Time Remains

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December 12, 2013

Abstract

How should one understand the implications of general covariance for the role of time in classical theories of gravity? On one popular view, the essential lesson is that change is relational in a strong sense, such that all it is for a physical degree of freedom to change is for it to vary with regard to a second physical degree of freedom. At a quantum level, this view of *change as relative variation* leads to a *fundamentally timeless* formalism for quantum gravity, with the universe eternally frozen in an energy eigenstate. Here we will start from at different interpretation of the classical theory, and, in doing so, show how one may avoid this acute 'problem of time' in quantum gravity. Under our view, *duration is still regarded as relative, but temporal succession is taken to be absolute*. This approach to the classical theory of gravity forms the basis for an alternative *relational quantization* methodology, such that it is possible to conceive of a genuinely dynamical theory of quantum gravity within which time, in a substantive sense, remains. This paper accompanies a more technical paper [1], with which it may be read in parallel.

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We should not be able to tell the story of our relations with another, however little we knew him, without registering successive movements in our own life. Thus every individual – and I myself am one of those individuals – measured duration by the revolution he had accomplished not only round himself but round others and notably by the positions he had successively occupied with relation to myself.

–Marcel Proust
Time Regained [2]

1 Introduction

1.1 The Problem of Time

A key feature of Einstein's theory of gravity is its invariance under arbitrary transformations of the coordinate labels used to refer to events in spacetime. This *general covariance* symmetry implies that only the coordinate-free information contained in the geometry has a physical basis within the theory. Unfortunately, it is not entirely clear how one should understand the implications of general covariance for the specific role of time in the theory. In the Lagrangian formulation, where the theory is expressed in terms of the Einstein-Hilbert action, this difficulty manifests itself in our inability to find a representation of time in terms of an action of the real numbers implementing time translations on the space of physical (i.e., coordinate invariant) solutions.[1] Similarly, in the Hamiltonian formulation of the theory, which is the basis for many modern approaches to the quantization of gravity, we find ourselves lacking a coordinate free means of representing time (see [4] for a classic or [5] for an updated review).

Imagine a loaf of bread that we can irregularly cut up into a sequence of slices. The loaf is spacetime and the slices are instantaneous spatial surfaces. A *foliation* is then a local parameterization of a spacetime by a time ordered series of spatial slices. General covariance implies that spacetimes described by general relativity which are related by *re*-foliations are physically equivalent. Within the Hamiltonian formulation, which dates back to Dirac [6], general covariance is implemented in two parts: i) spatial coordinate invariance; and ii) spacetime foliation invariance.[2] We thus have within the theory an ability to re-slice a spacetime into an infinite number of different decompositions of space and time without changing anything physical. It is the conceptual and technical complications involved in representing this symmetry that leads to the acute 'problem of time' within the formalism.

Foliation symmetry further implies that any observable quantity within the theory must not be dependent upon the local temporal labelling of spacetime.[3] This leads us directly to the question of how we should understand the *change* in physical quantities? In addition to not having a representation of time, we seem also to have lost a clear methodology for representing change! Our conceptual machinery appears in need of retooling.

According to the *correlation* or *partial observables* view of time in general relativity, the radical moral one should draw from general covariance is that *change is relational* in a strong sense, such that all that it is for a physical degree of freedom to change is for it to vary with regard to a second physical degree of freedom – and there is no sense in which this variation

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[1]See [3] for an extensive discussion of this and related points regarding the representation of time and change within Lagrangian field theories, including general relativity. With regard to the Hamiltonian framework, the present analysis differs on several key interpretational and formal points.

[2]Note that this does not constitute the full group of spacetime diffeomorphisms since neither large diffeomorphisms nor diffeomorphisms that fail to preserve the space-like embedding of hypersurfaces are represented canonically.

[3]Formally this is usually expressed in terms of the requirement that the functions which represent observable quantities should commute with the Hamiltonian constraints which implement foliation invariance [7].

can be described in absolute, non-relative terms.[4] This view implies that there is no unique parameterization of the time slices within a spacetime, and also that there is no unique temporal ordering of states. Furthermore, it implies a fundamentally different view of what a degree of freedom actually is – such parameters no longer have distinct physical significance since they can no longer be understood as being free to change and be measured independently of any other degrees of freedom. This means that all one-dimensional systems must be understood as stationary since a relational notion of change cannot be constituted: there is no degree of freedom for the system to change with respect to. A one-dimensional pendulum is thus, under this understanding of dynamics, a stationary system with no genuine degrees freedom. And a two-dimensional pendulum is to be understood as a one-dimensional system, with the change in the (arbitrarily chosen) free variable expressed in terms of the other, 'clock' variable.

On this first view, it should be no great surprise that when the equations of a classically foliation-invariant theory are quantized, one arrives at a timeless quantum gravity formalism[5] – since, in essence, this facet is already implicit within the classical theory. Both classically and quantum mechanically, the functions that faithfully parametrize the true degrees of freedom of the theory – the observables – are taken to be those which are completely independent of the local time parametrization and, both classical and quantum mechanically, these *perennials* cannot by definition vary along a dynamical trajectory. Thus, we see that this first response to the problem of time in classical and quantum gravity is essentially one of capitulation. We deal with the problem of representing time and change in a generally covariant manner by giving up on the concepts in a substantive sense. To us this seems unsatisfactory as a solution, and in the remains of this paper we will articulate an alternative.

1.2 Our Solution

The starting point of our approach is the conviction that the radical variant of relationalism with regard to change and time discussed above has gone a little too far. The lessons for time drawn from general covariance are more subtle, and imply that, *while duration is relative, succession is absolute*. In essence, by this we mean that our formalism should be such that both the change in a given degree of freedom and the ordering of such change along a dynamical history[6] are fundamental structures in the theory – it is only the labelling of change that is arbitrary, not the change itself. Such a *Machian* view of classical theory is simple both to motivate and to realise within particle models with global temporal re-labelling symmetry but far less easily constituted within the full general theory of relativity due to the need to define observables that respect foliation invariance. It is therefore understandable that the radical morals with regard to change and time discussed above are often drawn. *However*, it is still true that within general relativity there exist fundamental temporal structure relating to ordering in time. In canonical general relativity, such structure is encoded within fact that the arbitrary slicings are always in terms of a (local) *monotonically increasing* time parameter (as implied by the positivity of the lapse multiplier). It is also present in a more subtle sense within the Lagrangian theory due to the form of the Einstein-Hilbert action. This is because the relevant variational principle requires finding a curve that minimizes the integral of the scalar curvature over the curve, and these curves, by definition, require a parametrization by monotonically increasing parameter.

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[4]The correlation view is most closely associated with the work of Carlo Rovelli [8, 9] and is put forward in slightly different ways by many physicists working on canonical quantum gravity. For a more detailed appraisal of the strengths and weaknesses of this view in the classical gravity context see [10].

[5]This is the Wheeler–DeWitt-type 'frozen formalism', endemic within both the old quantum geometrodynamics approach [11], and modern variants of canonical quantum gravity [12].

[6]It is important to note here that by 'time ordering' we specifically *do not* mean anything like an arrow of time, or objective difference between past and future. Rather, 'time ordering' here, and in what follows, merely implies the existence of a monotonically increasing parameterization of time-slices which is, by definition, time-reversal invariant.

Thus, the formalism of general relativity should not be seen as telling us to dispense with time altogether.

Furthermore, on our view, that the (canonical) quantization of general relativity leads to a timeless formalism should be understood as a consequence of an incorrect treatment of the temporal symmetries of the classical theory. By treating local temporal labellings as *entirely* unphysical, and change as *entirely* relational, we do not retain in the quantum formalism the full classical dynamics or the implicit temporal ordering structure.[7] The question remains, however, if conventional quantization techniques cannot preserve the essential temporal structure of general relativity, how do we find a new methodology that can?

