PASCALIAN EXPECTATIONS AND EXPLORATIONS

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Edited by Roger Ariew and Yuval Avnur (Wiley-Blackwell)

Alan Hájek and Elizabeth Jackson

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Abstract: Pascal's Wager involves expected utilities. In this chapter, we examine the Wager in light of two main features of expected utility theory: utilities and probabilities. We discuss infinite and finite utilities, and zero, infinitesimal, extremely low, imprecise, and undefined probabilities. These have all come up in recent literature regarding Pascal's Wager. We consider the problems each creates and suggest prospects for the Wager in light of these problems.

Keywords: Pascal's Wager; expected utility; infinite utility; zero probability; low probability; infinitesimal probability; undefined probability; Pascal's mugging

1. Introduction

Pascal's Wager takes up a mere three or four pages of the *Pensées*, but it has generated countless responses. Indeed, if we regard the ratio of *number of citations* to *number of pages* as a measure of the punch of a philosophical work, it is hard to beat Pascal's Wager. Over the centuries it has spawned rich debates over the rationality of theism, the nature of God, the nature of infinity, the afterlife, the ethics of belief, voluntarism about belief, and decision theory, and these are all lively lines of inquiry nowadays. (For overviews of some of these debates, see Rota 2017, Hájek 2018, Jackson 2021, 2023a.) We clearly cannot pursue them all in detail in this chapter; since the Wager's impact on decision theory has been especially significant, and since this bears on the other debates, we focus mostly on it. Hacking (1975: viii) considers the Wager to be "the first well-understood contribution to decision theory", and it has inspired many contemporary developments in decision theory.

The first contribution to decision theory it may well be—but is it well understood? This chapter will put at center stage the numerous problems for decision theory that it raises, and various ways in which contemporary authors have grappled with them, which in turn enhance our understanding of the Wager itself. In **Section 2**, we introduce expected utility theory and the role it plays in Pascal's Wager. In the subsequent two sections, we focus on two central aspects of expected utility theory: utility and probability. **Section 3** covers infinite utility (3.1) and finite utility (3.2) and the relationship of each to the Wager. **Section 4** covers five types of probability: zero probability (4.1), infinitesimal probability (4.2), extremely low probability (4.3), imprecise probability (4.4) and undefined probability (4.5), and implications each has for the failure (or success) of Pascal's argument—we canvass and add to the contemporary literature on these topics. In **Section 5**, we discuss some recent alternatives to expected utility theory that have been inspired by Pascal's Wager, and in **Section 6**, we discuss the continuing influence of Pascalian reasoning, especially in the fields of normative and applied ethics. We conclude in **Section 7**.

2. Expected Utility Theory and Pascal's Wager

Expected utility theory is the standard approach to decision theory. As a normative theory, it tells us what we should decide to do when we are uncertain about facts relevant to those decisions. For example, you might use expected utility theory to decide whether to buy car insurance or whether to invest in a certain stock. You can also use it for more mundane decisions, like whether to carry around an umbrella if there's a chance of rain or whether to make a silly gamble.

Expected utility theory can (at least in principle) be applied to any decision. A decision problem is represented by a matrix that specifies a number of options (the rows) and a number of possible states (the columns) with associated probabilities. These probabilities are measured on the interval [0,1]. Each cell in the matrix specifies the utility (value) of the outcome of the corresponding option/state combination. We calculate an option's expected utility as follows: for each state, multiply the utility that the option produces in that state by the state's probability; then, add these numbers. According to expected utility theory, rationality requires you to perform an option that maximizes this quantity (if there is one).[1]

Let's say you're offered the following bet. A coin is flipped. If it lands heads, you receive \$10. If it lands tails, you have to pay \$1. If you decline the bet, no money changes hands. You now

[1] We set aside complications concerning rival formulations of decision theory—for example, *evidential decision theory* (Jeffrey 1965/1983) versus *causal decision theory* (Joyce 1999). See Briggs (2023) for more on expected utility theory.

