On Multiverses and Infinite Numbers*

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Abstract: A multiverse is comprised of many universes, which quickly leads to the question: How many universes? There are either finitely many or infinitely many universes. The purpose of this paper is to discuss two conceptions of infinite number and their relationship to multiverses. The first conception is the standard Cantorian view. But recent work has suggested a second conception of infinite number, on which infinite numbers behave very much like finite numbers. I will argue that that this second conception of infinite number is the correct one, and analyze what this means for multiverses.

When it comes to infinite number, the overarching question that I will address is: Which objects are the infinite numbers? How do I mean this question? In the finite case, you have the finite whole numbers, also called the natural numbers, numbers like 7, 13, and 106. I am interested in the question: How should these numbers, the finite natural numbers, be extended into the infinite?[1] I will address this question, and then talk about how that conception of infinite number bears on multiverses. As a preview, if you ask the question -- How many universes are there? -- then I think that any sensible finite answer is constrained to a finite natural number. You have to say that there are 7 universes, or that there are 106 universes. It doesn't make any sense to say that there are 2.5 universes, or that there are -2 universes, or that there are 3+2i universes. Now similarly I am going to argue that if you give an infinite answer to the question -- How many universes are there? -- then any sensible infinite answer is constrained to an infinite natural number. So that is why I think it is important to answer the question: Which objects are the infinite natural numbers?

That is a bit of an overview. Let us turn to the concept of infinite number. I am going to tell a bit of a story. This is a story in which the whole numbers were not moved into the infinite in the correct way. The standard view of infinite number is a Cantorian view and I am going to tell a story, involving concrete objects, that tries to highlight the sort of mistake that I think was made. Consider the following items (and concepts): male birds, female birds, and black cars in the United States. And somebody comes along, and he (his concepts) carves up the world as follows. When he sees a male bird, he says "There is a male bird." But he thinks that the black cars in the U.S. are the female birds. So when he sees a black car in the U.S. he says, "There is a female bird." Here is a table summarizing this:

Object	**Concept/Calls Them**
Male Birds	"Male Birds"
Female Birds	
Black Cars in the U.S.	"Female Birds" [wrong conception]

Now, I think that this person would run into all sorts of questions and problems. For example: Why can't male birds and female birds produce viable offspring? Why are female birds so large? Why, when female birds cross the border into Canada, do they disappear? There would be a whole host of

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questions that would arise. I suggest that it would be unwise to tackle these problems head-on. Instead, we should try to change this person's way of carving up the world.

I am just going to assume that there are better and worse ways for our concepts to carve up the world. I won't argue for that point. I do just think that that is true, and I suggest that the situation described above is not a good way to carve up the world. Now the question becomes: Is there any way of resolving this dispute, that is, is there any way of straightening this person out? I suggest that there is. We agree on male birds, that is, both I and this confused person point to a male bird and say, "There is a male bird." The disagreement is that I think that the female birds are the female birds, whereas this person thinks that the black cars in the U.S. are the female birds. Since we agree on male birds, the way to go is to tell this person, "Hey, the female birds are a lot more like the male birds than are black cars in the U.S." I think that this is the way out of the confusion. You begin from your point of agreement, in order to argue that the female birds are the female birds. This argument isn't profoundly deep and subtle. It is simply based on the idea that the more something walks like a duck and quacks like a duck, the more likely it is to be a duck.

