able to have perfectly unproblematic (from the point of view of well-foundedness) nested ordered pairs. Another modification is called for. We must take care in finding it, since any restriction with respect to Pair Abstraction may motivate analogous restrictions onto BP. For instance, *prima facie*, the very same grounds that motivate ruling out the ordered pair  $a$ , could also motivate ruling out true numerical identity statements such as ‘ $2 = N[x : x = 1 \vee x = 2]$ ’.<sup>165</sup> More generally, if we cannot make use of the numerical operator in the right-hand side of BP, then we cannot have nested number terms, thereby impeding the derivation of Frege’s Theorem. Thus what the right restriction should be is not a straightforward matter.

In any case, Hewitt has only hinted at a resolution of the problem, but has not made a specific proposal as to how to implement it. The existence of  $p = \langle p, p \rangle$  is still compatible with Pair Abstraction. I conclude that, as things stand, Pair Abstraction is unsatisfactory, as it has unintended models.

Moreover, another problem arises from the fact that pluralities are typically understood as being non-empty (recall the axiom of Non-Emptiness:  $\forall xx \exists x(x < xx)$ ). As discussed in Chapter 2, this is necessitated by the view that plural reference is ontologically innocent. Put in a somewhat metaphorical manner, once the objects are gone, so is their plurality. This fact proves troublesome when it comes to the present application of Plural Logic. The problem is that we no longer have available something analogous to an empty property, whose number would have been taken as the number 0.

*Prima facie*, a solution would be to assign ‘0’ a dummy object (which we can call ‘the null object’) as its referent. Unfortunately, since the null object would be arbitrary, we cannot be sure that it is not another number – i.e. that it is not the successor of some number. This blocks the derivation of the axiom of Infinity, thus ultimately preventing us from deriving

---

<sup>165</sup>Hereafter, an expression of the form  $[x : \phi(x)]$  is a plural definite description to be read ‘the objects which  $\phi$ ’.



Frege's Theorem altogether.<sup>166</sup> To see this, note that in order to show that every number has a successor, the usual strategy is to prove that every number  $n$  is such that it precedes the number  $m$  of numbers less than or equal to  $n$ : this shows that  $m$  is a successor of  $n$ . Since we cannot be sure that, for example,  $0 \neq 1$ , we cannot prove in generality that every number is such that it precedes the number of numbers less than or equal to it. Thus, at least *prima facie*, without empty pluralities, Infinity and hence Frege's Theorem are out of reach.

Another option would be to use Boolos' translation of second-order quantified statements as plurally quantified statements, which is especially designed to deal with the empty case, and thus prove the existence of zero as usual. Roughly speaking, 'zero' could be defined as the number of any uninstantiated property, the existence of which is derived from Boolos' translation of second-order expressions. According to Boolos' translation scheme,<sup>167</sup> instead of proving  $\exists x x \forall x (x < x x \leftrightarrow x \neq x)$ , we prove that  $\exists x x \forall x (x < x x \leftrightarrow x \neq x) \vee \forall x (x \neq x \leftrightarrow x \neq x)$ , which is satisfied even though there is no plurality of non-self-identical things. Shapiro and Weir (2000, p. 177) have shown that if we follow this strategy, then the right amendment of HP is:

$$\forall F \forall G ((N(F) = N(G) \leftrightarrow F1 - 1G) \wedge (\neg \exists x Fx \leftrightarrow N(F) = 0))$$

Even though this would suffice to prove a result analogous to Frege's Theorem, a problem with this strategy, as pointed out by them, is that we cannot use it to prove that there is a single zero. That is because we can assign each empty predicate a different object as its number, all of which would satisfy the modified HP. This would be in tension with the neo-Fregean's aims, since the introduction of zero in this way would not account for all uses mathematicians make of the term. For instance, one would not be able to derive the true statement that the number of the numbers less than 0 is also 0, i.e. *the same* 0. This is at odds with Frege's Constraint, the requirement that the basic elements of each mathematical theory are introduced by appeal to their paradigmatic applications.

Hewitt proposes a different strategy to bypass this difficulty. He suggests that we assume the existence of an arbitrary non-number in our domain. Let us call this object  $a$ . With  $a$  in hand, we can show that every number (starting from 1) precedes the number of the numbers less than or equal to it and  $a$ ; a result analogous to Frege's Theorem is back within our reach, since, given that  $a$  is not a number, we can be sure that in particular it is distinct from all successors.

This solution appears to have some drawbacks. Firstly, the proof that there are infinitely

<sup>166</sup>See Shapiro and Weir (2000, pp. 163-171).

<sup>167</sup>See Chapter 2, Section 2.2.

many natural numbers becomes conditional on there being a certain non-number in the first place. As pointed out by Hewitt, this not only violates ontological innocence (i.e. the condition that logic should hold even in an empty world), but it also violates ontological neutrality (i.e. the condition that logic should hold regardless of subject matter). Hewitt says that the former may not represent a big challenge for neo-Fregeanism; after all, it is plausible that no a posteriori investigation is needed to establish that something exists, that reasoning alone informs us of the existence of some things. However, the loss of ontological neutrality appears to make the claim to logicality less plausible.

Given that my proposal is based on the same conception of plural reference as Hewitt's, I will have to follow the same strategy to address the lack of an empty plurality. I will return to an assessment of the limitations of this strategy below, where I will argue that in terms of abiding by logicist standards, the two conditions just described (i.e. ontological innocence and ontological neutrality) are equally important. Thus, Hewitt's is not any more problematic than any approach which posits the existence of anything. Moreover, I will argue that, by following Hewitt's path, we still arrive at some valuable results.

### 7.3 Superplural Logicism

In this section I describe my proposal to prove Frege's Theorem from 3LPL\* (recall, Third-Level Plural Logic without non-logical predicates) and a superplural rendering of HP. Here I limit the discussion to the technical result. In the next section I argue that my approach is to be preferred to Hewitt's. In the rest of the chapter, I evaluate more generally the ontological and epistemological status of 3LPL\*.

My approach is in the same ballpark as Hewitt's in the sense that I make use of a principle akin to HP. However, instead of taking pairs as primitive (and introducing them via an abstraction principle), I shall avoid pairs altogether. Rather than defining equinumerosity via a dyadic relation, I shall take advantage of the fact that we can paraphrase HP so as to dispense with polyadic variables by making use, instead, of third-level plural variables.

In the proof of Frege's Theorem, full polyadic 2OL is not actually required. We can make do with M2OL extended with a restricted fragment of polyadic 2OL, namely, its dyadic fragment. This is the fragment I focus on. My strategy consists in replacing ordered pairs with second-level pluralities and relations with the third-level pluralities formed from those. To this end, I will make use of one of the set-theoretic definitions of ordered pair in its HLP version. Using the informal notation introduced in Chapter 3 to represent higher-level

pluralities, we can express a definition analogous to that of Kuratowski:

**(7.5) Definition**  $\langle x, y \rangle := (x, (x, y))$ <sup>168</sup>

The term ‘ $\langle x, y \rangle$ ’ is defined as the second-level plural which denotes the first coordinate of the pair, on the one hand, and the plurality of both coordinates, on the other.<sup>169,170</sup>

In order to state the superplural version of HP in a succinct manner, we will help ourselves to the following definitions. First, we introduce a shorthand for expressing the idea that a second-level plurality  $a^2$  is an ordered pair:

**(7.6) Definition (Ordered Pair)**  $OP(a^2) := \exists x \exists y [\exists x^1 < a^2 (x < x^1 \wedge \forall z < x^1 (z = x)) \wedge \exists x^1 < a^2 (x < x^1 \wedge y < x^1 \wedge \forall z < x^1 (z = x \vee z = y)) \wedge \forall x^1 < a^2 ((x < x^1 \wedge \forall z < x^1 (z = x)) \vee (x < x^1 \wedge y < x^1 \wedge \forall z < x^1 (z = x \vee z = y)))]$

Suppose  $OP(p^2)$ . Then we can define the predicates ‘is the first coordinate of  $p^2$ ’ and ‘is the second coordinate of  $p^2$ ’ as follows:

**(7.7) Definition (First Coordinate)**  $1C(a, p^2) := \forall x^1 < p^2 (a < x^1)$

**(7.8) Definition (Second Coordinate)**  $2C(a, p^2) := \exists x^1 < p^2 (a < x^1) \wedge \forall y^1 < p^2 \forall z^1 < p^2 (y^1 \neq z^1 \rightarrow \neg(a < y^1) \vee \neg(a < z^1))$

Equipped with these definitions we can express a superplural version of HP in which, instead of defining equinumerosity as the existence of a certain relation, we define it as the existence of a third-level plurality consisting of all the second-level pluralities which correspond to the ordered pairs of the relevant bijection:

<sup>168</sup>This is short for an expression in the language of Second-Level Plural Logic augmented with notation for HLP terms in the form of definite descriptions.

<sup>169</sup>Recall that the HLP hierarchy includes mixed higher-level pluralities, that is, pluralities which correspond to terms such as ‘Carla and her students’. Pairs, as defined here, are of this sort.

<sup>170</sup>While, as we shall see, dyadic 2OL suffices to derive the Peano Axioms, it is insufficient to prove facts about addition and multiplication, since the latter are typically conceived as triadic relations. (I am indebted to Øystein Linnebo for pointing this out to me). Nevertheless, I believe this does not really posit a problem for Superplural Logicism. As is well-known, triadic relations can be modelled as dyadic relations holding between pairs of objects. For instance, the triple  $\langle x, y, z \rangle$  can be modelled as the pair  $\langle x, \langle y, z \rangle \rangle$ . Thus, all we need in order to replace triadic relations in the superpluralist framework is to give a definition of those pairs in superplural terms. And this can be done by going two levels up the hierarchy of higher-level pluralities. Firstly, triples can be defined à la Kuratowski as fourth-level pluralities. For example,  $\langle x, \langle y, z \rangle \rangle$  would be defined as the fourth-level plurality  $(x, (x, (y, (y, z))))$ . In turn, triadic relations would be defined as fifth-level pluralities, namely, as pluralities of fourth-level pluralities. Since everything that I will argue in this chapter concerning the logicity of third-level plural logic applies equally well to its fifth-level extension, nothing is lost by helping ourselves to said extension. I leave this issue aside for the time being and focus on the derivation of the Peano Axioms.

**(7.9) Principle (Superplural HP)**  $\forall x^1 \forall y^1 \{N(x^1) = N(y^1) \leftrightarrow \exists p^3 [\forall q^2 < p^3 (OP(q^2) \wedge \forall x(1C(x, p^2) \rightarrow x < x^1) \wedge \forall y(2C(y, p^2) \rightarrow y < y^1)) \wedge \forall x < x^1 \exists q^2 < p^3 (1C(x, q^2) \wedge \forall r^2 < p^3 (1C(x, r^2) \rightarrow r^2 = q^2)) \wedge \forall y < y^1 \exists q^2 < p^3 (2C(y, q^2) \wedge \forall r^2 < p^3 (2C(y, r^2) \rightarrow r^2 = q^2))]\}$

Read informally, this principle says that any two pluralities, the  $x^1$ s and the  $y^1$ s, have the same number if, and only if, there is a third-level plurality which consists only of ordered pairs whose first coordinates are among the  $x$ s and whose second coordinates are among the  $y$ s and such that every  $x$  is the first coordinate of a single pair and every  $y$  is the second coordinate of a single pair.

