Essay Review
Conceptual Foundations of Yang-Mills Theories

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Alexandre Guay

CIRST, UQÀM, Case postale 8888, Succ. Centre-ville, Montréal (Qc) H3C 3P8, Canada

Gauging What's Real
The Conceptual Foundations of Contemporary Gauge Theories, R. Healey. Oxford University Press (2007), 229 pp., Hardcover

1 The book and a few commentaries

Richard Healey's *Gauging What's Real* is something not frequently seen on the landscape of philosophy of physics. It is not a book about the interpretation of quantum mechanics or quantum field theory, nor is it about space and time or causality. Rather it focuses on the conceptual foundations of specific theories of interaction, namely gauge theories of the Yang-Mills (YM) type (including electrodynamics).[1] Since three of the four fundamental interactions are modelled by theories of this kind, the importance of the subject is obvious for the elaboration of a scientific image of interaction. The book aims at a delicate balance of readability while being exhaustive. Most subjects already present in the philosophical literature are discussed, even if it is sometimes briefly. Healey takes remarkable steps to make his book accessible to a large community of researchers. For example there are no less than six pedagogical appendixes. Not surprisingly, in this context many questions would require a much more detailed discussion to satisfy the specialist.

Email address: `alexandre.guay@gmail.com` (Alexandre Guay).

[1] On many occasions Healey makes valuable points about General Relativity, another gauge theory. But GR is obviously not his main target. Most of the time comments are made in order to contrast GR with Yang-Mills theories. In this review I discuss only YM theories.

But as Healey says himself: “[T]he book does not represent the last word - or even my last word - on this topic ... If I have conveyed a sense of its intellectual interest and importance, while provoking some reader to do a better job, then I will be satisfied” (p. x). The relative rarity of books about fundamental interaction theories gives us more than enough reason to forgive some too short discussions.

The introductory chapter presents, among other things, the principal fibre bundle formalism. This geometrical framework is the main conceptual item in Healey’s toolbox when discussing classical YM theories. It is obviously an especially appropriate tool in this context. After this first chapter follow three chapters focusing mostly on classical YM theories. The first of these is about the Aharonov-Bohm (AB) effect, a quantum phenomenon predicted by a model of quantized charged particles subjected to the influence of a classical electromagnetic gauge potential. The importance of the AB effect should not be underestimated. In the philosophical literature it is clearly one of the main empirical reasons to reconsider the usual field ontology of classical electromagnetism. Moreover, even if it is a quantum effect, it has a significant impact on the interpretation of classical YM theories. The model of a quantized charged particle (for example represented by a wave function) subjected to a classical gauge potential can be modelled by a $U(1)$ principal fibre bundle. This geometrical construction is also the natural setting to represent the classical $U(1)$ YM theory (the classical theory associated with quantum electrodynamics). Therefore an interpretation of the AB effect can possibly be transposed to the $U(1)$ YM theory and vice versa. Moreover a convincing interpretation of the $U(1)$ YM theory can serve as a foundation for interpreting classical non-Abelian YM theories. About the AB effect, Healey proposes three interpretations, each transposable as an ontological interpretation of all classical YM interaction: (1) no new electromagnetic properties view (for YM theories, no gauge potential properties view), (2) new localized electromagnetic properties view (new localized gauge properties), and (3) new non-localized electromagnetic properties view (non-localized gauge potential properties view). For the AB effect, and for classical YM theories in general, Healey supports the third position.

Healey’s main argument against the first position is that it seems to bring back action at a distance, ostensibly a bad thing, to field theory (see the next section for a discussion about this point). But also it seems that in non-Abelian theories there may be physically distinct situations in a region even though the gauge field is the same throughout the region in each situation. New properties seem needed to distinguish these situations. Since there is no empirical application of a non-Abelian classical YM theory, this argument relies on a certain interpretation of the mathematical formalism that could be contested.

In the recent literature numerous philosophers defend one version or another of the second position, for example (Leeds, 1999) and (Maudlin, 1998). Most of Healey’s attacks on this position consist in reframing these positions in the formalism of the principal fibre bundle. If this geometrical formalism represents adequately classical YM theories (and we have no reason to believe the contrary), it excludes the possibility of localized gauge properties. For a recent enlightening analysis of the ontological implications of the principal fibre bundle formalism, see (Catren, 2008).

