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SEMANTIC ANALYSIS OF WH-COMPLEMENTS*

## 0. Introduction

This paper presents an analysis of wh-complements in Montague Grammar. We will be concerned primarily with semantics, though some remarks on syntax are made in Section 4. Questions and wh-complements in Montague Grammar have been studied in Hamblin (1976), Bennett (1979), Karttunen (1977) and Hauser (1978) among others. These proposals will not be discussed explicitly, but some differences with Karttunen's analysis will be pointed out along the way.

Apart from being interesting in its own right, it may be hoped that a semantic analysis of wh-complements will shed some light on what a proper analysis of direct questions will look like. One reason for such an indirect approach to direct questions is the general lack of intuitions about the kind of semantic object that is to be associated with them. A survey of the literature reveals that direct questions have been analyzed in terms of propositions, sets of propositions, sets of possible answers, sets of true answers, the true answer, properties, and many other things besides. As far as wh-complements as such are concerned, we do not seem to fare much better, but there is this clear advantage: we do have some intuitions about the semantics of declarative sentences in which they occur embedded under such verbs as know, tell, wonder. What kind of semantic object we may choose to associate with wh-complements is restrained by various facts about the semantics of these sentences.

This paper is organized as follows. In Section 1 we discuss a number of semantic facts concerning declarative sentences containing wh-complements, leading to certain conclusions regarding the kind of semantic object that is to be associated with wh-complements. In Section 2 we show that Ty2, the language of two-sorted type theory, gives suitable means to represent the semantics of wh-complements, and that Ty2 can take the place of IL in PTQ as a translation medium. In Section 3 we indicate how the analysis proposed can be implemented in a Montague Grammar and how the semantic facts discussed in Section 1 are accounted for. In Section 4 a possible syntax for wh-complements which suits our semantics is outlined in some detail. Section 5 deals with the coordination of complements, whilst in Section 6 we tie up some loose ends and make a speculative remark on the semantics of direct questions.

## 1. Semantic properties of wh-COMPLEMENTS

In this section a number of semantic properties of wh-complements will be traced by considering the validity of arguments in which sentences containing them occur. The conclusion of our considerations will be that there are good reasons to assume wh-complements to denote the same kind of semantic object as that-complements: propositions. The differences between the two kinds of complements will be explained in terms of differences in sense.

### 1.1. Whether-complements and That-Complements

Consider the following valid argument, of which one of the premisses contains a whether-complement and the conclusion a that-complement.
(I) John knows whether Mary walks

Mary walks
John knows that Mary walks
The validity of this type of argument reflects an important fact of sentences containing whether-complements and, by implication, of whether-complements themselves. As (I) indicates, there is a relation between the semantic object denoted by whether Mary walks and the proposition denoted by that Mary walks. Similarly, the validity of (II) is based on a relation between the semantic object denoted by whether Mary walks and the proposition denoted by that Mary doesn't walk.
(II) John knows whether Mary walks

Mary doesn't walk
John knows that Mary doesn't walk
Together, (I) and (II) indicate that the actual truth value of Mary walks determines whether the relation holds between whether Mary walks and that Mary walks, or between whether Mary walks and that Mary doesn't walk.

The following examples show that the validity of (I) and (II) does not depend on the factivity of the verb know:
(III) John tells whether Mary walks

Mary walks
John tells that Mary walks
(IV) John tells whether Mary walks Mary doesn't walk

John tells that Mary doesn't walk
Since $x$ tells that $\varphi$ does not imply that $\varphi$ is true, the validity of (III) and (IV) cannot be accounted for in terms of factivity, and neither should the validity of (I) and (II) if, as we do, one assumes that it has to be explained in a similar way.
The overall suggestion made by (I)-(IV) is that there is a relationship between sentences in which a whether-complement occurs embedded under verbs as know or tell and similar sentences containing a thatcomplement. The most simple account of this relationship would be to claim that whether $\varphi$ and that (not) $\varphi$ denote the same kind of semantic object. Taking that (not) $\varphi$ to denote a proposition, this amounts to claiming that whether $\varphi$ denotes a proposition too.

### 1.2. Index Dependency

Although on this account both that- and whether-complements denote propositions, they do this in different ways. The contrast between (I) and (III) on the one hand, and (II) and (IV) on the other, shows that which proposition whether $\varphi$ denotes depends on the actual truth value of $\varphi$. This marks an important difference in meaning between that- and whether-complements. The denotation of that-complements is index independent: at every index that $\varphi$ denotes the same proposition. The denotation of a whether-complement may vary from index to index, it is index dependent. At an index at which $\varphi$ is true it denotes the proposition that $\varphi$; at an index at which $\varphi$ is false it denotes the proposition that not $\varphi$. ${ }^{\prime}$ In other words, whereas the propositional concept which is the sense of a that-complement is a constant function from indices to propositions, the propositional concept which is the sense of a whether-complement (in general) is not. So, aithough, at a given index, a whether-complement and a that-complement may have the same denotation, their sense will in general be different.

### 1.3. Extensional and Intensional Complement Embedding Verbs

The difference in sense between that-complements and whether-complements plays an important role in the explanation of the semantic properties of sentences in which they are embedded. Embedding a complement under a verb semantically corresponds to applying the interpretation of the verb to the sense of the complement, i.e. to a
propositional concept. This is the usual procedure for functional application, motivated by the assumption that no context can, a priori, be trusted to be extensional. We speak of an extensional context if a function always operates on the denotation of its arguments, and not on their sense.

As a matter of fact, such verbs as know and tell are extensional in this sense, ${ }^{2}$ and moreover, the validity of the arguments ( I -(IV) is based upon this fact. Verbs such as know and tell operate on the denotations of their complements, i.e on propositions, and not on their sense, i.e. propositional concepts. The extensionality of these verbs will be accounted for by a meaning postulate which reduces intensional relations between individual concepts and propositional concepts to corresponding extensional relations between individuals and propositions.

However, there are also complement embedding verbs which do create truly intensional contexts. In terms of Karttunen's classification, inquisitive verbs (ask, wonder), verbs of conjecture (guess, estimate), opinion verbs (be certain about), verbs of relevance (matter, care) and verbs of dependency (depend on) count as such. The assumption that no extensional relation corresponds to the intensional one denoted by these verbs explains why arguments such as (I)-(IV) do not hold for them. That some of these verbs (e.g. guess, estimate, matter, care) can be combined with that-complements, while others (ask, wonder, depend on) cannot (at least not without a drastic change in meaning, cf. Note 9), is an independent fact that needs to be accounted for as well.

### 1.4. Constituent Complements

Consider the following arguments, of which one of the premisses contains a wh-complement with one or more occurrences of wh-terms such as who, what, which girl.
(V) John knows who walks

Bill walks
John knows that Bill walks
(VI) John knows which man walks

Bill walks
John knows that Bill walks
(VII) John knows which man which girl loves

Suzy loves Peter and Mary loves Bill
John knows that Suzy loves Peter and that Mary loves Bill

Given the usual semantics, these arguments are valid. ${ }^{3}$ Again, this can be explained in a very direct way if we take constituent complements to denote propositions. The validity of (V)-(VII) no more depends on the factivity of know than does the validity of (I) and (II). This will be clear if one substitutes the non-factive tell for know in (V)-(VII). The validity of all these arguments does depend on the extensionality of know and tell. As was the case with whether-complements, which proposition a constituent complement denotes depends on what is in fact the case. For example, which proposition is denoted by who walks depends on the actual denotation of walk. If Bill walks, the proposition denoted by who walks should entail that Bill walks; if Peter walks, it should entail that Peter walks. This index dependent character can more generally be described as follows. At an index i, who walks denotes that proposition $\mathbf{p}$, which holds true at an index $\mathbf{k}$ iff the denotation of walk at $\mathbf{k}$ is the same as its denotation at $i$.

### 1.5. Exhaustiveness

This more general description of the proposition denoted by who walks not only implies, as is supported by argument (V), that for John to know who walks he should know - de re - of everyone who walks that he does, but also implies that of someone who doesn't walk, he should not erroneously believe that she does. That this is right appears from the validity of the following argument:

> (VIII) John believes that Bill and Suzy walk Only Bill walks

## John doesn't know who walks

If only Bill walks and John is to know who walks, he should know that only Bill walks and he should not believe that someone else walks as well. We will call this property of propositions denoted by constituent complements their exhaustiveness.

Another way to make the same point is as follows. For a sentence John knows $\rho$, where $\rho$ is a wh-complement, to be true, it should hold that if one asks John the direct question corresponding to $\rho$, one gets exactly the correct answer. So, if only Bill walks and John knows who walks is to be true, John should answer: 'Bill' when asked the question: 'Who walks?', and not for example: 'Bill and Suzy do'. A similar kind of exhaustiveness is exhibited by whether-complements of the form whether $\varphi$ or $\psi{ }^{4}$ Consider the following argument:
(IX) John knows whether Mary walks or Bill sleeps Mary doesn't walk and Bill sleeps

John knows that Mary doesn't walk and that Bill sleeps
The validity of this argument illustrates that the proposition denoted by an alternative whether-complement is exhaustive too. At an index $\mathbf{i}$, whether $\varphi$ or $\psi$ denotes that proposition $p$ that holds at an index $\mathbf{k}$ iff the truthvalues of both $\varphi$ and $\psi$ at $\mathbf{k}$ are the same as at i .
In fact, one can distinguish different degrees of exhaustiveness of complements. Exhaustiveness to the lowest degree implies that for John to know who walks, he should know of everyone who walks that he/she does (and not merely of someone). This is the interpretation of exhaustiveness Karttunen defends (against Hintikka). Exhaustiveness to a stronger degree is used above. Not only do we require that John knows of everyone who walks that he/she does, but also that of no one who doesn't walk, John erroneously believes that he/she does. Exhaustiveness to at least this degree is required to explain the validity of arguments like (VIII). Since Karttunen only incorporates exhaustiveness to the lowest degree, he is unable to account for the validity of (VIII) and (IX). Whether he does consider these arguments to be valid is unclear to us. His analysis forces him to neglect stronger forms of exhaustiveness for a reason not related to this, which will be discussed in the next section.

We feel that an even stronger notion of exhaustiveness is called for. Suppose that John knows of everyone who walks that he/she does; that of no one who doesn't walk, he believes that he/she does; but that of some individual that actually doesn't walk, he doubts whether he/she walks or not. In such a situation, John would not say of himself that he knows who walks. We see no reason to override his judgement and to claim that in this situation, John does know who walks. This seems to suggest that for John to know who walks, he should not only know of everyone who walks that he/she does, but also of everyone who doesn't that he/she doesn't. This would mean that ( X ) (and its inverse) is a valid argument:

## John knows who walks

John knows who doesn't walk
In view of the plausible arguments for exhaustiveness given above, there seems to be only one type of situation in which knowing who walks may not turn out to be the same as knowing who doesn't, i.e. which gives rise
to counterexamples against (X). This is the type of situation in which the subject of the propositional attitude is not fully informed as to which set of individuals constitutes the domain of discourse. More in particular, only if a certain individual which in fact belongs to the domain of discourse and which in fact does not walk, does not belong to what John considers to be the domain of discourse, the situation can arise that John knows the positive extension of the predicate walk without also knowing its negative extension. Such a situation would be a counterexample against (X). (Of course, similar counterexamples can be constructed against the inverse of (X).)

In our formal analysis, we will not deal with cases like these, and consequently, we will accept the validity of (X), for the following reason. Incorporating into the framework of possible world semantics the type of situation in which individuals are not fully informed about what constitutes the domain of discourse is possible, for example by allowing the domain of discourse to vary with possible worlds, but at a cost. It creates a number of well-known problems, for which no definitive solution is yet available. We refrain from incorporating this aspect because of the problems it raises, and we feel free to do so because it is not inherent to an analysis of wh-complements. ${ }^{5}$

Another observation that somewhat weakens the significance of (X), is the following. That one must know the negative extension of a predicate as well as its positive extension, in order to know who satisfies it, appears less dramatic if one realizes that wh-terms, like all other quantifiers, are usually restricted to some, contextually or otherwise specified, subset of the entire domain of all entities. If someone asks who walks?, then he/she does not, or at least not usually, want a specification of all walkers on this earth, but rather a specification which exhausts the walkers in some restricted domain. Such restrictions are usually left implicit, but are there nonetheless. In fact, a contextual restriction functions as a 'hidden' common noun in the wh-term. In the next section, we will see that arguments similar to ( $X$ ) which contain wh-terms of the form which $\delta$ instead of who, unlike (X) are not always valid. Again, the phenomenon of contextual restriction is not specific for wh-complements, but occurs with every kind of quantification in natural language. We therefore feel free to ignore it in our formal analysis.

### 1.6. A De Dicto/De Re Ambiguity of Constituent Complements

Sentences in which constituent complements containing wh-terms of the form which $\delta$ occur exhibit a certain kind of ambiguity, which resembles
the familiar de dicto/de re ambiguity, and which will henceforth be referred to as such. For example, whether the following argument is valid or not depends on how the conclusion is read.

$$
\begin{equation*}
\frac{\text { John knows who walks }}{\text { John knows which girl walks }} \tag{XI}
\end{equation*}
$$

That (XI) is valid could be argued for as follows. Since the set of girls is a subset of the set of individuals, and since if one knows of a set which of its elements have a certain property, one also knows this of every subset of that set, it cannot fail to hold that John knows which girl walks if he knows who walks. Here the conclusion is taken de re.

On the other hand, one might point out that (XI) is not valid by presenting the following situation. Suppose that just one individual walks. Suppose further that it is a girl. If John knows of this individual that she is the one that walks, but fails to believe that she is a girl, then the premiss of ( $\mathrm{XI} \mathrm{)} \mathrm{is} \mathrm{true} ,\mathrm{but} \mathrm{its} \mathrm{conclusion} \mathrm{is} \mathrm{false}$. reasoning the conclusion is taken de dicto. It takes for granted that the conclusion should be read in such a way that if John is to know which girl walks, he should believe of every individual which is in fact a girl and walks, not only that she walks, but also that she is a girl. Within the first line of reasoning, this assumption is not made. So, whether (XI) is valid or not depends on how the conclusion is read. If we assign it a de re reading ( XI ) is valid, under a de dicto reading it is not. The de re reading of the conclusion of (XI) can be paraphrased as Of each girl, John knows whether she walks.

This de dicto/de re ambiguity also plays a role in an argument like (XII), which is analogous to argument (X) discussed in the previous section.
(XII) John knows which man walks

John knows which man doesn't walk
Even if we assume the domain of discourse to be the same for every possible world, i.e. if we exclude the kind of counterexample discussed with respect to (X), this argument, unlike its counterpart (X), is not valid as such. It is valid iff both the premiss and the conclusion are read de re, its inverse is then valid as well. Under all other possible combinations of readings (XII) is not valid. Consider e.g. the de dicto/de dicto combination. Suppose the premiss is true. This is compatible with there being an individual of which John erroneously believes that it is a man, but rightly believes that it does not walk. However, in such a situation, if
the conclusion is read de dicto, it is false. Similar examples can be constructed to show that (XII) is also invalid on the other two combinations of readings. This shows, by the way, that the de dicto and de re readings involved are logically independent.

Once we take into account the type of situation, described in the previous section, in which individuals are not fully informed as to which set of individuals constitutes the domain of discourse, arguments like (XII) are no longer valid, even if premiss and conclusion are read de re. For then, the same kind of counterexample as we outlined against (X) can be constructed. The same holds if we incorporate contextual restrictions on quantification in our semantic framework. Then again, arguments like (X), and (XII) read de re are no longer valid in view of the possibility that the subject of the propositional attitude may be mistaken as to which subset of the domain of discourse is determined by the contextual restriction. As we said above, such a contextual restriction functions as a 'hidden' common noun in the wh-term, thus allowing for de dicto readings with respect to it. The type of situation in which individuals are not fully informed about what constitutes the domain of discourse can be viewed in this way too (e.g. as misinformation about the denotation of the predicate entity). So, there are striking similarities between the three cases, which is also evident from the fact that the counterexamples that can be constructed in each case, are structurally the same. However, only the de dictolde re ambiguity of constituent complements is particular to an analysis of wh-complements, the other phenomena being of a more general nature.

The possibility of distinguishing de dicto and de re readings of constituent complements marks an important difference between Karttunen's analysis and ours. Karttunen can account only for de re readings. As a result, arguments like (XI) come out valid in his analysis. Nevertheless, (XII) is not a valid argument in Karttunen's theory. This is caused by the fact that he incorporates exhaustiveness only in its weakest form. He explicitly rejects stronger forms of exhaustiveness because, combined with the fact that his analysis accounts only for de re readings, this would make arguments like (X) and (XII) valid. ${ }^{6}$ Rejecting strong exhaustiveness, Karttunen is able to regard (XII) as invalid but for the wrong reason, as can be seen from the fact that (XI) still is valid in his analysis. Worse, he thereby deprives himself of the means to account for the validity of arguments like (VIII) and (IX). We believe that an analysis which can both account for exhaustiveness and for the fact that the validity or invalidity of (XI) and (XII) depends on how the conclusion is read, is to be preferred.

### 1.7. Implicatures Versus Presuppositions

From the previous discussion, in particular from Sections 1.4. and 1.5., it will be clear that we consider the following arguments to be valid ones:
(XIII) John knows who walks
Nobody walks

John knows that nobody walks
(XIV) John knows who walks

Peter and Mary walk
John knows that Peter and Mary walk
(XV) John knows whether Peter walks or Mary walks Neither Peter nor Mary walks

John knows that neither Peter nor Mary walks
(XVI) John knows whether Peter walks or Mary walks Both Peter and Mary walk

John knows that both Peter and Mary walk
One might object to the validity of these arguments by pointing out that John knows who walks presupposes that at least/exactly one individual walks, and that John knows whether Peter walks or Mary walks presupposes that at least/exactly one of the alternatives is the case. Therefore, one might continue, the first premiss of these arguments is semantically deviant in some sense, say lacks a truth value, if the second premiss happens to be true.

We adhere to the view, also advocated by Kartunen, that it is better to regard these phenomena as (pragmatic) implicatures and not as presuppositions in the strict semantic sense. More generally, we believe that many of the arguments put forward in Kempson (1975), Wilson (1975) and Gazdar (1979) showing that presupposition is a pragmatic notion hold for presuppositions of wh-complements as well. (See also the discussion in Section 5.)
In Karttunen's analysis, (XIII)-(XVI) are valid as well. The validity of (XIII) and (XV), however, has to be secured by a special clause in a meaning postulate relating know + wh to know that. The need for this special clause explains itself by the fact that the validity of (XIII) and (XV) is at odds with not incorporating exhaustiveness. One would expect that in an analysis in which (VIII) and (IX) of Section 1.5 are not valid, (XIII) and (XV) would not be valid either.