### 7.3.1 Superplural Frege's Theorem

In this section, I shall prove that an interpretation of PA2 can be derived in 3LPL\* augmented with Superplural HP (hereafter, 'SHP') and suitable definitions.<sup>171</sup>

In my proofs I will implicitly make use of the following lemma:

**(7.10) Lemma**  $\forall x^1 \exists x(x = N(x^1))$

*Proof.* Consider an arbitrary plurality  $a^1$ . And instantiate both prenex quantifiers in SHP with  $a^1$ . The right-hand side of the biconditional in SHP holds for  $a^1$ , since there is a third-level plurality  $a^3$  modelling the identity map from  $a^1$  to themselves. It follows that  $N(a^1) = N(a^1)$ , from which by singular existential and plural universal instantiation we obtain the desired result.  $\square$

From now on, I use  $m$  and  $n$  as variables ranging restrictedly over numbers (that is, over any object  $x$  for which there are  $x^1$  such that  $x = N(x^1)$ ).

As I said above, given that there is no empty plurality, we are going to need to help ourselves to an object to play the role of zero. Hereafter, we take  $a$  to be an arbitrary object. We know that there is such an object by existential instantiation on  $\exists x(x = x)$ , which is a theorem of 3LPL\*. Following Hewitt, we will also assume that  $a$  is not a number.

Next, we need some definitions. First, we define the number 1:

**(7.11) Definition (1)**  $1 := N[x : x = a]$

Next, we define 'immediate predecessor':

<sup>171</sup>I mainly follow Wright (1983), Boolos (1987) and Hewitt (unpublished).

**(7.12) Definition (Immediate Predecessor)**  $P(ab) := \exists x^1 \exists x(x < x^1 \wedge b = N(x^1) \wedge a = N[z : z < x^1 \wedge z \neq x])$

We are now ready to start proving the superplural version of Frege's Theorem. First, we prove that two different numbers have different successors.

**(7.13) Theorem**  $\forall m \forall n (\exists y (P(my) \wedge P(ny)) \rightarrow m = n)$

*Proof.* Take arbitrary  $m$  and  $n$  and suppose  $P(my)$  and  $P(ny)$ . Thus there are objects  $a^1$  such that  $y = N(a^1)$  and  $m = N[x : x < a^1 \wedge x \neq w]$ , where  $w < a^1$ . Likewise there are objects  $c^1$  s.t.  $y = N(c^1)$  and  $n = N[x : x < c^1 \wedge x \neq v]$ , where  $v < c^1$ . By SHP, there is a third-level plurality  $a^3$  encoding a bijection between  $a^1$  and  $c^1$ .

Now let  $r$  and  $s$  be the items such that  $(w, (w, r))$  and  $(s, (s, v))$  belong to  $a^3$ . And consider the third-level plurality  $b^3$  which is like  $a^3$  except that, if it doesn't already, it contains  $(s, (s, r))$  and  $(w, (w, v))$ . It can be checked that this is also bijection from  $a^1$  to  $c^1$ . Now, remove  $(w, (w, v))$  from  $b^3$  and call the resulting third-level plurality  $c^3$ . It is easy to check that  $c^3$  encodes a bijection from  $[x : x < a^1 \wedge x \neq w]$  to  $[x : x < c^1 \wedge x \neq v]$ .  $\square$

Next we need to define 'Ancestral':

**(7.14) Definition (Ancestral)**  $P^*(ab) := \forall x^1 [\forall z ((P(az) \rightarrow z < x^1) \wedge \forall x \forall y ((x < x^1 \wedge P(xy)) \rightarrow y < x^1)) \rightarrow b < x^1]$

We are now equipped to define the predicate of being a natural number:

**(7.15) Definition (Natural number)**  $Nat(n) := n = 1 \vee P^*(1n)$

From now on, the letters  $m$  and  $n$  range restrictedly over the natural numbers.

That one is a natural number follows immediately from the definition of 'natural number':

**(7.16) Theorem**  $Nat(1)$

The Principle of Induction can now be derived:

**(7.17) Theorem**  $\forall x^1 ((1 < x^1 \wedge \forall n (n < x^1 \rightarrow \forall m (P(nm) \rightarrow m < x^1))) \rightarrow \forall n (n < x^1))$

*Proof.* Assume the antecedent and take  $a^1$  as instantiating  $x^1$ . In order to show that every natural number  $p$  is among  $a^1$  it suffices, by the definition of natural number, that we consider two cases:

- (i)  $p = 1$ . This is immediate by the first conjunct of the antecedent.

(ii)  $P^*(1p)$ . By the antecedent of induction we have that  $1 < a^1 \rightarrow \forall z(P(1z) \rightarrow z < a^1)$ . And since we know that  $1 < a^1$ , we have that  $\forall z(P(1z) \rightarrow z < a^1)$ . Also by the antecedent of induction and some symbolic manipulation we have that  $\forall x \forall y((x < a^1 \wedge P(xy)) \rightarrow y < a^1)$ . These are the two conjuncts of the antecedent of the definition of ancestral. Thus, we can conclude that  $p < a^1$ .

We are also ready to prove that 1 is not the successor of any number:

**(7.18) Theorem**  $\neg \exists n P(n1)$

*Proof.* Assume, for reductio, that some number, say  $p$ , is such that  $P(p1)$ . It follows that there are some objects  $b^1$  s.t.  $p = N(b^1)$ . By definition of predecessor, we have that  $1 = N(c^1)$  for some  $c^1$  which consist of  $b^1$  and an extra object  $e$  which is not among  $b^1$ . But by definition of 1, it must be the case that  $c^1 = a$ . Absurd.  $\square$

Finally, we are prepared to prove that every natural number has a successor:

**(7.19) Theorem**  $\forall n \exists m P(nm)$

However, this proof is too complex to reproduce here. It relies on the following lemmas:

**(7.20) Lemma**  $\forall n(n = N[x : (Nat(x) \wedge P^*(xn) \vee x = a)])$

This lemma is required, given that zero is not a natural number and therefore we cannot identify natural numbers with the number of their ancestral predecessors. Instead, each number  $n$  is the number of the plurality consisting of all of  $n$ 's ancestral predecessors plus  $a$ .

The other necessary lemma is:

**(7.21) Lemma**  $\forall n(\neg P^*(nn))$

Details of this proof can be found in Wright (1983, 161-169).

With this brief reconstruction we have sketched a derivation of the superplural version of Frege's Theorem:

**(7.22) Theorem (Superplural Frege's Theorem)** An interpretation of PA2 can be derived in 3LPL\* together with SHP and suitable definitions.

### 7.3.2 Evaluation

How does this account compare to plural logicism via Pair Abstraction?

On the one hand, it overcomes the problem of non-well-founded pairs. Recall, the problem was that Pair Abstraction was not sufficiently informative to rule out intuitively defective pairs, in particular, non-well-founded ones. A very simple non-well-founded model satisfying it is the one element model consisting of  $p = \langle p, p \rangle$ . This could be seen as a particularly bad instance of the Caesar problem since it cannot be solved by appealing to ontological categories. My account overcomes this, as I take a pair as a special kind of second-level plurality; thus the well-foundedness of the relation of plural membership ensures well-foundedness for pairs, as defined here. More generally, although this proposal does face the Caesar problem (it obviously arises with regard to SHP), it does not introduce any additional instances thereof, since it does not make use of any additional abstraction principles.

On the other hand, our account inherits the problem Hewitt's faces with respect to the lack of an empty plurality. The need to posit the existence of an arbitrary non-number makes our proposal not entirely satisfactory from a logicist point of view (for it cannot be a matter of logic that a non-number exists). However, I believe that this approach is not without merit. It shows that from the existence of a single non-number (for instance, a concrete object), we can derive the existence of an  $\omega$ -sequence; something has been gained. Even though we have not met the desideratum of reducing arithmetic to pure logic, we appear to have met a less ambitious aim of the neo-Fregean programme.

One of the objectives of the neo-Fregean is to explain how we, concrete beings, can have epistemic access to the abstract and acausal objects of arithmetic, the numbers. This is one of the classic puzzles faced by any Platonistic philosophy of mathematics: how can we explain our knowledge of abstract mathematical objects without appealing to any Kantian-like mechanism of intuition? And this aim appears to be met by the route of positing a non-number, for, if our arguments are correct, we have proved the existence of the naturals from everyday knowledge (plus logical rules and definitions). Although plausibly not logical, our knowledge of the existence of things which belong to a domain disjoint from that of the natural numbers – physical objects, mental states, etc. – is part of our ordinary knowledge. And as such it is free of any suspicion of unnaturalness (unlike a Kantian-like route).

Moreover, if we focus on this epistemological desideratum, there does not appear to be a big leap between presupposing that something exists and presupposing that something other than a number exists. In other words, the loss incurred from violating ontological neutrality

on top of ontological innocence does not seem substantial with respect to this aim. After all, the same everyday means by which we gain knowledge of the former are those by which we gain knowledge of the latter.

Overall, given that it overcomes the problem of non-well-founded pairs, I believe Superplural Logicism represents progress. In the next section I will argue for the logical status of HLPL and, in particular, of 3LPL\*. This, assuming a critical view of the logical status of higher-order logic, should support the claim that our proposal represents an improvement with respect to the usual second-order route to Frege's Theorem.

## 7.4 The Prospects of Logicality

In this section I consider the question of the logical status of 3LPL\*, the formal system employed in our derivation of the superplural version of Frege's Theorem. My aim is to provide support for the thesis that 3LPL\* is purely logical. My general strategy will be to argue that HLPL without non-logical predicates (hereafter, 'HLPL\*') is purely logical. However, when necessary, I will focus exclusively on the former, which strictly speaking is all we need. Although the arguments advanced in this section are intended to provide support to the logicist proposal just put forward, they may be of interest in their own right.

In my assessment, I will follow the partial analysis of the notion of logicality in Linnebo (2003). He focusses on the logicality of PL, but analogous demands can be made on HLPL\*:

- *Universal Applicability*: HLPL\* can be applied to any realm of discourse, no matter what sort of or how many objects this discourse is concerned with.
- *Ontological Innocence*: The HLPL\* comprehension axioms are not ontologically committed to any entities other than those contained in the singular first-order domain.
- *Cognitive Primacy*: HLPL\* can be understood without a prior understanding of any extra-logical ideas.

For the sake of the argument, I take for granted that these three are necessary criteria for the logicality of HLPL\*. Let us examine them one at a time.



### 7.4.1 Universal Applicability

If HLPL\* were universally applicable, then one would expect it to be such that it can be applied to a domain of higher-level pluralities;<sup>172</sup> that it can be used to regiment discourse about higher-level pluralities themselves. However, the question whether we can use HLPL\* to regiment talk of higher-level pluralities in general must be answered negatively. This may be seen as a problem for the claim that HLPL\* is universally applicable, but I will argue that, despite appearances, it is not.