So it seems we are left with the third position. One of the more interesting theses of Healey's book is that the new non-localized gauge properties are captured by holonomies in the appropriate principal fibre bundle. Thus, the holonomy interpretation of classical YM theory asserts that the theory represents intrinsic holonomy properties of regions of space-time, each of which consists of all points on a loop. If this interpretation is correct, the main metaphysical consequence is the unavoidable non-separability of physical processes in classical physics.

Healey answers many objections to the holonomy interpretation but some questions remain:

(1) Holonomy is not the unique way to characterize the non-localized gauge properties. Other representations can be formulated. For example, if the asymptotic boundaries are empty Minkowskian we can use, as variables, gauge invariant line integrals coming from infinity. For other boundary conditions other choices are possible. Does this freedom in representation have an impact on the ontological conclusions?

(2) Holonomies for non-Abelian principal fibre bundle are not gauge invariant. As Healey explains, for holonomies to capture intrinsic properties of a principal bundle, one or more reference points in a particular fibre have to be chosen. The apparent space-time realism implied by the introduction of loops and reference points in the interaction ontology could be problematic if we aim, like many physicists, for a background free theory. The fact that the paradigmatic example of a locally defined entity, the matter field, cannot be considered realistically since it is not gauge invariant inclines us to look with suspicion on an interpretation relying on a realist position about space-time points.

(3) Knowing all this, are holonomies the appropriate variables to identify gauge orbits in the space of histories of the theory (presumably the physical variables)? Should we not look for a way to identify gauge orbits that involve, in a transparent way, matter degrees of freedom?

This said, even if we are suspicious about the loop interpretation, this does not in any way diminish Healey's claim about the fact that classical YM theories involve a non-localized interaction process. This is a significant assertion.

Then second half of Healey's book concentrates on quantum YM theories. Healey discusses an impressive number of subjects that often are presented for the first or almost for the first time in the philosophical literature. Not surprisingly many sections feel too compact. Let us briefly expose the main content of each chapter.

The fifth chapter is an introduction to the quantization of YM theories, mainly YM free field theories. This chapter is a good introduction but does not allow the reader to grasp all the complexity and the subtleties involved in the quantization of a gauge theory. For example, the fact that the BRST symmetry is not discussed when ghost fields are introduced is a weakness. Also, only two pages are devoted to theories with interacting terms with matter fields. This is unfortunate since one of the great strength of Healey's analysis of classical YM theories is that it does not loose sight of matter fields. In fact this is one of the advantages of using the principal fibre bundle formalism in which matter fields are easily represented.

Chapter six is a defence of the formal character of gauge symmetry. Familiar approaches to this question are extensively discussed, like the possible observation of gauge transformations and the empirical implication of the gauge argument. More interestingly, much less discussed approaches are exposed and criticized: the empirical status of ghost fields, the possible gauge dependence of the spontaneous gauge symmetry-breaking, the possible empirical distinction between large and small gauge transformations and finally the status of anomalies. In each case Healey argues in a compact way against the position that gauge symmetry is empirical.

The seventh chapter is an introduction to what a gauge reduced quantum YM theory would look like. The version the author is presenting is based on a loop representation of the YM fields. Healey forthrightly does not hide the problems encountered by this representation, especially when interaction with a matter field is included. Nevertheless, the potential significance of such representation is not to be underestimated. Philosophers, as Healey is rightly pleading, should take a particular interest in this area of research since these representations are presumably based on physical—in other words, gauge invariant—variables.

YM quantum theories are models of the more general quantum field theory (QFT). It is now a cliché to assert that the ontology implied by QFT is not obvious. Could this problem be clarified if we focus on YM theories? The author explores briefly this question in the eight and final chapter of the book, pushing his loop interpretation in the framework of Bohm, Copenhagen and modal interpretations. This chapter is not the strongest but it makes an interesting general point. Philosophers of physics have the tendency to focus on the more general theories. Healey suggests that specific theories, like YM theories, can give us a new perspective on interpretative questions about quantum physics. This chapter could induce new research to clarify this possibility. The remainder of the book consists in a concluding chapter, asserting the importance of acknowledging the possibility of non-localized properties, and of six useful appendixes. The next section discusses in more details the interpretation of the AB effect. The third section approaches the problem of the language in which conceptual foundations questioning is taking place.