### 1.8. Towards a Uniform Treatment of Complements

A distinctive feature of our analysis is that wh-complements are taken to be proposition denoting expressions. This is an important difference between our approach and that of others. To mention only two, in Karttunen's they denote sets of propositions, and in Hausser's they are of all sorts of different categories. From this difference other differences follow, e.g. the possibility of a uniform treatment of complements. For, besides the fact that it provides a simple and direct account of the validity of the various arguments discussed above, the hypothesis that that- and wh-complements denote the same kind of semantic object makes it possible to assign them to the same syntactic category. ${ }^{7}$ This seems especially attractive in view of the fact that it is possible to conjoin wh- and that-complements:
(1) John knows that Peter has left for Paris, and also whether Mary has followed him
(2) Alex told Susan that someone was waiting for her, but not who it was

Further, if both kinds of complements can belong to the same syntactic category, we are no longer forced to assume there to be two complement taking verbs know, of different syntactic categories, and of different semantic types: one which takes that- and one which takes wh-complements. We need not acknowledge two different relations of knowing which are only linked indirectly, i.e. by a meaning postulate. ${ }^{8}$ This happens for example in Karttunen's analysis. There wh-complements denote sets of propositions, and that-complements denote propositions. Consequently, there are two relations of knowing. Karttunen reduces the relation to sets of propositions to the relation to propositions by postulating that $x$ stands in the first relation to a set of propositions iff $x$ stands in the second relation to all the elements of this set. (Actually, his postulate is slightly more complex, but that is irrelevant here.) Not only is this a rather cumbersome way of accounting for our intuition that there is one verb know, it is also not at all clear whether a strategy like this is applicable in all cases. A case in point are truly intensional verbs which take both wh-complements and that-complements, such as guess and matter. If we categorize $w h$-complements and that-complements differently, the problem arises how to account for the obvious semantic relation (identity) between the two verbs guess (or matter, etc.) we are then forced to assume. In these cases one cannot reduce the one to the other, for obvious reasons. For example, John guesses who comes to dinner does not mean the same as for all $x$, if $x$ comes to dinner, then

John guesses that $x$ comes to dinner. ${ }^{9}$ In what other way the interpretation of the two verbs could be related adequately, is quite unclear. In the analysis proposed in this paper, there is no problem at all. Since wh-complements and that-complements are of the same syntactic category, no verbs need to be duplicated in the syntax. The extensionality of verbs such as know and tell can be accounted for by means of a meaning postulate. As for truly intensional verbs such as guess and matter, they express the same relation to a propositional concept, be they combined with a wh-complement or with a that-complement. The semantic differences between the two constructions are accounted for by the different properties of the propositional concepts expressed by wh-complements and that-complements respectively.

Of course, there are also verbs such as wonder, which take only wh-complements, and verbs such as believe, which take only thatcomplements. The relevant facts can easily be accounted for by means of syntactic subcategorization or, preferably, in lexical semantics, by means of meaning postulates.

## 2. Ty2 and the semantic analysis of wh-complements

In Section 1 we have sketched informally the outlines of a semantics for wh-complements. In particular, we argued that wh-complements denote propositions and do this in an index dependent way. The description of this index dependent character involves comparison of what is the case at different indices. This leads to the choice of a logical language in which reference can be made to indices and in which relations between indices can be expressed directly. The language of two-sorted type theory, Gallin's Ty2, is such a language. In this section we will show that it serves our purpose to express the semantics of $w h$-complements quite well.

Ty2 is a simple language. Rather than by stating the explicit definitions, we will discuss its syntax and semantics by comparing it with IL, the language of intensional logic of PTQ, thereby indicating how Ty2 can be put to the same use as IL in the PTQ system. We will also make some methodological remarks on the use of Ty2. For a formal exposition and extensive discussion of Ty2, the reader is referred to Gallin (1975).

### 2.1. Ty2, the Language of Two-Sorted Type Theory

The basic difference between IL and Ty2 is that $s$ is not introduced only in constructing more complex, intensional types, but that it is a basic
type, just like $e$ and $t$. Complex types can be constructed with $s$ in exactly the same way as with $e$ and $t$. As is to be expected, the set of possible denotations of type $s$ is the set of indices. Since it is a type like any other now, we will also employ constants and variables of type $s$. This means that it is possible to quantify and abstract over indices, making the necessity operator $\square$ and the cap operator ${ }^{\wedge}$ superfluous.

A model for Ty 2 is a triple $\langle A, I, F\rangle, A$ and $I$ are disjoint non-empty sets, $A$ is to be the set of individuals, $I$ the set of indices. $F$ is an interpretation function which assigns to every constant a member of the set of possible denotations of its type. Notice the difference with the interpretation function $F$ of IL-models, which assigns senses and not denotations to constants. The interpretation of a meaningful expression $\alpha$ of Ty 2 , written as $\llbracket \alpha \rrbracket_{M, g}$, is determined with respect to a model $M$ and an assignment $g$ only. (As usual, $g$ assigns to every variable a member of the set of possible denotations of its type.)

The important difference with interpretations in IL is that the latter also need an index to determine the interpretation of an expression. This role of indices as a parameter in the interpretation is taken over in Ty2 by the assignment functions. The effect of interpreting in IL. an expression with respect to an index $\mathbf{i}$ is obtained in Ty2 by interpreting expressions with respect to an assignment which assigns to a free index variable occurring in the expression the index $i$. To an index dependent expression of IL (an expression of which the denotation varies from index to index) there corresponds an expression in Ty2 which contains a free index variable. The result is an expression the interpretation of which varies from assignment to assignment. A formula $\phi$ is true with respect to $M$ and $g$ iff $\llbracket \phi \rrbracket_{M, g}=1 ; \phi$ is valid in $M$ iff for all $g, \phi$ is true with respect to $M$ and $g ; \phi$ is valid iff for all $M, \phi$ is valid in $M$.

### 2.2. Translating into Ty2

To illustrate the difference between IL and Ty2, consider first how the English verb walk translates into Ty2. Instead of simply translating it into a constant of type $f(I V)$, it is translated into the expression walk' $\left(v_{0, s}\right)$, in which walk' is a constant of type $\langle s, f(I V)\rangle$, and $v_{0, s}$ is a variable of type $s$, so the full translation of the verb is an expression of type $f(I V)$.

All translations of basic expressions will contain the same free index variable. For this purpose we use $v_{0, s}$, the first variable of type $s$, which from now on we will write as $a$. Therefore, the translation of a complex expression will be interpreted with respect to the index assigned to $a$ by the assignment function.

The rules for translating PTQ English into Ty2 can be obtained by using the fact that $\lambda a \alpha$ expresses the same function in Ty2 as ${ }^{\wedge} \alpha$ in IL, ${ }^{\wedge} \alpha$ is the same as $\alpha(a)$; and $\square$ corresponds to $\wedge a$. Consider the following examples of Ty2 analogues of (parts of) some PTQ translation rules, in which $\leadsto$ abbreviates 'translates into'.
(T:1) (a) If $\alpha$ is in the domain of $g$, then $\alpha \leadsto g(\alpha)(a)$.
With the usual exceptions, $g$ associates a basic expression $\alpha$ of category A with a Ty 2 constant $\alpha^{\prime}$ of type $\langle s, f(A)\rangle$, giving its sense. The full translation of $\alpha, \alpha^{\prime}(a)$, gives as usual its denotation.
(b) $\quad b e \leadsto \lambda \mathscr{P} \lambda x \mathscr{P}(a)(\lambda a \lambda y[x(a)=y(a)])$
(c) necessarily $\leadsto \lambda p \wedge a(p(a))$
(d) John $\leadsto \lambda P[P(a)(\lambda a j)]$
(e) $\quad h e_{n} \leadsto \lambda P\left[P(a)\left(x_{n}\right)\right]$
(T:2) If $\delta \in P_{C N}$, and $\delta \leadsto \delta^{\prime}$, then every $\delta \leadsto \lambda P \wedge x\left[\delta^{\prime}(x) \rightarrow P(a)(x)\right]$
(T:4) If $\alpha \in P_{T}, \delta \in P_{I v}, \alpha \leadsto \alpha^{\prime}$, and $\delta \leadsto \delta^{\prime}$, then $F_{4}(\alpha, \delta) \leadsto \alpha^{\prime}\left(\lambda a \delta^{\prime}\right)$.
Of course, the meaning postulates of PTQ can be translated into Ty2 as well. (Notice that the rigid designator view of proper names like John is already implemented in its translation.) The translation of a sentence is illustrated in (3):
(3)


### 2.3. That-Complements and Whether-Complements in Ty2

The proposition denoting expression which is to be the translation of a that-complement that $\varphi$ can be constructed from the translation of $\varphi$ by using abstraction over indices. For example, the sentence Mary walks
translates into the formula walk' $(a)(m)$; from this formula we can form the expression $\lambda a\left[\right.$ walk $\left._{*}(a)(m)\right]$. Its interpretation $\llbracket \lambda a\left[\right.$ walk $\left._{*}^{\prime}(a)(m)\right] \rrbracket_{M, g}$ is that proposition $\mathbf{p} \in\{0,1\}^{1}$ such that for every index $\mathbf{i}$ : $\mathbf{p}(\mathbf{i})=1$ iff $\llbracket \mathrm{walk}_{*}^{\prime}(a)(m) \rrbracket_{M, g[i / a]}=1$. So, $\lambda a\left[\right.$ walk $\left._{*}^{\prime}(a)(m)\right]$ denotes the characteristic function of the subset of the set of indices at which it is true that Mary walks.

Notice that $\lambda a\left[\right.$ walk $\left._{*}^{\prime}(a)(m)\right]$ does not contain a free index variable. This makes it the index independent expression it was argued to be in 1.1. and 1.2. Its sense, denoted by the expression $\lambda a \lambda a\left[\right.$ walk $\left._{*}^{\prime}(a)(m)\right]$, is a constant function from indices to propositions.

In Section 1.1. we circumscribed the denotation of whether Mary walks as follows: at an index at which it is true that Mary walks it denotes the proposition that Mary walks, and at an index at which it is false that Mary walks it denotes the proposition that Mary doesn't walk. Another way of saying this is that at an index i whether Mary walks denotes that proposition $\mathbf{p}$ such that for every index $\mathbf{k}, \mathbf{p}$ holds true at $\mathbf{k}$ iff the truth value of Mary walks at $\mathbf{k}$ is the same as at $\mathbf{i}$. In Ty2 this can be expressed by the index dependent proposition denoting expression (4), the interpretation of which is given in (4). By $g[x / y]$ we will understand that assignment $g^{\prime}$ which is like $g$ except for the possible difference that $g(y)=x$.

$$
\begin{align*}
& \lambda i\left[\text { walk }_{*}^{\prime}(a)(m)=\operatorname{walk}_{*}^{\prime}(i)(m)\right]  \tag{4}\\
& \llbracket \lambda i\left[\operatorname{walk}_{*}^{\prime}(a)(m)=\operatorname{walk}_{*}^{\prime}(i)(m)\right] \rrbracket_{M, \mathrm{~g}} \text { is that proposition } \\
& \mathbf{p} \in\{0,1\}^{1} \text { such that for every index } \mathbf{k} \in I: \mathbf{p}(\mathbf{k})=1 \text { iff } \\
& \text { [walk}{ }_{*}^{\prime}(a)(m)=\text { walk }_{*}^{\prime}(i)(m) \rrbracket_{M_{, g[k i] ~}}=1 \text { iff } \\
& \llbracket \text { walk }_{*}^{\prime}(a)(m) \rrbracket_{M, g \mid k i]}=\llbracket \mathbf{w a l k}_{*}^{\prime}(i)(m) \rrbracket_{M_{, ~ g l}[i]} \text { iff } \\
& \llbracket \text { walk }_{*}^{\prime}(a)(m) \rrbracket_{M, 8}=\llbracket \text { walk }_{*}^{\prime}(i)(m) \rrbracket_{M_{, ~ g / k / i] ~}}
\end{align*}
$$

So, at the index $g(a)$, the expression (4) denotes the characteristic function of the set of indices at which the truth value of Mary walks is the same as at the index $g(a)$. The index dependent character of whether-complements discussed in 1.1. and 1.2. is reflected by the fact that a free index variable occurs in their translation. The expression $\lambda a \lambda i\left[\operatorname{walk}_{*}^{\prime}(a)(m)=\operatorname{walk}_{*}^{\prime}(i)(m)\right]$, denoting the propositional concept which is the sense of whether Mary walks, does not denote a constant function. For different indices its value may be a different proposition.

### 2.4. Constituent Complements in Ty2

The kind of expressions which denote propositions in the required index dependent way can be constructed not only from formulas, such as
walk ${ }_{*}^{\prime}(a)(m)$ in (4), but from expressions of arbitrary type. Let $\alpha / a /$ and $\alpha / i /$ be two expressions such that where the first has free occurrences of $a$, the second has free occurrences of $i$, and vice versa. Then the expression (5) denotes a proposition in an index dependent way, as its interpretation given in ( $5^{\prime}$ ) shows. ${ }^{10}$

$$
\begin{align*}
& \lambda i[\alpha / a /=\alpha / i /]  \tag{5}\\
& \llbracket \lambda i[\alpha / a /=\alpha / i /] \mathbb{\mathbb { M }}_{M, g} \text { is that proposition } \mathbf{p} \in\{0,1\}^{I} \text { such that for } \\
& \text { every index } \mathbf{k} \in I, \mathbf{p}(\mathbf{k})=1 \text { iff } \llbracket \alpha / a / \mathbb{\rrbracket}_{M, g}=\llbracket \alpha / i / \mathbb{\rrbracket}_{M, g[\mathbf{k} / i]} \text {. }
\end{align*}
$$

Expressions serving as translations of wh-complements will always be of this form. The translation of a whether-complement has been given in (4). There $\alpha / a /$ is the formula walk $_{*}^{\prime}(a)(m)$. An example of an expression which will serve as the translation of a constituent complement is:

$$
\begin{equation*}
\lambda i\left[\lambda u\left[\operatorname{walk}_{*}^{\prime}(a)(u)\right]=\lambda u\left[\text { walk }_{*}^{\prime}(i)(u)\right]\right] . \tag{6}
\end{equation*}
$$

In this case, $\alpha / a /$ is $\lambda u\left[\right.$ walk $\left._{*}^{\prime}(a)(u)\right]$, an expression of type $(e, t\rangle$. At an index $g(a)$, (6) denotes that proposition which holds at an index $k$ iff $\llbracket \lambda u\left[\right.$ walk $\left._{*}^{\prime}(a)(u)\right] \rrbracket_{M, g}$ is the same set as $\llbracket \lambda u\left[\right.$ walk $_{*}^{\prime}(i)(u) \rrbracket \rrbracket_{M, g[k / i]}$. I.e. at an index $g(a)$, (6) denotes that proposition which holds true at an index $k$ iff the denotation of walk ${ }_{*}^{\prime}$ at that index $k$ is the same as at the index $g(a)$. And this is precisely the index dependent proposition which, in Section 1.4., we required to be the denotation of the constituent complement who walks.

### 2.5. Methodological Remarks on the Use of Ty2

In this section we will defend our use of Ty2 against some objections that are likely to be raised against it.

A first objection might be that translations in Ty2 are (even) less 'natural' than those in IL. In view of the fact that within a compositional semantic theory the level of translation, be it in Ty2 or in IL, is in principle dispensable, we do not see that there is empirical motivation for this kind of objection.

A second objection that is often raised against the use of a logical language which allows for reference to and quantification over indices, is that it involves stronger ontological commitments than a language in which the relevant phenomena are dealt with by means of intensional operators. We do not think that this objection holds. It is not the object language in isolation, but the object language together with the metalanguage in which its semantics is described that determines ontological commitments. Since the statement of the semantics of intensional operators involves reference to and quantification over indices as well,
the commitments are the same. The dispensability of the translation level even strengthens this point.
A more serious reason for preferring an operator approach to a quantificational approach might be that for some purposes one does not need the full expressive power of a quantificational language and therefore prefers a language with operators which has exactly the, restricted, expressive power one needs. In fact, in Section 6.2. we will point out that by the introduction of a new intensional operator to IL, one can get a long way in the semantic analysis of wh-complements. However, phenomena remain which escape treatment in this intensional language, an example is discussed in 6.1.

Taking the semantic analysis of tense into consideration as well, we think a lot can be said in favour of a logical language in which reference to and quantification over indices is possible. It appears that analyses set up in the Priorean fashion tend to become stronger and stronger, up to a point where if there is still a difference in expressive power with quantificational logic at all, this advantage is annihilated by the unintuitiveness and complexity of the language used. For an illuminating discussion of these points, see van Benthem (1978). In fact, we think that Ty2 provides a suitable framework for the incorporation of a semantic analysis of tense in the vein of Needham (1975) into a Montague Grammar as well.

## 3. Wh-complements in a Montague Grammar

In this section we will outline how the semantic representations of complements in Ty2, given in Section 2, can systematically be incorporated in the framework of a Montague Grammar. We will not present the syntactic part of our proposal in detail. In particular, the definitions of the various syntactic functions occurring in the syntactic rules will not be stated until Section 4. We will concentrate on the explanation of the semantic facts discussed in Section 1.

### 3.1. Whether-Complements and That-Complements

Complements are expressions which denote propositions. Therefore, they should translate into expressions of type $\langle s, t\rangle$. In PTQ there is no syntactic category which is mapped onto this type, ${ }^{11}$ therefore we add the following clauses to the definitions of the set of categories and the function $f$ mapping categories into types;

$$
\text { If } A \in C A T \text {, then } \bar{A} \in C A T ; f(\bar{A})=\langle s, f(A)\rangle
$$

So, $\bar{t}$ will be the category of complements. Complement embedding verbs, such as know, tell, wonder and believe will be of category $I V / \bar{t}$. As we remarked in Section 1.8, the categories $\bar{t}$ and $I V / \bar{t}$ will have to be subcategorized, since not all of these verbs take all kinds of complements. This can be done in an obvious way, with which we will not be concerned here.

In (7) an analysis tree of a sentence containing a that-complement is given together with its translation. Here and elsewhere, notation conventions and meaning postulates familiar from PTQ are applied whenever possible.

| John knows that Mary walks, $t$ $\operatorname{know}^{\prime}(a)\left(\lambda a j, \lambda a \lambda a\left[\right.\right.$ walk $\left.\left._{*}^{\prime}(a)(m)\right]\right)$ |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
| know, $I V / \bar{t}$  <br> know $^{\prime}(a)$ $\lambda a\left[\right.$ twalk $\left._{*}^{\prime}(a)(m)\right]$ |  |  |  |
| Mary walks, $t$ walk $_{*}^{\prime}(a)(m)$ |  |  |  |

The syntactic rule deriving a that-complement and the corresponding translation rule are:

$$
\begin{aligned}
& \text { (S:THC) If } \varphi \in P_{t} \text {, then that } \varphi \in P_{\bar{i}} \\
& (T: T H C) \quad \text { If } \varphi \leadsto \varphi^{\prime} \text {, then that } \varphi \leadsto \lambda a \varphi^{\prime} .
\end{aligned}
$$

The rule which embeds the complement under a verb is a simple rule of functional application. The corresponding rule of translation follows the usual pattern:

$$
\begin{array}{ll}
(S: I V / \bar{t}) & \text { If } \delta \in P_{I V / \bar{t}} \text { and } \rho \in P_{\bar{t}} \text {, then } F_{I V I t}(\delta, \rho) \in P_{I V} . \\
(T: I V / \bar{t}) & \text { If } \delta \leadsto \delta^{\prime} \text { and } \rho \leadsto \rho^{\prime} \text {, then } F_{I V I t}(\delta, \rho) \leadsto \delta^{\prime}\left(\lambda a \rho^{\prime}\right) .
\end{array}
$$

Sentence (7) expresses that an intensional relation of knowing exists between the individual concept denoted by $\lambda a j$ and the propositional concept denoted by $\lambda a \lambda a\left[\right.$ walk $\left._{*}^{\prime}(a)(m)\right]$. By means of a meaning postulate, to be given below, this intensional relation will be reduced to an extensional one.