Our solution involves two non-trivial steps, the motivation for which will be outlined at length below. The first relates to a fundamental re-description of gravity in terms of the *shape dynamics* formalism originally advocated by Barbour [13, 14] and collaborators [15, 16] and then brought into modern form in [17]. From the view of the current paper (see also the more detailed argument of [1]) the existence of shape dynamics as, in a precise sense, a *dual* to general relativity reveals classical gravity to be essentially *Janus-faced*. There are two distinct gravitational ontologies: the traditional 'Einstein ontology' of spacetimes invariant under four dimensional coordinate transformations; and a second, hitherto masked, ontology of sequences of *scale-invariant* three dimensional spatial surfaces (i.e, spatial geometries invariant under re-scallings of lengths). As shall be detailed later, this second ontology is closely related to a proposal for the interpretation of the degrees of freedom of gravity made in the nineteen eighties by James York [18], and so we will call it the 'York Ontology'.

Figure 1. Janus was the Roman god of gateways, transitions and time, and is usually depicted as having two, non-identical faces pointing in opposite directions as above. Thus, figuratively (and in fact geometrically) the two faces of Janus are an apt representation for the two faces of gravity.

Given this understanding of gravity as having two dual faces, when confronted with the problem of understanding the role of time in classical gravitation, one has the option of choosing whichever formalism – shape dynamics or general relativity – is formally and conceptually easier to work with. Here, we choose to use shape dynamics, and assume the particular characterization of the theory is given by the York ontology. It is from this basis that the second step in our proposal can be made. Originally in [19] and then more recently in [1], a procedure for the *relational quantization* of theories with temporal relabelling symmetries was outlined. Whereas, the first of these two papers served to offer a range of conceptual arguments leading to the need for relational quantization, the second served to place relational quantization within a formal framework for understanding symmetries in physical theory in general.

One of the principal motivations of the current paper is to explicate further the philosophical foundations of this approach to symmetries and time. In particular, in what follows we will: First, introduce a general methodology for the classification of symmetries and symmetry-related variables according to physically motivated criteria, see §2.1 ; Second, demonstrate that

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[7]Equivalently, in more formal language, if Hamiltonian constraints are treated as generating purely unphysical transformations one does not, in the quantum theory, retain their role in generating dynamics from a given initial data hyper-surface, nor in providing a temporal ordering.

our classification scheme leads naturally to a procedure for quantization via the introduction of dummy variables, see §2.2; Third, provide philosophical motivation for the Machian view of time discussed above, see §3.1; Fourth, show how our philosophical motivations for the treatment of time symmetries mesh with our general prescriptions for symmetry and lead to the procedure for relational quantization, see §3.2. These arguments establish a framework sufficient to motivate the relational quantization of gravity from the perspective of the York ontology, see §4.1 and §4.2, and in doing so provide a demonstration that, given suitable starting assumptions, time can remain in quantum gravity.

2 Understanding Symmetry

2.1 Freedom by Degrees

We can understand a physical theory as giving us a 'picture of the world' in that it associates a set of mathematical and empirical structures to a particular ontology. Generally, this is done only for a very specific type of phenomena – e.g., electromagnetic, thermodynamic, gravitational – and so the picture is in fact not really of 'the world' but rather of a very specific sub-division of 'the world'. This sub-division is identified with the system that the theory describes and to which – it is hoped – its predictive power can reliably be applied. In classical terms, the relationship between these systems and their theoretical representation is contained within the specification of three pairs of structures, one of each pair formal and one of each pair physical. These pairs relate in turn to degrees of freedom, dynamical laws and the specifiable initial (or boundary) data.

Explicitly, for a finite dimensional classical system, we can consider a physical system as being represented formally by: i) a configuration space with n-configuration variables and some pre-determined metric structure; ii) some specific *nomological* restrictions on the (geodesic) variation of curves; iii) a set of further initial (or end-point) conditions on the curves. Such a specification serves as a representation of: a) the physical degrees of freedom; b) the dynamical law which determines the evolution of the system; and c) the physical conditions on the preparation of the system. The representational pairings are then i-a, ii-b, iii-c. We will designate the formal side of these pairings the *formalism* of the system, and the physical side the *characteristic behaviour* of the system.

One can be more explicit with regard to the formalism side of the set up by defining the *Lagrangian formalism*. We first introduce a velocity variable for each configuration variable (formally these are tangent vector spaces), and then collect everything together to form a velocity-configuration space (tangent bundle). The combination of a special function on this space (the Lagrangian) and a variational principle telling us to minimise the integral of this function along any curve with fixed endpoints (the principle of least action) then picks out the dynamical curves. We now have a formal representation of physical degrees of freedom in terms of the velocity and configuration variables; the relevant nomological restrictions, in terms of the variational principle and Lagrangian; and the preparation conditions, in terms of the specification of the end points of the curves in the variation. An alternative, but generally equivalent, *Hamiltonian formalism* can then be derived by defining momentum variables (cotangent vector spaces) based upon the velocity variables and forming a phase space (cotangent bundle) in terms of positions and momenta variables which now play the role of representing the physical degrees of freedom. The relevant nomological restrictions, which represent the dynamical laws, are then given by the association of a direction to each point in phase space (vector field) defined in terms of the relevant energy function, the Hamiltonian. For the purposes of this formalism, the preparation conditions are given simply by specification of initial values of the positions and momenta variables.

All this is familiar to anyone with basic knowledge of mathematical physics. It is important,

however, to be clear regarding the relevant representational relationships. For a given system we have, on the one hand, the physical characteristics of the system contained in the nature of the degrees of freedom, the physical preparation of the system, and the law-like regularities in behaviour; and, on the other, we have the formal description of the system contained in the formal variables, the formal initial (or end-point) conditions, and nomological restrictions on these. The connection between these two triples is precisely what a physical theory gives us. In some very simple cases, such connections can be understood unambiguously; however, invariably, when we want to understand realistic systems, we often encounter representative redundancy.

How these redundancies arise and how we should deal with them is often not entirely clear. Classically, such issues are difficult enough, but quantum mechanically they can lead to severe problems since the procedure for dealing with a particular redundancy depends crucially upon its origin. What is needed are general principles that are based upon physical reasoning but lead to precise mathematical diagnoses. Unfortunately, from our perspective, what is provided by most existing approaches[8] are precise mathematical principles leading to a diagnostic schema that is not physically reliable, in that it leads to *extremely unintuitive* conclusions for an important class of systems. As shall be discussed extensively further down, for the case of redundancy born of the unphysical nature of temporal labellings in certain systems, the relevant mathematical principles seem to lead inevitably to an interpretation in terms of the extreme form of relationalism about time mentioned in the previous section. Furthermore, standard treatments are ill-suited for dealing with a recently discovered form of symmetry; the class of *hidden* symmetries, which will be discussed below, and which constitutes the basis of the duality between General Relativity and Shape Dynamics mentioned above. More on these points later. For the meantime, let us tackle the problem of finding the relevant general *physical* principles for distinguishing types of redundancy.

The first principle we can identify relies upon the notion of *action*. This is a quantity defined for any curve in a configuration space, and is given by the integral of a specific function (the Lagrangian) along the curve. As noted above, the action is directly connected to the nomological restrictions that allow the formalism of a theory to pick out physical dynamics. If we consider the variation of a curve in a particular direction in the configuration space and find that the action is invariant up to a total derivative, then the degree of freedom associated with that direction is related to a manifest form of symmetry. It's important to point out, for our considerations, that a manifest symmetry can either be a symmetry which depends on space or time (i.e., a *local* symmetry) or not (i.e., a *global* symmetry). This criteria relates to a property of the action itself and is indifferent to the properties of any variational principle used on the boundary, which can either be a boundary in time or in space. To fully classify a degree of freedom, we will need a second criteria, described below, which leads to a richer set of physically distinct cases than what is *explicitly* considered by standard textbook definitions of symmetry.

Physically speaking, a manifest symmetry implies that there will be multiple possible sequences of configurations (representing histories in the configuration space) which are either *physically indistinguishable* or correspond to different values of a conserved charge. While the first case is applicable either to the whole Universe or sub-systems of it, the second case can only arise when subsystems of the Universe are being described (for a nice description of such systems, see [**?**]). Furthermore, the physical indistinguishability of the histories could be due either to practical limitations within the particular experimental set-up being considered (as is nicely illustrated, for example in [**?**]) or to fundamental limitations within the system.