2 The Aharonov-Bohm effect

Most of the discussions about the interpretation of the Aharonov-Bohm effect follow a similar path. For example (Aharonov & Bohm, 1959), (Feynman et al., 1964), (Healey, 1997) and (Belot, 1998). Schematically, (1) since there is an observable effect (a phase shift in the interference pattern) on quantized charged particles, something, let us call it X, must have acted on them. (2) X cannot be the electromagnetic field because charged particles cannot reach non-zero electromagnetic field regions. Action at a distance is excluded. (3) Therefore new electromagnetic properties (localized or not) have to be invoked to explain the effect. This is why the AB effect is a pure quantum effect. (4) Furthermore, once new properties are believed to be necessary, in order to have a unified bearer of interaction, we interpret all electromagnetic interaction as the result of these new properties. In consequence the electromagnetic field becomes a derived entity.

First a remark about this common argument. A reader might worry about (2) because the localization of quantum particles is not a well defined concept. However this proposition, already present in (Aharonov & Bohm, 1959), can be expressed in a less problematic way. For example, in the Feynman functional quantization method, the AB effect can be predicted without including in the sum over histories any path crossing a region where the electromagnetic field is not zero. In consequence the effect does not depend on a local interaction with $F_{\mu\nu}$ but rather on a relative phase shift between path contributions that depends on the electromagnetic flux. Therefore no direct electromagnetic field interaction is involved in the production of the AB effect. Now I would like to present an alternative, but complementary, line of reasoning that, to my knowledge, is absent from the philosophical literature and from Healey's book.

There are many ways to modify, with an electromagnetic field, an interference pattern obtained by a two-slit scattering of charged particles. For example, (A) we can introduce a enclosed magnetic flux between the slits. In this case we obtain a fringe shift with respect to the unperturbed pattern, while the pattern envelope remains the same. This is a case of the magnetic AB effect. Or (B) we could add a uniform magnetic or electric field behind the slits and then obtain a displacement of the pattern without an envelope or relative phase change. Also (C) we could install an electromagnetic source, like a light, behind the slits. If the source is sufficiently intense, it can destroy the interference pattern. In this case the envelope and the pattern become identified. We could imagine other examples, but these will suffice for my purpose. The point is that only in cases B and C is a net change, caused by the addition of an electromagnetic field, of average momentum and energy recorded. In case A, momentum, energy and angular momentum are conserved as an average (Olariu & Iovitzu Popescu, 1985). In B, a net transfer of transversal momentum explains the displacement of the pattern. In C, the destruction of the interference pattern implies a strong enough electromagnetic interaction in order for the charged particles to behave classically. Thus the AB effect is a pure quantum (beyond classical) effect because it is a measurable effect that does not require the transfer of a physical quantity that we usually associate with classical interaction.

Before returning to the AB effect let us discuss briefly the ontological status of the electromagnetic field in classical physics. The best reason we have to include a field in our ontology is clearly provided by Einstein discussing the special theory of relativity:

> We now shall inquire into the insights of definite nature which physics owes to the special theory of relativity. (I) There is no such thing as simultaneity of distant events; consequently there is also no such thing as immediate action at a distance in the sense of Newtonian mechanics. Although the introduction of actions at a distance, which propagate with the speed of light, remains thinkable, according to this theory, it appears unnatural; for in such a theory there could be no such thing as a reasonable statement of the principle of conservation of energy. It therefore appears unavoidable that physical reality must be described in terms of continuous functions in space. (Einstein, 1970, p. 61)

For Einstein, some conservation principles are more fundamental than assumed ontology. Retarded action as a mode of electromagnetic interaction violates conservation of energy and momentum (Lange, 2002, chapter 5). Furthermore, violation of energy in this context entails a violation of determinism (Guay, 2004, chapter 5). The introduction in the theory ontology of the electromagnetic field solves all these problems. On the other hand, since a non-zero gauge potential does not always carry energy it is not necessary to add this entity to the ontology. It would be too strong to say that in classical physics to exist is to carry energy. Nevertheless, to justify the introduction of a new entity to the ontology of a theory, a very strong reason would be to save certain (local) conservation principles that we believe to be essential. In the same way, action at a distance is rejected not because it is unthinkable but rather because it puts in danger the conservation principles of energy and momentum.