Now let's turn to numbers. I do think that Cantor was trying to move the whole numbers into the infinite. Consider a quote of Cantor's:

> "I arrive at a natural extension, a continuation of the sequence of real, whole numbers which leads me successively and with the greatest security to the increasing powers [here he means the infinite ordinals] whose precise definition has failed me until today."[2]

There are a number of quotes along these lines. I'll just read one more so as not to multiply quotes too much. These two quotes are from Michael Hallett's, "Cantorian Set Theory and Limitation of Size" where, again, there are a number of quotes along these lines:

> "This I do in my transfinite number theory … with a definition based on the general concept of well-ordered set. Here I will show, in the clearest possible way, that we are here concerned with real concrete numbers in the same sense as 1, 2, 3, 4, 5, … that have been designated and looked upon as numbers from ancient times."[3]

So I do think that Cantor was trying to move the whole numbers into the infinite.[4]

What I am now going to suggest is that the Cantorian infinite (the infinite ordinals and cardinals) are not the infinite numbers. Below I have a table, meant to mirror the table above, but now involving finite numbers, infinite numbers (and here I mean infinite natural numbers in a nonstandard model of the reals), and the Cantorian infinite.

<table>
<tr><th>Object</th><th>Example</th><th>Concept/Calls Them</th></tr>
<tr><td>Finite Numbers</td><td>| | | | |</td><td>"Finite Number"</td></tr>
<tr><td>Infinite Numbers</td><td>| | | …   …| | | | |…   …| | |</td><td></td></tr>
<tr><td>Cantorian Infinite</td><td>| | | | | ...</td><td>"Infinite Number"   [wrong conception]</td></tr>
</table>

In this chart, finite numbers, infinite numbers, and the Cantorian infinite[5] are playing the roles, respectively, of male birds, female birds and black cars in the United States. So that when someone says, pointing at ω, “there is an infinite number,” that is exactly the same sort of mistake as when someone points at a black car in the United States and says “there is a female bird.”

What is the argument here? Essentially it boils down to the claim that the infinite numbers in a nonstandard model of the reals just behave, and are much more like, the finite numbers, than is the Cantorian infinite. So again, it’s the same idea that if we agree on male birds, I’ll just point out that the female birds are more like the male birds than are black cars in the U.S. Similarly, the infinite numbers in nonstandard model just behave much more like the finite numbers than does any example of the Cantorian infinite.[6]

Many people have not run across these nonstandard numbers, and so let us consider some examples of this similarity of structure and behavior. Any finite number, when you consider order, has a first element with a successor, a last element with a predecessor, and middle elements with both of these. Infinite numbers share this structure (e.g., see the example above, where I use strokes and dots to represent an infinite number). There is a first element that has a successor, a last element with a predecessor, and middle elements that have both predecessors and successors. Any infinite number (in a nonstandard model of the reals, which I sometimes drop for brevity) is even or odd. Any infinite number is prime or composite. If you add one to any infinite number, you get a larger number; it is larger by one. So I suggest that when we ask this overarching question -- which objects are the infinite numbers? – or even – what does an infinite number look like? -- the infinite numbers are infinite numbers in nonstandard model of the reals. They simply behave and look very much like finite numbers.[7]

Abraham Robinson developed these numbers in the 1960s. Robinson writes,

> “In the fall of 1960 it occurred to me that the concepts and methods of contemporary Mathematical Logic are capable of providing a suitable framework for the development of Differential and Integral Calculus by means of infinitely small and infinitely large numbers.”[8]

Recall that the overarching question we are considering is: Which objects are the infinite numbers? Another argument (that the infinite numbers in a nonstandard model of the reals are the infinite numbers) can be made simply by considering language. For example, Robison writes:

> “Thus any finite natural number is smaller than any infinite natural number.”[9]

To my knowledge, no one has ever been inclined to call any example of the Cantorian infinite an infinite natural number. So I think that argument can also just come directly out of language.[10]

Now let me talk a bit about the importance of this conception of infinite number outside of the context of multiverses, because I think there might be the concern that this is just a verbal issue. A person might come along and say, “That’s Ok. I’ll agree to call the infinite numbers in a nonstandard model ‘the infinite natural numbers.’ I’ll call the Cantorian infinite ‘the Cantorian infinite.’ So what’s the problem?” Well, in my initial example with birds and cars, these questions that arose could be tackled head on, the sort of silly questions I raised. But I think a better response to that person is to say, “you are just not carving up the world in the right way, you aren’t talking about female birds.” Similarly, in the case of infinite number, I think that certain issues arise, for example with paradoxes of the infinite in philosophy, or the continuum hypothesis in mathematics, where I don’t think the issues should be tackled head-on. I think the correct response is to say "you aren't talking about infinite numbers, you aren't carving up the world the right way."