In the following quote, Linnebo points to the kind of expressive limitation I am concerned about in this section:

We have seen that Boolos is driven to a theory where it is always possible to add new layers of even higher pluralities. It follows from Cantor's theorem that each new layer of pluralities gives us a domain that is larger than the previous one. Clearly, if we were to singularize these higher pluralities, that is, if we were to treat them as sets, we would surrender all hope of being able to quantify over absolutely all sets: each new layer of pluralities would then lead to a greater domain of sets. However, even if we decide not to singularize the higher pluralities, the situation won't be essentially different. For at no level of this theory will we be able to quantify over all the pluralities there are: There will always be higher levels. This situation exhibits a limitation of the same general kind as that which led Parsons to deny that we can quantify over absolutely all sets. (Linnebo, 2003, p. 88)

In the last sentence, Linnebo is referring to Parsons (1990, pp. 296-300), where Parsons argues that a Boolosian approach to the axiomatization of set theory is inadequate. Parsons argues that Boolos' plural regimentation of set-theoretic discourse amounts to extending ZFC with Morse-Kelley class theory<sup>173</sup> (hereafter, 'MK'). Moreover, Parsons adds, once MK is brought into the picture, we can go on to nominalize the second-order terms (in the sense of replacing them with co-referential first-order singular terms), thus expanding the first-order domain of sets with proper classes. Parsons claims that when attempting to talk about all sets we are always led to carry out this process of expressive and subsequent ontological

---

<sup>172</sup>As before, I use 'higher-level pluralities' as referring to pluralities of all levels, including first-level pluralities and individuals.

<sup>173</sup>We require Morse-Kelley due to the impredicativity needed to properly express Replacement and Separation.

expansion. Thus, he concludes, talk of all sets is out of reach.

However, if one takes seriously the intended interpretation of HLPL\*, then it seems that the limitations exhibited by each case are of a different nature. The ones exhibited by Parsons' picture are problematic in a way that the ones exhibited by HLPL\* are not.

First, note that when I speak of Parsons' picture I am referring to the expansion of set theory with proper classes, not to the plural axiomatization of set theory. For one thing, I do not think the two can be likened in the first place. So let us just contrast the expressive limitations of a proper class theory with those of HLPL\*.

The alleged limitations are that, on the one hand, we cannot quantify over all sets and, on the other, that we cannot quantify over all higher-level pluralities.

In the case of sets, we find ourselves with two options: (i) either we accept a typed picture of ever larger set-like entities (this is what Linnebo dubs 'Inexhaustibility'), or (ii) we bring proper classes into the domain of sets via singularization (this is what Linnebo dubs 'Ontological Proliferation'). The former goes against the idea that sets are all the set-like entities there are; the latter goes against the idea that there is a fixed range of sets and leads to the view that the collection of all sets is affected by some sort of indefinite extensibility. This is because once proper classes have been included in the first-order domain of quantification, there is no reason why we should not be able to form classes of the objects in the resulting domain, which triggers a new expansion of the ontology. Since stopping this process at any given point would appear to rest on an arbitrary restriction, this process appears to go on indefinitely. In both cases the result is the same: we lose the capacity to quantify over all set-like entities. In the former, because of type restrictions. In the latter, because of the lack of a determinate domain of sets.

Does the same occur with respect to HLPL\*? In my view, only part of it occurs, but even that does not have the unwanted consequences that it has for set theory.

On the one hand, note that Ontological Proliferation does not arise in the case of HLPL\*. This is because Parsons' step of singularization is not applicable to HLP reference. As Linnebo seems to acknowledge by leaving it as an optional step, if one takes seriously the intended interpretation of HLPL\*, then one cannot employ first-order terms to pick out higher-level pluralities, for there are no such things (despite our inevitably singularizing way of talking about them in English).

On the other hand, there is a sense in which Inexhaustibility does occur in the HLPL\*

case, but it is a sense in which it is harmless. Once again, if one takes seriously the claim that a higher-level plurality is not an object, then, that it is impossible to quantify over all higher-level pluralities means that there are no terms which generalize over all higher-level plural forms of reference, which *accumulate* all of them. But this result is in consonance with our typed approach, which was philosophically motivated in the first place. With it I intended to capture the fact that HLP reference is not reference to higher-level pluralities (thus liable to be nominalized – which would remove part of the motivation behind a typed approach), but rather a collection of ever more expressive forms of reference (due to cumulativeness). HLPL\* was never intended as a theory of all higher-level pluralities, but rather as a language consisting of infinitely many modes of reference and quantification. As such, we do not lose anything by not having all-encompassing HLP terms. In fact, it is a result one would expect.

Nevertheless, one may complain that, although this limitation may be harmless for HLPL\*, it is detrimental for the understanding of our informal talk of higher-level pluralities, which, indeed, appears to involve generalizations about all higher-level pluralities. This is an important point and it is the objection to which I now turn.

**Objection: Discourse about semantic values** There is another way of objecting to the alleged universal applicability of HLPL\*: the fact that HLPL\* does not have quantification over all higher-level pluralities puts the higher-levelist in an uncomfortable position, as she appears to be committed to claims that do involve this kind of quantification. In particular, these are claims that belong to the meta-semantics of HLPL\*. This has been argued in Linnebo (2006) with respect to type theory, but the same complaint can be raised in the present context. The starting point of this objection is the observation that the higher-levelist is committed to claims such as:

*Infinity*: There are infinitely many different kinds of semantic value.

*Unique Existence*: Every expression of every syntactic category has a semantic value which is unique, not just within a particular level, but across all levels.

*Compositionality*: The semantic value of a complex expression is determined as a function of the semantic values of the expression's simpler constituents.

Given that these claims involve quantification across all HLP levels, HLPL\* cannot properly regiment them. To put it differently, if it is true that not being able to speak of higher-level pluralities in general is not problematic by the lights of the higher-levelist, how does

she explain the reference, in the statement of this very claim, to them? Isn't this claim self-refuting?

I believe this objection can be circumvented by following Krämer (2014), who has addressed it in connection with Higher-Order Logic. As usual, the discussion can be brought into our terrain. Krämer argues that all of these statements presuppose a general notion of reference and a corresponding cross-level notion of semantic value which the higher-levelist rejects, hence, despite appearances, she is not committed to these claims. However, as Krämer himself highlights, the fact that the claims seem plausible is in need of explanation. Thus we must explain their plausibility without employing an over-arching notion of HLP reference.

The way in which Krämer does this is by describing the common role that the metalinguistic interpretation function symbols play. Even though they each belong to a different level, they all play an analogous role and this is what gives the statements above plausibility. Very briefly put, their common role is that they are all quasi-disquotational with respect to their associated expressions – when attached to the name of an expression of the right category they give rise to another expression equivalent to it (relative to a model):<sup>174</sup>

(T)he truth-predicate has a kind of disquotational function, in that appending 'is true' to the quotation-name of a given interpreted (object language) sentence  $S$  results in a meta-language sentence which is equivalent to (a meta-language translation of)  $S$ . Something similar can be said about 'the denotation of': prefixing it to the quotation-name of a name  $n$  produces a name that is equivalent to  $n$ . And the phrase 'applies to' has the analogous feature with respect to first-order monadic predicates: appending it to the quotation-name of such a predicate  $P$  produces a predicate that is equivalent to  $P$ . (Krämer, 2014, pp. 721-2)

This commonality is what explains the intuition behind the claims above. However, strictly speaking, they are not true by the lights of the higher-levelist; thus HLPL\* is not affected by the expressive limitations that would arise from the impossibility of regimenting them.

In fact, this commonality is also what explains general talk of higher-level pluralities in this dissertation. General terms that refer to the semantic values of HLP terms (i.e. to higher-level pluralities) are, not surprisingly, ubiquitous along these pages. Krämer has argued, in

<sup>174</sup>See Krämer (2014, pp. 720-3) for more details.

my view convincingly, that they should not be taken at face value, but rather as picking out the common role of the respective interpretation functions.<sup>175</sup>

**Objection: Discourse about sets** Another challenge for HLPL\*’s claim to Universal Applicability arises from a certain stance on the relation between pluralities and sets. In particular, from a specific answer to the question: which pluralities form sets?

The question, more precisely, is: which pluralities are such that for each of them there is a set which has the members of that plurality as its only members? This issue has usually been raised in contexts where only basic plural reference was assumed to be available. In this section, I bring it to the present context, where higher levels of plural reference are also available.

In particular, the view on the relation between pluralities and sets which poses a threat to Universal Applicability is the one according to which all pluralities form sets. The reason why this is problematic is that it follows from it that there is no such thing as the plurality of all sets (or else there would be a set of all sets); thus we cannot plurally quantify over all sets, hence closing the door to using plural quantification to regiment discourse about sets (à la Boolos, for example). The applicability of Plural Logic turns out to be limited. As a consequence, so does the applicability of HLPL\*.

In my examination of this issue I focus on a specific variant of this view, the one endorsed in Linnebo (2010), Linnebo (2013) and Linnebo (2016). In slogan form, Linnebo’s view is that all pluralities *can* form sets. He sees plural reference as a mechanism at play in the

---

<sup>175</sup>One may complain that this introduces a tension into the higher-levelist’s views. Recall that she takes speaker’s intuitions seriously and tries to capture them in her semantic theories. Here, however, she would appear to violate her own guiding principle. Why should we take a term like ‘the Cheerios’ at face value, but not ‘the higher-level pluralities’? Isn’t there a tension lurking behind this asymmetry? I believe not, for the cases are not symmetric to begin with. We introduced terms like ‘second-level plurality’, etc. as translations of HLP foreign expressions. We made this convention clear thereby making them exempt from being taken at face value, that is, as singularly referring expressions. It is thus no surprise that ‘the higher-level pluralities’, a term which intends to generalize over those expressions, is not taken at face value, but rather as pointing towards the common contribution all of those other terms make to the sentences in which they appear. Note that there could be a natural language rendering of ‘the higher-level plurality’. Some ordinary language could have the means to refer to objects restricted to any clusters whatsoever, to put it in terms of clusters. I have not found any evidence of this, but it is nonetheless possible. This would probably motivate a non-typed approach to HLP reference à la Oliver and Smiley, in which case the objection considered in this section would not even arise.

More generally, recall that the idea that speakers’ intuitions should be taken at face value is applicable only in cases in which speakers’ intuitions are more or less uncontroversial. The case of ‘the higher-level pluralities’ is certainly not one of these and as such I believe we are free, in this case, not to take appearances for granted.

formation of sets, which he takes to be a potential process. Without intending to give a detailed description of his views on set theory, let me expound the part most relevant to us.

Linnebo offers an interesting new diagnosis of the paradoxes of set theory. According to it, naïve set comprehension should not be abandoned but rather modified. When analyzed as the conjunction of two different principles, it becomes obvious that it is only when these two principles are taken at face value that contradiction ensues. Instead, he argues, we should always understand one of those two principles as being implicitly modalized.