In (8) an analysis tree and its translation of a sentence containing a whether-complement are given:


The rule which forms a whether-complement from a sentence, and the corresponding translation rule are as follows. (An asterisk indicates that a rule will later be revised.)
(S:WHC*) If $\varphi \in P_{i}$, then whether $\varphi \in P_{i}$.
( $T: W^{*} C^{*}$ ) If $\varphi \leadsto \varphi^{\prime}$, then whether $\varphi \leadsto \lambda i\left[\varphi^{\prime}=\left[\lambda a \varphi^{\prime}\right](i)\right]$.
Whether-complements can be generated by a more general rule ${ }^{12}$ :
(S:WHC) If $\varphi_{1}, \ldots, \varphi_{n} \in P_{t}$, then whether $\varphi_{1}$ or $\ldots$ or $\varphi_{n} \in P_{\bar{t}}$.
(T:WHC) If $\varphi_{1} \leadsto \varphi_{1}^{\prime}, \ldots, \varphi_{n} \leadsto \varphi_{n}^{\prime}$, then whether $\varphi_{1}$ or $\ldots$ or $\varphi_{n} \leadsto$ $\lambda i\left[\varphi_{1}^{\prime}=\left[\lambda a \varphi^{\prime}\right](i) \wedge \ldots \wedge \varphi_{n}^{\prime}=\left[\lambda a \varphi_{n}^{\prime}\right](i)\right]$.

Obviously, (S:WHC*) and (T:WHC*) are special cases of (S:WHC) and ( $T: W H C$ ).

In general, whether-complements of the form whether $\varphi_{1}$ or $\ldots$ or $\varphi_{n}$ are ambiguous between an alternative and a yes/no reading. The following two trees and their translations illustrate this ambiguity.
(9) whether John walks or Mary walks, $\bar{t}$

whether John walks or Mary walks, $\bar{t}$
$\lambda i\left[\left(\right.\right.$ walk $_{*}(a)(\mathrm{j}) \vee$ walk $\left._{*}^{\prime}(a)(m)\right)=\left(\operatorname{walk}_{*}^{\prime}(i)(j) \vee\right.$ walk $\left.\left._{*}^{\prime}(i)(m)\right)\right]$
John walks or Mary walks, $t$
walk $_{*}^{\prime}(a)(j) \vee$ walk $_{*}^{\prime}(a)(m)$

### 3.2. Extensional and Intensional Complement Embedding Verbs

In Section 1.3. we stated that verbs such as know and tell are extensional. The meaning postulate guaranteeing this reads as follows:
$(M P: I V / \bar{t}) \quad \vee M \wedge x \wedge r \wedge i[\delta(i)(x, r)=M(i)(x(i), r(i))]$
$M$ is a variable of type $\langle s,\langle\langle s, t\rangle,\langle e, t\rangle\rangle\rangle ; x$ of type $\langle s, e\rangle ; r$ of type $\langle s,\langle s, t\rangle\rangle ; i$ of type $s$; and $\delta$ is the translation of know, tell, etc.

Requiring this formula to hold in all models guarantees that to certain intensional relations between individual concepts and propositional concepts, extensional relations between individuals and propositions correspond. We extend the substar notation convention of PTQ as follows:
$(\mathrm{SNC}) \quad \delta_{*}=\lambda a \lambda p \lambda u[\delta(a)(\lambda a p)(\lambda a u)]$

$$
p \text { is a variable of type }\langle s, t\rangle, u \text { of type } e .
$$

Combining (MP:IV/ $\bar{t}$ ) with (SNC) we can prove that (11) is valid: ${ }^{13}$

$$
\begin{equation*}
\wedge i\left[\delta(i)(x, r)=\delta_{*}(i)(x(i), r(i))\right] . \tag{11}
\end{equation*}
$$

If we apply (11) to the translations of (7) John knows that Mary walks and (8) John knows whether Mary walks, we get the following results:

$$
\begin{align*}
& \operatorname{know}_{*}^{\prime}\left(j, \lambda a\left[\operatorname{walk}_{*}^{\prime}(a)(m)\right]\right)  \tag{7'}\\
& \operatorname{know}_{*}^{\prime}\left(j, \lambda i\left[\operatorname{walk}_{*}^{\prime}(a)(m)=\text { walk }_{*}^{\prime}(i)(m)\right]\right) .
\end{align*}
$$

Formula ( $7^{\prime}$ ) expresses that the individual John knows the proposition that Mary walks. In ( $8^{\prime}$ ) it is expressed that John knows the proposition denoted by $\lambda i\left[\right.$ walk $_{*}^{\prime}(a)(m)=$ walk $\left._{*}^{\prime}(i)(m)\right]$. As has been indicated in Section 2.2., which proposition is denoted by this expression at $g(a)$ depends on the truth value of walk $_{*}^{\prime}(a)(m)$ at $g(a)$. More generally, we can prove that the following holds: ${ }^{14}$

$$
\llbracket \lambda i[\varphi / a /=\varphi / i /] \rrbracket_{M, B}=\left\{\begin{array}{l}
\llbracket \lambda i[\varphi / i /] \rrbracket_{M, g} \text { if } \llbracket \varphi / a / \rrbracket_{M, g}=1  \tag{12}\\
\llbracket \lambda i[7 \varphi / i /] \rrbracket_{M, g} \text { if } \llbracket \varphi / a / \rrbracket_{M, g}=0 .
\end{array}\right.
$$

Given (12), it is obvious that the arguments (I) and (II) of Section 1.1. are valid. Their translations are:

$$
\begin{align*}
& \left.\operatorname{know}_{*}^{\prime}(a)\left(j, \lambda i\left[\text { walk }_{*}^{\prime}(a)(m)=w^{\prime}\right\lrcorner \mathbf{l k}_{*}^{\prime}(i)(m)\right]\right) \\
& \frac{\text { walk }_{*}^{\prime}(a)(m)}{\operatorname{know}_{*}^{\prime}(a)\left(j, \lambda a\left[\text { walk }_{*}^{\prime}(a)(m)\right]\right)}
\end{align*}
$$

( $\mathrm{II}^{\prime}$ )

$$
\begin{aligned}
& \operatorname{know}_{*}^{\prime}(a)\left(\mathrm{j}, \lambda i\left[\text { walk }_{*}^{\prime}(a)(m)=\text { walk }_{*}^{\prime}(i)(m)\right]\right) \\
& \frac{7 \text { walk }_{*}^{\prime}(a)(m)}{\left.\operatorname{know}_{*}^{\prime}(a)\left(\mathrm{j}, \lambda a[ \urcorner \text { walk }_{*}^{\prime}(a)(m)\right]\right)}
\end{aligned}
$$

Since (MP:IV/ $\bar{t}$ ) also holds for tell, the arguments (III) and (IV) are rendered valid in exactly the same way. And precisely because (MP: IV/ $\bar{t}$ ) does not hold for intensional verbs, arguments like (I)-(IV) cannot be constructed for them. The relations expressed by these verbs are not extensional in object position, their second argument is irreducibly a propositional concept.

Argument (IX), concerning the exhaustiveness of alternative whethercomplements, is discussed in Section 3.4. The arguments (XV) and (XVI) of Section 1.7. are left to the reader.

### 3.3. Single Constituent Complements with Who

First we consider constituent complements which contain just one occurrence of the wh-term who. An example of an analysis tree of a sentence containing such a complement, together with its translation is:
(13) John knows who walks, $t$


Constituent complements are formed from sentences containing a syntactic variable, but in an indirect way. First a so-called abstract is formed, an expression of category $t / / / e$. The wh-term $w h o(m)$ is placed at the front of the sentence, certain occurrences of the variable are deleted, others are replaced by suitable pro-forms. For details see

Section 4. In fact, our use of the phrase 'wh-term' is rather misleading. Unlike the wh-terms in Kartunen's analysis for example, they do not belong to a fixed syntactic category. In this they are like their logical language counterpart, the $\lambda$-abstraction sign. Why this is necessary is explained in Section 3.8. This rule of abstract formation and its translation are:

```
( \(\mathrm{S}: \mathrm{AB} 1\) ) If \(\varphi \in P_{t}\), then \(F_{A B \mid, n}(\varphi) \in P_{t / / / / \mathrm{e}}\).
\((T: A B 1)\) If \(\varphi \leadsto \varphi^{\prime}\), then \(F_{A B 1 . n}(\varphi) \rightsquigarrow \lambda x_{n}\left(\varphi^{\prime}\right)\).
```

The translation of an abstract is a predicate denoting expression. From these abstracts constituent complements are formed. The syntactic rule that does this is a category changing rule. The corresponding translation rule turns predicate denoting expressions into proposition denoting expressions in the way indicated in (5) in Section 2.4.

```
(S:CCF*) If \(\chi \in P_{t / / f e}\), then \(F_{C C F}(\chi) \in P_{i}\).
( \(T: C C F^{*}\) ) If \(\chi^{\leadsto \rightarrow \chi^{\prime}}\), then \(F_{C C F}(\chi) \leadsto \lambda i\left[\chi^{\prime}=\left[\lambda a \chi^{\prime}\right](i)\right]\).
```

The intermediate level of abstracts is not strictly needed for single constituent complements, but, as shall be argued in Section 3.8., it is essential for a correct analysis of constituent complements that contain more than one occurrence of a wh-term. (Moreover, an attractive feature of our analysis is that another kind of wh-construction, relative clauses, can both syntactically and semantically be treated as abstracts as well, see Section 4.5.)

We are now able to show that argument (V) of Section 1.4. is valid. Its translation is:

$$
\begin{align*}
& \operatorname{know}_{*}^{\prime}(a)\left(j, \lambda i\left[\lambda u\left[\operatorname{walk}_{*}^{\prime}(a)(u)\right]=\lambda u\left[\operatorname{walk}_{*}^{\prime}(i)(u)\right]\right]\right) \\
& \operatorname{walk}_{*}^{\prime}(a)(b) \\
& \operatorname{know}_{*}^{\prime}(a)\left(j, \lambda a\left[\operatorname{walk}_{*}^{\prime}(a)(b)\right]\right)
\end{align*}
$$

From $\llbracket \operatorname{walk}_{*}^{\prime}(a)(b) \rrbracket_{M, 8}=1$, it follows that $\llbracket \lambda u\left[\right.$ walk $_{*}^{\prime}(a)(u) \rrbracket \rrbracket_{M_{, 8}}\left(\llbracket b \rrbracket_{M, 8}\right)=$ 1. So, at every index $k$ such that $\llbracket \lambda i\left[\lambda u\right.$ walk $\left._{*}^{\prime}(a)(u)\right]=$ $\left.\lambda u\left[\operatorname{walk}_{*}^{\prime}(i)(u)\right]\right]_{\mathbb{M}_{M, g}}(\mathbf{k})=1$, it also holds that $\llbracket \lambda u\left[\operatorname{walk}_{*}^{\prime}(i)(u)\right] \rrbracket_{M, 8[k i]}$ $\left(\llbracket b \rrbracket_{M, g[\mid i l i}\right)=1$. I.e. at every such index $\mathbf{k}:\left[\lambda a\left[\operatorname{walk}_{*}^{\prime}(a)(b)\right] \rrbracket_{M, s}(\mathbf{k})=1\right.$. Under the not unproblematic, but at the same time quite usual assumption that to know a proposition is to know its entailments, this means that ( $\mathrm{V}^{\prime}$ ) is valid. The assumption in question can be laid down in a meaning postulate in a straightforward way.

### 3.4. Exhaustiveness

It is easy to see that argument (VIII) of Section 1.5, illustrating the exhaustiveness of the proposition denoted by a constituent complement is valid too. Its translation is:
$\left(\right.$ VIII' $\left.^{\prime}\right) \quad \operatorname{believe}_{*}^{\prime}(a)\left(\mathbf{j}, \lambda a\left[\right.\right.$ walk $_{*}^{\prime}(a)(b) \wedge$ walk $\left.\left._{*}^{\prime}(a)(s)\right]\right)$

$$
\wedge u\left[b=u \leftrightarrow \operatorname{walk}_{*}^{\prime}(a)(u)\right]
$$

$$
\neg \operatorname{know}_{*}^{\prime}(a)\left(j, \lambda i\left[\lambda u\left[\operatorname{walk}_{*}^{\prime}(a)(u)\right]=\lambda u\left[\operatorname{walk}_{*}^{\prime}(i)(u)\right]\right]\right) .
$$

Suppose the conclusion is false and the second premiss is true. Then $\llbracket \lambda u$ walk $_{*}^{\prime}(a)(u) \rrbracket_{M, g}$ is (the characteristic function of) the unit set consisting of $\llbracket b \rrbracket_{M, g}$. From this it follows that $\llbracket \operatorname{know}_{*}^{\prime}(a)(j, \lambda a[\wedge u[b=$ $\left.\left.\left.u \leftrightarrow \operatorname{walk}_{*}^{\prime}(a)(u)\right]\right]\right)_{M, 8}=1$. Under the assumption that knowing implies believing, also to be laid down in a meaning postulate, it follows that the first premiss is false. So, (VIII') is valid. We leave it to the reader to verify that the similar arguments (XIII) and (XIV) of Section 1.7. are valid too.

Argument (IX), showing the exhaustiveness of whether-complements, translates as follows:

$$
\begin{align*}
& \operatorname{know}_{*}^{\prime}(a)\left(j, \lambda i\left[\left(\text { walk }_{*}^{\prime}(a)(m)=\text { walk }_{*}^{\prime}(i)(m)\right) \wedge\right.\right.\left(\operatorname{sleep}_{*}^{\prime}(a)(b)\right.  \tag{IX'}\\
&\left.\left.\left.=\operatorname{sleep}_{*}^{\prime}(i)(b)\right)\right]\right) \\
& \frac{\urcorner \text { walk }_{*}^{\prime}(a)(m) \wedge \operatorname{sleep}_{*}^{\prime}(a)(b)}{\text { now } \left._{*}^{\prime}(a)\left(j, \lambda a[ \urcorner \operatorname{walk}_{*}^{\prime}(a)(m) \wedge \operatorname{sleep}_{*}^{\prime}(a)(b)\right]\right) .}
\end{align*}
$$

From the truth of the second premiss it follows that for every index $\mathbf{k}$ such that $\llbracket \lambda i\left[\left(\operatorname{walk}_{*}^{\prime}(a)(m)=\operatorname{walk}_{*}^{\prime}(i)(m)\right) \wedge\left(\operatorname{sleep}_{*}^{\prime}(a)(b)=\right.\right.$ sleep $\left._{*}^{\prime}(i)(b)\right) \rrbracket \rrbracket_{M, g}(\mathbf{k})=1$ it holds that $\left.\llbracket\right\urcorner$ walk $_{*}^{\prime}(a)(m) \wedge$ sleep $_{*}^{\prime}(a)(b) \rrbracket_{M, g \mid k / a]}$ $=1$ and thus that for every such index $\mathbf{k}$ it holds that $\llbracket \lambda a\left[7\right.$ walk $_{*}^{\prime}(a)(m) \wedge$ $\left.\operatorname{sleep}_{*}^{\prime}(a)(b)\right]_{M, g}(\mathbf{k})=1$.

As we already indicated in our discussion of exhaustiveness in Section 1.5., argument ( X ), which translates as ( $\mathrm{X}^{\prime}$ ), comes out valid in our formal analysis.

$$
\frac{\operatorname{know}_{*}^{\prime}(a)\left(j, \lambda i\left[\lambda u\left[\operatorname{walk}_{*}^{\prime}(a)(u)\right]=\lambda u\left[\operatorname{walk}_{*}^{\prime}(i)(u)\right]\right]\right)}{\left.\left.\operatorname{know}_{*}^{\prime}(a)\left(j, \lambda i\left[\lambda u[ \urcorner \operatorname{walk}_{*}^{\prime}(a)(u)\right]=\lambda u[ \urcorner \operatorname{walk}_{*}^{\prime}(i)(u)\right]\right]\right)}
$$

As we argued in Section 1.5., the fact that ( $\mathrm{X}^{\prime}$ ) is valid is not due to the incorporation of exhaustiveness, but is a consequence of the fact that the only type of situation which can give rise to counterexamples to ( $\mathrm{X}^{\prime}$ ), the situations in which the subject of the propositional attitude is not fully informed as to what constitutes the domain of discourse, is not
dealt with in the semantic framework used here. Situations of misinformation about what subset of the domain is determined by a contextual restriction on the range of who, can be regarded as a subtype of this kind of situation. Once either one of these two aspects, which being of a general nature need to be built into the semantic framework anyway, is incorporated, counterexamples to ( $\mathrm{X}^{\prime}$ ) can be constructed which are structurally the same as those discussed in the next section with regard to argument (XII).

### 3.5. Single Constituent Complements with Which

The analysis of constituent complements in which one occurrence of a wh-term of the form which $\delta$ occurs is illustrated in the following example:

## John knows which man walks, $\mathfrak{t}$

know $_{*}^{\prime}(a)\left(j, \lambda i\left[\lambda u\left[\operatorname{man}_{*}^{\prime}(a)(u) \wedge\right.\right.\right.$ walk $\left._{*}^{\prime}(a)(u)\right]$


John, T
know which man walks, IV
$\lambda P[P(a)(\lambda a j)] \quad \operatorname{know}^{\prime}(a)\left(\lambda a \lambda i\left[\lambda u\left[\operatorname{man}_{*}^{\prime}(a)(u) \wedge \operatorname{walk}_{*}^{\prime}(a)(u)\right]\right.\right.$

which man walks, $t / / / e$


Again, the complement is formed in two steps. First, from a sentence containing a syntactic variable, and a common noun phrase an abstract is formed. The syntactic function which does this is quite similar to the one forming abstracts with who. The syntactic rule and the translation rule are:

```
( \(\mathrm{S}: \mathrm{AB} 2\) ) If \(\varphi \in P_{t}\) and \(\delta \in P_{C N}\), then \(F_{A B 2, n}(\delta, \varphi) \in P_{t| | l e}\)
(T:AB2) If \(\varphi \leadsto \varphi^{\prime}\) and \(\delta \leadsto \delta^{\prime}\), then \(F_{A B 2, n}(\delta, \varphi) \leadsto \lambda x_{n}\left(\delta^{\prime}\left(x_{n}\right) \wedge \varphi^{\prime}\right)\).
```

The translation is a complex predicate denoting expression. It denotes the conjunction of the predicate denoted by the common noun phrase and the predicate that can be formed from the sentence.