Some examples are as follows. In Thompson's lamp,[11] a lamp begins on, and the button is pressed to toggle the lamp on and off at times a half, three quarters, 7/8ths, time 15/16ths, and so on…. The button presses are happening faster and faster, each within half the time it took to make the previous button press. Then the structure of the button presses is the structure of ω, that is, the structure of the positive integers. There is a first, a second, a third, a fourth button press, and so on. The question arises (Thompson tried to create a problem by asking): What is the state of the lamp after these infinitely many button presses? Many people think Benacerraf successfully replied to Thompson by saying that the state of the lamp is not determined at two minutes.[12] But I think the correct response is to say, "That's not infinite many, that's not an infinite number of button presses. If you are talking about infinitely many, you have to be talking about an infinite natural number in a nonstandard model of the reals." Note that any such number has to be even or odd. So if the button was pressed an even infinite number of times, then the lamp is in it starting state; if the button is pressed an odd infinite number of times, then the lamp in its opposite its starting state. When you actually talk about infinite numbers, not only is there no paradox, there is not even any lingering confusion.[13]

Another example is a Zeno sphere. This would be an object centered at an origin that has shells of increasing radius. There is a spherical shell of radius a half, and also has a shell of radius three quarters, a shell of radius 7/8th, and so on…. These shells go on approaching one, getting closer and closer and closer but never getting to one. The shells get increasingly thin so that no two shells touch each other. Note that there is no outermost shell. People ask questions like: What happens when two Zeno spheres collide? What happens when you shine a light on a Zeno sphere? I tend to think that the literature on this topic is a bit tortuous.[14] I also think the correct response here is to say, "that is not an infinite number of shells. That's not a possible thing you are describing." I don't that that ω exists in any actual determined sense, so there is a very Aristotelian feel to this view. If you want to talk about an object with an infinite number of shells, then you need to be talking about an object that has an infinite number of shells. And then there will be an outer shell, and there is no problem with these things colliding or with light bouncing off them. Again, no paradox or puzzle remains.

A third example is the spaceship paradox. A space ship travels one mile in an half an hour, another mile in a quarter of an hour, a mile in an eighth of an hour, a mile in a sixteenth of an hour, and so on…. So it keeps travelling miles but faster and faster and faster. You can ask the question: After these infinitely many trips, where is the spaceship? There is no good answer here. But again I think that the correct response here is that you are not talking about infinitely many. ω is not an infinite number. If you do talk about an infinite number it has to be an infinite number. Call it M. Then if you make infinitely many, M many, trips of 1 mile, then there is no problem -- you are M miles away. So I do think that you dissolve a number of, or maybe all, paradoxes of the infinite by recognizing the correct conception of infinite number.[15]

To take stock, when you are talking about infinitely many, it has to be an infinite integer. Any infinite integer has a certain structure, including having a last element. You get rid of many problems and puzzles of the infinite.[16] Now perhaps it is an overstatement to write, as I just did, that when talking about infinitely many then "it has to be an infinite integer" involved. A person can carve up the world so that the black cars in the United States are called and conceived of as the "female birds." But confusion ensues. Similarly, correctly identifying the infinite numbers need not be done, but it should be done. Arriving at the correct conception of the infinite numbers leads out of a great deal of confusion. What are the costs? I see none. ω can still be used, discussed, and explored. Nothing is lost. The Cantorian confusion is to think that ω is an infinite number, and that it is an actual, determined, static entity, so that, for example, a sphere can have ω shells. Rather, ω is merely potentially infinite. There is no benefit in wracking one's brain in attempting to determine what happens when two Zeno-spheres collide, because a Zeno-sphere cannot exist.[17] By contrast, it makes perfect sense to use ω to talk about limit processes; certainly the sequence 1/1, 1/2, 1/3, 1/4, 1/5, … has a limit of 0. No legitimate uses of ω are barred.[18]