Given an orbit on configuration space in which the action is *not* invariant, there are two further possibilities for the relevant degree of freedom: it might be the case that there are *no symmetries* associated with it, or it might be the case that there are *hidden symmetries*

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[8]The most comprehensive book on gauge theory and its quantization is [20]. A useful selection of philosophy papers on gauge theory can be found in [21].

associated with it. This third case has not been explored until recently, but will prove important for gravity. We will return to its detailed consideration later.

The second principle we can identify relates specifically to the nature of the variational principle used to vary the relevant variables in the action. As was noted above, it is essential to remember that the abstract initial (or end-point) conditions used in the variational principle are part of a formalism which stands in a possibly non-unique representative relationship with a class of physical systems defined in terms of their characteristic behaviour (i.e., physical observables within the system, physical preparations of the system, and dynamical laws obeyed by the system). Let us again consider the variation of the action based upon the variation of a configuration space curve in a direction associated with a particular degree of freedom – but in this case focus upon the details of the variational principle used along the *entire* history of the system in question, including the boundary (in space and time).

If the characteristic behaviour – which is fixed by the actual physical degrees of freedom, the physical preparation of the system, and the dynamical law – places no restriction upon such a variation, then we say that it is a *free variation*.[9] Clearly, this type of variation implies that the relevant degree of freedom is an otiose formal artefact since the notion in 'the world' places a restriction upon relevant variable's value. Nothing in the characteristic behaviour of the system fixes anything in the mathematical formalism, and so it must be a facet of redundancy within our representation. The alternative is that, for a given direction and associated degree of freedom, restrictions are placed on the variation. In such a situation, we say that we have a *fixed variation*, and we expect that the relevant degree of freedom has some representative relationship to something physical.

It is important to note why one would expect that it is the second, and not the first principle, which is decisive in the categorisation of a symmetry-related degree of freedom as inherently redundant or not. Although the first principle does relate to the action, it does not relate specifically to the variation principle. It is precisely the variational principle that fixes the full characteristic behaviour of the system. Thus, it is only the variation principle that can ultimately be sensitive to the difference between redundancies that are linked to dynamical conservation properties, and those that are entirely due to our use of excessive coordinates within the instantaneous representation of a physical state.

Keeping this important point in mind, and given our two *physically motivated* principles, we can set about categorising types of redundancy according to a *physically motivated* diagnosis. One would then hope that the mathematical exactitude of standard techniques will be recoverable where these techniques have proven physically reliable. This indeed proves to be the case if we consider the most blatantly unphysical form of redundancies: those connected to manifest symmetries and free variations. In such situations, the relevant degree of freedom clearly encodes no dynamical information, and defines a direction which is, by definition, superfluous to the representation of the world. Such variables are *gauge variables* and the relevant symmetries are *gauge symmetries*.

Our categorisation is sufficient, *although not necessary*, to recover the conventional mathematical categorisations of gauge symmetries. These are generally in terms of the symmetry being associated with either a local (i.e., functionally dependent on space and time) symmetry group, particular restrictions on the map (the Legendre transformation) between the velocity-configuration and phase space formalisms (i.e., it must fail to be invertible), or certain restrictions upon the phase space function associated with the degree of freedom (i.e., it must be a *first class primary constraint* [22]). This means that for all the standard cases where there is no perceived ambiguity about the cause and interpretation of redundancy – e.g., electromagnetism, Yang-Mills theories, the standard model (all in the presence of no spatial boundaries) – our definition will coincide with standard definition. However, as we shall see, for the case of

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[9]Formally, this means that the variation of the action, including boundary terms, is invariant *locally* under the symmetry.

time labelling symmetries, and indeed many other symmetries which which will not be discuss at length here,[10] our scheme still allows for an alternative, physically motivated categorisation.

Here, we should note an important point for the purposes of our discussion: those degrees of freedom identified as gauge within the classical theory are always (in some sense) eliminated within the process of constructing the quantum theory. Since the degree of freedom is not representing anything about the physics of the system, the quantum correlate of this degree of freedom must not be associated with an observable operator in the quantum formalism. In essence, the methodology for ensuring faithful treatment of gauge degrees of freedom is always the same: treat the direction associated with the degree of freedom as non-physical.

A further, more subtle form of redundancy derives from the presence of a degree of freedom associated with a manifest symmetry, but a fixed variation. Since the variation is fixed we know that the relevant variable *is* connected to something physical, and in general we can understand this 'something physical' as the conservation of some empirically determinable quantity throughout the system's evolution. For this reason, we call the relevant symmetries *conservation symmetries*.[11] Again our definition allows us to recover the parts of the standard scheme that are physically well motivated: all symmetries associated with Noether's first theorem are, under our definition, conservation symmetries.[12]

An example particularly relevant to the considerations of this paper is that our classification scheme non-standardly directs us to categorise symmetries associated with temporal relabelling as conservation symmetries and, as well shall discuss later, this proves absolutely pivotal for understanding the role of time in *relational* quantum theories, including prospective theories of quantum gravity.[13] The key point is that, in general, conservation symmetries, since they are tied into the characteristic behaviour of the system, must be treated entirely differently from gauge symmetries when constructing the quantum formalism. They are associated with physical directions, and, thus, the quantum theory must allow for states which depend upon a superposition of the values of the relevant constants of motion. This is entirely unlike gauge symmetries, where the associated constants must be set to zero in the quantum formalism.

A simple case of a conservation symmetry will illustrate this point well. Let us consider a standard Newtonian point particle system treated as an isolated sub-system of the Universe consisting of three particles starting in different positions which evolve under the force of gravity.

In our terms, such a translation is precisely a manifest symmetry since it corresponds to a global variation of configuration space curves in a direction along which the action is invariant. We can further classify the symmetry as manifest-fixed, because, for a Newtonian system, redefinition of the variational principle behind the system in terms of an infinitesimal change in the endpoints which define the boundary conditions of relevant variation *is not something we can do freely* – the physics of the system places definite restrictions such that only some spatially translated variations are equivalent. In this case, these restrictions are just the conservation of

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[10]Note that asymptotically flat GR and Yang–Mills theory in the presence of spatial boundaries which break gauge invariance are examples of theories that also *do not* fall into the category of pure gauge theories by our classification, although they would by standard treatments. We believe that our classification scheme is more appropriate for these cases because a notion of background is introduced by the relevant boundary conditions.

[11]By this terminology, we do not wish to imply that *all* conserved quantities are related to conservation symmetries (e.g., electric charge is a conserved quantity arising in gauge theories and is not a conservation symmetry by our definition), although conservation symmetries necessarily have conserved charges associated with them. Their distinguishing feature, which is relevant for us here, is that, in the quantum theory, conservation symmetries allow for superpositions of states with different values of the conserved charge.

[12]Our scheme also allows us to capture classical symmetries exhibiting conserved charges that would not normally fall under the treatment of Noether's theorem. For example, General Relativity with asymptotically flat boundary conditions is locally invariant under spacetime diffeomorphisms. Nevertheless, it still has conserved charges associated to it (the ADM momenta [**?**]) because the boundary variation is performed in a fixed way, using our terminology, due to how the asymptotic boundary conditions are imposed. Thus, the Poincaré invariance on the asymptotic boundary is a conservation symmetry by our definition.

[13]The conserved quantity associated with relabelling symmetry is the Hamiltonian function itself, which, as one can easily show, is a conserved quantity of the classical evolution.

linear momentum and the relevant constant of motion is just the total linear momentum of the system. Thus, we have a symmetry which is fixed and manifest – a conservation symmetry in our terminology. Standard quantum mechanics, which is the quantum theory defined based upon Newtonian mechanics, is then such that we can have superpositions of momentum eigenstates, as one would expect from our general prescription. If, on the other hand, one misclassified the spatial translations as gauge symmetries (i.e., manifest-free), then the quantum formalism that resulted would be a quantum theory of a single momentum eigenstate – which is clearly not a faithful quantization of Newtonian theory. What is lost in this analysis is the ability to treat the centre-of-mass velocity of the system as an operator in the quantum theory, since forcing the system to a momentum eigenstate forces this observable to be precisely zero. However, if the three particle system is an isolated sub-system of the Universe, then the behaviour of the centre-of-mass velocity clearly has meaning as part of the characteristic behaviour of the system, and such a misclassification would fail to capture the full behaviour of the system. It is precisely this form of categorisation error that we hold to be behind the idea that 'time disappears in quantum gravity', in complete analogy to how 'centre-of-mass velocity disappears' in the example above.