The second step is to apply the category changing rule ( $S: C C F^{*}$ ) which turns abstracts into complements. This way of constructing complements like which man walks gives rise to the de dicto reading discussed in Section 1.6. The proposition $\llbracket \lambda i\left[\lambda u\left[\operatorname{man}_{*}^{\prime}(a)(u) \wedge\right.\right.$ walk $\left._{*}^{\prime}(a)(u)\right]=\lambda u\left[\operatorname{man}_{*}^{\prime}(i)(u) \wedge\right.$ walk $\left.\left.\left._{*}^{\prime}(i)(u)\right]\right]\right]_{M, s}$ holds at an index $\mathbf{k}$ iff the intersection of the set of men and the set of walkers at $\mathbf{k}$ is the same as at $g(a)$. If John knows this proposition, it is implied that if a certain individual is a walking man, John knows both that it is a man and that it walks. In view of this, (XII'), the translation of (XII) with both the premiss and the conclusion in the de dicto reading is not valid:

$$
\begin{aligned}
& \text { (XII') } \quad \operatorname{know}_{*}^{\prime}(a)\left(j, \lambda i\left[\lambda u\left[\operatorname{man}_{*}^{\prime}(a)(u) \wedge \operatorname{walk}_{*}^{\prime}(a)(u)\right]\right.\right. \\
& \left.\left.=\lambda u\left[\operatorname{man}_{*}^{\prime}(i)(u) \wedge \operatorname{walk}_{*}^{\prime}(i)(u)\right]\right]\right) \\
& \text { know }_{*}^{\prime}(a)\left(j, \lambda i\left[\lambda u\left[\operatorname{man}_{*}^{\prime}(a)(u) \wedge 7 \text { walk }_{*}^{\prime}(a)(u)\right]\right.\right. \\
& \left.\left.\left.=\lambda u\left[\operatorname{man}_{*}^{\prime}(i)(u) \wedge\right\urcorner \operatorname{walk}_{*}^{\prime}(i)(u)\right]\right]\right) .
\end{aligned}
$$

A counterexample can be constructed as follows. Suppose that for some assignment $g$ and for some individual $d$ it holds that: $\llbracket \operatorname{walk}_{*}^{\prime}(a) \rrbracket_{M, g}(d)=$ $\llbracket \operatorname{man}_{*}^{\prime}(i) \rrbracket_{M_{, 8}}(\mathbf{d})=\llbracket \mathrm{walk}_{*}^{\prime}(i) \rrbracket_{M_{, 8}}(\mathbf{d})=0$, and $\llbracket \operatorname{man}_{*}^{\prime}(a) \rrbracket_{M, 8}(\mathbf{d})=1$. Then we can construct a model in which the proposition which is the argument in the premiss holds at $g(i)$, whereas the proposition which is the argument in the conclusion does not. So, the proposition in the premiss does not entail the proposition in the conclusion, which, given the usual semantics of know would be the only way in which the premiss could imply the conclusion. By a similar argument it can be shown that the inverse of (XII') is not valid either.

### 3.6. De re Readings of Constituent Complements

In Section 1.6. we argued that (XII) is valid iff both its premiss and its conclusion are read de re (excluding situations in which individuals may not be fully informed about the domain of discourse). This means that a second way to derive sentences containing constituent complements should be added to the syntax. In this derivation process common noun phrases are quantified into sentences containing a common noun variable one $e_{0}$, one $e_{1}, \ldots$, which translate into variables $o_{0}, o_{1}, \ldots$ of type $\langle\langle s, e\rangle, t\rangle$. The rule of common noun quantification and the corresponding translation rule are as follows:
( $S: C N Q$ ) If $\varphi \in P_{t}$ and $\delta \in P_{C N}$, then $F_{C N Q, n}(\delta, \varphi) \in P_{t}$.
(T:CNQ) If $\varphi \rightsquigarrow \varphi^{\prime}$ and $\delta \leadsto \delta^{\prime}$, then $F_{C N Q, n}(\delta, \varphi) \rightsquigarrow \lambda o_{n} \varphi^{\prime}\left(\delta^{\prime}\right)$.
The sentence John knows which man walks can now also be derived as follows:

## John knows which man walks



The translation of (XII) with both premiss and conclusion raed de re is now:
$\left(\mathrm{XII}^{\prime}\right) \quad \mathrm{know}_{*}^{\prime}(a)\left(\mathrm{j}, \lambda i\left[\lambda u\left[\operatorname{man}_{*}^{\prime}(a)(u) \wedge \operatorname{walk}_{*}^{\prime}(a)(u)\right]\right.\right.$

$$
\begin{aligned}
\operatorname{know}_{*}^{\prime}(a)\left(\mathrm{j}, \lambda i\left[\lambda u \left[\operatorname{man}_{*}^{\prime}(a)(u)\right.\right.\right. & \left.\wedge 7 \operatorname{walk}_{*}^{\prime}(a)(u)\right] \\
& \left.\left.=\lambda u\left[\operatorname{man}_{*}^{\prime}(a)(u) \wedge 7 \text { walk }_{*}^{\prime}(i)(u)\right]\right]\right) .
\end{aligned}
$$

The proposition denoted by the complement in the premiss at $g(a)$ is the same as the one denoted by the complement of the conclusion at $g(a)$. The first proposition holds true at an index $\mathbf{k}$ iff the intersection of the set of men at $g(a)$ and the set of walkers at $g(a)$ is the same as the intersection of the set of men at $g(a)$ and the set of walkers at $\mathbf{k}$. Clearly, this is the case iff the intersection of the set of men at $g(a)$ and the set of non-walkers at $g(a)$ is the same as the intersection of the set of men at $g(a)$ and the set of non-walkers at $k$, i.e. iff the second proposition holds true at $\mathbf{k}$. So, both (XII') and its inverse are valid arguments.

We leave it to the reader to satisfy her/himself that (XI) with its conclusion read de dicto is not valid, whereas with the conclusion read de re it is.

### 3.7. Multiple Constituent Complements

In this section we will outline our treatment of constituent complements in which more than one wh-term occurs. The construction of multiple constituent complements starts out with a sentence containing more than one syntactic variable. By using one of the abstract formation rules given above, an abstract is obtained from such a sentence. From this abstract, a 'higher level' abstract is formed. This process can be repeated as long as there are variables left, each time resulting in an abstract of one level higher. This means that there is not just one category of abstracts, but a whole set of abstract categories. The definition of this set and of the corresponding set of abstract types are as follows:
(a) $A B$ is the smallest subset of CAT such that
(i) $t / / / e \in A B$,
(ii) if $A \in A B$, then $A / e \in A B$,
(b) $A B^{\prime}$ is the smallest subset of TYPE such that

$$
\text { if } A \in A B \text {, then } f(A) \in A B^{\prime}
$$

To the two rules which formed abstracts from sentences, one for who and one for which $\delta$, there correspond two rules, or better rule schemata, which from an abstract form an abstract of one level higher:
(S:AB3) If $\chi \in P_{A}, A \in A B$, then $F_{A B 3, n}(\chi) \in P_{A / e}$.
(S: AB4) If $\chi \in P_{A}, A \in A B$, and $\delta \in P_{C N}$, then $F_{A B 4, n}(\delta, \chi) \in P_{A / l e}$.
The two syntactic functions of this pair of rules differ from those of the former pair. In particular, the wh-term is not placed in front of the abstract, but is substituted for a certain occurrence of the syntactic variable. As a matter of fact, this is the main reason for distinguishing the two pairs of rules; the new translation rules follow the same pattern as the old ones. This is most obvious in the case of who:
(T:AB3) If $\chi^{\leadsto} \chi^{\prime}$, then $F_{A B 3, n}(\chi) \leadsto \lambda x_{n} \chi^{\prime}$.
Like the syntactic rule, the translation rule is a rule schema, making use of the fact that the syntactic rule of the logical language forming $\lambda$-abstracts is a rule schema as well: abstracts $\lambda x \alpha$ can be formed from a variable $x$ and an expression $\alpha$ of arbitrary type.

For which $\delta$ the situation is slightly more complicated. The old translation:

$$
\lambda x_{n}\left[\delta^{\prime}\left(x_{n}\right) \wedge \varphi^{\prime}\right]
$$

cannot be used as such in case $\varphi$ is not a sentence, but an abstract. The
conjunction sign $\wedge$ does not have the variable character that the $\lambda$-abstractor has.
We therefore extend our logical language with a new kind of expressions which do have this flexible character. These expressions are called restricted $\lambda$-abstracts and are of the form $\lambda x\lceil\alpha\rceil \beta$. The abstraction is restricted to those entities which satisfy the predicate denoted by $\alpha$. We will use these new expressions in the translation rule ( $T: A B 4$ ) as follows:

$$
\text { (T:AB4) If } \delta \leadsto \delta^{\prime} \text { and } \chi^{\leadsto>} \chi^{\prime} \text {, then } F_{A B 4, n}(\delta, \chi) \leadsto \lambda x_{n}\left\lceil\delta^{\prime}\right\rceil \chi^{\prime} \text {. }
$$

So, the translation is a restricted $\lambda$-abstract, where the abstraction is restricted to the individual concepts which satisfy the translation of the common noun phrase $\delta$ in which $\delta$.

The new clause in the definition of the logical language and its interpretation are as follows:
(RA) If $x \in V A R_{a}, \alpha \in M E_{(a, t)}$ and $\beta \in M E_{b}, b \in A B^{\prime}$, then $\lambda x\lceil\alpha\rceil \beta \in M E_{\langle a, b\rangle}$
$\llbracket \lambda x\lceil\alpha\rceil \beta \rrbracket_{M, g}$ is that function $\mathbf{h} \in D_{M,\langle a, b\rangle}$
such that for all $\mathbf{d} \in D_{M, a}$

$$
\begin{aligned}
\mathbf{h}(\mathbf{d}) & =\llbracket \beta \rrbracket_{M, g[x]]} & \text { if } \llbracket \alpha \rrbracket_{M, 8}(\mathbf{d})=1, \\
& =\operatorname{zero}_{b} & \text { if } \llbracket \alpha \rrbracket_{M, 8}(\mathbf{d})=0,
\end{aligned}
$$

where $^{\text {zero }_{t}}=0 ;$ zero $_{(a, b)}$ is the constant function from $D_{M, a}$ to zero $_{b}$.

The expressions $\beta$ are restricted to expressions of abstract types, i.e. they are $n$-place predicate expressions ( $n \geq 1$ ). A more general definition of restricted $\lambda$-abstraction for arbitrary types is possible, if we are prepared to have zero elements of type $e$ and type $s$ as well. The expression $\lambda x\lceil\alpha\rceil \beta$ is an abstract of one level higher than $\beta$, i.e. an $n+1$ place predicate expression. When applied to an argument $d$ of which the one-place predicate denoted by $\alpha^{\prime}$ is true, $\llbracket \lambda x\lceil\alpha\rceil \beta \rrbracket_{M_{, ~}}(\mathrm{~d})$ denotes the same $n$-place predicate as the unrestricted abstract $\llbracket \lambda x \beta \rrbracket^{M, g}$ applied to d. When $\alpha$ is false of $\mathbf{d}, \llbracket \lambda x\lceil\alpha\rceil \beta \rrbracket_{M, 8}(\mathbf{d})$ denotes a zero $n$-place predicate: a predicate which invariably gives the value 0 , no matter to which arguments it is applied.

The category changing rule ( $\mathrm{S}: \mathrm{CCF}^{*}$ ) which formed constituent complements from expressions of abstract category $t / / / e$, can now be generalized to a constituent complement formation rule scheme ( $\mathrm{S}: C C F$ ) which applies to expressions of arbitrary abstract category. The corresponding translation rule ( $T: C C F$ ) remains essentially the same as the old one:
( $S: C C F$ ) If $\chi \in P_{A}, A \in A B$, then $F_{C C F}(\chi) \in P_{i}$.
( $T: C C F$ ) If $\chi^{\wedge} \rightarrow \chi^{\prime}$, then $F_{\mathrm{CFF}}(\chi) \leadsto \lambda i\left[\chi^{\prime}=\left[\lambda a \chi^{\prime}\right](i)\right]$.

The following analysis trees are examples of the derivation of sentences containing multiple constituent complements with who and which:


It can in general be proved that if $\beta$ is an $n$-place predicate expression, taking arguments of type $a_{1}, \ldots, a_{n}$, and $x_{1}, \ldots, x_{n}$ are variables of type $a_{1}, \ldots, a_{n}$ respectively, then $\lambda x\lceil\alpha\rceil \beta$ is equivalent to $\lambda x \lambda x_{1}, \ldots, \lambda x_{n}\left[\alpha(x) \wedge \beta\left(x_{1}, \ldots, x_{n}\right)\right]$. This means that the translation of the second line of (17) is equivalent to: $\lambda x_{0} \lambda x_{1}\left[\operatorname{girl}^{\prime}(a)\left(x_{0}\right) \wedge \operatorname{man}^{\prime}(a)\left(x_{1}\right) \wedge\right.$ $\operatorname{love}^{\prime}(a)\left(x_{0}, x_{1}\right)$. So the top line of (17) is equivalent to:

$$
\begin{align*}
& \lambda i\left[\lambda u \lambda v\left[\operatorname{girl}_{*}^{\prime}(a)(u) \wedge \operatorname{man}_{*}^{\prime}(a)(v) \wedge \operatorname{love}_{*}^{\prime}(a)(u, v)\right]\right.  \tag{17'}\\
& \left.\quad=\lambda u \lambda v\left[\operatorname{girl}_{*}^{\prime}(i)(u) \wedge \operatorname{man}_{*}^{\prime}(i)(v) \wedge \operatorname{love}_{*}^{\prime}(i)(u, v)\right]\right] .
\end{align*}
$$

This means that it is possible to reformulate ( $T: A B 2$ ) in terms of restricted $\lambda$-abstraction. (The same holds for ( $T: A B 1$ ) and ( $T: A B 3$ ) if that turns out to be necessary, cf. the remarks on argument (X) in Sections 3.4. and 1.5.) We leave it to the reader to verify that the arguments (VI) and (VII) of Section 1.4. are valid. The proof of their validity runs parallel to that of $\left(\mathrm{V}^{\prime}\right)$, given in Section 3.3.

The analysis of constituent complements presented here can easily be extended to cover complements with expressions like why, where, when, etc. as well. What is needed are syntactic variables that range over the proper kinds of entities. Further the set of abstract categories has to be extended, to cover abstraction over these variables. The syntactic and the corresponding translation rules have the same form as the rules discussed above.

### 3.8. Why Abstracts Are Necessary

As we already stated in Section 3.3., the level of abstracts is not strictly needed for the analysis of single constituent complements, they could be formed directly from sentences. However, abstracts (or some similar distinct level of analysis) seem to be essential for a correct analysis of multiple constituent complements. The reasons behind this can be outlined as follows.
Without the intermediary level of abstracts, one would need a syntactic rule which forms (multiple) constituent complements by introducing a (new) wh-term into a complement. On the semantic level such a rule would have to transform an expression of the form (a) into one of the form (b):
(a) $\lambda i[\alpha / a \mid=\alpha / i /]$
(b) $\lambda i[\lambda x[(\ldots \alpha \ldots) / a /]=\lambda x[(\ldots \alpha \ldots) / i /]]$.

The problem is to make this transition in a compositional way. A possibility that might suggest itself is to treat wh-terms not as a kind of abstractors, but as a kind of terms that can only be introduced by means of a quantification rule. We might translate who as in (c), and formulate a quantification rule which, when applied to a wh-term $\beta$ and a complement $\rho$, translates as (d):

$$
\begin{equation*}
\lambda P \wedge x[P(a)(x)] \tag{c}
\end{equation*}
$$

(d) $\quad \lambda j\left[\beta\left(\lambda a \lambda x_{n}(\rho(\mathrm{j}))\right)\right]$, where $\beta$ translates a wh-term and $\rho$ a complement and $x_{n}$ is the variable quantified over

If we apply (d) to the term (c) and a complement of the form (a), the result is (e), which is equivalent to (f). The expression (f) is of the form (b), so in this case we have succeeded in making a transition from an expression of the form (a) to an expression of the form (b) in a compositional way.

$$
\begin{align*}
& \lambda j \wedge x\left[\lambda x_{n}[\alpha / a \mid=\alpha / j /](x)\right]  \tag{e}\\
& \left.\lambda i\left[\lambda x_{n} \alpha / a /=\lambda x_{n} \alpha / i\right]\right] . \tag{f}
\end{align*}
$$

However, this approach is only possible as long as we do not take wh-terms of the form which $\delta$ into consideration. A term of the form which $\delta$ would translate as (g). Applying (d) to a term of the form (g) and a complement of the form (a) results in (h):

$$
\begin{equation*}
\lambda P \wedge x[\delta(x) \rightarrow P(a)(x)] \tag{g}
\end{equation*}
$$

(h) $\quad \lambda j\left[\wedge x\left[\delta(x) \rightarrow\left(\lambda x_{n}[\alpha / a /=\alpha / j /]\right)(x)\right]\right]$.

The expression ( h ) is equivalent to ( $\mathbf{i}$ ):

$$
\begin{equation*}
\lambda i\left[\lambda x_{n}\left[\delta\left(x_{n}\right) \wedge \alpha / a /\right]=\lambda x_{n}\left[\delta\left(x_{n}\right) \wedge \alpha / i /\right]\right] . \tag{i}
\end{equation*}
$$

But, since both occurrences of $\delta$ in (i) contain a free occurrence of $a$, this results only in de re readings of complements, not in de dicto ones. Result (i) is not of the required form (b). The de dicto reading would be expressed by ( j ):

$$
\begin{equation*}
\lambda i\left[\wedge x\left[\left[\delta(x) \wedge\left(\lambda x_{n} \alpha\right)(x)\right] / a l=\left[\delta(x) \wedge\left(\lambda x_{n} \alpha\right)(x)\right] / i /\right]\right] . \tag{j}
\end{equation*}
$$

This formula ( j ) is equivalent to one of the form (b), but it seems impossible to obtain ( j ) from (a) and (g) in a compositional way. Although we lack a formal proof, we are convinced that there is no way to proceed from (a) and (g) to an expression which gives de dicto readings. Consequently, we feel that the level of abstracts is indeed necessary, it is necessary to account for de dicto readings of multiple constituent complements. ${ }^{15}$

In a nutshell, this is the reason why Karttunen's approach, being a quantificational one, can only account for the de re readings. The fact that Karttunen uses existential rather than universal quantification is not essential. It has to do with the fact that in his analysis complements denote sets of propositions instead of single propositions and with the fact that he does not take into account the exhaustiveness of whcomplements.
This is also the reason why it is impossible to treat wh-terms as terms, i.e. as expressions of (a subcategory of) the category T. In a quantificational approach like Karttunen's, wh-terms can be treated as 'normal' terms. From a syntactic point of view, this may be an advantage. However, as we hope to have shown, the quantificational approach has important semantic shortcomings. And it seems that semantic considerations lead us to the abstractor view of $w$-terms. This means that wh-terms have to be treated as syncategorematic expressions (or, alternatively, as expressions belonging to the whole range of categories $(t / / / e) / t,((t / / e) / e) /(t / / / e)$, etc.).