must choose whether to take the bet or not take the bet. (We assume that all you care about here are the monetary amounts, and that you value them at their face value—you value each dollar the same whether you are richer or poorer.) This matrix represents how you could use expected utility theory to make this choice:

	Heads (pr = 0.5)	Tails (pr = 0.5)	Expected utility
Take the bet	10	-1	4.5
Don't take the bet	0	0	0

Recall that to calculate the expected utility of each option, you multiply the probability of each state (heads, tails) by the utility (or value) that corresponds to each option. Then you add these products across each row to calculate the expected value of each option. You *multiply down, add across*. So we calculate the expected utility of taking the bet:

Take the bet: $(0.5 \times 10) + (0.5 \times -1) = 5 - 0.5 = 4.5$

We compare that to the expected utility of not taking the bet:

Don't take the bet: $(0.5 \times 0) + (0.5 \times 0) = 0$

So the expected utility of taking the bet is 4.5, and the expected value of not taking it is 0. Since 4.5 is greater than 0, according to expected utility theory, you should take this bet.

Pascal's Wager applies expected utility theory to the question of whether you should wager for God.[2] There are different ways of wagering, including what Jackson (2023b) calls the *doxastic* wager, which involves believing in God, and the *acceptance* wager, which involves acting as if God exists or making a religious commitment. We'll remain neutral between these possibilities and refer to their disjunction as "wagering" on God.

Contemporary receptions of the Wager include contemporary *formulations* of the Wager in decision-theoretic terms. We will follow this formulation, consisting of three premises: the first

[2] In fact Pascal gives four arguments for wagering for God in *Pensées* §233. Three are *dominance* arguments—see Hacking (1972) and Hájek (2011, 2018). But it is the argument from expected utilities that is usually called "Pascal's Wager", and we will focus on it.

concerns the decision matrix of utilities, the second concerns your probability for God's existence, and the third is the central principle of expected utility theory that we have just presented.

1. Either God exists or God does not exist, and you can either Wager for God or Wager against God. The utilities of the relevant possible outcomes are as follows, where f_1, f_2, f_3 and f_4 are finite numbers.

	God exists (pr = p > 0)	God does not exist (pr = 1– p)	Expected utility
Wager for God	∞	f_1	∞
Don't wager for God	f_2	f_3	f_4

2. Rationality requires you to assign positive probability to God's existence.
3. Rationality requires you to maximize expected utility.

Conclusion. Rationality requires you to wager for God.

The conclusion seems to follow from the premises. The expected utilities are as follows:

Wager for God: $(p \times \infty) + ((1-p) \times f_1) = \infty$ (since $p > 0$ by premise 2).

Don't wager for God: $(p \times f_2) + ((1-p) \times f_3) = f_4$

$\infty > f_4$, hence by premise 3, rationality requires you to wager for God.

Expected utilities are sums of products of utilities and probabilities. Recent literature raises various problems for expected utility calculations associated with Pascal's Wager: infinite utility, zero probability, infinitesimal probabilities, extremely low probabilities, probabilities that are imprecise over an interval that includes 0, and probability gaps. We'll consider each of these in turn.

3. Pascal's Wager and Utility

3.1 Infinite Utility

Lakatos (1974) quipped that "all theories are born refuted", and it is indeed striking that decision theory was inaugurated by one of its most controversial applications. Philosophical and mathematical problems begin with the very first entry in premise 1's decision matrix: ∞, the putative utility of salvation. How should we understand this magnitude, and can it represent a decision-theoretic utility? This is not merely a *potential infinity* (which an Aristotelian might recognize) but an *actual infinity* that is fully realized (of which an Aristotelian is skeptical). Pascal speaks of "an infinity of infinitely happy lives". The extended real number line adds two infinite elements to the real number system: ∞ and $-\infty$. We will interpret Pascal as claiming that salvation's utility is the former: one's reward for wagering for God if God exists.[3] However, it is controversial whether expected utility theory allows such a utility.