More specifically, naïve set comprehension can be seen as the result of accepting the following two principles:

**(7.23) Principle (Collapse)**  $\forall xx \exists y \text{Form}(xx, y)$

The predicate  $\text{Form}(xx, y)$  is read as ‘the objects  $xx$  form the set  $y$ ’. Hence Collapse is read as saying that for any objects, there is a set that has as elements all and only those objects.

**(7.24) Principle (Plural Comprehension)**  $\exists xx \forall x (x < xx \leftrightarrow \phi(x))$

Russell’s paradox follows since Plural Comprehension is unrestricted – i.e. there is the plurality of all sets – and Collapse says that for any objects, there is the set which has those objects as all and only members – i.e. there is a set of all sets. The standard response to this situation is to abandon Collapse – not all pluralities form sets. However, Linnebo argues that Collapse is an attractive principle and that it is preferable to keep it, since there is an alternative solution to the paradox. In particular, he argues that we should conceive of the hierarchy of sets as being potential: sets are introduced at different stages and there is no stage at which all of them are finally introduced. In order to capture this, he holds that in most contexts Collapse must be read as being implicitly modalized as follows:

**(7.25) Principle (Collapse<sup>◇</sup>)**  $\Box \forall xx \Diamond \exists y \text{Form}(xx, y)$

The modal operators at play are primitive, but their intended interpretations are as follows: ‘ $\Box\phi$ ’ is read ‘no matter what sets we go on to form it will remain the case that  $\phi$ ’ and ‘ $\Diamond\phi$ ’ is read ‘it is possible to go on to form sets so as to make it the case that  $\phi$ ’. Thus, Collapse<sup>◇</sup> says that no matter what sets we go on to form, it will be possible to form a set  $y$  whose members are exactly  $xx$ . Under this framework, one starts with some urelements and goes on to form all their pluralities (or, rather, to refer to them plurally in every possible way). At the next stage, the sets corresponding to all those pluralities are formed.<sup>176</sup> Next,

<sup>176</sup>Linnebo (2016, p. 24) considers the possibility that not all available pluralities are turned into sets at

we can form pluralities of the resulting sets, whose sets can be subsequently introduced. And so on. Russell's Paradox does not arise, since there is no stage from which we cannot go on to form further sets, thus there is no stage containing the plurality of all sets.

Fortunately for the pluralist and the higher-levelist, there are ways to challenge this view. I begin by considering the difficulties it seems to face independently of the arguments advanced in this thesis.<sup>177</sup> I finish by turning to the higher-level context and raising an additional objection.

The first objection to Linnebo's approach to set theory concerns the effectiveness of one of his main motivations: that the naïve conception of sets is intuitive and should be retained. The problem with this line appears to be that even though there is a clear sense in which the naïve view is appealing, there is an alternative explanation as to why that is, namely, that we find it appealing because it is a confused expression of the view that any objects form a plurality. This appears to be indeed an irresistible truism and it is plausible that we have been misled by a singularized version of Plural Comprehension, which retained part of its initial appeal. Under this view, naïve comprehension is appealing to us because of a prior singularizing habit – the habit to paraphrase plural expressions as singular ones. That we indeed have such a habit is clear given our tendency, as English speakers, to singularly nominalize various grammatical categories when quantifying into them.<sup>178</sup> Once one frees herself of this linguistic habit, the plausibility of naïve comprehension can be seen to diminish.

In fact, the arguments provided in the rest of this thesis should have added some plausibility to the idea that we indeed have a singularizing habit – it has proven very difficult to find a principled reason why limit ourselves to the use of singular expressions. The fact that there are independent reasons to think that singularization is simply an unjustified habit only adds plausibility to the claim that this is indeed what is affecting our judgement in this case. Accordingly, the resulting diagnosis of the set-theoretic paradoxes would be that it is singularization what brings them about and thus that Collapse should be abandoned. The fact that there is an alternative explanation as to the apparent plausibility of Collapse undermines to some extent the motivation for the view above.

Another drawback of Linnebo's view seems to concern the modality at play. Even though

---

each stage. However, he shows that given that the relevant accessibility relation is directed (i.e. such that  $\forall w_1 \forall w_2 \exists w_3 (w_1 \leq w_3 \wedge w_2 \leq w_3)$ ), that does not really matter: whenever we can choose between different sets to individuate, the order in which we do it is irrelevant.

<sup>177</sup>For the first three objections I largely follow Hewitt (2015).

<sup>178</sup>We saw this in Chapter 2, Section 2.2.2.2, when considering the objection to pluralism that quantification is always ontologically committal.

it is taken as primitive, Linnebo offers some elucidations of the nature of his invoked modality. Firstly, as he himself points out, it cannot consist of metaphysical modality since we want to preserve the view that sets exist of metaphysical necessity. For obvious reasons, it cannot be logical modality either. In Linnebo (2010), he refers to it as ‘individuation’ and describes it as follows:

To individuate a mathematical object is to provide it with clear and determinate identity conditions. This is done in a stepwise manner, where at any stage we can make use of objects already individuated and use this to individuate the set with precisely those objects as elements. (Linnebo, 2010, p. 16)

Although this description has some immediate appeal, as things stand, the exact meaning of ‘individuate’ remains obscure. It has a surprising constructivist feel to it, but that cannot be an option if we are to adhere to the view that sets exist of metaphysical necessity. Nevertheless, according to Linnebo, we can elucidate individuation in a way which dispels any suspicion of constructivism. Individuation must be understood as the possibility to expand one’s language in a certain way.<sup>179</sup> In particular, I can individuate an object  $x$  such that  $\phi(x)$  if, and only if, there is an extension of my language which includes a singular term  $a$  such that ‘ $\phi(a)$ ’ is true.

Hewitt (2015) points to two problems with this idea. Firstly, it is not clear that we are entitled to assume that such extensions of the basic language will be available so as to ensure that the whole iterative hierarchy of sets is (potentially) recovered. Second, this notion of individuation makes the existence of sets hinge on linguistic facts, thus ultimately veering towards a seemingly anti-realist view of sets.

Leaving this aside, I would like to point to another objection. When considering at which stage of this set-individuating process we find ourselves, Linnebo writes:

Set theorists do not generally regard themselves as located at some particular stage of the process of forming sets but rather take an external view on the entire process. It would therefore be wrong to assign ourselves any particular stage of the process. (Linnebo, 2010, sec. 6)

This points to a further problem with the elucidation of the relevant modality. Given how ‘individuation’ is understood, there should be a fact of the matter as to at which stage we

---

<sup>179</sup>This is laid out in detail in Linnebo (2012).



find ourselves. As we have seen, whether a set is actual turns on the actual extension of our language – i.e. how many sorts of singular terms it includes at the moment. Hence, if there is a fact of the matter concerning the actual extension of our language, then there must be a fact of the matter concerning in which stage of the set-individuation process we find ourselves. But if this were the case, then the view would incur an element of revisionism with respect to set-theoretic practice, which, as Linnebo acknowledges, places itself outside of any such stage.

Whether this form of revisionism is seen as problematic will depend on one's prior views on the role of philosophy with respect to mathematics and science more generally. However, we have an independent reason to reject such form of revisionism in the present context: it introduces a tension in the views of the pluralist. Recall that the pluralist intends to retain as much as possible of the intuitions of ordinary speakers about their own language in her regimentations and semantic theorizing. This includes specialized domains such as that of set theory. Thus underestimating the set-theorists' intuitions would force her to violate her own stricture.

Another potential problem with Linnebo's modality arises from the assumption that the process of extending the language is likewise potential. According to Linnebo, we can individuate a set if, and only if, we can speak as though we are referring to that set, and we can only do that once we can express its identity conditions, that is, once we have singular terms to denote its elements. But what is the relevant language at play? If the relevant language is ordinary language, then it is doubtful that, as a matter of fact, it expands at all and if so that it expands in the steady way Linnebo describes. By contrast, if it is an artificial language, then supposedly it would have to be an extension of PL with set-theoretic expressions. In that case, a story as to in what sense it expands in the way it does seems to be called for, since in the usual way of thinking of artificial languages, they are described in their entirety at once as soon as their basic elements and formation rules (and possibly an interpretation) are provided.

Leaving aside issues having to do with the nature of the modality at play, another weakness of Linnebo's approach to sets seems to be that he needs to allow for the existence of an empty plurality if he is to obtain an empty set. In Chapter 2, I argued that this take on plural reference is unsatisfactory. Note that this does not affect the issue that interests us now (which pluralities form sets), but rather its dual: which sets are formed from pluralities. However, it is yet another sign that plural talk and set talk come apart in important ways.

I conclude that Linnebo's view faces some challenges, even in the absence of the ad-

vances we have made in the rest of this thesis, that is, even if we focus exclusively on first-level plural reference. However, my arguments regarding higher-level plurals (if successful) give rise to an additional difficulty for it. If I am correct that acceptance of plural reference leads to acceptance of HLP reference, then a new question arises: what is the relation between higher-level pluralities and sets in the potential hierarchy of sets envisioned by Linnebo? In particular, is it the case that once we can refer to some objects higher-level plurally the analogous set can be individuated – where the set in question has as members all the sets formed from the members of the higher-level plurality? More precisely, consider a certain higher-level plurality  $A$  and consider the smallest higher-level plurality containing it and closed under plural membership,  $A^*$ . Now consider the set  $B$  which is such that its transitive closure,  $B^*$ , is analogous to that of  $A$  in that for any member of  $A^*$ ,  $B^*$  has as member the corresponding set (and  $B^*$  has no other members). The question is then whether for any higher-level plurality  $A$ , it is possible to individuate such a set  $B$ .

According to Linnebo, the existence of a set is only conditional on the existence of its elements.<sup>180</sup> According to this formulation of the process of set formation, it seems that being able to refer higher-level plurally to some objects does not give rise to the corresponding set, since we need to first establish the existence of each of the members of the set (as sets, that is). However, Linnebo's specific formulation is based on the assumption that there are no modes of nominal reference other than singular and plural. Once we accept the legitimacy of HLP reference, since all that is required to individuate a new entity is that we have the means to specify its identity, HLP reference would seem to be suited to do the job of individuating sets. An HLP term carries all the information needed in order to determine which are the members of the transitive closure of a certain set. In fact, the legitimacy of HLP reference shows that plural reference to the members of a set is not the only available route to determining the identity of a set (pace the empty set, singleton sets and all the sets whose transitive closure contains the empty set or a singleton): equipped with HLP devices, we can also proceed via HLP reference to the relevant urelements.<sup>181</sup>

The objection is now almost immediate: given that all levels of HLP reference are available at any given stage (since they do not rely on the existence of any entities other than the individuals, which are available from the start), the process of individuating non-empty sets, non-singleton sets and sets that do not have the empty or a singleton set in their transitive closure could be exhausted already at the second stage of the potential hierarchy (in the first

---

<sup>180</sup>Where this existence claim must be understood in the linguistic sense sketched above, along the lines of: on the condition that we are able to express the identity conditions of those elements.

<sup>181</sup>In the absence of urelements, we would need some dummy object to serve as the empty set.

stage we form all the corresponding higher-level pluralities; in the second one we go on to individuate those). If HLP reference is indeed legitimate, then its incorporation into the process of individuation is at odds with the potentialist picture devised by Linnebo. Moreover, it has as a consequence that, at the fourth stage we could be introducing proper classes into the hierarchy of sets (by individuating the set of all non-self-membered non-singletons, for instance).