## 4. Details of a possible syntax for wh-COMPLEMENTS

### 4.1. Background Assumptions

In Section 3 we explained how the semantic analysis of wh-complements proposed in this paper can be incorporated systematically in the framework of Montague Grammar. There we did not bother about the syntactic details. In this section we will try to be a little bit more explicit. We will sketch one possible syntax of wh-constructions which is suitable for our semantics. The syntax presented here is in the line of the modifications of Montague's original syntax as proposed by Partee (see Partee, 1976, 1979a and 1979b) and others. Some of its aspects will remind the reader of work done in transformational grammar. Of course, we do not claim that the analysis of wh-constructions presented here is new. Moreover, we do not attempt to solve all of the notoriously difficult syntactic problems in this area. We merely wish to show in this section that our semantic analysis of wh-complements can be combined with a feasible syntactic analysis.

In what follows the following assumptions concerning the syntax are made. The syntax produces not plain strings, but labelled bracketings (or, equivalently, phrase structure trees). The labelled bracketings account for the intuitions about the constituent structure of expressions and contain all the information which is needed for syntactic purposes. The constituent structure of an expression is, in general, not enough to determine its semantic interpretation. The semantic interpretation of an expression is determined by its derivation, which is encoded in its analysis tree.

Further it is assumed that the facts concerning pronominalization, reflexivization and 'wh-movement' are to be accounted for in terms of structural properties, i.e. properties of labelled bracketings, such as Reinhart's notion of c-command (see Reinhart, 1976). For an analysis of pronominalization and reflexivization in terms of structural properties in the Montague framework the reader is referred to Landman and Moerdijk (1981). Their paper also contains an analysis of some wh-constructions which, like the one presented here, uses structural properties, but differs from our analysis in several other respects.

## 4.2. 'Wh- Preposing' and 'Preposable Occurrences'

We will concentrate on the rules which build abstracts. There are four of them, two 'preposing' rules, ( $S: A B 1$ ) and ( $S: A B 2$ ), and two 'substitution' rules ( $S: A B 3$ ) and ( $S: A B 4$ ). We start with ( $S: A B 1$ ), the rule
which produces abstracts with preposed who $(m)$. We want this rule to produce structures such as (18b)-(21b) from structures such as (18a)(21a):
(a) ${ }_{t}\left[T\left[h_{o}\right]_{T V}[w a l k s]\right]$
(b) ${ }_{A B}\left[{ }_{w H T}[w h o]_{t}\left[{ }_{w H T}[]_{I V}[\right.\right.$ walks $\left.\left.]\right]\right]$
(a) ${ }_{[\mathrm{T}}[\mathrm{John}]_{\text {IV }}\left[\right.$ TV $\left.\left.[\text { loves }]_{\mathrm{T}}\left[\mathrm{him}_{o}\right]\right]\right]$
(b) ${ }_{A B}\left[{ }_{W H T}[\text { whom }]_{t}[\text { [John }]_{I V}\left[\right.\right.$ TVV $\left.\left.[l o v e s]_{W H T}[]\right]\right]$
(a) ${ }_{t}\left[{ }_{t}\left[\mathrm{he}_{o}\right]_{I V}[\right.$ walks $\left.]\right]$ and ${ }_{t}\left[\mathrm{~T}\left[\mathrm{he}_{o}\right]_{I V}[\right.$ talks $\left.\left.]\right]\right]$
(b) ${ }_{A B}\left[{ }_{W H T}[w h o]_{t}\left[w_{H T}[]_{I V}[\right.\right.$ walks $\left.]\right]$ and ${ }_{t}\left[T[]_{I V}[\right.$ talks $\left.\left.]\right]\right]$

( $S: A B 1$ ) operates on sentential structures containing one or more occurrences of a syntactic variable hen. It creates a structure labelled $A B$ by 'preposing' the wh-term who( $m$ ), substituting a trace (i.e. empty node) for some, 'preposable', occurrences of $h e_{n}$ and anaphorizing the others. The occurrences of $h e_{n}$ which are replaced by a trace share certain structural properties. They are called the wh-p-antecedent occurrences of $h e_{n}$. One of these occurrences is replaced by a WHTtrace, the others by $T$-traces. Traces are left because in order for pronominalization, reflexivization and abstract formation to work properly, the structural properties of certain expressions in the original structure have to be recoverable. In effect, leaving traces is nothing but building into the structure those aspects of derivational history which continue to have syntactic relevance.

We add two general remarks. First, notice that labels like $A B$ and WHT are not category labels. $A B$ acts as a variable over category labels, WHT labels expressions which are introduced syncategorematically. The use of such labels does not present semantic problems since it is the derivational history, and not the structure, of an expression that determines its meaning. Second, as structures (21) show, the output of a category changing rule no longer contains the original category label: the complement of know is of the form ${ }_{i}\left[\mathrm{wHT}^{[ }[\mathrm{who}] \ldots\right]$ and not of the form ${ }_{i}\left[{ }_{A B}\left[w_{H T}[w h o] \ldots\right]\right]$. This is based on the assumption that information about the old category is no longer syntactically relevant. Nothing in our analysis, however, depends on this assumption.
The notion of wh-p-antecedent occurrence is not only needed to
distinguish those occurrences of $h e_{n}$ which are to be replaced by a trace, it will also be used to determine whether a given structure is a proper input for ( $S: A B 1$ ). Before giving a definition, let us point out what will be understood by an occurrence. Formally, an occurrence of an expression $\alpha$ in a structure $\beta$ is an ordered pair $\left\langle n,{ }_{X}[\alpha(-)]\right\rangle$, where $n$ defines a position in $\beta, X$ is the label of $\alpha$ and ( - ) is the set of features that determines the morphological form. In what follows we will not use the term 'occurrence' so strictly. For example we will write ${ }_{\tau}\left[\mathrm{him}_{o}\right]$ instead of ${ }_{T}\left[\mathrm{he}_{o}(\mathrm{acc})\right]$, etc. The notion of $w h-p$-antecendent occurrence is defined as follows:
(WH-P)
The wh-p-antecedent occurrences of $h e_{n}$ in $\phi$ are those occurrences $\alpha$ of $h e_{n}$ in $\phi$ such that:
(i) $\alpha$ is not $c$-commanded by another occurrence of $h e_{n}$ in $\varphi$;
(ii) $\alpha$ is not dominated by a node $t$ such that that node is directly dominated by a node $A$ : $A \neq t$;
(iii) if $\alpha$ occurs in a coordinate structure in $\phi$ then for every coordinate $\psi$ there is a wh-p-antecedent occurrence of $h e_{n}$ in $\psi$.

We will give a few examples to illustrate this definition. In these examples only the relevant aspects of the structures are represented. First consider (22):

$$
\begin{align*}
& \mathrm{he}_{\alpha} \text { loves him }{ }_{o} \text { self }  \tag{22}\\
& \beta
\end{align*}
$$

$\alpha$ is a wh-p-antecedent occurrence of he $e_{o}$, but $\beta$ isn't, since $\beta$ is $c$-commanded by $\alpha$. So, (22) will give rise to (22a) but not to (22b):
(22) (a) ${ }_{A B}\left[\right.$ who $_{t}[$ wht [] loves himself] $]$
(22) (b) ${ }_{A B}\left[\right.$ who $_{T}\left[\right.$ ] loves $\left.\left.{ }_{W H T}[]\right]\right]$.

Next consider (23):

$$
\left.\left.\begin{array}{l}
\text { he }_{o} \text { says }_{\bar{t}}\left[\text { that }_{t}[\text { Mary loves him }\right. \\
\beta
\end{array}\right]\right]
$$

Again $\alpha$ is a wh-p-antecedent occurrence, and $\beta$ is not. Not only because $\beta$ is $c$-commanded by $\alpha$, but also because $\beta$ is dominated by a $t$ which is directly dominated by a $\bar{t}$. So, (23) will lead to (23a), but not to (23b):
(23) (a) ${ }_{A B}\left[w^{2} \mathrm{ho}_{t}[\right.$ whr [] says that Mary loves him] $]$
(23) (b) ${ }_{A B}\left[\right.$ whom $m_{t}[]$ says that Mary loves wht [ $\left.\left.]\right]\right]$.

Another example illustrating condition (ii) is (24):

John says ${ }_{\bar{i}}\left[\right.$ that $_{t}\left[\right.$ he $_{o}$ loves Mary]]
$\alpha$
$\alpha$ is not a wh-p-antecedent occurrence, because it is dominated by a $t$ which is directly dominated by $\overline{\mathrm{t}}$. Thus (24a) will not be derivable from (24):
(24) (a) ${ }_{A B}\left[\right.$ who $_{t}\left[\mathrm{John}^{2} \operatorname{says}_{t}\left[\operatorname{that}_{t}\left[{ }_{w H T}[]\right.\right.\right.$ loves Mary $\left.\left.\left.]\right]\right]\right]$.

Notice that condition (ii) excludes any occurrence of a syntactic variable in an embedded clause. As (25a) indicates, this is too strong:
(25) (a) ${ }_{A B}\left[\right.$ whom $_{t}\left[\right.$ John says ${ }_{t}\left[\right.$ that $_{t}\left[\right.$ Mary loves $\left.\left.\left.\left.{ }_{w h t}[]\right]\right]\right]\right]$.

This would have to be derived from the structure (25):
(25) John says ${ }_{i}\left[\right.$ that $_{t}\left[\right.$ Mary loves him ${ }_{o}$ ]].

## $\alpha$

If we weaken condition (ii) by adding:
$\ldots$ unless the case of $\alpha \neq$ nominative and $A=\bar{t}$-that
then $\alpha$ in (25) counts as a wh-p-antecedent of $h e_{o}$. Notice that $\beta$ in (23) is still excluded by condition (i). By $\bar{t}$-that, of course, we mean to label the subcategory of that-complements. That the above weakening should be restricted to that-complements is made clear by (26):
(26) $\quad *_{A B}\left[\right.$ whom $_{t}\left[\right.$ John wonders ${ }_{i}\left[\right.$ whether $_{t}[$ Peter loves wht []$\left.\left.\left.]\right]\right]\right]$.

Another example illustrating condition (ii) involves a subordinate clause:

$$
\begin{equation*}
\text { the fact }{ }_{\bar{t}}\left[\text { that }_{t}\left[\mathrm{he}_{o} \text { is ill }\right]\right] \text { bothers him }{ }_{o} \tag{27}
\end{equation*}
$$ $\alpha \quad \beta$

$\alpha$ is not a wh-p-antecedent occurrence, $\beta$ is. So, from (27) we can obtain (27a), but not (27b):
(27) (a) ${ }_{A B}\left[\right.$ whom $_{t}\left[\right.$ the fact ${ }_{i}\left[\right.$ that $_{t}[$ he is ill] $]$ bothers $\left.\left._{\text {WHT }}[]\right]\right]$
(27) (b) ${ }_{A B}\left[\right.$ whom $_{t}\left[\right.$ the fact $_{i}\left[\right.$ that $_{t}\left[{ }_{w H T}[\right.$ ]is ill] $]$ bothers $\left.\left._{T}[]\right]\right]$.

As a last example, consider (28):
(28) $t_{t t}\left[\text { Mary loves } \operatorname{him}_{o}\right]_{t t t}\left[\right.$ if ${ }_{t}\left[\right.$ Suzy hates $\left.\left.\left.\operatorname{him}_{o}\right]\right]\right]$
$\alpha$ $\beta$
$\alpha$ is a wh-p-antecedent occurrence, $\beta$ is not, which predicts that (28a) can result from (28), but not (28b): ${ }^{16}$
(28) (a) ${ }_{A B}\left[\right.$ whom $_{t}\left[L_{t}\left[\right.\right.$ Mary loves $w_{H T}[]_{t / t}\left[\right.$ if ${ }_{t}[$ Suzy hates him $\left.\left.\left.]\right]\right]\right]$
(28) (b) ${ }^{*}{ }_{A B}\left[\text { whom }_{t}{ }^{[t}[\text { Mary loves him }]\right]_{t / t}\left[\right.$ if ${ }_{t}[$ Suzy hates $\left.\left.\left.w H T[]]\right]\right]\right]$.

The coordinate structure constraint (iii) prevents the derivation of (29a) from (29):

```
    t[t[\mp@code{he walks] and t[Peter talks]]}]
    (a) **AB [who t[t[wHT}[] ]walks] and t[Peter talks]]].
```

Notice that in case we weaken condition (ii) as indicated above, there is a wh-p-antecedent occurrence of he in (30), but not in (31) according to (iii):
(30) John says ${ }_{i}\left[\right.$ that $_{t}\left[{ }_{t}\left[\right.\right.$ Peter loves him $\left.{ }_{0}\right]$ and ${ }_{t}\left[\right.$ Mary kisses him $\left.\left.\left.{ }_{o}\right]\right]\right]$

John says ${ }_{i}\left[\right.$ that $_{t}\left[{ }_{t}\left[\right.\right.$ Peter loves him $\left._{o}\right]$ and ${ }_{t}[$ Mary kisses Bill $\left.\left.]\right]\right]$.
Notice further that (32) does not contain a wh-p-antecedent occurrence of $h e_{o}$ since, although $\alpha$ and $\beta$ are dominated by a node $t$ which is directly dominated by another node $t$, they also occur in a $t$ (i.e. the entire coordinate structure) which is directly dominated by $\bar{t}$ :

$$
\begin{equation*}
\text { John says }_{i}\left[\text { that }_{t}\left[t\left[\text { he }_{o} \text { walks }\right] \text { and }{ }_{t}\left[\text { he }_{o} \text { talks }\right]\right]\right] . \tag{32}
\end{equation*}
$$ $\alpha \quad \beta$

All those occurrences of $h e_{n}$ in $\phi$ which are not wh-p-antecedent occurrences according to (WH-P) we call wh-p-anaphor occurrences of $h e_{n}$ in $\phi$. The formulation of the syntactic rule ( $S: A B 1$ ) now runs as follows:
( $S: A B 1$ ) If $\phi \in P_{t}$, then $F_{A B 1, n}(\phi) \in P_{t / / / l}$.
Condition: $\phi$ contains one or more $w h-p$-antecedent occurrences of $h e_{n}$, all of which have the same case $c$.
$F_{A B 1, n}(\phi)={ }_{A B}\left[w_{W T}[w h o(c)]_{t}\left[\phi^{\prime}\right]\right]$, where $\phi^{\prime}$ comes from $\phi$ by performing the following operations:
(i) if $c=$ nominative then replace the first, else replace the last, wh-p-antecedent occurrence of $h e_{n}$ in $\phi$ by wht [ ];
(ii) delete all other wh-p-antecedent occurrences of $h e_{n}$ in $\phi$, i.e. replace them by ${ }_{T}[]$;
(iii) anaphorize all wh-p-anaphor occurrences of $h e_{n}$ in $\phi$.

The examples (18)-(32) illustrate the working of this rule. The condition which restricts the application of ( $S: A B 1$ ) deals with the familiar cases of case-conflict. It would become superfluous once a theory of features, e.g. in the line of Landman and Moerdijk (1981), is incorporated. Clause (i) is stated in terms of case, we do not want to exclude the possibility to formulate it in terms of structural properties. The anaphorization operation in (iii) here comes to simply removing indices.

The second 'wh-preposing' rule, which preposes wh-terms of the form which $\delta$, is a minor variation of the one just given. It reads as follows:
(S:AB2) If $\phi \in P_{t}$ and $\delta \in P_{C N}$, then $F_{A B 2, n}(\delta, \phi) \in P_{t / \mid / l e}$.
Condition: as in (S:AB1).
$F_{A B 2 . n}(\delta, \phi)={ }_{A B}\left[w_{H T}[\text { which } \delta(c)]_{[ }\left[\phi^{\prime}\right]\right]$, where $\phi^{\prime}$ comes from $\phi$ by performing the following operations:
(i) and (ii) as in ( $\mathrm{S}: \mathrm{AB} 1$ );
(iii) as in ( $S: A B 1$ ), taking into account the (number and) gender of $\delta$.

Examples similar to the ones already given for ( $\mathrm{S}: A B 1$ ) can easily be constructed.

### 4.3. Wh-Reconstruction

Interesting cases of application of ( $S: A B 2$ ) are those in which the common noun $\delta$ is not lexical, but itself complex and contains an occurrence of a syntactic variable, e.g.:
${ }_{A B}\left[\right.$ which poem of him ${ }_{\alpha}{ }_{\beta}{ }_{\beta}^{[\text {he }}{ }_{o}$ likes best wHTL $\left.\left.]\right]\right]$
${ }_{A B}$ [which man who loves $\operatorname{him}_{\alpha}{ }_{o}\left[{ }_{\beta}{ }_{o}\right.$ likes best wht [ ] $]$ ].

Notice that in both structures $\alpha$ and $\beta$ do not $c$-command each other. If it were the case that $\beta c$-commanded $\alpha$, then this could be used to explain why (35a) and (36a) are acceptable, whereas (35b) and (36b) are not (on coreferential readings, of course):
(35) (a) ${ }_{A B}$ [which poem of him ${ }_{t}$ [every poet likes best $\left.\left.{ }_{\text {wht }}[]\right]\right]$.
(35) (b) $*_{A B}$ [which poem of every poet ${ }_{t}$ [he likes best wht [ ]]].
(36) (a) ${ }_{A B}$ [which man who loves her ${ }_{t}\left[\right.$ every girl likes best $\left.\left.{ }_{w H T}[]\right]\right]$.
(36) (b) ${ }_{A B}\left[\right.$ which man who loves every girl ${ }_{t}\left[\right.$ she likes best $\left.\left.{ }_{\text {wHT }}[]\right]\right]$.

A natural condition (see Reinhart, 1976, 1979) on antecedent-anaphor relations is that an anaphor does not $c$-command its antecedent. Notice that although $\beta$ does not $c$-command $\alpha$, it does $\boldsymbol{c}$-command the trace of the wh-term in which $\alpha$ occurs. It seems that in the process of deriving (35a) from (33) structural relations such as $c$-command are not determined on (33) as such, but on what is called the wh-reconstruction of (33). ${ }^{17,18}$ This notion is defined as follows:
(WH-R) The wh-reconstruction of a structure $\phi$ is that structure $\phi^{\prime}$ which is the result of replacing, bottom up, each substructure of the form $\left[w_{H r}[\gamma]_{t}[\psi]\right]$ by $\left[t\left[\psi^{\prime}\right]\right]$, which is the result of substituting the wh-term $\gamma$ for its trace in $\psi$.

Notice that the existence of a unique trace for each occurrence of a wh-term is guaranteed by the direction of the reconstruction process (bottom up) and the nature of the preposing rules ( $S: A B 1$ ) and ( $S: A B 2$ ).

For every structural property $P$ we define a corresponding structural property $P^{\prime}$ as follows:
(RSP) $\alpha$ has the structural property $P^{\prime}$ in the structure $\phi$ iff $\alpha$ has the structural property $P$ in the wh-reconstruction of $\phi$.

From now on we will refer to structural properties $P^{\prime}$ as $P$, e.g. from now on $c$-command stands for $c$-command ${ }^{\prime} .{ }^{19}$

At this point a remark on the nature of WHT-traces is in order. In fact a WHT-trace is nothing but a $T$-trace in a special structural position. So, WHT-traces are marked $T$-traces. However, whether or not a $T$-trace is in this special structural position, can always be determined, so the special marking is not essential.