How does this conception of infinite number bear on multiverses? One answer is that this position has implications for the mathematics of multiverses. In particular, if there are infinitely many universes, then there is some infinite (natural) number of universes. The mathematics then becomes very like the finite case. It's almost exactly the same. And so if M is an infinite integer, and you have M universes of value 1, then the total value is M times 1, or M. If you then add in another universe of value 1, the total value is greater, namely M+1. Adding a good universe to a multiverse always increases the total value.

Adding a good universe may, however, decrease the average value. If you have M universes of value 1, then the average value is 1. Let's say that you then add a universe of value 0.5. The average value is now lower, it is infinitesimally less than 1, namely (M+0.5) / (M+1). The average value has gone down.[19]

Let us consider one more example to highlight the fact that the mathematics of multiverses, on what I claim is the correct conception of infinite number, is very much like the finite case. Imagine that you have universes numbered 1, 2, 3… and so on through ...M-2, M-1, M. You can ask: What is the total value? Now if M is a finite number, the answer is given by the formula M(M+1)/2. So if m is 4, and so there are universe of value 1, 2, 3, and 4, then the formula gives a correct total value of 4*5/2 = 10. And to see that this is correct, note that 1 + 2 + 3 + 4 = 10. The same formula holds in the infinite case. If M is an infinite integer, you have universes valued 1, 2, 3...M-2, M-1, M, and you ask for the total, then it is still given by that formula M(M+1)/2. If you ask for the average value, this case is again like the finite case, with the average value given by (M+1)/2. So again, the summary is that the mathematics becomes very much like the finite case. The mathematics of multiverses is greatly simplified on the correct conception of infinite number. With the math just like the finite case, it is possible to talk about total values, average values, and so forth. In the Cantorian infinite, matters are not so clear.

Now, what does this mean for multiverses and theism? Certainly that depends on the specific theistic position under consideration and also on the multiverse theory that is proposed. For example, here is a theory of a multiverse: the multiverse begins with one universe and then splits into some finite number of universes. At any point in time, a single universe can split into a finite number of universes. The conception of infinite number that I have proposed does not have any bearing on this type of multiverse, because at any given time there are only finitely many universes. For example, there may be one universe, then two, then four, then eight, etc. The number of universes doubles and therefore grows without bound, but at no time do an infinite number of universes exist.