The classical formulations of expected utility theory do not. Jeffrey (1983: 150) observes that Pascal's Wager is "outside the scope" of Bayesian decision theory. Von Neumann & Morgenstern (1953), Savage (1954), Jeffrey and others all have a version of a *continuity* axiom that prohibits infinite utilities. Suppose that you regard option C as at least as preferable as option B, and option B as at least as preferable as option A. We may write this as:

$$A \preceq B \preceq C.$$

Then continuity requires that you are indifferent between B and some gamble between A and C, with real-valued probabilities of yielding each. Imagine a probability dial that controls how likely it is that you will receive C rather than A. Setting the dial at 1 is at least as good for you as B. Now we slowly twiddle the dial, gradually reducing the probability of C and increasing the probability of A. At some point you must hit a setting that you regard as *exactly* as good as B.

However, if the utility of salvation is ∞, continuity will be violated. For example, you prefer salvation to \$1 and prefer \$1 to \$0. There is no gamble between salvation and \$0 that you regard as exactly as good as \$1. As we gradually reduce the probability of salvation, you keep preferring the gamble to \$1, until we finally reduce the probability of salvation to 0, at which point you suddenly prefer \$1—how much you value the gamble discontinuously 'jumps' below the value of \$1. More generally, continuity requires that sufficiently small changes in the probability of salvation should yield correspondingly small changes in the value you attach to the gamble at salvation. But there is a gulf between the value you attach to *no* chance of salvation and to *some*

[3] Some commentators interpret him as claiming that damnation's utility is the latter: one's punishment for *not* wagering for God if God exists. But we find this hard to square with Pascal's writing (1662: section III): "The justice of God must be vast like His compassion. Now justice to the outcast is *less vast* … than mercy towards the elect" (our emphasis).

chance of salvation, however small that chance. See Jeffrey (1965/1983/1990), McClennen (1994), Sorensen (1994), and Hájek (2014) for further discussion.[4]

This is symptomatic of the swamping effect of infinity—it overwhelms anything finite in the expected utility calculations. Multiplying ∞ by any positive, finite probability again yields ∞. Let us call this property of ∞ *reflexivity under multiplication* (by such a probability). Of course, this is Pascal's Wager's great virtue: anyone who assigns positive probability to God's existence, however small, should feel its full force.

But it is also arguably its great vice—see Duff (1986) and Hájek (2003a). It seems that by Pascal's lights, one need not wager for God to enjoy infinite expected utility. Any option that delivers a positive probability that one will wager for God also has infinite expected utility. Suppose that you let a coin toss determine whether you will wager for God or not: if the coin lands heads you will, if it lands tails you won't. With probability ½ the coin will land heads, and your expected utility is ∞ at that point; with probability ½ the coin will land tails, and your expected utility is finite at that point—by Pascal's lights. The expected utility of this option is:

$$(\tfrac{1}{2} \times \infty) + (\tfrac{1}{2} \times f_4) = \infty.$$

Now suppose you let a lottery with a million tickets determine whether you will wager for God or not: if ticket #1 wins you will, otherwise you won't. The expected utility of this option is:

$$(1/1000000 \times \infty) + (999999/1000000 \times f_4) = \infty.$$

And so it goes for any option that results in your wagering for God with probability $q > 0$, however small q is:

$$(q \times \infty) + ((1 - q)) \times f_4) = \infty.$$

Hájek argues that this may include any option you may perform. The upshot is that Pascal's Wager is invalid. Granting him all his premises, it does not follow that rationality requires you to wager for God. These alternative 'mixed strategies' also maximize expected utility.[5]

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[4] It is consistent with the continuity axiom that utility is unbounded: there is no finite number below which the absolute value of utility must remain. This is compatible with an Aristotelian view that allows it to be potentially infinite. Hardin (1982) and Joyce (1999: 37) insist that utility is bounded; this challenges all the more premise 1's appeal to infinite utility. However, see Fishburn (1970), Nover & Hájek (2004), and Russell & Isaacs (2021) for a defense of unbounded utility.