We could do without WHT-traces and only use T-traces. The whreconstruction is then defined as follows:
(WH-R') The wh-reconstruction of a structure $\phi$ is that structure $\phi^{\prime}$ which is the result of replacing, bottom up, each substructure of the form $\left[{ }_{w H T}[\gamma]_{t}[\psi]\right]$ by $\left.\dot{[ }_{t}\left[\psi^{\prime}\right]\right]$, which is the result of substituting for the first $T$-trace in $\psi$ if $\gamma$ has nominative case, and for the last $T$-trace in $\psi$ otherwise.

Of course, if one extends the present analysis to the more difficult cases involving pied-piping etc., the definition of wh-reconstruction might become more complicated. However, we feel that a reconstruction in terms of structural positions of $T$-traces will always be possible. In fact it has to be since this seems to be the only explanation for the fact that language users are able to interpret wh-constructions at all. A language user is capable of recognizing a hole in a structure (i.e. a trace), he will be capable of determining its category and its structural properties, but it seems unlikely that he is able to distinguish between subcategories of holes, if the subcategory information in question represents structural information which is not also present in the structure itself.

### 4.4. Wh-Substitution and Substitutable Occurrences

Other cases where we need wh-reconstruction than the ones discussed above, involve the other two abstract formation rules, the wh-substitution rules. These rules form abstracts from abstracts by substituting who( $m$ ), which $\delta$, for an occurrence of a syntactic variable. They are highly parallel to the previous two. However, they operate on a type of occurrences of syntactic variables which is a bit less constrained than wh-p-antecedent occurrences. The difference is that the substitution rules are allowed to operate on occurrences which are inside a complement. Consider the following three examples:
(37) (a) ${ }_{A B}\left[\right.$ who $\left.{ }^{1}\left[{ }_{w H T}[] \mathrm{knows}_{t}\left[\mathrm{who}_{t}\left[{ }_{w H T}[] l o v e s \mathrm{him}_{o}\right]\right]\right]\right]$.

(38) (a) ${ }_{{ }_{B}}\left[\right.$ who $_{t}\left[{ }_{w H T}[] \mathrm{knows}_{t}\left[\right.\right.$ whether $_{t}\left[\mathrm{he}_{o}\right.$ walks $\left.\left.\left.]\right]\right]\right]$.
(38) (b) ${ }_{A B}\left[\right.$ who $_{t}\left[w H T[] k n o w s_{t}\left[w^{2} e t h e r_{t}[\right.\right.$ which girl walks] $\left.\left.]\right]\right]$.
(39) (a) ${ }_{A B}\left[\right.$ who $_{t}\left[\mathrm{whr}^{[ }[] \mathrm{knows}_{[ }\left[\right.\right.$that $_{t}\left[\mathrm{He}_{o}\right.$ walks $\left.\left.\left.]\right]\right]\right]$.
(39) (b) ${ }_{A B}\left[\right.$ who $_{t}\left[w_{H r}[] k_{n o w s}^{[ }\left[\right.\right.$that ${ }_{t}[$ which girl walks $\left.\left.\left.]\right]\right]\right]$.

The multiple constituent complements in the (b)-sentences can be constructed from the single constituent complements in the (a)-sentences. To see that the substitution rules are more liberal than the preposing rules, compare (38) with (26) and (39) with (24). This leads to the following notion of wh-s-antecedent occurrence:
(WH-S) The wh-s-antecedent occurrences of he $e_{n}$ in $\phi$ are those occurrences $\alpha$ of he $_{n}$ in $\phi$ such that:
(i) $\alpha$ is not $c$-commanded by another occurrence of $h e_{n}$ in $\phi$;
(ii) $\alpha$ is not dominated by a node $t$ such that that node is directly dominated by a node $A: A \neq t, \bar{t}$;
(iii) if $\alpha$ occurs in a coordinate structure in $\phi$ then for every coordinate $\psi$ there is a wh-s-antecedent occurrence of he $e_{n}$ in $\psi$.
(WH-S) only differs from (WH-P) in that in clause (ii) A may be either $t$ or $\bar{t}$. So occurrences within subordinate clauses other than complements are still out of bounds. As an example consider (40):

$$
\begin{equation*}
{ }_{A B}\left[\text { which } \operatorname{man}_{R C}\left[\text { who }\left[w_{H T}[] l o v e s \operatorname{him}_{\alpha}\right]\right]_{\mathrm{H}}[\mathrm{wHT}[] \text { walks }]\right] \text {. } \tag{40}
\end{equation*}
$$

According to (WH-S) $\alpha$ is not a wh-s-antecedent occurrence of $h e_{o}$, since $R C \neq t, \bar{t}$. (In Section 4.5. we will identify $R C$ as a subcategory of $t / / l e$.) The $w h-s$-anaphor occurrences of $h e_{n}$ in $\phi$ are those which are not $w h-s$-antecedent occurrences of $h e_{n}$ in $\phi$. The two $w h$-substitution rules
can now be formulated as follows:
(S:AB3) If $\chi \in P_{A}, A \in A B$, then $F_{A B 3, n}(\chi) \in P_{A / e}$.
Condition: $\chi$ contains one or more wh-s-antecedent occurrences of $h e_{n}$, all of which have the same case $c$.
$F_{A B 3, n}(\chi)=\chi^{\prime}$ where $\chi^{\prime}$ comes from $\chi$ by performing the following operations:
(i) if $c=$ nominative then replace the first, else the last, wh-santecedent occurrence of $h e_{n}$ in $\chi$ by wht [who( $c$ )];
(ii) delete all other wh-s-antecedent occurrences of $h e_{n}$ in $\chi$, i.e. replace them by ${ }_{T}[]$;
(iii) anaphorize all wh-s-anaphor occurrences of he ${ }_{n}$ in $\chi$.
(S: AB4) If $\chi \in P_{A}, A \in A B$, and $\delta \in P_{C N}$, then $F_{A B 4, n}(\delta, \chi) \in P_{A / e}$.
Condition: as in (S: AB3).
$F_{A B 4, n}(\delta, \chi)=\chi^{\prime}$, where $\chi^{\prime}$ comes from $\chi$ by performing the following operations:
(i) if $c=$ nominative then replace the first, else replace the last, wh-s-antecedent occurrence of $h e_{n}$ in $\chi$ by wнt [which $\delta(c)$ ] as in (S:AB3);
(iii) as in (S:AB3), taking into account the (number and) gender of $\delta$.

Given these rules (37b)-(39b) can be derived from the corresponding (a)-structures. Two other examples are: ${ }^{20}$
(41) (a) ${ }_{A B}\left[\right.$ who $_{t}\left[{ }_{t}\left[\right.\right.$ wht []$\left.l o v e s \operatorname{him}_{o}\right] \operatorname{and}_{t}\left[{ }_{T}[]\right.$ kisses him $\left.\left.\left.{ }_{0}\right]\right]\right]$.
(41) (b) ${ }_{A B}\left[\right.$ who $_{t}\left[{ }_{t}\left[\right.\right.$ wht $_{T}[]_{l o v e s}^{T}[$ ] $]$ and ${ }_{t}[T[$ kisses whom $\left.]]\right]$.
(42) (a) ${ }_{A B}\left[\right.$ which $\operatorname{girl}_{t}\left[t\left[\mathrm{he}_{o} \operatorname{loves}_{T}[]\right] \operatorname{land}_{t}\left[\right.\right.$ he $_{o} \operatorname{kisses}_{W H T}[$ ] $\left.\left.]\right]\right]$.
(42) (b) ${ }_{A B}\left[\right.$ which $\operatorname{girl}_{t}\left[t\left[\right.\right.$ which man loves $_{T}[$ ]] $\left.\left.\operatorname{and}_{t}\left[{ }_{T}[] \operatorname{kisses}_{\text {whT }}[]\right]\right]\right]$.

The notion of wh-reconstruction plays an essential role in determining the wh-s-antecedent occurrences of a syntactic variable and thereby in the way in which ( $S: A B 3$ ) and ( $S: A B 4$ ) function. Consider again (33): ${ }_{A B}$ [which poem of $\operatorname{him}_{o}$ the ${ }_{o}$ likes best $\left.\left.w_{H T}[]\right]\right]$.

$$
\begin{equation*}
\alpha \quad \beta \tag{33}
\end{equation*}
$$

If the structural notions like $c$-command were not redefined as in (RSP), then both $\alpha$ and $\beta$ would count as wh-s-antecedent occurrences. Together with the 'same case'-condition this means that we could not derive (43):
(43) $A_{A B}\left[\right.$ which poem of $\operatorname{him}_{t}\left[\right.$ which poet likes best $\left.\left.{ }_{W H T}[]\right]\right]$.

However, given the fact that the $c$-command notion used in (WH-S) is redefined as in (RSP), in fact only $\beta$ counts as a $w h-s$-antecedent occurrence in (33), since $\beta$ c-commands (in the old sense) $\alpha$ in the wh-reconstruction of (33). This means that (43) can be derived from (33).

### 4.5. Relative Clauses

We will end Section 4 by indicating how another type of wh-constructions, that of relative clauses, can be treated in this framework. Observe that the kind of expressions formed by ( $S: A B 1$ ) can not only be used to form complements from, but can also be used as relative clauses. Relative clauses are constructed in exactly the same way and are subject to exactly the same constraints (in English at least). So all the relevant examples given above apply here too.

Semantically we can regard relative clauses as abstracts, i.e. predicate denoting expressions, too. So, relative clauses are taken to be constructed from sentences containing a wh-p-antecedent occurrence of a syntactic variable by the first abstract formation rule ( $S: A B 1$ ). This means that the category $t / / / e$, the category of expressions produced by the two preposing abstract formation rules ( $S: A B 1$ ) and ( $S: A B 2$ ), has to be split into two subcategories, $(t / / e) 1$, which contains the results of ( $S: A B 1$ ), and ( $t / / / e$ )2, which contains the results of ( $\mathrm{S}: A B 2$ ). Expressions of the first subcategory can then be used as input in two rules which combine them with a common noun or a term. These rules can be formulated as follows:

$$
\begin{aligned}
& (S: R R C) \text { If } \delta \in P_{C N}, \chi \in P_{(t \mid l l e) 1} \text {, then } F_{R R C}(\delta, \chi) \in P_{C N}, \\
& \text { where } F_{R R C}(\delta, \chi)=\delta \chi .
\end{aligned}
$$

(T:RRC) If $\delta \leadsto \delta^{\prime}, \chi^{\leadsto \rightarrow} \chi^{\prime}$, then $F_{R R C}(\delta, \chi) \leadsto \lambda x\left[\delta^{\prime}(x) \wedge \chi^{\prime}(x)\right]$.
( $S: N R C$ ) If $\alpha \in P_{T}, \chi \in P_{(t / f e) 1}$, then $F_{\text {NRC }}(\alpha, \chi) \in P_{T}$,

$$
\text { where } F_{\text {NRC }}(\alpha, \chi)=\alpha \chi \text {. }
$$

(T:NRC) If $\alpha \rightsquigarrow \alpha^{\prime}, \chi \rightsquigarrow \chi^{\prime}$, then $F_{\text {NRC }}(\alpha, \chi)$

$$
\leadsto \lambda P\left[\alpha^{\prime}\left(\lambda a \lambda x\left[P(a)(x) \wedge \chi^{\prime}(x)\right]\right)\right] .
$$

Rule ( $S: R R C$ ) produces restrictive relative clause constructions, ( $S: N R C$ ) non-restrictive relative clause constructions. Both rules do not, as they stand, account for the necessary agreement in number and gender. This could be handled either by a theory of features as proposed by Landman and Moerdijk (1981) or by a mechanism of subcategorization as proposed by Janssen (1980b).

The two translation rules are straightforward. In fact, the analysis of restrictive relative clause constructions can be regarded as an analysis of
the CN-S type, with this difference that ( $S: R R C$ ) does not take a sentence as such, but an abstract formed from a sentence (see Janssen, 1981, for extensive discussion of the various types of analyses of restrictive relative clause constructions). The semantic part of the analysis of non-restrictive relative clause constructions is in essence the one given by Rodman (1976).

The fact that both types of wh-constructions, viz. relative clause constructions and constituent complements, at a certain level of analysis can be regarded as constructions of the same category, in our opinion supports the existence of the level of abstracts as a separate level of analysis.

## 5. COORDINATION OF COMPLEMENTS

### 5.1. The Need for Complement-Level Terms

In Section 1.8. we argued that the fact that wh-complements and that-complements can be coordinated is an argument in favour of treating them as belonging to the same syntactic category. We have not yet shown how the coordination of complements is to be carried out. The reason for this is that a proper account involves complications which might have obscured the basic principles of our analysis of the semantics of wh-complements. In order to give a proper account of the coordination of complements, one needs to analyze them as a kind of terms, as expressions denoting not propositions as such, but sets of properties of propositional concepts. This 'higher level' analysis is needed to ensure that the following three types of complements come out as they should:
(a) whether $(\phi$ and $\psi) \quad$ 'conjunctive complement';
(b) whether $\phi$ and whether $\psi$ 'conjunction of complements';
(c) whether $\phi$ or $\psi \quad$ 'alternative complement'.

The relation between alternative complements and disjunctive complements, i.e. complements of type whether ( $\phi$ or $\psi$ ), has already been discussed in Section 3.1., examples (9) and (10). A fifth type of complement is disjunction of complements, i.e. complements of type whether $\phi$ or whether $\psi$. They will not be discussed since they are analogous to conjunctions of complements.

The difference between conjunctive complements and conjunctions of complements is clear from the difference in meaning between sentences of the form (44) and (45):
(44) Bill wonders whether ( $\phi$ and $\psi$ ). Bill wonders whether $\phi$ and whether $\psi$.

Whereas (45) implies that Bill wonders whether $\phi$, (44) does not. In other words, (45), but not (44), is equivalent to (46):
(46) Bill wonders whether $\phi$ and Bill wonders whether $\psi$.

This means that conjunctions of complements should be analyzed in such a way that complement taking verbs distribute over the complements which are their conjuncts.

The difference between conjunctions of complements and alternative complements may be a little harder to grasp. At first they may seem equivalent, but we will argue that they are not. Consider the following sentence forms:
(47) Bill wants to know whether $\phi$ or $\psi$.
(48) Bill wants to know whether $\phi$ and whether $\psi$.
(49) Bill knows whether $\phi$.

Obviously, (48) is false if (49) is true. It may seem that this holds for (47) too. However, in our opinion this is not the case without further qualification. The truth of (49) as such does not imply the falsity of (47). That it seems to do so is caused by the implicature carried by alternative complements that (according to the subject) exactly one of the alternatives holds. If (Bill assumes that) either $\phi$ or $\psi$ is true, but not both, then it would indeed follow from (49) that (47) is false. As we already argued in Section 1.7., however, we are dealing here with an implicature, and not with an implication. That it is an implicature is also clear from the fact that it can be cancelled, as is illustrated in the following example:

Bill wanted to know whether Mary, or John, or Peter, or Harry or $\left\{\begin{array}{l}\text { all four of them } \\ \text { several of them }\end{array}\right\}$ witnessed the murder.

Sentence (50) contains an alternative complement of the form whether $\phi_{1}$, or $\phi_{2}$, or $\phi_{3}$, or $\phi_{4}$, or $\phi_{5}$. It is not a contradiction, which means that the implicature that exactly one of the alternatives is true, is cancelled in (50). This means that the truth of (51):
(51) Bill knew that Mary witnessed the murder
is compatible with the truth of (50), as is shown by (52), which is not contradictory:
(52) Already having concluded that Mary witnessed the murder, Bill wanted to know whether Mary, or John, or Peter, or Harry, or all four of them, witnessed the murder
Sentence (52) is not necessarily false. But, to be sure, uttering it one would strictly speaking violate the Gricean maxims. On the other hand, (53) is a contradiction:

Already having concluded that Mary witnessed the murder, Bill wanted to know whether Mary and whether John and whether Peter and whether Harry witnessed the murder.
This means that alternative complements and conjunctions of complements, despite their seeming similarity, express different propositions. The similarity is explained by the fact that if the implicature is not cancelled, then on the assumption of its truth, (49) implies that (47) is false.

An indirect argument which leads to the same conclusion, involves the relation between constituent complements and alternative complements. Semantically, constituent complements are equivalent to alternative complements. In case one deals with a finite (sub)domain and $d_{1}, \ldots, d_{n}$ name all the elements, the alternative complement corresponding to a constituent complement can be written down, as the following pair of sentences illustrates:
(54) Bill investigated who did it.
(55) Bill investigated whether $d_{1}$ did it, or $\ldots$, or $d_{n}$ did it.

Clearly, (54) and (55) are equivalent. Now, again, (56) is not a contradiction:
(56) Already having established that Peter didn't do it, Bill investigated who did it.
Given the equivalence of (54) and (55), this means that (57) isn't a contradiction either:
(57) Already having established that Peter didn't do it, Bill investigated whether Mary did it, ..., or Peter did it, or ....
Like (52), (57), though not necessarily false, may violate the Gricean maxims. Notice that ( 56 ) is much less likely to be in conflict with these maxims than (57). On the other hand, (58) is contradictory:
(58) Already having established that Peter didn't do it, Bill investigated whether Mary did it and whether Harry did it... and whether Peter did it . . . .
And this leads to the same conclusion as above: despite their seeming similarity, which can be explained in terms of implicatures, alternative
complements and conjunctions of complements express different propositions.

### 5.2. Analyzing Complements As Complement-Level Terms

The facts discussed in Section 5.1., in particular the fact that complement taking verbs distribute over the complements which make up a conjunction of complements, point towards a 'higher level' analysis of complements. For different reasons, such a higher level analysis of that-complements is proposed in Delacruz (1976). He argues that thatcomplements are to be analyzed in terms of sets of properties of propositions. In our analysis this comes to considering complements to be expressions which denote sets of properties of propositional concepts. It should be noted that kicking complements upstairs in this way does not change anything fundamental in our semantic analysis. The rule which transforms complements 'old style' into complement terms, i.e. expressions of category $t /(t / \bar{t})=C T$, is as follows:
( $S: C T F$ ) If $\rho \in P_{i}$, then $F_{\text {CTF }}(\rho) \in P_{\text {CT }}$.
(T:CTF) If $\rho \leadsto \rho^{\prime}$, then $F_{C T F}(\rho) \rightsquigarrow \lambda R\left[R(a)\left(\lambda a \rho^{\prime}\right)\right]$, where $R$ is a variable of type $\langle s,\langle\langle s,\langle s, t\rangle\rangle, t\rangle\rangle$.
The reason to keep the intermediary stage of expressions of category $\bar{t}$, is that they are needed as input for a rule which quantifies terms into complements (see Section 4.3.).