On other theories of multiverses, an infinite number of universes exist. On these theories, I claim, the number of universes in a multiverse must be an infinite number. Let me now discuss how the conception of infinite number outlined above relates to this sort of multiverse and theism. Our universe is a rather remarkable thing. The vastness of the universe, the solar system, Earth, humans, morality, beauty, order -- these are all things that seem to cry out for an explanation. Many explanations are possible.[20] Some theists argue to God's existence (or at least to the existence of a designer) from the fine-tuning of the universe. For example, it is often pointed out that if physical constants had been slightly different, then humans could not exist. The universe is fine-tuned for life, which may raise the probability that the universe was designed to be able to support life. An atheist might then reply that there is a multiverse, and so it is not surprising that we find ourselves in one of the universes able to support the existence of human life. And indeed if there are infinitely many universes, then the claim that fine-tuning provides support for the existence of God is undercut. Here I think we are in the realm of probability, where we can ask the question: what is more likely, the existence of God or the existence of infinitely many universes? Though I won't argue for these claims here, I believe that smaller numbers are simpler than larger numbers, and that simplicity raises the (prior) probability of a hypothesis. Richard Swinburne has argued these two points.[21] It follows that it is most likely that 1 universe exists. The atheist in this dialogue can attempt to argue that the infinite is simple, because the infinite is a sort of endpoint or maximal entity. This move attempts to raise the probability of a multiverse that contains infinitely many universes. However, the Cantorian position on infinite number may seem simple, but if my view of infinite number is correct, then a specific infinite number itself is not simple – any infinite number is mysterious in its specificity. This then detracts from the (prior) probability that there is a multiverse. Any infinite number is just as mysterious and specific as a finite number. By this I mean that if you say that there are 56 universes, questions arise. Why not 57? Why not 55? Why not any other number? Why an even number? All these questions also arise when you say that there are an infinite number of universes. If there are M universes, where M is an infinite integer, then M is even or odd. Questions arise: Why not M+1 universes? Why not M-1? Why not any other number? If M is even, why an even number? And so I think that a problematic feature of this position on infinite number for multiverses is that you lose this appeal to simplicity, which then affects prior probabilities. Thus, if we focus on multiverses versus a designer as hypotheses explaining order and fine-tuning, this view of infinite number makes the multiverse less likely, and therefore makes the existence of a designer correspondingly more likely. The overriding point is that the hypothesis of one universe is far more likely than the hypothesis of any larger specific number of universes, whether finite or infinite. And so this position on infinite number, I believe, provides some support for theism and the existence of one universe.

Though above I discussed two competing explanations for the existence of our universe, I certainly did not mean to imply that the list was exhaustive, nor did I mean to imply that theism and a multiverse are contradictory. For example, perhaps God exists and creates all and only those universes that are worthy of creation.[22] I think that the considerations outlined above lower the probability of there being a multiverse (relative to the probability of there being one universe), as any infinite number is not simple. Ultimately, strong reasons would have to be presented in order to have a successful argument for the existence of a multiverse. I think that it is currently an open question whether there are any such strong reasons. The numerical considerations presented above make it most likely that one universe exists, ours.

Before summarizing and concluding, I will tie up a few points and try to anticipate a few objections. One key point: I don't want to overstate the claim that I am making. I talked before about how I was going to tell a story where the whole numbers were not moved into the infinite correctly, Cantor did not undertake the task correctly. And indeed I believe that the infinite numbers have been misidentified. When you say “there is an infinite number” or are talking about “infinitely many,” you should be referring to be an infinite integer. But I don’t mean to detract from Cantor’s contribution. I am not claiming that ω is completely useless. Nothing of the sort. My claim is a limited one about the correct conception of infinite number. ω is certainly infinite, insofar as it is not finite, but it is not an infinite number. I think that ω is a potential infinity, and so, for example, of course you can talk about infinite sums. The summation of $1/2^i$, for i ranging from 0 to infinity equals 2. This is a true, meaningful, and important claim. But I think that it is also important to keep in mind the underlying meaning. The underlying meaning is that for any epsilon, no matter how small, I can tell you where, from some point on, the partial sums are within epsilon of 2. It is not somehow that you really get to 2. This fact provides evidence that ω is a potential infinity.[23] This paper, in this regard and as I have mentioned, has an Aristotelian feel. In contrast when you take a hyperfinite sums, so you add up infinitely many things, where the sum runs from 1 to M and M is an infinite integer, then I think that you are dealing with an actual infinity.[24] And your sum really does sum to a specific number. The sum from i=1 to M of 1/M really sums to 1. The sum from 1 to M of $1/2^i$ is actually infinitesimally shy of 1, so there is no sort of potential infinity in these cases involving the correct infinite numbers. This bolsters the claim that these objects really are the infinite numbers.