[5] However, at least it follows that you should not wager against God outright (this 'pure' strategy has finite expected utility).

To be sure, these alternative routes to wagering for God are less likely to result in infinite utility than outright wagering for God, by Pascal's lights. To shore up the Wager against this objection, we may add a further premise that one should maximize the probability that one will wager for God (see Schlesinger 1994). This raises further issues about what one's options really are: are they simply wagering for God or not, or rather other options that may result in one's wagering for God or not with higher or lower probabilities? And if they are the latter, what is the option that maximizes this probability? Is it *trying to wager for God*, *intending to wager for God*, or perhaps something else? (Notice that these were not the options in the original Wager.) In any case, the Wager is no longer as simple as Pascal's original version.

Hájek (2003a: 34–49) offers many valid reformulations of the Wager in response to this objection—we will see a couple of them shortly. Several of them involve replacing the ∞ utility of salvation with a quantity that is *not* reflexive under multiplication: multiplying it by ½, or 1/1000000, or $p > 0$ makes a difference—for the worse. Along similar lines, Jackson and Rogers (2019: 64–65) provide several thought experiments that suggest that, at least in these cases, the relevant infinities are not reflexive under multiplication. For example, suppose you are offered a choice between a 0.1 chance at getting infinite utility or a 0.9 chance at getting infinite utility. If we treat infinities as reflexive under multiplication, you should be indifferent, since any positive probability times infinity is infinity. However, clearly you should prefer the 0.9 chance at infinite utility. Along similar lines, we should prefer the pure strategy of simply wagering on God to any mixed strategies. In general, even when dealing with infinity utilities, probability should matter, as it obviously does when dealing with finite utilities. Indeed, it should matter *more* in the former case, since the stakes are so much higher!

While Hájek agrees that denying that salvation is reflexive under multiplication provides a solution to the mixed strategies objection, he worries that such reformulations are not faithful to Pascal's theology. In particular, Pascal (1662) writes: "Unity added to infinity adds nothing to it." Let us call this property of ∞ *reflexivity under addition*. In other words, $\infty + 1 = \infty$; adding any real number to infinity makes no value difference. Because Pascal thought that salvation was supposed to be the best thing possible, it makes sense that Pascal would want the infinity associated with salvation to be reflexive under addition. However, then we are in a predicament of denying that the infinity associated with salvation is reflexive under multiplication, but affirming that it is reflexive under addition (a particularly pressing problem given the close relationship between the two arithmetic operations). It's not clear that we can both parry the mixed strategies objection and maintain that salvation is the best possible thing.

Given this dilemma, some authors have suggested that we ought to relinquish Pascal's theological commitments. For example, consider these two claims:

1. You ought to prefer a 0.9 chance at ∞ to a 0.1 chance at ∞.
2. Salvation is the best thing possible.

Jackson and Rogers (2019: 66) argue that (1) is much more plausible than (2), so if we have to choose, we should give up on (2) (see also Jackson 2016: 95). Rota (2017: fn. 4) points out that salvation might be the best thing possible for humans as they are, even if greater goods are conceivable. Wenmackers (2018: 304) points out that "it is not clear that the existence of larger numbers on the utility scale entails the existence of states or rewards corresponding to them."

The downside of these contemporary responses, is, of course, that they are at odds with Pascal's view that salvation is reflexive under addition, as we quoted above. This raises the methodological question of how firmly we ought to remain true to Pascal's own views—we are interested in *Pascal's* Wager, after all—in the face of worrisome objections. Insofar as we prioritize preserving Pascal's original intentions, we may prefer the former. However, a more contemporary approach to the Wager may put less weight on Pascal's theological commitments. Hájek himself (2012, 2018) suggests further reformulations of the Wager that appear to be valid. And in fact, in his (2018), Hájek even suggests that there's a way to preserve Pascal's idea that salvation is the best thing possible, while figuring in a valid argument for wagering for God: on this formulation, the utility of damnation is $-\infty$. But this seems to relinquish another of Pascal's theological commitments: God's "justice to the outcast is *less vast* … than mercy towards the elect."