The syntactic rule is a category changing rule. The translation rule shows that the complement term formed from a complement $\rho$ denotes the set of properties of the propositional concept expressed by $\rho$. Complement-embedding verbs are now of a higher level too, of course. They are expressions of category IV/CT. The complement-embedding rule remains a simple rule of functional application. Sentence (8) of Section 3.1. is now analyzed as follows:
(59) John knows whether Mary walks, $t$

(59) expresses that an intensional relation of knowing holds between an individual concept and the intension of a set of properties of a propositional concept. The following meaning postulate reduces this highlevel intensional relation to a low-level extensional one, i.e. to a relation between an individual and a proposition.

$$
\begin{aligned}
(M P: I V / C T-E) \quad & \vee M \wedge x \wedge \mathscr{R} \wedge i[\delta(i)(x, \mathscr{R}) \\
& =(\mathscr{R}(i)(\lambda i \lambda r[M(i)(x(i), r(i))]]],
\end{aligned}
$$

$M$ is a variable of type $\langle s,\langle\langle s, t\rangle,\langle e, t\rangle\rangle\rangle ; x$ of type $\langle s, e\rangle ; \mathscr{R}$ of type $\langle s,\langle\langle s,\langle\langle s,\langle s, t\rangle\rangle, t\rangle\rangle, t\rangle\rangle ; i$ type $s ;$ $r$ of type $\langle s,\langle s, t\rangle\rangle$ and $\delta$ is the translation of know, tell, etc.

The substar notation convention is now extended as follows:
$(S N C) \delta_{*}=\lambda i \lambda p \lambda u[\delta(i)(\lambda i u, \lambda i \lambda R[R(i)(\lambda i p)])]$, $p$ is a variable of type $\langle s, t\rangle ; u$ of type $e ; R$ of type $\langle s,\langle\langle s,\langle s, t\rangle\rangle, t\rangle\rangle ; p$ of type $\langle s, t\rangle$.

Combining (MP:IV/CT-E) with (SNC) one can prove that (60) is valid:

$$
\begin{equation*}
\wedge i\left[\delta(i)(x, \mathscr{R})=\left(\mathscr{A}(i)\left(\lambda i \lambda r\left[\delta_{*}(i)(x(i), r(i))\right]\right)\right] .\right. \tag{60}
\end{equation*}
$$

Applying (60), we get the following reduced translation of (59):

$$
\begin{equation*}
\operatorname{know}_{*}^{\prime}(a)\left(j, \lambda i\left[\operatorname{walk}_{*}^{\prime}(a)(m)=\operatorname{walk}_{*}^{\prime}(i)(m)\right]\right) . \tag{59'}
\end{equation*}
$$

This is exactly the same result as we obtained in our low-level analysis. For those verbs, such as wonder, which are extensional in subject position, but intensional in object position, we propose the following meaning postulate which reduces the high-level intensional relation expressed by these verbs to a low-level intensional one.

$$
\begin{aligned}
(M P: I V / C T-I) & \vee N \wedge x \wedge \mathscr{R} \wedge i[\delta(i)(x, \mathscr{R})=\mathscr{R}(i)(\lambda i \lambda r[N(i)(x(i), r)))], \\
& N \text { is a variable of type }\langle s,\langle\langle s,\langle s, t\rangle\rangle,\langle e, t\rangle\rangle .
\end{aligned}
$$

Further, we introduce the following notation convention:
$(C N C) \delta_{+}=\lambda i \lambda r \lambda u[\delta(i)(\lambda i u, \lambda i \lambda R[R(i)(r)])]$.
Combining (MP: IV/CT-I) with (CNC) one can prove that (61) is valid:

$$
\begin{equation*}
\wedge i\left[\delta(i)(x, \mathscr{R})=\mathscr{R}(i)\left(\lambda i \lambda r\left[\delta_{+}(i)(x(i), r)\right]\right)\right] . \tag{61}
\end{equation*}
$$

Given (61) the following is the reduced translation of Bill wonders whether Mary walks:

$$
\begin{equation*}
\text { wonder }_{+}^{\prime}(a)\left(b, \lambda a \lambda i\left[\operatorname{walk}_{*}^{\prime}(a)(m)=\operatorname{walk}_{*}^{\prime}(i)(m)\right]\right) . \tag{62}
\end{equation*}
$$

### 5.3. Complement Coordination

Let us now turn to complement coordination, which necessitates this move to the complement term level (we restrict ourselves to conjunction, the rule for disjunction is completely analogous):
( $\mathrm{S}: \mathrm{CTCO}$ ) If $\Sigma, \Theta \in P_{C T}$, then $\Sigma$ and $\Theta \in P_{C T}$.
( $T: C T C O$ ) If $\Sigma, \Theta \leadsto \Sigma^{\prime}, \Theta^{\prime}$, then $\Sigma$ and $\Theta \leadsto \lambda R\left[\Sigma^{\prime}(R) \wedge \Theta^{\prime}(R)\right]$.
These rules can be illustrated by considering the derivation of the three types of complements (a), (b) and (c):
(a') whether $(\phi$ and $\psi), C T$
$\lambda R[R(a)(\lambda a \lambda i[(\phi|a| \wedge \psi / a /)=(\phi / i / \wedge \psi / i)])]$
whether $(\phi$ and $\psi), \bar{t}$
$\lambda i[(\phi|a| \wedge \psi / a /)=(\phi|i| \wedge \psi \mid i /)]$
(b') whether $\phi$ and whether $\psi, C T$

whether $\psi, C T$
$\lambda R[R(a)(\lambda a \lambda i[\psi / a /=\psi / a /])]$
whether $\psi, \bar{t}$
$\lambda i[\psi|a|=\psi \mid i /]$
(c') whether $\phi$ or $\psi, C T$
$\lambda R[R(a)(\lambda a \lambda i[(\phi / a \mid=\phi / i /) \wedge(\psi|a|=\psi / i /)])]$
whether $\phi$ or $\psi, \bar{t}$
$\lambda i[(\phi|a|=\phi \mid i /) \wedge(\psi|a|=\psi \mid i /)]$
It can be proved that the complement terms ( $a^{\prime}$ ), $\left(b^{\prime}\right)$ and ( $c^{\prime}$ ) denote different sets of properties of propositional concepts. Sentences of the form (44) and (45) are now translated as follows:
(44') Bill wonders whether $(\phi$ and $\psi)$, t wonder ${ }^{\prime}(a)(\lambda a b, \lambda a \lambda R[R(a)(\lambda a \lambda i[(\phi|a| \wedge \psi / a /)=(\phi|i| \wedge \psi / i /)])])$
(45') Bill wonders whether $\phi$ and whether $\psi, t$ wonder' $(a)(\lambda a b, \lambda a \lambda R[R(a)(\lambda a \lambda i[\phi|a|=\phi \mid i /])$

$$
\wedge R(a)(\lambda a \lambda i[\psi / a /=\psi / i /])])
$$

If we apply (MP:IV/CT-I) to these translations, we get the following results:
(44') $\quad$ wonder ${ }_{+}^{\prime}(a)(b, \lambda a \lambda i[(\phi / a / \wedge \psi / a /)=(\phi / i / \wedge \psi / i /)])$

$$
\text { wonder }_{+}^{\prime}(a)(b, \lambda a \lambda i[\phi / a /=\phi / i)
$$

$$
\wedge \operatorname{wonder}_{+}^{\prime}(a)(b, \lambda a \lambda i[\psi / a /=\psi / i /])
$$

Of course, (45') is also the translation of (46).
(46) Bill wonders whether $\phi$ and Bill wonders whether $\psi$

This illustrates that complement-embedding verbs distribute over a conjunction of complements, but the fact that ( $44^{\prime \prime}$ ) does not imply ( $45^{\prime \prime}$ ) shows that they do not distribute over a conjunctive complement.

The difference between (44") and (45') can also be illustrated using the following meaning postulate:

$$
\begin{aligned}
&(M P: I N Q) \wedge x \wedge r \wedge i\left[\delta(i)(x, r) \rightarrow \neg \text { know }_{*}^{\prime}(i)(x, r(i)]\right. \\
& \text { where } \delta \text { is wonder } \\
&+
\end{aligned} \text {, investigate } e_{+}, \text {ask }_{+}, \text {etc. }
$$

Given (MP:INQ), which captures a central part of the meaning of inquisitive verbs, (44') and (45') imply (63) and (64) respectively:

$$
\begin{align*}
& \urcorner \operatorname{know}_{*}^{\prime}(a)(b, \lambda i[(\phi / a / \wedge \psi / a /)=(\phi / i / \wedge \psi / i /)])  \tag{63}\\
& \left.\neg \operatorname{know}_{*}^{\prime}(a)(b, \lambda i[\phi / a /=\phi / i /]) \wedge\right\urcorner \operatorname{know}_{*}^{\prime}(a)(b, \lambda i[\psi / a /=\psi \mid i /]) \tag{64}
\end{align*}
$$

Using the same meaning postulate we can also illustrate the difference between (47) and (48). Using (MP:INQ), (47) implies (65), whereas (48) implies (64):

$$
\begin{equation*}
\urcorner \operatorname{know}_{*}^{\prime}(a)(b, \lambda i[(\phi / a /=\phi / i /) \wedge(\psi / a /=\psi / i /)]) \tag{65}
\end{equation*}
$$

One might think that not just (65), but also the stronger (64) follows from (47). This is, however, again a matter involving implicatures. Although (64) is not an implication of (47), it is an implication of (48). And, as we have seen above, (48) in its turn follows from (47) on the assumption of the truth of implicature that exactly one of the alternatives holds. But that means that (64) follows from (47) too, if this implicature is true.

To sum up, treating complement coordination like we do enables us to account for the differences in meaning between (a), (b) and (c). The facts discussed above show that (45) implies (47) which in its turn implies (44). An interesting fact to note is that in this respect too there is a difference between intensional and extensional complement embedding verbs. Consider (66)-(68):
(66) Bill knows whether John walks and Mary walks.
(67) Bill knows whether John walks and whether Mary walks.
(68) Bill knows whether John walks or Mary walks.

It turns out that (67) and (68) are equivalent and that both imply (66). The equivalence of (67) and (68) may at first sight seem counterintuitive since there are clearly differences between them. However, as we argued above, in Section 1.7., these differences do not concern truth conditional aspects of meaning, but are of a pragmatic nature.

## 6. Two loose ends and one speculative remark

### 6.1. A Scope Ambiguity in Wh-Complements

In this section we will show how a certain type of scope ambiguity can be accounted for in our analysis. A prime example is the ambiguity of sentence (69), extensively discussed in Karttunen and Peters (1980):
(69) Bill wonders which professor recommends each candidate.

In order to facilitate the exposition we will discuss a simpler sentence, (70), and return to (69) at the end of this section:
(70) Bill wonders whom everyone loves.

Following Karttunen and Peters we claim that (70) has three different readings. Two of them, (70a) and (70b), can be obtained in a straightforward way with the rules already available:

$$
\begin{align*}
\text { wonder }_{+}^{\prime}(a)\left(b, \lambda a \lambda i\left[\lambda v \left[\wedge u\left[\operatorname{love}_{*}^{\prime}(a)(u, v)\right]\right.\right.\right. &  \tag{70a}\\
& \left.\left.=\lambda v\left[\wedge u \operatorname{love}_{*}^{\prime}(i)(u, v)\right]\right]\right)
\end{align*}
$$

'Bill wonders who is loved by everyone'.

$$
\begin{align*}
& \wedge u\left[\text { wonder } _ { + } ^ { \prime } ( a ) \left(b, \lambda a \lambda i\left[\lambda v \operatorname{love}_{*}^{\prime}(a)(u, v)\right]\right.\right.  \tag{70b}\\
&\left.\left.\left.=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)(u, v)\right]\right]\right)\right]
\end{align*}
$$

'For each person Bill wonders ${ }^{\prime}$ who is loved by that person'.
(70a) can be obtained by direct construction, (70b) by quantifying everyone into the sentence Bill wonders whom he loves. Given (MP:INQ), (70b) implies that for each person Bill does not know who is loved by that person. This predicts that the following is a contradiction:
(71) Bill knows that Suzy loves only John, but he still wonders whom everyone loves.

Following Karttunen and Peters we assume that (71) is not necessarily false. This means that (70) also has a reading which has a weaker implication than (70b), viz. that Bill doesn't know for each person who is loved by that person. The obvious way to try to obtain readings like this
is to quantify terms not only into sentences but also into complements. For this purpose we add the following rule:
( $\mathrm{S}: Q C$ ) If $\alpha \in P_{T}, \rho \in P_{\bar{i}}$, then $F_{\mathrm{QN} . n}(\alpha, \rho) \in P_{\bar{i}}$.
(T:QC) If $\alpha, \rho \leadsto \alpha^{\prime}, \rho^{\prime}$, then $F_{Q C, n}(\alpha, \rho) \leadsto \lambda i\left[\alpha^{\prime}\left(\lambda a \lambda x_{n}\left[\rho^{\prime}(i)\right]\right)\right]$.
Given these rules a third reading of (70) can be obtained as follows:
(70c) Bill wonders whom everyone loves, $t$ wonder ${ }_{+}^{\prime}(a)\left(b, \lambda a \lambda i\left[\wedge u\left[\lambda v\left[\operatorname{love}_{*}^{\prime}(a)(u, v)\right]\right.\right.\right.$

$$
\left.\left.\left.=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)(u, v)\right]\right]\right]\right)
$$

whom everyone loves, $\bar{t}$


Universal quantification semantically amounts to a (possibly infinite) conjunction. Suppose we are dealing with finite cases so that we can write these conjunctions down. (This is of course not an essential restriction.) Then (70) (a)(b)(c) are equivalent to the conjunctions (70) $\left(\mathrm{a}^{\prime}\right)\left(\mathrm{b}^{\prime}\right)\left(\mathrm{c}^{\prime}\right)$ (in which $d_{1}, \ldots, d_{n}$ name all the individuals):

$$
\begin{align*}
& \text { wonder }_{+}^{\prime}(a)(b, \lambda a \lambda i {\left[\lambda v \left[\operatorname{love}_{*}^{\prime}(a)\left(d_{1}, v\right) \wedge \ldots\right.\right.} \\
&=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)\left(d_{1}, v\right) \wedge \ldots \wedge \operatorname{love}_{*}^{\prime}(a)\left(d_{n}, v\right)\right] \\
&\left.\left.\left.(i)\left(d_{n}, v\right)\right]\right]\right) .
\end{align*}
$$

$$
\begin{gather*}
\text { wonder }_{+}^{\prime}(a)\left(b, \lambda a \lambda i\left[\lambda v\left[\operatorname{love}_{*}^{\prime}(a)\left(d_{1}, v\right)=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)\left(d_{1}, v\right)\right]\right]\right)\right. \\
\\
\wedge \ldots \text { wonder }_{+}^{\prime}(a)\left(b, \lambda a \lambda i\left[\lambda v\left[\operatorname{love}_{*}^{\prime}(a)\left(d_{n}, v\right)\right]\right.\right. \\
\left.\left.=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)\left(d_{n}, v\right)\right]\right]\right) .
\end{gather*}
$$

(70c') wonder ${ }_{+}^{\prime}(a)\left(b, \lambda a \lambda i\left[\left(\lambda v\left[\operatorname{love}_{*}^{\prime}(a)\left(d_{1}, v\right)\right]=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)\left(d_{1}, v\right)\right]\right)\right.\right.$

$$
\left.\left.\wedge \ldots \wedge\left(\lambda v\left[\operatorname{love}_{*}^{\prime}(a)\left(d_{n}, v\right)\right]=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)\left(d_{n}, v\right)\right]\right)\right]\right) .
$$

It can be proved that $\left(70 a^{\prime}\right),\left(70 b^{\prime}\right)$ and ( $70 c^{\prime}$ ) express different propositions. In connection with this, it may be useful to point at the correspondence between (70a') and conjunctive complements, between (70b') and conjunction of complements, and between (70c') and alternative complements.

The implications resulting from application of (MP:INQ) to (70) (a)(b)(c) reflect the intuitions about the differences between the three readings of (70):

```
(70a") \(\neg \mathrm{know}_{*}^{\prime}(a)\left(b, \lambda i\left[\lambda v\left[\wedge u \operatorname{love}_{*}^{\prime}(a)(u, v)\right]\right.\right.\)
    \(\left.\left.=\lambda v\left[\wedge u \operatorname{love}_{*}^{\prime}(i)(u, v)\right]\right]\right)\).
\(\left(70 \mathrm{~b}^{\prime \prime}\right) \quad \wedge u\left[7 \operatorname{know}_{*}^{\prime}(a)\left(b, \lambda i\left[\lambda v\left[\operatorname{love}_{*}^{\prime}(a)(u, v)\right]=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)(u, v)\right]\right]\right)\right]\).
\(\left(70 \mathrm{c}^{\prime}\right) \quad \neg \mathrm{know}_{*}^{\prime}(a)\left(b, \lambda i\left[\wedge u\left[\lambda v\left[\operatorname{love}_{*}^{\prime}(a)(u, v)=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)(u, v)\right]\right]\right]\right)\right.\).
```

It is interesting to note that, like in Section 5.3. and of course for the same reasons, there is a difference between extensional and intensional complement embedding verbs. If the matrix verb is extensional the (c)-reading collapses into the (b)-reading. This result is in accordance with the fact that (71), in contrast with sentence (70) has only two readings:

Bill knows whom everyone loves.
The results of quantifying into the sentence and the complement respectively are:
(71b) $\wedge u\left[\operatorname{know}_{*}^{\prime}(a)\left(b, \lambda i\left[\lambda v\left[\operatorname{love}_{*}^{\prime}(a)(u, v)\right]=\lambda v\left[\operatorname{love}_{*}^{\prime}(i)(u, v)\right]\right]\right)\right]$.
(71c) $\operatorname{know}_{*}^{\prime}(a)\left(b, \lambda i\left[\wedge u\left[\lambda v\left[\operatorname{love}_{*}^{\prime}(a)(u, v)\right]=\lambda v[\operatorname{love}(i)(u, v)]\right]\right]\right)$.
We leave it to the reader to verify that (71b) and (71c) are indeed equivalent, stressing the fact that this equivalence is essentially due to the fact that (71b) and (71c) concern relations between individuals and propositions, and not, as (70b) and (70c) do, relations between individuals and propositional concepts.

This difference between extensional and intensional complement embedding verbs also accounts for the fact that (72) is equivalent with (73) and with (74) on the reading where everyone has widest scope (but see the remarks in Sections 1.5. and 3.4.), whereas (75) is not equivalent with (76) (nor with (77) on the reading with everyone having widest scope):
(72) Bill knows who walks.
(73) Of everyone, Bill knows whether he/she walks.
(74) Bill knows whether everyone walks.
(75) Bill wonders who walks.
(76) Of everyone, Bill wonders whether he/she walks.
(77) Bill wonders whether everyone walks.

Notice that despite the equivalence of (72) and (73), (78) and (79) need not be equivalent:
(78) Bill knows which man walks.
(79) Of every man, Bill knows whether he walks.
(78) and (79) are equivalent only if (78) is read de re. Analogously, (70),
on its reading ( 70 c ), is equivalent to (80), but (82) is equivalent to (81), on its third reading, only if (82) is read de re:
(70) Bill wonders whom everyone loves.
(80) Bill wonders whom who loves.
(81) Bill wonders whom every man loves.

Bill wonders whom which man loves.
This means that quantifying a term into a complement always results in a de re reading of the common noun contained in the term (if any). So our approach predicts that (69) is equivalent to one reading of (83), viz. the one in which which candidate is read de re:
(69) Bill wonders which professor recommends each candidate.

Bill wonders which professor recommends which candidate.
Whether this is a completely satisfactory result is, to be honest, beyond the scope of our intuitions.