But why, an objector may ask, reject other conceptions of infinite number? The reason is that I am asking a specific question, and I think that there is a single correct answer. In particular, the infinite numbers, correctly identified, are the infinite numbers in a nonstandard model of the reals. By way of comparison, imagine that there was 120 years of confusion on the bird/car issue, so that people see a black car is the U.S. and say “there is a female bird.” I think that this is simply wrong. The female birds are the female birds. Similarly, the infinite natural numbers should be properly identified. But why, the objector may continue, is there only one correct conception of infinite number? And why must the finite numbers be our guide to determining what the infinite numbers are? To the second question I simply ask: Why else do we have? If there are any other suggestions as to how to arrive at the infinite numbers, let them be produced.[25] In reply to the first question, note that the person who thinks that black cars are the female birds can ask the same question, namely: why is there only one correct conception of female bird? Why not have black cars in the U.S. and female birds fall under the concept of female bird? Whatever response is given here can be used in the case of infinite number. The response, at its core, is simply that this way of breaking the world into concepts is not a good one. It most likely will lead to confusion. In the context of infinite number, I have suggested two examples of such confusion (that arises from thinking of the Cantorian infinite as being one example of infinite numbers): philosophical paradoxes involving the Cantorian infinite and the continuum hypothesis.

Another potential objection is: What if the universe really does go on infinitely far in all directions? And here my response is that I don’t think that this is describing a genuine possibility. Just as ω is not an actually infinite number, so too a ray is not any actually determined distance. If you want an infinite universe, then it has to be contained within an actually infinite distance, and any actually infinite distance has endpoints. You can then embed open-ended space within that, in a copy of the hyperreals (in one dimension). A very similar objection, returning to the context of number, is: What if there just really is a multiverse that has universes numbered 1, 2, 3,... and so on? But here again my response is that I don’t think that that is describing anything. There is no set of all finite numbers. ω is not an actual determined thing, and so I don’t think that that is a real possibility.[26]

One final objection: Why does there have to be a number of universes? Someone may say that there not only doesn’t have to be a number of universes, but furthermore the universes don't even need to form a set. Here my reply is that I think that the finite has to be our guide. And so I mentioned before that any finite answer (to the question: how many universes are there?) is constrained to a finite number. Similarly, any sensible infinite answer is constrained to an infinite number.

To summarize and conclude, there are better and worse ways for our concepts to carve up the world. I do not believe that the infinite numbers have been properly identified. I think that the infinite numbers are as I've described. Sensible infinite answers to the question -- how many? -- must be infinite numbers. This view of infinite numbers simplifies the mathematics of multiverses, but detracts from the probability of there being any specific number of universes in a multiverse. In the context of the competing explanations of God versus a multiverse to explain fine-tuning, this lowers the probability of a multiverse, and may thereby bolster the probability of theism insofar as the existence of only one universe is more likely than the existence of many universes.

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Notes

[1] We might also ask the questions: What does one infinite number look like? And in what sort of system does it operate? These questions are also addressed in Gwiazda (2011) and (2012).

[2] Quoted in Hallett 60.

[3] Quoted in Hallett 51.

[4] Though perhaps this is overstated. It might be more accurate to say that Cantor was concerned with a particular mathematical problem, which then led him to the ordinals. He then thought that these ordinals were the correct extension of the finite natural numbers into the infinite. It is this last point that is important – Cantor thought that his ordinals answered the question: What are the infinite (natural, whole) numbers?

[5] I should maybe say a bit about how I think of these numbers. I really do just think of them in terms of strokes and dots. And so five, above, is an example of a finite number -- it is five strokes. It does have a certain structure to it. So for example, order is involved, so in 5, or | | | | |, there is a first element, that has a successor, a last element has a predecessor, and middle elements with both predecessors and successors.

[6] I said above that the argument was not profoundly deep or subtle. And with all my talk of birds and cars, I worry that it is easy to dismiss my argument. But I am claiming that people, for over 100 years, have not correctly identified the infinite numbers. And just as misidentifying the female birds would lead to all sorts of confusion, I claim that people have been led into all sorts of confusion based on this misidentification of the infinite numbers. The argument may not be deep or subtle, but I do suggest that the conclusion is correct and important.

[7] Just like female birds behave and look very much like male birds.