The simplest case in our classification scheme is where there is no redundancy in the relevant representative relationship between a physical degree of freedom and its formal correlate. This is the case where there is no manifest symmetry and the variation is fixed. This case corresponds to a conventional dynamical degree of freedom. Its initial or boundary conditions are specified by the variational principle and, in a phase space formalism, it simply evolves according to the flow of the energy function or Hamiltonian. Quantum mechanically, these degrees of freedom correspond to observable operators that can exist in the appropriate superpositions.

The most non-trivial case is if there is no manifest symmetry and the variation is free. In this case, it is possible that there is a *hidden* symmetry in the system. This can only happen if there is another manifest symmetry in the theory that has a particular type of formal relationship with the one at hand (it is *second class* with respect to it – i.e., the Poisson brackets of the constraints generating these symmetries is not weakly zero). If this is the case, the elements of the formalism can be modified (without changing the physical predictions of the theory) in such a way that the first symmetry becomes manifest. This is called *symmetry trading* and has been used to construct the *shape dynamics* formalism introduced in [17]. The general theory of symmetry trading is developed in [23]. We will give an intuitive introduction to this idea in the context of general relativity further down. The quantum mechanical implications of trading hidden for non-hidden symmetries are subtle yet potentially very powerful. As shall be outlined below, one of the major possible benefits of symmetry trading is that it allows us to exchange one symmetry – which we are unsure how to quantize, for another for which there are available techniques. Finally, if there is no manifest symmetry, the variation is free and there is no symmetry which can be traded, then the situation is more complicated and the system is very likely to be inconsistent.

We can now collect together all the possible types of degrees of freedom classified as distinct within our scheme, as well as the appropriate prescription for treating the quantum mechanical equivalents in Figure 3 below.

The table makes clear both the generality and potential physical importance of our scheme. If a degree of freedom is misclassified then not only will the interpretation of its role within the classical formalism be incorrect, the quantum formalism derived will fail to capture the characteristic behaviour of the classical system in the appropriate limit. A mistake at this stage will lead to an incorrect quantum theory. The key claim that will be defended later in this paper is that precisely such a misclassification has been made for the case of gravity, and that the so-called timelessness of quantum gravity is actually a manifestation of this mistake, and not the absence of basic temporal structure in the relevant system class.

Variation	Symmetry in Action	Classification	D.o.f in Quantum Formalism
Free	Manifest	Gauge Symmetry	Fixed to zero
Fixed	Manifest	Conservation Symmetry	Operator obeying identity
Fixed	None	Conventional d.o.f	Operator with no restriction
Free	Hidden	Tradable Symmetry	Represented in new way
Free	None	Possible Inconsistency	Quantization blocked

Figure 3. Classification of different degrees of freedom using physical principles.

2.2 Voluntary Redundancy

In the previous section, we outlined a classification scheme for the redundancies that can occur within the representative relationship between classical mechanical formalisms and classical physical systems. One of the most important applications for this scheme is to ensure a physically well-motivated quantization of the theory in question – i.e., one leading to a quantum formalism where the relevant quantum mechanical analogues to the classical degrees of freedom are faithfully represented. Unfortunately, for some classical systems, standard techniques for quantization *do not* lead to quantum formalisms with such properties.[14] Thus, even if we correctly classify the symmetries in the classical formalism, we may lose track of them during the process of quantization. Furthermore, while, for simple systems and symmetries, it can be a straightforward task to isolate the degree of freedom associated with a particular symmetry, the general case is famously problematic when explicit systems are considered (as an example, compare electromagnetism to Yang–Mills theory). In order to prevent such problems, we recommend a general formal procedure that provides a concrete method for explicitly isolating the degree of freedom associated to the symmetry in question. The description we will give here for this procedure will be rather more intuitive than explicit. More technically inclined readers may refer to §3 of the companion paper [1] for details.

The procedure we recommend for dealing with this problem involves introducing *even more* redundancy into the formalism in the form of auxiliary fields that artificially parametrize the symmetry in question. These auxiliary fields, called *compensator* fields, are introduced in such a way that they 'compensate' for the symmetry in question making them tightly bound to the sought-after degree of freedom. This degree of freedom can then either be varied in a *free* way or *fixed* way by either imposing or not imposing a specific functional restriction, which is referred to as the *best-matching constraint*, onto the corresponding compensator field. Although the use of these compensator fields is often a matter of finding a convenient way to mathematically isolate a degree of freedom associated with some symmetry, for the case of reparametrization symmetry – which is the primary case of interest to us here – the introduction of a compensator field is a *mandatory* technical step in being able to faithfully represent the symmetry.

To illustrate what the best-matching conditions achieve, we will briefly describe the role of the compensator fields in the formalism. These fields are simply symmetry group parameters that represent an active transformation of the configuration variables of the system. In the case where this symmetry group $\mathcal{G}$ has a particular form (i.e., it can be represented as a Lie group) the formalism becomes straightforward to describe. The active transformation of the configuration variables, q, can be written by exponentiating the contraction of the compensator fields, θ, with the elements, $t \in \mathfrak{g}$, of the algebraic structure associated with the group (i.e., the generators of the relevant Lie algebra). This gives a set of actively transformed quantities $\bar{q} = e^{\theta \cdot t} q$ depending on the compensator fields θ. It is clear from this definition what these barred coordinates represent: the difference between them and the original q's is just given by motion in the direction associated with the symmetry group orbit. Thus, they can be used to

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[14]This issue is over-and-above the occurrence of anomalies – which we will not discuss here.

absorb the symmetric degree of freedom we are looking for.

The compensator fields are used to implement either a fixed or free variation in a two-step process. First, we define the momenta p, conjugate to q, and π_θ, conjugate to θ and perform a coordinate transformation from the original coordinates $(q, \theta; p, \pi_\theta)$ to a set of barred coordinates $(\bar{q}, \bar{\theta}; \bar{p}, \bar{\pi}_\theta)$. This sort of change of variables on phase space is a fundamental freedom that one always has when working with canonical theories provided that, after the change of variables, the barred momenta $(\bar{p}, \bar{\pi}_\theta)$ really do correspond to the momenta of the barred configurations variables $(\bar{q}, \bar{\theta})$, respectively. If this condition is met, then the change of coordinates is completely analogous to changing coordinates from cartesian coordinates to polar coordinates on the plane.[15] Furthermore, this restriction, and the additional requirement $\bar{\theta} = \theta$, precisely fix the form of this transformation (for details see [1]). The effect of this transformation is to effectively mix the compensator field with the symmetric degree of freedom in such a way that the barred momentum, $\bar{\pi}_\theta$, represents the momentum conjugate to the symmetric degree of freedom, as expressed in terms of the new variables.

The second step is to perform the actual variation. How we do this depends crucially upon the free vs. fixed distinction. If the variation is fixed, we know that the degrees of freedom in the original phase space, (q, p), were all physical. We should therefore treat only the directions associated with the introduction of the compensator fields (and their momenta) as corresponding to surplus representative structure. This equates to making the variation independent of two degrees of freedom for each compensator field, and can be done explicitly by enforcing the canonical restrictions $\pi_\theta = 0$. This condition can be treated using standard gauge theory methods developed by Dirac [22]. Although the details are not important here, the main result is that the restriction $\pi_\theta = 0$ eliminates the extra redundancy introduced into the theory when adding the compensator fields.[16]

If the variation is free, then an additional step must be taken since the original theory already had non-physical redundancies. This step involves adding an additional constraint to the system that guarantees that the action is independent of the velocities of the $\bar{\theta}$s. This will ensure that the theory is independent of any freely specifiable information associated with the symmetric degree of freedom, independently of when the endpoints of the variation are specified. Formally, we can express the relevant requirement as the disappearance of the transformed momentum variable to the compensator field, i.e., via imposing the *best-matching* constraint equation: $\bar{\pi}_\theta = 0$.