Thus the legitimacy of HLPL\* provides a reason to abandon the thesis that pluralities always determine sets. The question what sets there are (at any given stage) cannot be determined by the possibility of talking about some things plurally, for that legitimizes the analogous claim regarding HLP reference. Our linguistic resources seem to be too rich to be compatible with the potentialist view of sets.

Overall, I hope to have made a case for the claim that the right answer to the question which pluralities form sets is that only some do. If I am right about this, this brings us closer to Universal Applicability: we can talk about sets and indeed all of them plurally (and higher-level plurally too: e.g. ‘the ZFC sets and the natural numbers form two different mathematical structures’).<sup>182</sup>

## 7.4.2 Ontological Innocence

The second aspect taken as necessary for logicity is ontological innocence. This is the condition that our theories do not make any substantial demands as to what there is. Logical truths should hold regardless of metaphysical facts, in particular, regardless of whether anything exists at all; thus they should have the minimum amount possible of ontological commitments. If they do not have any ontological commitments other than those of a first-order domain of objects, then one typically says that they are ontologically innocent – singular First-Order Logic is usually taken as the paradigm of ontological innocence (even though in its standard formulation the first-order domain is required to be non-empty).

Before we may check our theories for ontological innocence this idea needs to be made precise and a specific mechanism for reading off ontological commitments from a given formalism needs to be laid out. Since a detailed investigation into the topic of ontological

---

<sup>182</sup>The question whether there is a clear-cut criterion to determine which pluralities determine sets has been tackled by Hewitt (2015), where he proposes a ‘naturalist’ approach to the question: all and only those pluralities form sets which our best theory of sets tells us do. I find his arguments appealing, but I must leave this issue out of the present investigation.

commitment falls outside the scope of this dissertation, I will examine HLPL\* in the light of some commonly held assumptions in neighbouring debates, such as those revolving around the logicity of Plural Logic and Higher-Order Logic.

In these debates, the ontological commitment of a formal language is taken to be an aspect of the truth-conditions of the sentences of that language. The interpretation that will do the job in our case is, of course, the higher-levelist semantics described in Chapter 3. Thus the question which interests us is whether HLPL\* under a higher-levelist reading is committed to any entities other than those in its first-order domain. In trying to stay in consonance with the other debates we will take quantified statements to be the crux of the ontological commitments of a formal language. In this sense, our view on the matter is Quinean in spirit. Recall Quine's criterion (in a formulation adapted to the present context):

**(7.26) Principle (Quinean Criterion)** *A singular first-order sentence carries commitment to objects which  $F$  just in case objects which  $F$  must be counted amongst the values of the variables in order for the sentence to be true, where  $F$  is a plural distributive predicate.*

However, our view cannot be fully Quinean, since our final criterion must apply to languages that go beyond singular First-Order Logic. We follow Rayo (2007), who has described the following generalization of the Quinean Criterion:

**(7.27) Principle (Semi-liberalized Quinean Criterion)** *A singular or plural first-order sentence carries commitment to objects which  $F$  just in case objects which  $F$  must be counted amongst the values of the (singular or plural) variables in order for the sentence to be true, where  $F$  is a plural distributive or collective predicate.*

The novelty of this criterion is that where for Quine  $F$  had to be a distributive predicate, there is now no restriction as to its reading; it can also be collective.

Bricker (2016) makes the following remark with respect to the semi-liberalized criterion:

Within the framework of plural logic, however, there are two distinctively different ways that a theory can be ontologically committed to entities, or kinds of entity, corresponding to the distinction between distributive and collective predication (see Rayo (2007, pp. 435-7)), who calls the latter 'plethological commitment'. (If ' $K$ ' is a predicate that applies collectively, then the plural quantification 'there are some things such they are  $K$ s' is committed to  $K$ s in a distinctively plural way that, in general, cannot be captured in terms of its ontological commitments. (*Ibid.*, sec. 1.7.2))

If I interpret this passage correctly, at first Bricker seems to mark a distinction between the singular/distributively plural and the collectively plural way of being ontologically committed to some objects, but in the last sentence he seems to suggest that not only these are different ways of being ontologically committed to individual objects, but that they are associated with different kinds of commitment. I believe we should refrain from conflating these two ideas. The fact that we have more than one way to demand that the world contain the same objects must be taken seriously if one accepts the legitimacy of plural reference. In fact, that was the aim of liberalizing the Quinean criterion in the first place. That is why I believe that calling the commitments of plural collective predications ‘plethological commitments’, as Rayo does, may lead to confusion. The sort of commitments arising from each criterion are the same from an ontological point of view. What differs is only the way in which we acquire those commitments.

Nevertheless, we are not where we want to be yet. We need to liberalize further the schematic  $F$  if our criterion is to apply to HLPL\*. However, for the higher-levelist this is only one step away: let  $F$  be an HLP predicate. We have already argued for the legitimacy of this kind of predicates both as ordinary idioms and as part of a formalism, thus nothing deters us from using them in the statement of our definitive criterion:

**(7.28) Principle (Liberalized Quinean Criterion)** *A higher-level plural first-order sentence carries commitment to objects which  $F$  just in case objects which  $F$  must be counted amongst the values of the (higher-level plural) variables in order for the sentence to be true, where  $F$  is a higher-level plural (distributive of collective) predicate.*

Where the semi-liberalized criterion relied on the acceptance of plural collective predication, the liberalized one relies on the acceptance of HLP collective predication. According to this criterion, HLP sentences are committed only to the referents of the HLP terms in them. We have shown that those should be taken as primitive and as such that there is no need to invoke sets, groups or any other collectivizing entities as their semantic values. Provided one accepts this semantic approach, according to this criterion, HLPL\* is not committed to anything other than the objects of its first-order (and only) domain of quantification.

To sum up, provided one accepts the Liberalized Criterion – and those of a Quinean inclination convinced by our previous arguments should be convinced by it – and provided one is willing to accept our homophonic approach to higher-level plurals, then they should conclude that HLPL\* is ontologically innocent too. In particular, given the fact that my previous arguments are built upon the assumptions and motivations of the pluralists, this argument for ontological innocence is a conditional one as well. If one thinks that PL\* is ontologically innocent, the very same reasons that led one to conclude that should led one to

the same conclusion with respect to HLPL\*.

**Objection: Syntactic types and ontological commitment** As is to be expected, our optimistic conclusions are met with some resistance. For instance, Hewitt (2015) has raised a complaint against the ontological innocence of HLPL\*. Although he does so in relation to a transfinite version of HLPL\*, his objection also applies to our case. Hewitt argues that once we move beyond the plural level and thus accept additional ideological commitments, additional ontological commitments are in turn inevitably incurred.

The problem appears to arise as soon as we require infinitely many sorts of variables. The key question is how we are to defend the claim that we have available HLP terms corresponding to each level without appealing to the existence of abstract types. We can of course provide a token term for any arbitrarily high level, thus showing that the corresponding type is available to us. However, it is impossible for us to provide tokens of all types. Hewitt's claim appears to be that we need to rely on the existence of those types independently of their being instantiated. In our case, this amounts to a commitment to as many abstracts as there are natural numbers.

However, I believe that a number of considerations can be advanced against this argument. First of all, this is one of the points in which the fact that our derivation of Superplural Frege's Theorem requires only 3LPL\* and not the full hierarchy of levels of HLPL\* is significant. If Hewitt's objection is to the effect that it is the usage of infinitely many *sorts* of expressions, then it would appear not to be available for 3LPL\*, since the entities present in the syntax of 3LPL\* can be identified with first-level, second-level and third-level plural token terms. Thus, the specific project put forward in this chapter is safe in this respect.

Nevertheless, there is an interpretation of Hewitt's objection which applies to 3LPL\*. However, it is an interpretation which applies to seemingly innocent theories like First-Order Logic, as well. It is typically assumed that a first-order language has a countably infinite number of first-order singular variables. Even though we often only make use of a few of them and thus make do with using a different letter for each of them, strictly speaking, we need an infinite stock of terms – thus in order to provide a larger number of tokens, we will eventually need to resort to numerical indices or a similar mechanism. More generally, it turns out that in this case we cannot identify the syntactic entities making up our language with concrete tokens either, for there are too many of them. We, being confined finite beings, could never produce all of them. Thus the objection is readily applicable to the first-order case too. However, this goes counter the widely held assumption that First-Order Logic

is ontologically innocent in the sense that it is only committed to a first-order domain of entities which, of course, may be finite (or even empty, if the logic is free). This suggests that something has gone wrong with this objection.

One way to block the objection may be to note that if we stick to the Liberalized Criterion of ontological commitment, then the types of terms are not to be counted as commitments of HLPL\*, but rather of a theory as to the existence of the expressions of HLPL\*. More generally, we need to distinguish between the commitments of the theories, which arise only once the language of the theory has been interpreted, from the commitments of the background theory about the syntax of the language. One may reply that such a theory must be taken for granted when employing the corresponding object language, however this does not seem right. What is taken for granted is the availability of the formulas of HLPL\*, not a theory to this effect. Ontological commitments have to do with the contents of the theory in question and abstract linguistic types are not among those.

Nevertheless, if we were to move on to a less strict notion of ontological commitment, one that also counted as part of the ontological commitments of HLPL\* the commitments of background theories – such as those of the theory of the availability of its linguistic items –, this objection may gain some force. After all, it is not until we have the syntax of our theory that we can do something with it.

However, even then, this does not seem to be a problem for the neo-Fregean. The question is not how we can have available the tokens we need to derive Frege's Theorem. The tokens can simply be produced by us, nothing else is needed. This allows us to derive arithmetic without incurring new commitments. A different question, however, is how we are going to make justice to the separate claim that there are infinitely many syntactic entities. One solution is to appeal to infinitely many abstract types. This would not be available to a nominalist, but the neo-Fregean has no problem with positing infinitely many entities to stand for syntactic entities. For instance, she can take the natural numbers themselves. No vicious circle seems to be incurred. The tokens used in our proofs of Frege's Theorem do not depend for their existence on some abstract entities. They depend for their existence on their having been produced or not. No extra entities are appealed to here.<sup>183</sup>

---

<sup>183</sup>Moreover, one may even be able to develop a theory about the syntax of HLPL\* which does not posit infinitely many entities. One could say something along the lines that it is sufficient for a certain syntactic item to exist that we are able, in principle, to produce a token thereof when needed. The difficulty with this will be to explain what 'in principle' means in this context.

### 7.4.3 Cognitive Primacy

Finally, let us briefly tackle the issue of the cognitive status of HLPL\*. Cognitive Primacy is the condition on logicity that is perhaps most difficult to assess. Recall that its statement makes explicit appeal to extra-logical ideas. Of course, there is no clear-cut divide between logical and extra-logical notions, thus a proper assessment of Cognitive Primacy does not seem available to us. However, as we have done with the very notion of logicity we will rely on a partial analysis thereof. Our aim will be the more modest one of showing that our formalism does not presuppose clearly mathematical notions, such as those of set theory or other branches of mathematics.