How is this discussion in classical physics relevant to the interpretation of the quantum AB effect? First, the AB effect is modelled as the influence of *classical* electromagnetism on quantized charged particles. The interaction is not theoretically understood in a quantized way. Second, as already said, the AB setting is a model of a $U(1)$ principal fibre bundle, the geometrical setting of the $U(1)$ *classical* YM theory. In many aspects the AB effect is giving us information about classical interaction in a semi-classical context. Authors like Belot (1998), Lyre (2004) and Healey go as far to assert that the AB effect is a good base to discuss the ontology of classical electromagnetism. Consequetly, in this context an Einsteinian ontological approach is acceptable.

Since the AB effect does not imply a modification of average momentum or energy of particles, the necessity to reject action at a distance as a mean of interaction is not clear in this case. Thus an electromagnetic interaction can take two forms:

- If there is a transfer of energy the bearer of the interaction is the electromagnetic field. This entity is still necessary to guarantee the conservation principles even in semi-classical contexts (Olariu & Iovitzu Popescu, 1985).
- If there is no average transfer of energy or momentum the interaction is mediated by a retarded action at a distance of $F_{\mu\nu}$ (or possibly from the charges themselves).

So there are two available stories. If your interaction concept implies some kind of locality condition that excludes action at a distance, the AB effect justifies the addition of new gauge properties. If your concept of a bearer of interaction only implies the local conservation of energy you have the option to not introduce new gauge properties and resort to retarded action at a distance. I do not deny that the imposition of a locality condition on interaction seems reasonable but I place the emphasis on the point that a locality condition on interaction which is not grounded in the local conservation of energy is not naturalistic and therefore must be grounded in some metaphysical conception of causality. Such a discussion is not present in papers about gauge interaction.

3 The choice of a formal language

Face with the plurality of not clearly related problems discussed in the second half of Healey's book, it seems appropriate to long for a Carnap-style philosophical approach. By that I mean the reframing of the different problems and questions in the same ideal language that allow clear identification of their relations, to expose and eliminate pseudo-problems, and to give tools to transpose interpretative solutions from one theory to another. In fact, for quantum YM theories we need the equivalent of what the principal fibre bundle formalism is doing for Healey when he discusses classical gauge theories. This formal apparatus should fill a few requirements:

(1) Naturalistic: it should come from physics itself.
(2) Not realm specific: it should give a transparent transition from classical to quantum YM theories.
(3) General: it should be flexible enough to treat all applications of YM theory in quantum physics.

Let us briefly examine each of these requisites. The first is designed to comply with methodological naturalism. The language for studying conceptual foundations of science should come from science itself. Having said that, it is not necessary to adhere to naturalism to aim for a formal framework coming from science. Recent history shows many examples where technical formalisms of scientific origin enrich in a significant way philosophical discourse. For example, the use of geometrical models in (Earman, 1989), of fibre bundle in (Maudlin, 2007) or of a mathematical approach to objectivity in (Nozick, 2001). Moreover since concepts like *gauge* and *field* are heavily theory laden, a scientific formal framework may be better able to represent them.

The second requisite would allow discussing in the same framework interpretations of classical and quantum YM theories. Ideally we want our philosophy of classical and quantum physics not to be completely separate. Some difficulties seem specific to quantum physics, like renormalization and anomalies, but a discussion of the variables of a gauge reduced classical theory could inform the interpretation of the associated quantum YM theory. If the passage from a classical gauge theory to its quantized version is transparent enough it is a clear advantage.

Finally, a good technical formalism must be general enough to the discussion of many theories in a similar way. For example a technical framework that would only be able to represent well $U(1)$ YM theory and not non-Abelian YM theories would be interesting but could in fact mislead us. As a requisite the chosen technical formalism should be able to represent all YM theories but also be a good framework to examine more specific features like the vacuum, renormalization and anomalies.