Now, let us try and understand more clearly the role played by the compensator fields *after* the transformation has mixed them with the symmetric degrees of freedom. The manifest symmetry requirement states that the action is invariant under the symmetry in question. In terms of the transformed compensator fields, this property, together with the Euler–Lagrange equations of the system, implies that $\bar{\pi}_\theta = \text{constant}$. This means that, in the manifest case, we always have a conserved quantity. This quantity is called the *Noether charge* associated to a global symmetry.

In a fixed variation, this charge is determined by the initial conditions of the system. For example, if the symmetry in question is represented by linear translations in space, the momenta of the compensator fields correspond to the total linear momentum in each of the three spatial directions. Such quantities are, of course, conserved in the evolution of any isolated system and we thus see that the conservation of the Noether charge relevant to linear translations simply is the expression of conservation of linear momentum. Classically, the value of a conserved charge is something definitely determined for once and for all time by the initial state of the relevant system. Quantum mechanically however, things are more flexible. Systems can, and

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[15]For technically inclined readers, we are referring to symplectomorphisms on phase space, which are simply diffeomorphisms (arbitrary changes of coordinates) that preserve the symplectic structure (the condition that the barred q must be canonically conjugate to the barred p's).

[16]More explicitly, the condition $\pi_\theta = 0$ is a first class constraint which can be gauge-fixed by the condition $\theta = \bar{\theta} = 0$, thus eliminating two phase space degrees of freedom, as outlined in, for example, [20].

generally do, exist in *superpositions* of different values of the relevant charge. This is a direct manifestation of the fact that conservation symmetries are rooted in physical dynamics rather than redundant representative structure.

In the free case, however, there is no way to fix the corresponding charge. Instead, this is done by the best-matching conditions, which force the charge to vanish.[17] This has important implications for the quantum theory. The application of the quantum analogue of the best-matching conditions forbids the existence of superpositions of eigenstates of that charge, since only the zero eigenvalue is allowed. Below, we will see how this seemingly innocuous point becomes essential to understanding time and its denial in the context of quantum theories of gravity.

3   Understanding Time

3.1   Change and Order

Things change, and time – whatever it may be – certainly is, at a minimum, a means for describing this change. Newton was father to a notion of time that gives us much more than just a measure of change, since his absolute time 'of itself, and from its own nature, flows equably without relation to anything external' [24]. Such a Newtonian time can be taken to constitute an *absolute temporal background* against which both the temporal order of, and the temporal distance between, events is defined irrespective of any changes that may take place. Thus, in a universe of no change, it makes sense to distinguish both the order of and time between events that are, other than their absolutely determined temporal position, entirely identical. More precisely, we can say that on the Newtonian view, both the *metric* (distance) and *topological* (ordering) structure of time are fixed absolutely irrespective of changes in the material universe.

At the other end of the spectrum from a Newtonian notion of time, we can conceive of a radically relationalist conception along the lines discussed in Section 1. Recall that, on this correlation view of time all that it is for a physical degree of freedom to change is for it to vary with regard to a second physical degree of freedom – and there is no sense in which this variation can be described in absolute, non-relative terms. Within this picture of the world, we explicitly deny *both* time's metric and topological structures. The notion of time one may recover from change is an inherently arbitrary and approximate one. We are free to choose any degree of freedom as an internal clock, and such a clock may, for some finite interval, give us a useful means of marking both the duration between and order of events as defined by correlations between the other degrees of freedom. However, such an internal clock may start to run backwards, and so generally will not give us an ordering of events in 'time' that is globally defined in terms of a linear sequence: the parameterization is not *monotonically increasing*. Moreover, such a method of distinguishing one variable as 'the clock' inevitably involves neglecting the dynamics of precisely that variable: once the internal clock choice is made we are no longer able to describe the change in that clock degree of freedom. This is, in effect, to reduce the dimensionality of our system by one. Thus, in the context of this strong relationalism about time: a one-dimensional system is always static, a two dimensional system is *really* a one dimensional system, and so on.

One of the primary justifications given for this view on time is a formal one. It is argued that, for theories in which we have local or global temporal relabelling symmetries, radical relationalism is forced upon us by the mathematical structure of the theories in question.[18] In particular, since the time symmetries present in these theories are judged, according to the standard classification scheme, to be gauge symmetries, it is presumed that we are compelled to

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[17]In electrodynamics, the best-matching constraint reduces to the usual Gauss constraint, so the terminology is a bit confusing: this 'charge' does not correspond to the usual electrodynamic 'charge'.

[18]On this see in particular,[9, §3.2.4].

take the interpretational step of strong relationalism about time. As indicated above, to us the logic of such arguments seems a strange one. Reliance upon a formal classification scheme to determine our interpretation of the ontology associated with a physical formalism seems to put the cart before the horse. Moreover, it can actually be proved that the scheme in question *does not* apply to at least the case of theories where global temporal relabelling is a symmetry [25]. Our suggestion is that one should remain agnostic as to the 'true nature of time' implied by a physical theory until after one has carefully considered the relationship between the formalism of that theory and the class of systems which it is taken to represent. In particular, one must have an understanding of the *physical basis* of the relevant temporal symmetries in terms of how the characteristic behaviour of the system class are related to the abstract degrees of freedom, the boundary and initial (or end-point) conditions, and the nomological restrictions on the evolution. Thus, on our view, it is only *after* applying something like our physical prescription for classifying symmetries that one is able to consider the interpretation of the formalism in terms of a temporal ontology.

Let us do this explicitly for the case of a simple finite dimensional system where global temporal relabelling is a symmetry – we will consider the more difficult case of infinite dimensional systems with local temporal relabelling symmetry, i.e., general relativity, in the following section. The situation is then this. We have a class of physical systems which have a finite number of physical degrees of freedom and for which there is no physical difference between the system passing through a sequence of physical states at different rates. Such a system may be represented via slightly adapted version of Newtonian mechanics called Jacobi's Theory.[19] This theory can be constructed in terms of a configuration space with an identical number of degrees of freedom as a the corresponding Newtonian configuration space but with an action which displays an extra symmetry. This symmetry is *reparameterization invariance*, and is equivalent to redefining the time parameter used to mark change between states within the theory. Thus, Jacobi's theory clearly does not include a formal representation of an absolute temporal background in the full Newtonian theory. The key question is then: what kind of symmetry is reparameterization invariance – is it manifest? and is it free or fixed?

The answer to the first question is fairly straightforward. Since, by construction, reparameterization invariance is a symmetry of the action, it can only be considered a manifest symmetry. What is interesting to note, however, is that the phase space direction associated with the symmetry is in fact given by the energy function or Hamiltonian. Thus the 'symmetry direction' is also the 'dynamics direction'.

The next step is to determine whether we are dealing with a conservation or gauge symmetry by distinguishing whether we have a free or fixed variation. Again this is a fairly simple question to answer since, again by construction, we have a configuration space where all the abstract degrees of freedom directly correspond to physical degrees of freedom – those of the corresponding Newtonian system. Explicitly, since infinitesimal change to the endpoints of the variational principle is in every direction parameterized by physical degrees of freedom, such variation cannot be done freely. It is fixed by the characteristic behaviour of the system, in particular the preparation conditions. Thus, reparameterization invariance, or temporal relabelling symmetry, is a conservation and not a gauge symmetry.[20] This is precisely as one would expect since, as we have just noted, the phase space direction associated with the symmetry is precisely the direction of dynamical change within the theory – thus, if the symmetry *were* a gauge symmetry, then we should not expect to have any physical dynamics whatsoever since the 'dynamics direction' would be an otiose representative structure. In that eventuality, the only

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[19]This is after the great German mathematician Carl Jacobi. See [26] for details of both the formalism and its historical context. In [9], Rovelli calls Jacobi's theory 'relativistic mechanics'.