First of all, note that this issue cannot be settled by reflecting on one's impression of one's own cognitive processes in understanding HLP quantification. For two reasons: (i) one may very well be wrong about one's own cognitive processes; (ii) even if one is right, this does not show that those processes are required in a strong enough sense. But what do we mean by a strong enough sense of 'prior understanding'? To begin with, the following would be a case in which prior cognitive requirements are clearly not strong enough to be relevant for logicity. When learning a second language, say Finnish (for an English speaker), speakers appear to go through a phase of having to translate the Finnish expressions into English before they can understand the former. Of course, speakers of different languages will proceed via different translations of the Finnish phrases. These are clear cases in which speakers need to understand something else in order to understand the Finnish expressions. However, they are also clear cases in which the secondary notions, in this case the Finnish phrases, are not considered any less cognitively basic than the understanding of the primary notions, their translations. Thus, clearly, not all cases of understanding in which there is a prior cognitive requirement should be revoked of cognitive primacy. We need a stronger sense of 'demanding a prior understanding of something else'. We need this demand to be necessary. It is not sufficient that this demand be always accidentally present. Thus the question which interests us is whether in order to understand HLPL\* we necessarily need to understand something else (in particular, some part of mathematics, such as set theory).

In my view, we have already done all that can be expected to support a negative answer to this question. We have shown that some people appear to understand ordinary HLP expressions primitively, as they are part of their mother tongue. Moreover, we have reinforced this by showing that HLP expressions cannot be in general paraphrased away or interpreted in a reductionist way. It seems this is the best one can do to support the claim that a certain language or fragment of a language is cognitively primary; nothing else can be reasonably



asked of the advocate of any formal language in this respect. Unfortunately, it is insufficient, since there is still the possibility of unconscious necessary cognitive demands which may not even be captured by our best semantic theories. However, I do not know of any way to find out about these – any empirical investigation into our actual processes of thought will not do the job, for that can at most account for contingent demands.

If we have indeed successfully shown all of this, then any argument based on the fact that we can understand HLP quantification in other terms loses its force when it comes to undermining cognitive primacy. The burden of proof is not on the higher-levelist to show that HLPL\* cannot be understood in terms of anything else,<sup>184</sup> but rather on its opponent to show that HLPL\* can only be understood via something else.

To finish, let me address an objection to Cognitive Primacy.

**Objection: Combinatorics and the notion of arbitrary subplurality** Linnebo (2003) has raised an objection to the logicity of Plural Logic, which, if successful, would also undermine the logicity of HLPL\*. He has argued that in order to understand what the plural quantifiers mean, one needs to grasp the notion of arbitrary sub-plurality of a given plurality (a notion needed given the impredicativity of the plural comprehension axioms) and, in order to do so, one must have a basic grasp of combinatorics. Since combinatorics is a branch of mathematics, this means that we must understand some mathematics in order to understand what an arbitrary sub-plurality is. Incidentally, Linnebo uses this as an argument to the effect that one can make sense of superplurals,<sup>185</sup> but this is irrelevant for the present discussion. The aspect of this argument which interests us is the claim that PL and its higher-level extensions do not enjoy cognitive primacy, since non-logical notions need to be grasped in the first place in order to understand them. Our response is in line with what we just argued: the fact that we can understand the HLP comprehension axioms as claims about combinatorics does not show that HLPL\* is not cognitively primary, for the converse claim obtains as well: we can understand claims about combinatorics via HLPL\*. Thus it is not clear which of these two theories is primary. In other words, since the mere possibility to understand something via something else is insufficient to prove facts about cognitive dependency, Linnebo's argument only succeeds if we presuppose that combinatorics is indeed cognitively more fun-

<sup>184</sup> Although I also tried to show that when arguing that non-homophonic semantics for HLPL\* are not fully satisfactory.

<sup>185</sup> Roughly speaking, the idea is that if combinatorics is all it takes, in terms of cognitive resources, to form pluralities of objects, since combinatorial operations can be iterated, we can legitimately pass from pluralities to higher-level pluralities.

damental than HLPL\*, but this is precisely what is at stake. However, note that, despite the fact that, if my argument were correct, the objection would not go through, we appear to find ourselves in a stand-off, for it is analogously not clear that combinatorics is not more cognitively fundamental than HLPL\*.

## 7.5 Conclusions

In this chapter, I have outlined a proposal to use 3LPL\* to strengthen the neo-Fregean project. I have argued that this approach fares better than an alternative pluralist variant and I have defended the logicity of HLPL and, thus, of 3LPL\*.

As we have seen before in these sort of debates, no decisive arguments appear to be available for the conclusion that 3LPL\* (and, more generally, HLPL\*) is universally applicable, ontologically innocent and cognitively primary. Ultimately, whether these three conditions are seen to obtain hinges on whether one accepts a homophonic treatment of the relevant language in the first place. This may not seem satisfactory. However it is all one can do, since the reaction according to which, despite its technical merits (if it has any), the grammatical form of HLP expressions is misleading and thus does not correspond to their logical form is always available. Any positive argument for logicity we can offer is hopeless in the face of this form of scepticism.

Fortunately, the sceptical response leaves some argumentative room for us: we can at least make a case against the objections to the logicity of HLPL\*. In the evaluative section of this chapter I have attempted to take advantage of this room.

This chapter should serve as an extension of the conditional claim put forward throughout the other chapters: those who endorse pluralism should endorse higher-levelism, since the considerations which typically lead to the former also lead to the latter. Here, by arguing that those who accept our semantic treatment of higher-level plurals have reasons to consider HLPL\* as pure logic, we have extended that argument to the stronger one that those who take Plural Logic to be pure logic should be inclined to see its higher-level extensions in the same light.



# Conclusions

**What I set out to do** This dissertation has been an investigation into the legitimacy and merits of Higher-Level Plural Logic. As such, one of its underlying themes has been the question of what it would take, in general, to legitimize a certain understanding of a formal language. Let me stop to consider this more general question for a moment in order to shed some light onto the achievements and limitations of this dissertation.

An interpreted formal language is legitimate only if its interpretation is suitable (prima facie there is no reason to think that a formal language can only be suitably interpreted in one way); and a certain semantic theory is suitable only if (i) it is accessible (i.e. expressed in a well understood ordinary language) and (ii) it captures the meaning of the object language correctly. In my thesis, I set out to show that a higher-levelist reading of HLPL meets these two conditions. In order to show that a certain language is adequately interpreted by being re-used (or rather by employing its informal counterparts) in the metalanguage with which we express its semantics, one way to proceed is to show that ordinary language does contain these expressions and that it does so in an indispensable way. Importantly, proceeding in this way shows simultaneously that the semantic theory meets both condition (i) and (ii). On the one hand, it shows that the homophonic interpretation is accessible for native speakers of those languages, who have a direct understanding of it (and, as a consequence, indirectly, that it can be accessed via translations of the foreign idioms into an extension of one's own mother tongue). On the other, it shows that the semantics correctly captures the meaning of the object language, assuming that the formal language intends to encapsulate the meaning of its informal counterpart.

In my thesis, I set out to do just that with respect to HLPL under a higher-levelist reading. Moreover, I tried to go beyond this aim by developing a non-homophonic semantics for higher-level plurals with the aim to get more theorists on board. Finally, I aimed to motivate the interest towards HLPL by showing that not only it is a legitimate interpreted formalism, but that it can be a useful tool for philosophers, especially those working in the foundations of mathematics.

**What I think I didn't do** Resnik expressed the sceptical challenge with respect to plurals in the following words:

[N]either the intelligibility of plural quantifications nor their prima facie lack of commitment to collections is sufficient to demonstrate that they never commit us to collections. Determining whether they do involves representing them in an acceptable and suitably interpreted logical notation. Thus, those of us who find plural quantification in need of logical analysis will not be enlightened by formal explications such as Boolos's, which presuppose plural quantification for their interpretation. [...] Until Boolos can show us that construing it as quantifying over collections renders it plainly false or more controversial than it already is, my intuitions and his concerning it and the Geach-Kaplan sentence are at a standoff. (Resnik, 1988, p. 80)

In this quote, Resnik rightly points towards the limitations of any attempt to argue for the legitimacy of PL under a pluralist reading. The analogous complaint can be raised with respect to higher-level plurals and indeed, unsurprisingly, it stands almost untouched by this thesis, precisely because we have endorsed, like Boolos, ordinary HLP quantification to account for the meaning of formal HLP quantification. There is bound to be a disagreement hinging on contrary intuitions with respect to the ordinary idioms involved.

There are various reasons why one may disagree with our use of metalinguistic higher-level plurals. Firstly, if there were no higher-level plurals in natural language, one could think that we do not have the means to understand them – in other words, that they could not even belong to an alien language which we could learn. There is little that can be said against this view. However, I believe to have shown that there are higher-level plurals in natural language. In spite of that, one may still hold a sceptical position about our using them for semantic purposes. In particular, one would have to argue either that their grammatical form is misleading, or that, even though they are syntactically indispensable, they must nevertheless be understood in terms which are not higher-level plural. Against the former claim, one typically shows that paraphrase is not always available without it incurring some undesirable consequences. However, the standoff is likely to appear again when assessing what counts as an undesirable consequence. For instance, we have appealed to counterintuitive results regarding absolutely general quantification at this point, but the sceptic is likely to claim that those are simply unavoidable and apparently absolutely general sentences should not be taken at face value. We disagree, but again, little can be argued at this point. The analogous metalinguistic move displays a similar entanglement. We think ordinary language cannot be

analyzed in certain ways because of cardinality problems (among others); the sceptic disagrees because she thinks we should not take that bit of ordinary language at face value in the first place.

Hence, having shown that higher-level plurals appear in natural language in an indispensable way, whether my remaining arguments are convincing hinges on one's attitude towards the significance of ordinary speakers' intuitions with respect to the expressions at hand. This is the point from which my arguments are only compelling for those who share my view towards this issue. Importantly for us, among these, we find the pluralists (or most of them).

Despite not meeting the sceptical challenge, I believe to have helped higher-levelism make progress in various ways.

**What I think I did** Firstly, I believe to have helped settle the debate on higher-level plurals at the same point where the debate around plurals seems to have settled. In other words, I believe to have brought HLPL to the same sort of standoff PL faces. To do this I have argued that there is nothing incoherent about the notion of HLP reference and I have shown that there are ineliminable higher-level plurals in natural language.