### 6.2. Wh-Complements in an Extension of IL

In Section 2.5. we said that one can get a long way in the analysis of complements by adding a new intensional operator to IL. As a matter of fact, one could come quite as far as the end of Section 5 , since the phenomena that resist an adequate treatment in such an intensional language are phenomena like those discussed in the previous Section 6.1.

The new operator, called $\Delta$, can be introduced in IL as follows:

$$
\begin{align*}
& \text { If } \alpha \in M E_{a} \text {, then } \Delta \alpha \in M E_{(s, 1)}  \tag{i}\\
& \llbracket \Delta \alpha \rrbracket_{M, k, g} \text { is that } \mathbf{p} \in\{0,1\}^{I} \text { such that for every } \mathbf{i} \in I: \\
& \qquad \mathbf{p}(\mathbf{i})=1 \text { iff } \llbracket \alpha \rrbracket_{M, k, g}=\llbracket \alpha \rrbracket_{M, i, k} .
\end{align*}
$$

With the aid of $\Delta$, the translations of the complement formation rules discussed in Section 3 can be formulated as follows:

```
( \(T: T H C\) ') If \(\varphi \leadsto \varphi^{\prime}\), then that \(\varphi \leadsto{ }^{\prime} \varphi\) '.
( \(T: W H C^{\prime}\) ) If \(\varphi \leadsto \varphi^{\prime}\), then whether \(\varphi \leadsto \Delta \varphi^{\prime}\).
( \(T: W H C^{\prime}\) ) If \(\varphi_{1}, \ldots, \varphi_{n} \leadsto \varphi_{1}^{\prime}, \ldots, \varphi_{n}^{\prime}\), then whether \(\varphi_{1}\), or \(\ldots\), or
    \(\varphi_{n} \leadsto \rightarrow \Delta \lambda p\left[{ }^{\wedge} p \wedge\left[p={ }^{\wedge} \varphi_{1}^{\prime} \vee \ldots \vee^{\wedge} \varphi_{n}^{\prime}\right]\right]\).
( \(T: C C F\) ) If \(\chi^{\leadsto>} \chi^{\prime}\), then \(F_{C C F}(\chi) \leadsto \Delta \chi^{\prime}\).
```

The phenomena that cause this approach to fail have in common that their treatment requires the possibility to quantify terms into complements. An example of such a phenomenon is the 'third reading' of sentence (20), mentioned in Section 6.1. Another example is the reading
of (84)
John will tell whether every president walks,
in which the term every president has narrow scope with respect to the tense, but wide scope with respect to the complement. On this reading (84) is true if at some time in the future John tells of every individual which at that time is a president whether he or she walks or not.

In order to obtain these readings, we need to be able to quantify terms into complements. This rule of quantification ( $S: Q C$ ) and its translation rule ( $T$ : QC) were stated in Section 6.1.:

```
( \(R: Q C\) ) If \(\alpha \in P_{T}\), and \(\rho \in P_{\bar{t}}\), then \(F_{Q C, n}(\alpha, \rho) \in P_{\bar{t}}\).
( \(T: Q C\) ) If \(\alpha, \rho \leadsto \alpha^{\prime}, \rho^{\prime}\), then \(F_{Q C . n}(\alpha, \rho) \leadsto \lambda i\left[\alpha^{\prime}\left(\lambda \alpha \lambda x_{n}\left[\rho^{\prime}(i)\right]\right)\right]\).
```

The difficulty in formulating a translation rule in IL $+\Delta$ is that we cannot express the equivalent of $\rho^{\prime}(i)$. We can only express the equivalent of $\rho^{\prime}(a)$, namely, ${ }^{\vee} \rho^{\prime}$. (Notice that ${ }^{\vee} \Delta \alpha$ expresses the proposition that is true at every index.) In $I L+\Delta$ we could only arrive at the translation rule:

$$
\left(T: Q C^{\prime}\right) \text { If } \alpha, \rho \leadsto \alpha^{\prime}, \rho^{\prime} \text {, then } F_{Q C^{\prime}, n}(\alpha, \rho) \leadsto \wedge\left[\alpha^{\prime}\left(\wedge \lambda x_{n}\left[{ }^{\vee} \rho^{\prime}\right]\right)\right] .
$$

If $\psi^{\prime}$ is of the form $\Delta \alpha$, the resulting expression denotes a proposition that holds true at every index, instead of denoting a proposition in the required index dependent way.

### 6.3. Remark on the Semantics of Direct Questions

At the beginning of this paper, we expressed the hope that an adequate semantics of wh-complements might give a clue to the semantics of direct questions as well. At first sight, it seems that little or nothing speaks against simply associating direct questions with the same semantic objects we associated wh-complements with. An objection that might come to mind is this. Suppose $\varphi$ is true. Then the direct questions Does John know whether $\varphi$ ? and Does John know that $\varphi$ ? denote the same proposition. Wouldn't this mean that asking the first question comes to the same thing as asking the second one? No, no more than that asserting a declarative sentence $\varphi$ comes to the same thing as asserting a declarative sentence $\psi$ in case $\varphi$ and $\psi$ happen to have the same truth value. Although the denotations of the two questions are the same, their senses still are different.

Another interesting issue is to what extent we could consider the proposition denoted by a question to be the proposition expressed by an answer to it. At first sight, it seems to make a good deal of sense to say
that the proposition denoted by a question at a given index, is the proposition expressed by a true answer to that question at that index, and that hence the sense of a question could be described as a function from indices to true answers. However, things are more complicated. Compare the following sentences:
(85) Who won the Tour de France in 1980?
(87) The one who ended second in 1979 won the Tour de France in 1980.

Of course, (86) is a true answer to (85). However, in many cases (87) counts as a true answer as well. But it cannot be the case that both (86) and (87) express the proposition denoted by (85), since (86) and (87) clearly express different propositions. In our analysis, (86) expresses the proposition denoted by (85). In order to grant (87) the status of answerhood as well, one would need some property, in between 'denoting the same truth value' and 'expressing the same proposition', which (86) and (87) share. Such a property requires something in between truth values and possible worlds. It could very well be that the notion of possible fact, in the sense of Veltman (1981), is what is needed. One might then take a declarative sentence to be an answer to a question iff the possible fact expressed by the sentence is in some way related to the proposition denoted by the question. Then (86) and (87) would both qualify as answers to (85), since although they do not express the same proposition they do presumably express the same possible fact. It should be noted that this would not involve a change in the semantics of questions, it would be a refinement of the semantics needed for a satisfactory account of the property of answerhood (and probably of many other things besides).

So, we conclude that it is misleading to interpret the proposition denoted by a question as the unique true answer to it. ${ }^{21}$ Both (86) and (87) should count as answers to (85). In fact, we believe that (86) should not even be granted a special status, even though it expresses the same proposition as (85) actually denotes. For there are situations in which (87) is a better answer to (85), for example by being more informative, than (86) is. In our opinion, this holds quite generally. Within the semantic limits set by the denotation of a question, what counts as a good answer is determined by pragmatic factors. These concern, among other things, the information available to the hearer, the information of the speaker about the information of the hearer, etc. ${ }^{22}$
Pragmatic considerations again are all important in the following
example:
(88) Where can one buy Italian newspapers?
(89) At the Centraal Station (one can buy Italian newspapers).
(90) At the Atheneum Newscentre (one can buy Italian newspapers).

Clearly, there are situations in which each of (89) and (90) on its own constitutes a proper answer to (88). But the propositions expressed by (89) and (90) are only part of (entailments of) the proposition denoted by (88). Some have taken this to show that questions are ambiguous between an existential (examplificatory) and a universal (exhaustive) reading. This runs counter to the exhaustiveness, even to the lowest degree, which we ascribe to wh-complements. Like Karttunen, we feel that again this is a pragmatic rather than a semantic phenomenon. Whether a question asks for a complete answer or for an incomplete one, depends on the needs of the one asking it. For example, (88) when asked by an Italian tourist is properly answered, at least in most cases, by indicating one place where Italian newspapers are sold: what the tourist wants is a newspaper. (This does not mean that (89) and (90) in every such situation are equally good; other pragmatic factors, such as the acquaintance of the questioner with the various locations, etc. may be involved.) But when (88) is asked by someone who is interested in setting up a distribution network in Amsterdam for foreign newspapers, clearly an exhaustive answer to (88) is called for. So again, what counts as an answer is determined by pragmatic factors within the limits set by the semantics of the question.

Of course, these are just a few, rather speculative remarks, and a lot more has been (and still should be) said on these matters. But they seem to lead us to the conclusion that no semantic theory on its own can be expected to provide a satisfactory account of question-answer relations. Evidently, a pragmatic theory is called for. However, such a theory should be based on an adequate semantic theory. It is our hope that the semantic theory of wh-complements developed in this paper contributes to the survey of the semantic space within which pragmatic factors determine the question-answer relationship.

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## NOTES

[^0]Johan van Benthem, Theo Janssen, Lauri Karttunen and the anonymous referees of Linguistics and Philosophy for their comments and criticisms on earlier versions, which have led to many improvements.
' We are told by one of the referees that David Lewis has developed a similar idea concerning whether-complements in an unpublished paper. We have not seen the paper. therefore we are unable to draw a comparison.
[Added in proof: In the meantime we have obtained a copy of a recent version of Lewis' 1974 note, which under the title 'Whether' report is to appear in a Festschrift of which the publication data are not known to us. In this paper, Lewis discusses the index dependent character of whether-complements and proposes an analysis in terms of double indexing. We cannot argue for it here, but we feel that Lewis' analysis, in which whether-complements are taken to be expressions of sentence type, is less natural and less general than ours, in which they are considered to denote propositions. In particular, by taking the sense of complements to be propositional concepts, our analysis solves the problems with intensional (see Section 1.3.) complement embedding verbs which Lewis' proposal runs into.]
${ }^{2}$ In order to avoid terminological confusion, let us point out that the way we use the terms 'extensional' and 'intensional' here, is a generalization of the terminology used in PTQ which does not fully conform to the traditional use. So, know is extensional in our sense of the term since it operates on the denotation of the complement that is its argument. But it is intensional in the traditional sense since the denotation of a complement is an intensional entity, viz. a proposition.
${ }^{3}$ If their conclusions are read de re, these arguments are valid. If their conclusions are read de dicto, however, they are not. It turns out that the combination of treating proper names as rigid designators and verbs such as know as relations between individuals and propositions does not make it possible to distinguish a de dicto reading of the conclusions of these arguments. This is not correct, it should be possible to distinguish a de dicto reading of these sentences, while maintaining a rigid designator view of proper names at the same time.
${ }^{4}$ Complements of this form are ambiguous between an alternative and a yes/no reading. The latter might be indicated as whether ( $\varphi$ or $\psi$ ). In Section 3.1. we show how this ambiguity is accounted for. In (IX) the alternative reading is meant.
${ }^{5}$ That this is so, can be seen from the fact that the same phenomenon can be observed with other types of sentences. For example, it is not unreasonable to distinguish between a de dicto and a de re reading of the sentence John believes that everyone walks. Its de re reading would be true iff John believes of every individual that is in the domain of discourse that he/she walks, whereas its de dicto reading would be true iff John believes of every individual that according to him is in the domain of discourse that he/she walks. Yet within a possible world semantics, this distinction can be made only if one allows for varying domains in some sense. Since we are dealing here with a general problem of the semantics of propositional attitudes within an intensional semantic framework, and not with a problem that is specific to finding a correct semantics for wh-complements, and since this paper is about the latter and not about the former, we will not try to solve it here.
${ }^{6}$ Karttunen discusses argument (X). His reasons for not accepting (X) as valid accord with our remarks in the previous section on the type of situations that can give rise to counterexamples against (X). However, unlike Karttunen, we do not interpret the possibility of counterexamples as an argument against strong exhaustiveness.
${ }^{7}$ For a proposal which makes it possible to consider infinitival complements to be proposition denoting expressions as well, see Groenendijk and Stokhof (1979).
${ }^{8}$ There still remains the verb know which takes NP's, as in John knows Mary. An argument in favour of regarding this verb to be different from the one taking complements might be that in such languages as German and Dutch the difference is lexicalized. On the other hand, in a sentence like John knows Mary's phone number, the verb know seems to be quite like the complement taking know in many respects. (See also Note 10.)
${ }^{9}$ As a matter fact, Karttunen argues against Hintikka's analysis (in Hintikka, 1976) by pointing out that John wonders who came cannot be paraphrased, as Hintikka would have it, as Any person is such that if he came then John wonders that he came. Unlike such verbs as guess and matter, wonder seems to be a truly ambiguous lexical item (in other languages, e.g. in Dutch, the difference in meaning is lexicalized). What arguments like the one used in the text and the one used by Kartunen in our opinion really show is that there is an essential difference between extensional and intensional complement embedding verbs, and that Hintikka's analysis fails for the intensional ones.
${ }^{10}$ The possibility of constructing these proposition denoting expressions from expressions $\alpha$ of arbitrary type is quite interesting also in view of sentences like John knows Mary's phone number, mentioned in Note 8. If we simply apply procedure (5) with the translation of the term Mary's phone number substituted for $\alpha / a /$ we seem to obtain exactly the proposition John needs to know if he is to know Mary's phone number. The point was brought to our attention by Barbara Partee.
${ }^{11}$ Notice that in PTQ complements are in fact taken to be of category $t$. When embedded under complement taking verbs, we semantically apply the interpretation of the verb to the sense of the complement. This makes that proposition denoting expressions do occur in PTQ translations. Because of this, one might think that the new category $\bar{t}$ is superfluous. But it is not, since we want complements to denote propositions and to have propositional concepts as their sense.
${ }^{12}$ For those who find it unbearable, c.q. unnatural, that the translation of whether $\varphi$ or $\psi$ does not contain a disjunction, we present the following equivalent alternative:

$$
\left(\begin{array}{rl}
\left(T: W H C^{\prime}\right) & \lambda i\left[\lambda p\left[p(a) \wedge\left[p=\lambda a \varphi_{i}^{\prime} \vee \ldots \vee p=\lambda a \varphi_{n}^{\prime}\right]\right]\right. \\
& \left.=\lambda p\left[p(i) \wedge\left[p=\lambda a \varphi_{i}^{\prime} \vee \ldots \vee p=\lambda a \varphi_{n}^{\prime}\right)\right]\right] .
\end{array}\right.
$$

${ }^{13}$ For those complement embedding verbs for which (MP:IV/ $\bar{t}$ ) is not defined (i.e. the intensional ones), (11) holds trivially in case they are combined with a that-complement, since the sense of a that-complement is a constant propositional concept.
${ }^{14}$ As (12) shows, whether-complements resemble if then else statements of certain programming languages. In Janssen (1980a) the latter are used as counterexamples to the validity of cap-cup elimination in IL. It seems that wh-complements are natural language counterexamples. If $\rho$ translates a wh-complement, then $\lambda a(\rho(a)) \neq \rho$, i.e. " $\rho \neq \rho \rho$.
${ }^{15}$ Engdahl in Engdahl (1980) presents a modification of Karttunen's framework in which a kind of de dicto readings can be obtained by means of a special storage mechanism. However, it turns out that, in order to obtain correct results, restrictions on the order of quantification of ordinary terms and wh-terms are necessary. But this means that in her framework too, a special level of analysis in between sentences and complements has to be distinguished.
${ }^{16}$ Notice that condition (ii) allows the derivation of (i) (a) from (i), though it blocks (i) (b):
The man $_{R C}\left[\right.$ whot $\left[\right.$ wht $\left[\right.$ ]loves him $\left.{ }_{0}\right]$ kisses him ${ }_{o}$.

(i) (b) ${ }_{A B}\left[\right.$ whom $_{t}\left[\right.$ the $\operatorname{man}_{R C}\left[\right.$ who $\left.\left.\left._{t}\left[w H T[] l o v e s s_{T}[]\right]\right] \mathrm{Kisses}_{W H T}[]\right]\right]$.

Structures like (i) (a) are not generally considered to be well formed. These are problematic cases having to do with cross-over phenomena, which are not dealt with here and Wjich, to our knowledge, present a problem to any account of wh-constructions.

Of course, there is more to the antecedent-anaphor relation than c-command (see Landman and Moerdijk (1981) for an extensive discussion within the Montague framework). In the case discussed here, a consequence of using c-command and wh-reconstruction is that (i):
(i)

Which picture that John saw, he likes best
cannot be obtained with coreferentiality of John and he. How these and related problems are to be solved, is quite unclear.
${ }^{18}$ It is sometimes claimed, e.g. in Engdahl (1980), that a structure like (35a) has to be ambiguous, since the related direct question allows for two different kinds of answers: functional ones like his last, and pair-list ones like: Gorter, 'Mei'; Kouwenaar, 'Elba'; Gerhardt, 'In tekenen'. For a long time we have thought, following Bennett (1979), that functional readings could be regarded as a kind of shorthand for pair-list ones, and that only the latter would have to be accounted for in the semantics. However, in view of Engdahl's arguments and in view of such expressions as (i) and (ii):
(i) which woman no man loves
(ii) which woman few men love
which do not have a pair-list reading, but only a functional one (beside the direct reading), we are convinced now that functional readings are independent of pair-list ones. Moreover, they do not only occur with structures like (35a), but as (i) and (ii) show, are a quite general phenomenon. In Groenendijk and Stokhof (to appear), we propose to analyze functional readings by means of Skolem-functions. Abstract (35a) for example is then translated as (35a') and (i) as ( $i^{\prime}$ ):

$$
\begin{align*}
\left(\mathrm{a}^{\prime}\right) & \lambda f[  \tag{35}\\
& \left.\wedge u\left[\operatorname{poem-of}_{*}^{\prime}(a)(u, f(u))\right] \wedge \wedge u\left[\operatorname{poet}_{*}^{\prime}(a)(u) \rightarrow \text { like-best } t_{*}^{\prime}(a)(u, f(u))\right]\right] .  \tag{i'}\\
& \left.\wedge u\left[\operatorname{woman}_{*}^{\prime}(a)(f(u))\right] \wedge \wedge u\left[\operatorname{man}_{*}^{\prime}(a)(u) \rightarrow 7\left[\operatorname{love}_{*}^{\prime}(a)(u, f(u))\right]\right]\right] .
\end{align*}
$$

In these formulas $f$ is a variable ranging over functions from individuals to individuals. Complements are formed from these expressions in the usual way.
${ }^{19}$ Our notion of wh-reconstruction thus serves syntactical purposes only. In this respect it seems to differ from related notions, e.g. the one proposed in van Riemsdijk and Williams (1980), where it plays a role in establishing the logical form of wh-constructions.
${ }^{20}$ Actually clause (i) in (S:AB3,4) may be a bit too strict, since who loves whom and kisses him is well-formed, but cannot be derived here.
${ }^{21}$ Belnap calls this 'the unique answer fallacy' (see Belnap, 1982). We agree with him that it is a mistake to think that every question has in every situation a unique true answer. But we have a different diagnosis as to how and where this has to be accounted for. We cannot do justice here to the many interesting arguments Belnap puts forward, but as will become clear from what follows, we feel that there is far more pragmatics between questions and answers than is accounted for in Belnap's theory.
${ }^{22}$ A framework in which this kind of information of language users can be formally represented can be found in Groenendijk and Stokhof (1980) and van Emde Boas et a!. (1981).

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