[20] The conserved quantity associated with it varies in interpretation depending on the system in question. For timelike geodesics in spacetime, it corresponds to the mass of the test particle. For Jacobi mechanics, it corresponds to the energy of the system. In all cases, it is associated with the constrained Hamiltonian of the system.

option for an interpretation of the relevant temporal ontology *would* be the radical relationalism mentioned above. However, by the lights of our scheme, such a move is formally unjustified. We *do not* see temporal relabelling as a gauge symmetry and so can license an interpretation of theories with such a symmetry in terms of a more substantive notion of time.

One important point regarding the conservation symmetry classification of reparameterization invariant theories relates to the interpretation of the role of energy. As noted above, all conservation symmetries have associated conserved quantities or charges. For the case in hand, the charge will be equivalent to the energy of the system. This means that the energy of the universe is interpreted as a constant of motion. This is in contrast to many existing views whereby, in a reparameterization invariant theory, energy is a constant of nature. Classically, this difference has no empirical implications, but, quantum mechanically, it implies that we should expect superpositions of energy eigenstates – as we would usually for conserved charges in the quantum version of a theory with conservation symmetry. We will return to this feature of what we call 'relational quantum theories' in the following subsection.

Before then, we must first consider what kind of temporal ontology our conservation symmetry approach to classical reparameterization invariant theory can lead to. As noted, the theories in question clearly are also not so amenable to the Newtonian absolute notion of time, since temporal relabelling should not be a symmetry in the context of such an interpretation.[21] Rather, we need to look for a 'middle way' such that the status of temporal relabelling as a conservation symmetry is preserved faithfully within our interpretation. Thankfully, we need not strike out entirely on our own here, a number of thinkers have espoused views on time well suited to the case in hand. Ernst Mach, in particular, both criticised absolute notions of time on epistemic grounds *and* put forward a positive account of the kind of temporal structure we are presented with. According to Mach it is 'utterly beyond our power to measure the changes of things by time...quite the contrary, time is an abstraction at which we arrive through the changes of things.' [28]. Thus, on the Machian view, a consistent notion of time can be abstracted from change such that the inherently interconnected nature of every possible internal measure of time is accounted for. According to the Mittelstaedt-Barbour [29, 30] interpretation of Mach, we can understand this second Mach's principle as motivating a relational notion of time that is not merely internal but also equitable, in that it can be derived uniquely from the motions of the entire system taken together. Thus, any isolated system – and, in fact, the universe as a whole – would have its own natural clock emergent from the dynamics. Significantly, for a notion of time to be relational in this sense, it is not enough to be merely internal – it must also be unique and equitable. We cannot, therefore, merely identify an isolated subsystem as our relational clock, since to do so is not only non-unique but would also lead to an inequitable measure, insensitive to the dynamics of the clock system itself. Thus, the Machian view gives us a philosophically well-motivated alternative to both the radical relationalism about time and the Newtonian absolutest view.

This is not, however, from our view, a complete interpretation of temporal structure of the reparameterization invariant theories we are dealing with. Above, we mentioned that the we can think about the Newtonian view on time as involving *two* absolute structures: the metric (distance) and topological (ordering) structure of time. The first is clearly inimical to both the Machian interpretation and the reparameterization invariant formalism. The second, however, seems at least compatible with the first, and, to an extent, *implied* by the second. Briefly, putting things on a more formal setting for a moment: in the Hamiltonian formulation of reparametrization invariant theories, the Hamiltonian is a constraint, and the multiplier associated with it (the lapse) is positive definite. This means that the arbitrary parameter associated with the constraint (i.e., time) is monotonically increasing along all trajectories in phase space. This implies an absolute temporal ordering.

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[21] This is unless of course one where to adopt some sophisticated form of 'temporal substantivalism' via the introduction of anti-Haecceitist reasoning about temporal points. See [27, 10].

The interpretation we have been naturally led towards is one in which duration is relational in a Machian sense, but temporal ordering (time's topology) still plays the role of an absolute background structure. Here, there is an obvious epistemological worry: there seems to be no possible way for us to ever gain access to this 'absolute ordering structure' and so its adoption seems to involve rather strong metaphysical act of faith. There seems to be at least three ways of addressing this worry:

First, rather pragmatically, we can simply note that time ordering does appear to correspond to part of our physical formalism, and so, unless we can find an empirically adequate re-formulation without it, we have no good cause to question its status. There is, of course, a rather impressive precedent for the use of background structures to solve pressing theoretical problems, namely Newton's use of absolute space to give a coherent formulation of the principle of inertia [31, 32, 33]. And, in any case, if metaphysical minimalism is taken to be the main motivation for the elimination of non-empirical backgrounds, then accepting the fairly thin notion of a time ordering background is far more palatable than a full-strength Newtonian-style notion of time.

A second option is to situate the notion of temporal ordering within a perspectival context. It seems reasonable to argue that in physics we are always describing a physical system from an anthropic perspective within which there is necessarily a notion of succession. It is simply a requirement of an adequate representative framework that succession can be included at a basic elemental level, and the best way to do this consistently is to graft onto the relevant dynamical change a structure encoding temporal ordering. Thus, the absolute topology of time is not really a feature of the ontology, rather it is a necessary feature of our representative language. From this perspective, asking why we do not have epistemic access to the fundamental ordering structure behind temporal succession is simply the wrong kind of question. A similar such argumentative move has been deployed in the context of explaining the *arrow of time* in cosmology and themodynamics by appeal to the fact that our human perspective is situated inside an entropy gradient [34]. An exploration of the connection between these two 'problems of time' constitutes an interesting avenue for future research.

More ambitiously, we could accept that time ordering should be founded in accessible features of our theory but that perhaps the arena for doing this is quantum and not classical theory. It is possible that the process by which temporal ordering in classical physics *emerges* is connected to the classical limit of a fundamental quantum theory or even a broken symmetry in such a theory. A (rather technical) example that suggests the plausibility of this idea is in quantum field theory where a monotonic ordering is naturally encoded in the renormalization group (RG) flow near a conformal fixed point. Thus, the RG flow equation of a such a field theory could be reinterpreted as a time evolution equation in a shape dynamics theory. A simple toy model featuring such behaviour was studied in [35]. Furthermore, there are exciting indications that a similar scenario could be used to reproduce certain models of inflation [36]. However, many interesting open questions remain and such suggestions are still very tentative.

Given these three prospective justificatory strategies, let us accept, for the time being at least, that our middle-way, *succession-as-absolute-and-duration-as-relative*, ontology of time can be coherently philosophically defended. What fruits can it bear when brought back into the domain of physical theory? Can it give us new insights into the nature of time in relational quantum theories?

3.2   Quantization and Succession

In the previous section, we defended both the classification of temporal relabelling symmetries as conservation symmetries and the interpretation of theories with such symmetries in terms of a temporal ontology within which duration is relative but succession absolute. Each of these moves gains significance for the future development of physical theory when seen in the context of quantization. This is particularly clear when considering the so-called problem of time that

arises within attempts to quantize general relativity, but is also the case for simple globally reparametrization invariant models. The temporal relabelling symmetries of such models should, according to conventional classification schemes, be understood as gauge symmetries. This means that quantization, whether achieved via the voluntary redundancy route detailed above or otherwise, leads to a quantum theory in which the *gauge directions* are treated as unphysical and, correspondingly, the quantum system is restricted to a zero eigenstate of the relevant charge. For the case of globally reparametrization invariant models, this is equivalent to treating dynamical directions as unphysical and to restricting the system to a single zero energy eigenstate. Thus, the classification of global temporal relabelling symmetries as gauge symmetries leads directly to a frozen quantum formalism. The only ontology we can associate to such a picture is that of radical relationalism – and we are left without time.

The alternative, for which we are arguing, is to treat global temporal relabelling as a conservation symmetry. The problem is then how to quantize the theory such that both the reparametrization symmetry and absolute temporal succession structure is retained. This is where the methodology of voluntary redundancy comes into its own. To our knowledge, the *only* way to achieve quantization of a classical model such that time remains in the sense desired, is to use this method. Explicitly what we do (see companion paper [1] for more details) is choose a particular expansion of the phase space such that our single configuration compensator field, τ, has a canonical conjugate, π_τ, proportional to the energy of the system. Our single constraint is then constructed by the combination of the original Hamiltonian plus the new momentum variable, so we have $H(q^i, p_i) + \pi_\tau = 0$. As discussed in §2.2 above, imposition of this constraint leads to a variational principle dependent upon all but two of the degrees of freedom – i.e., precisely the number we started with. The quantum theory we reach by applying standard methods then preserves these physical degrees of freedom faithfully in that it allows their quantum analogues to change independently of each other.