By doing this, I believe to have shown that HLPL, just as PL, are in a certain sense in a better place than their higher-order counterparts – which are, in many contexts, their main competitors. The reason why this is so is that PL and HLPL have a better claim to have ordinary language counterparts than second-order and, more generally, higher-order logic do. Whereas an ordinary reading of quantification into predicate position (one which does not incur singularization – one that is not of the form 'there is a property' or 'there is a set') does not seem to be available,<sup>186</sup> ordinary plural and HLP quantification is within our reach. On the one hand, plural quantification is obviously present in English. On the other,

---

<sup>186</sup>Against this claim, Rayo and Yablo (2001) have made a proposal to capture predicate quantification in ordinary language. The problem with their proposal, however, seems to be that their translations capture either nominal or adverbial quantification, rather than predicate quantification. Their translation of monadic predicate quantification is 'an object is something such that...' and their translation of dyadic predicate quantification is 'somehow things relate such that...'. Firstly, the dissimilarity between the two cases is suspicious. Moreover, while the former appears to be nominal quantification (they argue that 'something' can be understood as an adjectival quantifier, but this seems controversial – once again, it looks like a singularly nominalized reading of adjectival quantification), the latter appears to be adverbial quantification. They appear to have previously reduced relational quantification into adverbial quantification: '*a* is taller than *b*' must be understood as saying that *a* and *b* are related in a being-taller-than way. These two appear to be two different sorts of claims: one says of *a* and *b* that they have the collective property of being related (and that they have it in a particular way), whereas the former is not a collective predication, but rather a dyadic one.

ordinary HLP quantification seems to be less problematic than predicate quantification in this respect. Firstly, we argued that ordinary Finnish may even contain one such quantifier. And secondly, since HLP quantification is a species of nominal quantification – it is into name position, albeit into HLP name position – construing such forms of quantification is straightforward in an extension of English with the translations of the foreign HLP terms. Simply attach an HLP noun to a nominal quantifier: ‘any objectses’ and ‘some objectses’ (note that ‘any’ and ‘some’ do not mark number, thus, they appear to be available to be attached to HLP nouns).

Moreover, I have attempted to transmit the meaning of higher-level plurals to the sceptic in various ways. Not only have I described their main features, but I have provided several examples throughout this dissertation and I have developed a non-homophonic semantic approximation to them based on the idea of reference restricted to clusters. All of these should serve to illuminate the meaning of higher-level plurals to those who, as of yet, do not speak an HLP language.

Furthermore, I have shown that the pluralist is likely to be committed to higher-levelism. My tentative diagnosis of the fact that some pluralists reject HLPL is that their view results from a lack of engagement with ordinary languages other than English, the language in which this debate has mostly taken place (as far as I am aware). That plurals and higher-level plurals stand or fall together will be seen as good news by some and bad news by others. The good news is that there is an increase in expressive power; the bad news is that with it, a suspicion of a loss of logicity arises. We have tried to defend HLPL from this seemingly bad news in the last chapter.

Finally, I believe my contribution will assist in bridging the gap between the linguistic and the philosophical literature on this topic. As we have seen, formal semanticists are typically aligned with the sceptics towards higher-level plurals. This seems to be at least in part because of two reasons: (i) formal semanticists are not interested in questions of ontology<sup>187</sup> and (ii) formal semanticists are often not interested in maximally general discourse. The former lack of interest seems to me to be justified given their aims. However, it is not so clear that they should not be concerned about properly capturing the intuitive meaning of general expressions. I believe this is one reason why linguists may be compelled by some of the arguments offered here.

Overall, if my arguments are successful, I believe that the burden of proof has been

---

<sup>187</sup>Possibly with the exception of Landman (1989b), who sees ontological proliferation as a reason to abandon Link’s mereological and group-based account.

shifted towards the sceptic. The challenge is now on the sceptic to explain why, unlike ordinary higher-level plurals, other ordinary idioms are suitable for semantic theorizing. What does ordinary singular first-order quantification have to be suitable for semantics that ordinary HLP quantification does not have? Until a clear reason for admitting one and not the other is laid out, I believe the higher-levelist has an advantage. In my view, the sceptic must admit that the standoff of intuitions affecting higher-level plurals arises with respect to ordinary language expressions which are typically considered adequate for semantic theorizing or else provide an explanation as to what is different in this case.

**What I would do next** To wrap things up, let me point to a couple of research avenues that had to be left out of this thesis and that I think would be worthwhile pursuing.

Firstly, I believe that the notion of restricted reference should be further investigated independently of its connection with HLP reference. As far as I am aware, there exists little research on this seemingly fruitful notion. In particular, I can think of another place within the debate around plurals where the notion could prove useful. Although we have taken pluralities to be rigid in the sense that each plurality has its members necessarily, this conception of pluralities only accounts for some uses of ordinary plural terms. Sometimes, plurals appear to be used in a way that violates rigidity. For instance, think of ‘The fourth graders went to the museum today’. In this sentence ‘the fourth graders’ is most likely used in a way that allows variation among its referents without change in truth value. Suppose Alba is among the fourth graders, but she did not go to the museum. In most contexts, this fact would not falsify the sentence. An interesting question is whether we can provide pluralist semantics for these uses of plural terms too and thus whether an ontologically innocent view of these uses of plurals is defensible. Here is where I think restricted reference could come into the picture. Perhaps it is possible to use plural reference restricted to a special class of aspects to account for cases like the one above. The idea would be that ‘the fourth graders’ above picks out the kids who actually went to the museum (i.e. not Alba), thus a plurality as understood by us, but it does so under their aspect of being the fourth graders, where this aspect is different from the ones we considered above – it opens the door to there being truth-preserving variability among the referents of the term it restricts. Pluralities restricted to these sorts of aspect would have the following two characteristics: on the one hand, Extensionality would fail for them (they can account for lecturers-graders cases) and, on the other, so would rigidity (they can account for cases like the one just sketched). If something along the lines of this proposal were successfully developed, that would have two positive outcomes: first, it would show that we can give an ontologically innocent account of



those uses of plurals; and, second, it would show that we can give a unified semantic account of a greater variety of fragments of natural language.

Another topic I would like to look further into is that of using plurals to provide semantics for predication. The significance of predicates has been left aside in this thesis for the most part. For one thing, it was not directly relevant to our aims; for another, as I mentioned in passing, I suspect predicate and plural talk should be kept separate. Although I think they can be likened in some contexts, there are areas where retaining both modes of signifying can be at least fruitful and at most indispensable. The following are some of the ways in which they appear to come apart. I have touched on some of these throughout this thesis. Firstly, Plural Comprehension has a better claim to being necessarily impredicative than Second-Order Comprehension does. Secondly, plural reference appears to forbid the existence of empty pluralities, whereas its second-order counterpart does not. Thirdly, second-order talk is open to an intensional reading, but plural talk is not equally suited to that end. A property may have been instantiated by different objects, but some objects could not have been some other objects (unless something like the proposal we sketched above worked). Finally, plural talk does not seem to be capable of capturing higher-order predication properly. The problem is that under a pluralist reading, collective predications of pluralities and second-order predications of first-order properties would be assigned the same truth-conditions, but intuitively, it takes different worldly contributions to make some objects have a certain property and to make a property have a certain property. This is in fact confirmed by the fact that, while  $\forall xx\phi(xx)$  implies  $\forall x\phi(x)$ , the translated second-order formulae are not so related:  $\forall X\phi(X)$  does not imply  $\forall x\phi(x)$ . The reason why the latter does not go through is that the property-based reading of second-order logic is typically not cumulative and thus predicates admit only arguments of a single type. This restriction can be dropped, but if so a property-based reading loses part of its rationale – intuitively, a property of first-order properties cannot also hold of individuals. This list informs nothing more than a suspicion. I believe that a deeper investigation into the ways in which these two notions go separate could lead to interesting results. In particular, we may be able to discern in which contexts (if there are any) these differences can be ignored and the mutual translations between the two languages exploited. Although some research in this area exists, most of it has been exclusively directed to discussing Boolos' approach. I believe that a more systematic and over-arching research into this issue is due.

## References

- Appleyard, D. L. (1987). A grammatical sketch of Khamtanga–I. *Bulletin of the School of Oriental and African Studies, University of London*, 50(2), 241–266.
- Arapinis, A., Carrara, M., & Moltmann, F. (2016). *Unity and plurality. philosophy, logic, and semantics*. Oxford University Press.
- Asher, N. (2006). Things and their aspects. *Philosophical Issues*, 16(1), 1–23.
- Ben-Yami, H. (2013). Higher-level plurals versus articulated reference, and an elaboration of *salva veritate*. *Dialectica*, 67(1), 81–102.
- Black, M. (1971). The elusiveness of sets. *Review of Metaphysics*, 24(4), 614–636.
- Boccuni, F. (2011). On the consistency of a plural theory of Frege's *Grundgesetze*. *Studia Logica*, 97(3), 329–345.
- Boccuni, F. (2013). Plural logicism. *Erkenntnis*, 78(5), 1051–1067.
- Boolos, G. (1984). To be is to be a value of a variable (or to be some values of some variables). *Journal of Philosophy*, 81(8), 430–449.
- Boolos, G. (1985). Nominalist Platonism. *Philosophical Review*, 94(3), 327–344.
- Boolos, G. (1987). The consistency of Frege's Foundations of Arithmetic. In J. Thomson (Ed.), *On being and saying: Essays in honor of Richard Cartwright* (pp. 3–20). MIT Press.
- Bricker, P. (2016). Ontological commitment. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Winter 2016 ed.). Metaphysics Research Lab, Stanford University.
- Burgess, J. P. (2004). E pluribus unum: Plural logic and set theory. *Philosophia Mathematica*, 12(3), 193–221.
- Cartwright, R. L. (1994). Speaking of everything. *Noûs*, 28(1), 1–20.
- Corbett, G. G. (2000). *Number*. Cambridge University Press.
- Cotnoir, A. J. (2013). Composition as general identity. In K. Bennett & D. W. Zimmerman (Eds.), *Oxford studies in metaphysics volume 8* (pp. 294–322). Oxford University Press.
- Davidson, D. (1968). On saying that. *Synthese*, 19(1-2), 130–146.
- Fine, K. (2000). Neutral relations. *Philosophical Review*, 109(1), 1–33.
- Florio, S. (2010). *The semantics of plurals: A defense of singularism* (Unpublished doctoral dissertation). Ohio State University.
- Florio, S. (2014a). Semantics and the plural conception of reality. *Philosophers' Imprint*, 14(22), 1–20.
- Florio, S. (2014b). Untyped pluralism. *Mind*, 123(490), 317–337.
- Florio, S., & Linnebo, O. (2015). On the innocence and determinacy of plural quantification. *Noûs*, 49(1), 1–19.