I believe two formal apparatuses fill all these requirements: Dirac quantization (DQ) based on the representation of a YM classical theory as a constrained Hamiltonian[2] and Feynman functional sum over histories quantization (FQ) which starts with a Lagrangian representation of the classical YM theory. Both formalisms come from physics itself. Both formalisms express the transition from classical to quantum theory in a relatively clear manner. Schematically, DQ is an extension of canonical quantization techniques. In the classical theory the gauge transformations correspond to first-class Hamiltonian constraints. During the quantization process the first-class constraints are promoted to operators on a Hilbert space that can identify vectors in the physical sector. In FQ the quantization process consists in producing a functional sum over possible classical histories of the studied system. Each history contributes through its associated action to a probability amplitude. I will return to this shortly. Finally, both formalisms are general in two senses. First, they can accommodate all YM theories and provide a representational framework to discuss more specific questions, like the status of the vacuum. Second, they can serve as formal language to formulate an interpretation of quantized gauge theories. In their actual form, these formalisms are far more than simple quantization tools. Unfortunately, you would not notice this reading Healey's book. Since the landmark publications of (Dirac, 1950) and (Feynman, 1948), these formalisms became sophisticated apparatuses able to frame in an interesting way most of our questions about YM theories. See for example (Henneaux & Teitelboim, 1992) or (DeWitt,

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[2] Healey uses the constrained Hamiltonian formalism to argue effectively certain points but he does not do it in a systematical way.

2003).

It is not the place here to discuss in depth all questions that can be clarified by adopting one or the other of these formalisms. For examples about the use of DQ I refer the reader to (Earman, 2003) and (Pons, 2005). As for FQ, since this formalism is almost absent from philosophical literature I will briefly present two examples where FQ can help to clarify some aspects of quantum YM theories. As a first example let us discuss briefly the status of holonomies in quantum YM theory. Healey pushes strongly for a holonomy interpretation of classical YM theories. He also maintains that the holonomy concept is essential to understand the AB effect because this phenomenon can be model as a $U(1)$ principal fibre bundle. The FQ formalism can help to defend this point and generalize it. In the context of non-relativistic quantum mechanics the transition probability amplitude of a system, let us say of one charged particle, is computed by summing the phase contributions of all possible trajectories (histories). If we add electromagnetism to the picture we can note that the relative change of phase between two paths, caused only by electromagnetic interaction, is exactly equal to the holonomy computed for a closed curve formed by the two already mentioned paths, with the electromagnetic potential playing the role of the connection. In other words, since in FQ a quantum phenomenon is the result of the interference between contributions of different histories, holonomy is the relevant entity to qualify *in general* electromagnetic interaction in semi-classical contexts.

The way to extend this reasoning to a fully quantized YM theory with matter fields is not obvious. However, if we limit ourselves to models built on a lattice, Wilson (1974) has shown that the gauge interaction can be characterized in significant contexts by the trace of what, in a geometrical setting, would be called holonomies. In consequence holonomies are not just a nice tool to characterize classical gauge properties but also are a fundamental mathematical entity to characterize gauge interaction in non-relativistic quantum physics and in certain models of quantum field theory on a lattice.

The second example is more technical. To quantize a non-Abelian YM theory is a tricky business. First, we have to get under control the gauge surplus. If we just fix the gauge the obtained quantized theory is not unitary. As explained by Healey (p. 167) one of the convenient ways to quantize a non-Abelian YM theory is to add to the gauge fixed Lagrangian terms involving new fields, called ghosts, that guarantee unitarity and renormalizability.[3] Physicists consider these fields fictive. Using the FQ formalism, Bryce DeWitt proposes a proof that shows that this quantized theory (with gauge fixing and ghost terms) is equivalent to an eventual quantization of a

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[3] This procedure is equivalent to imposing a new global symmetry, called the BRST symmetry, to the YM theory (Becchi et al., 1976). The role of this symmetry in classical YM theory is discussed in (Catren, 2008).

gauge reduced theory.[4] In other words the added degrees of liberty coming from the ghost fields are just what is needed to neutralize the effect in the quantum theory of the added degrees of liberty coming from the gauge surplus. To my knowledge such a proof exists only in the FQ. The implications of this proof for the status of the gauge principle, as a pragmatic principle, are discussed in (Guay, 2008).

Even if this Carnap style approach is appealing, an important question remains about the compatibility of these two frameworks. If it is proven that they do not produce compatible quantum theories, starting with the same classical theory, a choice will have to be made. But in the mean time both should be exploited in a systematic way by philosophers.

4 Conclusion

In spite of the few already mentioned weaknesses, the reading of *Gauging What's Real* is very stimulating. Since the measure of any book is the questions it makes us ask, Healey's work is exemplary. The diversity of problems and approaches discussed in the book will undeniably provoke responses and comments from many readers. Is this not what a philosophy book should do?

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[4] The proof can be found in (DeWitt, 2003, chapter 24).

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