Furthermore, as expected, the quantum theory we arrive at is such that we can represent states in superpositions of eigenvalues of energy since total energy is the Noether charge associated with our conservation symmetry. Our quantum formalism can therefore accommodate fundamental temporal structure associated with succession via the parametrization chosen to distinguish the distinct energy eigenstates. However, since this parametrization is arbitrary, unlike in conventional quantum theory, there is no preferred classical temporal background which fixes a notion of duration. Time remains, but only in the form of succession. Because time is retained in a relational sense, we call this procedure for the quantization of theories with global reparamterization symmetries *Relational Quantization*. From our perspective it is one of the crucial ingredients in the construction of a genuinely dynamic theory of quantum gravity as is consistent with the Machian view of time.

4   Time and Gravitation

4.1   The Two Faces of Classical Gravity

Our description of the universe is replete with different scales – from the unimaginably small distances of particle physics, up to the unimaginably big distances involved in modern astronomy and cosmology. The theories relevant to these domains have one important feature in common: they treat such scales as an absolute background structure. Thus, in almost all modern physical theory, if we uniformly double the lengths involved, the phenomena will change.[22] Such scale dependence is also part of our best theory of gravity: general relativity. Although the theory incorporates a huge amount of descriptive freedom, it still privileges length scales. Interestingly however, the type of argument that drove Einstein to try and eliminate coordinate dependence from Newton's theory of gravity also motivates us to eliminate scale dependence from Einstein's.

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[22]More specifically, the Higgs and gravitational sectors are not conformally invariant.

Just as he argued that we have no empirical access to absolute coordinate structures – only relative ones; we may argue that we have no empirical access to absolute scale structures – only relative ones. What remains to be seen, however, is to what extent local scale invariance can be implemented within a physically viable theory. Weyl's early attempts at a four dimensionally scale-invariant theory of gravity [37] were ultimately unsuccessful, in terms of leading to a theory of quantum gravity, because of strong indications that the theory is unstable. Although work in this vein is ongoing, (e.g., in the work of Mannheim [38] or 't Hooft [39]) there are still significant issues to be overcome.

Here we will outline a proposal, different to Weyl's, which seeks to implement *both* a Machian notion of time, and a three dimensional version of local scale invariance. The two ideas are in fact naturally connected. As has been argued by Barbour [40], Mach's general stance of epistemic scepticism with regard to non-relational concepts, should lead us to the conclusion that it is the local scale invariant 'shapes' of instantaneous configurations of the universe that should be taken as fundamental. These shapes can be determined by local observers through measurements of angles, which Barbour takes to be fundamental. In order for this definition to be meaningful, there must exist a preferred notion of global time through which the instantaneous configurations can be defined. Thus, we can see intuitively that the local temporal relabelling symmetry, which is fundamental to gravity, is in conflict with three dimensional scale invariance.

Here we will examine this conflict and offer conceptual foundations for its resolution (following the more formal arguments of [17]). The first step in our reasoning relies upon the notion of hidden symmetry which has already been discussed briefly above. To recap, the idea is to identify, for a particular theory, a direction in which there is no manifest symmetry and the variation is free. In this case, it is possible that there is a hidden symmetry in the system. This can only happen if there is another manifest symmetry in the theory that has a particular type of formal relationship with the one at hand.[23] If this is the case, the elements of the formalism can be modified (without changing the physical predictions of the theory) in such a way that the first symmetry becomes manifest. Remarkably, it has been proved for the case of canonical gravity that, given certain reasonable simplicity assumptions, the unique set of hidden symmetries can be identified as three dimensional transformations of scale which preserve volume [41]. By definition, hidden symmetries can only be identified when there exists another symmetry which is manifest and has the required formal relationship such that the two sets can be understood as dual to each other. For the case in hand, the relevant dual to the scale symmetry is *almost all* of the foliation symmetry. This means that if we *symmetry trade* such that the hidden volume preserving scale transformation symmetry becomes manifest, we simultaneously switch to a theory with merely global, rather than local, time relabelling symmetry (this corresponds to a single, non-local Hamiltonian constraint).

There is, thus, a unique formal move that allows us to re-describe gravitational systems in a fundamentally different way. Conceptually, we can provide a good motivation for this transformed formalism via a proposal due to York. Above we argued that, in order to classify the symmetries relevant to a class of systems consistently, we must first specify precisely what the relevant physical degrees of freedom and preparation conditions are. This translates into asking what degrees of freedom are independently specifiable on the boundary of the variation or, in the case in hand, 'What is fixed on the boundary in the action principles of General Relativity?' This question was posed by Wheeler to York, and is addressed in [18]. Our view corresponds to that taken in Section 4 of that paper: what is fixed on the boundary is: i) a three geometry invariant under scale and coordinate labelling symmetries; and ii) the mean of the 'York time' variable (which is canonical conjugate to the *spatial* volume). We will refer to this identification of the independent degrees of freedom of gravity as *York's ontology*. We take these variables to faithfully parametrize the characteristic behaviour of gravity and, thus, take

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[23]The formal requirement is that the constraint surfaces corresponding to the symmetries are 'orthogonal', i.e., second class, on phase space. For the general theory behind symmetry trading, see [23].

their variation to be of the fixed kind, while variation with respect to all other variables is free. York's identification of a locally scale-invariant three geometry as a variable to be fixed in the variational principle of gravity is consistent with the principle of scale-invariance just argued for on the basis of Mach's principles. However, one might note that York's second requirement: to keep the variable conjugate to the spatial volume fixed, is in direct conflict with the global principle of scale-invariance. Our view on this will be rather pragmatic at this stage. The fixing of this 'York time' can be motivated by the directly observable red-shift, which is undeniably part of the characteristic behaviour of gravity. However, on Machian grounds, one might expect the red-shift to result as an emergent phenomenon from a fully scale-invariant theory. In this eventuality, we would still expect York's proposal to be valid in some effective limit in the quantum regime. However, since a concrete proposal where such a scenario is realized has not yet be developed, we will consider York's ontology directly.

From the York perspective, canonical general relativity has the rather undesirable feature of neither having manifest invariance under volume preserving conformal transformations, nor being a conventional gauge theory with respect to diffeomorphisms, nor varying the York time in fixed manner. The first two difficulties can be resolved by noting that the volume preserving scale transformations are a hidden symmetry of the canonical version of general relativity (as we have just noted). The last difficulty can be dealt with using our proposal for Relational Quantization procedure detailed in the previous section.

Before we embark on the final phase of our analysis and detail our specific proposal for time in a substantive sense, to remain within a theory of quantum gravity, let us take a moment to consider the philosophical consequences of symmetry trading on the way we should think of the ontology of classical gravity. The traditional understanding of general relativity (in canonical terms or otherwise) is as a theory of spacetimes invariant under four dimensional coordinate transformations, or diffeomorphisms. This is essentially the ontology for gravity that was implied by Einstein's seminal work in the early part of last century, and which is still one key pillar of the scientific understanding of the universe. That this theory has a unique and robust formal correspondence, or duality, to another theory of gravity, which (under certain restrictions) has the same physical consequences is highly non-trivial. As mentioned in the introduction, on our view, it is taken to imply that gravity is essentially *Janus-faced*. From this perspective, we should see the Einstein ontology of diffeomorphism invariant four dimensional spacetime geometries as only one face of gravity. The other, newly unveiled face being constituted by the ontology proposed by York: sequences of three dimensional spatial geometries accompanied by the specification of the York time variable and invariant under both diffeomorphism and scale transformations.