- Florio, S., & Nicolas, D. (2015). Plural logic and sensitivity to order. *Australasian Journal of Philosophy*, 93(3), 444–464.
- Forbes, G. (1990). The indispensability of sinn. *Philosophical Review*, 99(4), 535–563.
- Forbes, G. (1997). How much substitutivity? *Analysis*, 57(2), 109–113.
- Frege, G. (1892/1952). On sense and reference. In P. Geach & M. Black (Eds.), *Translations from the philosophical writings of Gottlob Frege*. Philosophical Library.
- Gillon, B. S. (1987). The readings of plural noun phrases in English. *Linguistics and Philosophy*, 10(2), 199–219.
- Gillon, B. S. (1992). Towards a common semantics for English count and mass nouns. *Linguistics and Philosophy*, 15(6), 597–639.
- Hale, B. (1988). *Abstract objects*. Blackwell.
- Hale, B., & Wright, C. (2001). *Reason's proper study: Essays towards a neo-Fregean philosophy of mathematics*. Oxford University Press UK.
- Hazen, A. P. (1993). Against pluralism. *Australasian Journal of Philosophy*, 71(2), 132–144.
- Hazen, A. P. (1997). Relations in Lewis' framework without atoms. *Analysis*, 57(4), 243–248.
- Hewitt, S. (2012). The logic of finite order. *Notre Dame Journal of Formal Logic*, 53(3), 297–318.
- Hewitt, S. (2015). When do some things form a set? *Philosophia Mathematica*, 23(3), 311–337.
- Hewitt, S. (in press). Tuples all the way down? *Thought: A Journal of Philosophy*.
- Hewitt, S. (unpublished). *Frege's theorem in plural logic*.
- Hossack, K. (2000). Plurals and complexes. *British Journal for the Philosophy of Science*, 51(3), 411–443.
- Hurford, J. R. (2003). The interaction between numerals and nouns. In *Noun phrase structure in the languages of Europe, volume 20-7 of Empirical approaches to language typology*.
- Kanamori, A. (2003). The empty set, the singleton, and the ordered pair. *Bulletin of Symbolic Logic*, 9(3), 273–298.
- Klement, K. C. (2014). Early Russell on types and plurals. *Journal for the History of Analytical Philosophy*, 2(6), 1–21.
- Krämer, S. (2014). Semantic values in higher-order semantics. *Philosophical Studies*, 168(3), 709–724.
- Kreisel, G. (1967). Informal rigour and completeness proofs. In I. Lakatos (Ed.), *Problems in the philosophy of mathematics* (pp. 138–157). North-Holland.

- Kripke, S. A. (1980). *Naming and necessity*. Harvard University Press.
- Landman, F. (1989a). Groups, I. *Linguistics and Philosophy*, 12(5), 559–605.
- Landman, F. (1989b). Groups, II. *Linguistics and Philosophy*, 12(6), 723–744.
- Laserson, P. (2006). Plurality. In K. Brown (Ed.), *Encyclopedia of language and linguistics (second edition)* (Second Edition ed., pp. 642–644). Oxford: Elsevier.
- Lewis, D. (1991). *Parts of classes*. Blackwell.
- Link, G. (1983). The logical analysis of plurals and mass terms: A lattice-theoretic approach. In P. Portner & B. H. Partee (Eds.), *Formal semantics - the essential readings* (pp. 127–147). Blackwell.
- Link, G. (1984). Hydras: On the logic of relative clause constructions with multiple heads. In F. Landman & F. Veltman (Eds.), *Varieties of formal semantics: Proceedings of the Fourth Amsterdam Colloquium*. Foris Publications.
- Link, G. (1998). Ten years of research on plurals – where do we stand? In F. Hamm & E. Hinrichs (Eds.), *Plurality and quantification* (pp. 19–54). Dordrecht: Springer Netherlands.
- Linnebo, O. (2003). Plural quantification exposed. *Noûs*, 37(1), 71–92.
- Linnebo, O. (2006). Sets, properties, and unrestricted quantification. In G. Uzquiano & A. Rayo (Eds.), *Absolute generality*. Oxford University Press.
- Linnebo, O. (2010). Pluralities and sets. *Journal of Philosophy*, 107(3), 144–164.
- Linnebo, O. (2012). Reference by abstraction. *Proceedings of the Aristotelian Society*, 112(1pt1), 45–71.
- Linnebo, O. (2013). The potential hierarchy of sets. *Review of Symbolic Logic*, 6(2), 205–228.
- Linnebo, O. (2016). How to harness Basic Law V. In A. A. Massimiliano Carrara & F. Moltmann (Eds.), *Unity and plurality: Logic, philosophy, and linguistics* (pp. 19–32). Oxford: OUP.
- Linnebo, O. (2017). Plural quantification. In E. N. Zalta (Ed.), *Stanford encyclopedia of philosophy*. The Metaphysics Research Lab, Stanford University.
- Linnebo, O., & Nicolas, D. (2008). Superplurals in English. *Analysis*(68), 186 – 1974.
- Linnebo, O., & Rayo, A. (2012). Hierarchies ontological and ideological. *Mind*, 121(482), 269–308.
- MacBride, F. (2003). Speaking with shadows: A study of neo-logicism. *British Journal for the Philosophy of Science*, 54(1), 103–163.
- MacBride, F. (2005). The particular-universal distinction: A dogma of metaphysics? *Mind*, 114(455), 565–614.
- MacBride, F. (2006). Predicate reference. In B. C. Smith (Ed.), *The Oxford handbook of*

- philosophy of language* (pp. 422–475). Oxford University Press.
- MacBride, F. (2007). Neutral relations revisited. *Dialectica*, 61(1), 25–56.
- McKay, T. J. (2006). *Plural predication*. Oxford University Press.
- Moltmann, F. (2016). Plural reference and reference to a plurality. In M. Carrara, A. Arapinis, & F. Moltmann (Eds.), *Unity and plurality. philosophy, logic, and semantics*. Oxford University Press.
- Nicolas, D. (2007). *Can mereological sums serve as the semantic values of plurals?* (Unpublished)
- Nicolas, D. (2008). Mass nouns and plural logic. *Linguistics and Philosophy*, 31(2), 211–244.
- Ojeda, A. (2015). The semantics of number in Arabic. *Semantics and Linguistic Theory*, 2, 303.
- Oliver, A., & Smiley, T. (2001). Strategies for a logic of plurals. *Philosophical Quarterly*, 51(204), 289–306.
- Oliver, A., & Smiley, T. (2005). Plural descriptions and many-valued functions. *Mind*, 114(456), 1039–1068.
- Oliver, A., & Smiley, T. (2013). *Plural logic*. Oxford University Press.
- Oliver, A., & Smiley, T. (2016). *Plural logic: Second edition, revised and enlarged*. Oxford University Press.
- Parsons, C. (1990). The structuralist view of mathematical objects. *Synthese*, 84(3), 303–346.
- Pietroski, P. M. (1996). Fregean innocence. *Mind and Language*, 11(4), 338–370.
- Pleitz, M. (2017). Two accounts of pairs. In P. Arazim & T. Lavika (Eds.), *The Logica Yearbook 2016*. London: College Publications.
- Prior, A. (1971). *Objects of thought*. Clarendon Press.
- Quine, W. V. O. (1950). *Methods of logic*. Harvard University Press.
- Quine, W. V. O. (1953). *From a logical point of view*. Harvard University Press.
- Quine, W. V. O. (1970). *Philosophy of logic*. Harvard University Press.
- Rayo, A. (2006). Beyond plurals. In A. Rayo & G. Uzquiano (Eds.), *Absolute generality* (pp. 220–54). Oxford University Press.
- Rayo, A. (2007). Ontological commitment. *Philosophy Compass*, 2(3), 428–444.
- Rayo, A., & Uzquiano, G. (1999). Toward a theory of second-order consequence. *Notre Dame J. Formal Logic*, 40(3), 315–325.
- Rayo, A., & Uzquiano, G. (2006). *Absolute generality*. Oxford University Press.
- Rayo, A., & Yablo, S. (2001). Nominalism through de-nominalization. *Noûs*, 35(1), 74–92.
- Renisch, L. (1884). *Die Chamirsprache in Abessinien*. Gerold.

- Resnik, M. D. (1988). Second-order logic still wild. *Journal of Philosophy*, 85(2), 75–87.
- Rosen, G., & Dorr, C. (2002). Composition as a fiction. In R. Gale (Ed.), *The Blackwell Companion to Metaphysics* (pp. 151–174). Blackwell.
- Rumfitt, I. (2005). Plural terms : Another variety of reference? In J. L. Bermúdez (Ed.), *Thought, reference, and experience: Themes from the philosophy of Gareth Evans* (pp. 84–123). Clarendon Press.
- Russell, B. (1903). *The principles of mathematics*. Cambridge University Press.
- Russell, B. (1905). On denoting. *Mind*, 14(56), 479–493.
- Russell, B. (1918). The philosophy of logical atomism. *The Monist*, 28(4), 495–527.
- Salmon, N. U. (1986). *Frege's puzzle*. Ridgeview.
- Saul, J. M. (1997). Substitution and simple sentences. *Analysis*, 57(2), 102–108.
- Schwarzschild, R. (1993). Against groups. In *Proceedings of the Seventh Amsterdam Colloquium, part 2, Institute for Language, Logic and Information, University of Amsterdam* (pp. 475–594).
- Schwarzschild, R. (1996). *Pluralities*. Springer.
- Shapiro, S. (1991). *Foundations without foundationalism: A case for second-order logic*. Oxford University Press.
- Shapiro, S. (1993). Modality and ontology. *Mind*, 102(407), 455–481.
- Shapiro, S. (2000). Frege meets Dedekind: A neologicist treatment of real analysis. *Notre Dame Journal of Formal Logic*, 41(4), 335–364.
- Shapiro, S., & Weir, A. (2000). ‘Neo-logicist’ logic is not epistemically innocent. *Philosophia Mathematica*, 8(2), 160–189.
- Simons, P. (1982). Number and manifolds. In B. Smith (Ed.), *Parts and moments: Studies in logic and formal ontology* (pp. 160–98). Philosophia Verlag.
- Simons, P. (1997). Higher-order quantification and ontological commitment. *Dialectica*, 51(4), 255–271.
- Simons, P. (2016). The ontology and logic of higher-order multitudes. In A. A. Massimiliano Carrara & F. Moltmann (Eds.), *Unity and plurality: Logic, philosophy, and linguistics* (pp. 55–69). Oxford: OUP.
- Soames, S. (2002). *Beyond rigidity: The unfinished semantic agenda of Naming and Necessity*. Oxford University Press.
- Stump, G. T. (1989). A note on Breton pluralization and the elsewhere condition. *Natural Language & Linguistic Theory*, 7(2), 261–273.
- Tennant, N. (2008). Natural logicism via the logic of orderly pairing. In S. Lindström (Ed.), *Logicism, intuitionism, and formalism: What has become of them*. Springer.
- Urbaniak, R. (2013). Plural quantifiers: A modal interpretation. *Synthese*, 191(7), 1–22.

- Uzquiano, G. (2003). Plural quantification and classes. *Philosophia Mathematica*, 11(1), 67–81.
- Uzquiano, G. (2004). Plurals and simples. *The Monist*, 87(3), 429–451.
- van Cleve, J. (1994). Predication without universals? A fling with Ostrich Nominalism. *Philosophy and Phenomenological Research*, 54(3), 577–590.
- Williamson, T. (2003). Everything. *Philosophical Perspectives*, 17(1), 415–465.
- Wright, C. (1983). *Frege's conception of numbers as objects*. Aberdeen University Press.
- Wright, C. (2000). Neo-Fregean foundations for real analysis: Some reflections on Frege's constraint. *Notre Dame Journal of Formal Logic*, 41(4), 317–334.
- Yi, B. (2005). The logic and meaning of plurals. Part I. *Journal of Philosophical Logic*, 34(5-6), 459–506.
- Yi, B. (2006). The logic and meaning of plurals. Part II. *Journal of Philosophical Logic*, 35(3), 239–288.