[8] Robinson xiii.

[9] Robinson 51.

[10] I also think that the same point also applies to infinite distance. I don’t think there is any distance to a ray or even a real number line. I don’t think those are examples of infinite distance. I think that if you want to talk about infinite distance, it has to be infinite distance between two points, and the structure of this is going to be very similar to the structure of an infinite integer. Just as infinite integers have a certain structure, so too infinite distance has this structure:

|----------->   …   <-------------->   …   <---------------|

[11] Thomson 1954.

[12] Benacerraf 1962.

[13] This topic is addressed in Gwiazda 2013a.

[14] An actual, determined, static Zeno-sphere is physically impossible, I believe. For an example of an argument involving Zeno-spheres, which to my mind does not adequately explain what happens when light shines on such an object, see Peijnenburg and Atkinson 2010.

[15] In fact, I think that an interesting challenge is to try to arrive at any paradoxes, while referring to the correct conception of infinite number. That is, I am suggesting that perhaps no paradoxes of the infinite remain.

[16] I also think that there is potential importance to correctly identifying the infinite numbers to physics and math. For example in cosmology, if the universe is infinite, then it has some infinite distance. Just as any finite distance (in say, one dimension) is bound by endpoints, so too a universe of infinite distance must be bound. Cosmologists should consider the possibility that the universe is infinite and

bound. In math, I think that something like the continuum hypothesis is just a nonstarter. I don't think that there is any number of positive integers, and so it is no surprise that when you ask about the positive integers and the powerset of the positive integers, and ask the question -- Are there sizes in the middle? -- it is no surprise that you don't get a clean answer when there is not even any number of positive integers.

[17] Just as it is most likely not fruitful to explore why a black car and a male bird cannot mate. How our concepts carve up the world matters.

[18] Legitimate uses are precisely those where it is recognized that ω is merely a potential infinity.

[19] Though perhaps something of a tangent, I do find it an interesting question: Which value matters more in the context of multiverses, the average value or the total value. Here I admit that I do not have strong intuitions one way or the other. If Picasso painted a painting that was worth painting, but it brought down the average value of his paintings, it's unclear to me whether this is good or bad, or maybe more detail is needed. Perhaps producing paintings and producing universes is disanalogous, but in neither case do I have a sense of whether the total or the average is the relevant consideration.

[20] Of course, there might be no explanation. The existence of the universe might simply be a brute fact.

[21] Swinburne 2004.

[22] For one argument to this conclusion, see Kraay 2010.

[23] This topic is discussed in Gwiazda 2013b.

[24] For a test to determine whether a structure is potentially infinite, see Gwiazda 2013b.

[25] To push this point a bit further, if someone points at his cat and says "There is an infinite number," that seems wrong. But if you give up using finite numbers to arrive at the infinite numbers, it is genuinely unclear to me how to argue that this person is wrong. Again the question is, if we give up finite numbers, what do we then use to arrive at the infinite numbers?

[26] When it comes to the size of ω, there is no number of natural numbers. What is the argument? I think that two things are the case. The first has been the main theme of the talk: infinite numbers, properly understood, are infinite numbers in a nonstandard model. Second, I also think that the correct way to judge the relative sizes of infinite numbers is via what is sometimes called SUBSET, where if A is a proper subset of B, then A is smaller than B. So if you add 1, then you have a larger set, larger by 1. This contradicts judging sizes based on BIJECTION, or what is sometimes called Hume's Principle. If you grant these two claims, then it turns out that ω is too large to be a finite number, as any finite number is a proper subset of ω. But ω is also not an infinite number, as ω, structurally, is the initial segment of any infinite number. ω is this strange middling beast that is too large to be finite in number, but too small to be infinite in number. That is why I think that there is no number of natural numbers, and for example, why there cannot be ω many universes in a multiverse. See Gwiazda 2011 and Gwiazda 2012 to see this argument in more detail.