The situation of dual theories which are empirically equivalent, yet ontologically radically different is one of great interest within the philosophy of science since it seems to imply a particularly pernicious species of underdetermination.[24] Interestingly, the case which has garnered the most interest in the recent literature [45, 46, 47], that of the AdS/CFT correspondence in string theory/conformal field theory, is also one in which the two duals theories are respectively a coordinate invariant theory of gravity and a theory invariant under scale transformations.[25] However, one would not have expected that such striking, and perhaps worrying, underdetermination scenarios could crop up in one of our most established physical theories. Yet, for our purposes, this seeming theoretical vice will prove a virtue. It is only by recognising the second, scale-invariant face of gravity, that we can forge a new path towards quantization without sacrificing Time.

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[24]See [42] for a excellent overview and analysis of various notions of 'metaphysical underdetermination' leading towards a motivation of the philosophical viewpoint of 'ontological structural realism'. See [43] and [44, §19] for discussion of the problems that the radically different ontologies found within theories of gravity may pose for the position.

[25]Indeed, it has recently been argued [48] that the 'bulk-bulk' equivalence of shape dynamics and general relativity can be used as an *explanation* for certain limiting regimes of the AdS/CFT correspondence.

4.2 Retaining Succession in Quantum Gravity

As was argued for extensively in Section 2, correct identification and treatment of the physical degrees of freedom, and in particular their boundary variation behaviour, is essential to a faithful quantization of any physical theory. One of the major impediments to the quantization of gravity has been that, in canonical form, the symmetry and dynamics of the theory are 'deeply entangled'.[26] In the language of Section 2, this corresponds to the 'fixed' and 'free' aspects of the manifest symmetry being mixed together. Here we make no attempt to tackle the fearsome, and as yet unsolved, problem of a disentangling the physical from gauge variations within the theory. Rather, given the 'gravity is Janus-faced' revelation detailed above, we may simply turn to the alternative scale-invariant formalism for classical gravity and hope for a simpler representation of the relevant symmetries more amenable to quantization.

Starting with the canonical, or ADM, formulation of general relativity, symmetry trading yields a theory with the following manifest symmetries: a) three dimensional spatial coordinate invariance; b) three dimensional (spatial volume[27] persevering) spatial scale invariance; and c) invariance under one dimensional global time relabelling (or reparameterization). If we interpret this formalism in York's terms as discussed above, then we arrive at a unique and unambiguous specification of which of these symmetries correspond to free variations of the physical degrees of freedom at the boundary. These degrees of freedom are therefore unphysically grounded gauge symmetries, their associated symmetries correspond to fixed variations, and they are, therefore, physically grounded conservation symmetries. By construction, the York specification of boundary data is insensitive to variation of the (volume persevering) conformal and coordinate modes of the three dimensional metric tensor that characterises three dimensional spatial geometries. Thus, symmetries a) and b) are identified as gauge symmetries. Then we find, in correspondence with the discussion of Section 3, that the reparameterization symmetry c), is of the fixed kind. Thus, our prescription for quantization implies we should introduce two global degrees of voluntary redundancy that parametrize the direction associated with the function that generates the reparameterization symmetry. As for the simple particle case, this function is a global Hamiltonian, and this means that our extra variables are a time-ordering label and a conserved charge associated with the 'total energy'. We then append the second of these to the Hamiltonian to get an extended, but physically equivalent, formalism, which then can be quantized (at least in formal terms) via standard methods.

What we have gained in this rather circuitous route of symmetry trading, arbitrary extension and quantization, is simple to state. We are now equipped to represent the state of the universe *at different times*. This is because, as for the particle case, we are able to take superpositions of energy eigenstates, and consider the independent evolution of observable operators. And yet, we *have not* introduced a Newtonian-style background time into the theory. Time labellings are encoded in the arbitrary parameter which was introduced into the theory during the extension procedure, and so are neither fundamental nor observable. Rather, the temporal structure which this formalism for quantum gravity contains is precisely the temporal succession structure we associated with the ideas of Mach above. As was noted in the opening section, we should not think of such a 'temporal topological background' as being entirely alien to general relativity since it is in fact implicit within the canonical formalism at least (in terms of the positivity requirement on the lapse multiplier). Moreover, it is only by retaining such structure that, in the case of gravity, we can hope to preserve genuine change and avoid radical relationalism with

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[26]Formally, this facet is encoded within the non-trivial structure of the Dirac-Bergmann constraint algebra, in particular that the bracket between two Hamiltonian constraints only closes with structure functions is indicative of the dual dynamics-symmetry aspect of these constraints. The constraint algebra of shape dynamics (and the relationally quantized theory) on the other hand is a genuine Lie algebra, and so a clear formal distinction between symmetry and dynamics can be made.

[27]For the case of open spatial topology, this global restriction turns into a specification of asymptotic boundary conditions [23].

regard to time, as we have been advocating.

Our proposal for the quantization of gravity thus involves two substantive interpretative moves. Firstly, the switch from the Einstein ontology (as implied by the general relativity formalism for gravity) to the York ontology (as implied by the shape dynamics formalism of gravity). Secondly, the promotion of time ordering (or topological) structure from an implicit formal feature to an explicit background. Both individually, and as a package, we can provide a range of motivations for these non-trivial steps.

Most straightforwardly, there is the motivation from pragmatism: by following our prescription one opens up new strategies for theory development, and this, in the end, might be argued to be the true goal of foundational research. It remains to be seen precisely what lasting value the much vaunted 'spirit of general covariance' will prove to have as a heuristic for *future* theory construction. It may prove pivotal, or it may prove to have been misleading. Thus, if new and viable theoretical avenues can be opened up by reinterpreting symmetry in the context of gravity, we would be churlish to entirely ignore them since they do not sit conformably with an exciting abstract principle – no matter how fundamental it may currently appear to be. We should not let fetishism for four-dimensional spacetime coordinate invariance be a bar to potential progress.

Furthermore, over and above the conceptual novelty of our proposal, it provides several notable formal advantages. In traditional approaches to the quantization of gravity (i.e. the 'Wheeler-DeWitt-type' approaches), the resultant quantum formalism is such that only one energy eigenvalue is allowed. Evolution of the quantum states can then only be obtained by deparametrizing with respect to a degree of freedom, in the choice of which one must make an arbitrary decision. The definition of the functions used to represent observable quantities in the theory depend on this choice and, even for simple models, can lead to extremely complicated expressions. Through our approach, we arrive at a formalism where there can be superpositions of energy eigenstates, and the evolution of the full state can be given with respect to the auxiliary time label. Thus, the evolution does not depend on any arbitrary choice of auxiliary time label. The identification of the relevant observables is then also non-arbitrary, and is technically much easier (because of the time-independence of the Hamiltonian).

Finally, in addition to conceptual novelty and formal tractability, there is the simplest motivation of all: the motivation from time. It seems to be a basic requirement that, in one way or another, we are able to abstract some concept of time from our physical formalism – without it our physics would simply fail to be descriptively adequate. If the only approach to the quantization of gravity were via a timeless formalism, then it would perhaps be fair to insist that we must make do with the conceptual paucity of time merely as relative variation. However, given that there is a viable alternative route towards quantization, via symmetry exchange and relational quantization, the conceptual cost incurred by taking it should be counted as naught, next to the benefit of retaining a minimal, yet substantive, concept of time.

5   Conclusion

The aim of this paper was to argue that there are strong formal and philosophical reasons to expect time to remain within any theory of quantum gravity. Although the temporal symmetries of classical gravity are subtle, such that the redundant and physical aspects of the formalism are entangled, there does exist a precise formal recipe for making the unambiguous distinction needed for a faithful quantization. This recipe relies not just upon the technical notions of symmetry trading and relational quantization discussed here (and presented more formally elsewhere), but also upon two quite general and simple philosophical morals. First, that physics is not mathematics: it is our understanding of how the physical formalism relates to the world that should govern our interpretation of its mathematical structures and not vice-versa. Second, that, at base, time has two aspects: metric and topological. While the first *does* seem in conflict

with the relational, and 'background free' aspects of time in general relativity, the second appears implicitly even within the Einstein formalism. Furthermore, when seen in the context of our shape dynamics plus relational quantization proposal, topological or time ordering structure plays an important and unambiguous role: it is *The Remains of Time* in quantum gravity. As such, the virtues of a program towards its conservation are self-evident.

Acknowledgements

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