3.2 Finite Utility

Another contemporary response to these problems with infinite utility is instead to appeal to arbitrarily large but finite utilities. We might call Pascal's utility of salvation an *actual* infinite magnitude. Aristotle was skeptical of such magnitudes, but he believed in *potential* infinite magnitudes. In contemporary literature, some defenders of the Wager advocate for a version of the Wager that appeals to a large finite utility for salvation (see Byl 1994; Jordan 2006; Rota 2016; Franklin 2018). Some detractors of the Wager also consider its reformulation with a large finite utility (e.g. Mougin and Sober 1994: 386; Hájek 2003a; Bottomley and Williamson MS).

It might seem that these so-called "finite wagers" would lack the swamping effect that Pascal desired. Since there is no infinite utility to trump the finite utility involved if God does not exist, some conclude that you should wager only if your credence in God's existence is sufficiently high. Michael Rota (2016), for example, argues that you should wager on Christianity if your credence in Christianity is at least 0.5.

However, a "finite wager"' may gain the swamping effect of infinite utility without any appeal to infinity whatsoever. Pascal's Wager promised to work for all positive probabilities for God's existence, however small. In this sense, it promised to catch in its net all possible rational agents. (Those who assign 0 probability are irrational according to premise 2.) But note that it is really addressed to *people* like us, and it will have served its purpose if it catches all the rational members of its potential audience. That potential audience is finite. As such, there is the *smallest* positive probability that any of its members assigns to God's existence. The Wager will have done its job if it works for this probability; all the more, it will work for all larger probabilities. But then the utility of salvation need not be infinite; a sufficiently large finite utility $\boldsymbol{f}$ will do. Traditional decision theory can take it in its stride—in particular, there is no violation of continuity. So two problems for the original Wager are solved at once: the utility itself is no longer problematic, and the reformulated Wager is valid. See Hájek (2003a, 2018) for more details.

Traditional decision theory assumes that utilities and probabilities are *precise*, represented by real numbers that are perfectly sharp. Soon we will relax this assumption, allowing probabilities to be imprecise, in keeping with a large and ever-growing literature. There is less of a literature on imprecise utilities, but these seem appropriate for an agent who may be modeled as attaching a *range* of values to some outcome. How much do you value your neighbor having a can of tuna in their pantry? It's not obvious that you have an infinitely precise utility for this.[6] We might instead model you with a utility *interval*. In the case of Pascal's Wager, we may even allow the utility of salvation to be imprecise over an interval of the form $[\boldsymbol{f}, \infty]$, where $\boldsymbol{f}$ is a sufficiently large utility, as per the previous paragraph. We could also allow the interval to be open at the upper end, $[\boldsymbol{f}, \infty)$, more in keeping with an Aristotelian 'potential infinity'. Or we could allow the upper end to be

[6] This point remains even after we recognize that utility is not measurable on an absolute scale.

any finite value greater than $\boldsymbol{f}$. Any viable decision theory for imprecise utilities should deliver the result that Pascal wants.

4. Pascal's Wager and Probabilities

4.1 Zero Probabilities

Infinity is the highest possible utility; zero is the lowest possible probability. Infinity is the great uplifter: multiply anything positive by it and we get infinity; zero is the great leveler: multiply anything by it and we get zero. Infinity and zero are yin and yang to each other; what happens when these titans clash? Zero wins: $0 \times \infty = 0$. This suggests a way to resist the force of Pascal's Wager: assign probability 0 to God's existence (see Oppy 2006: ch. 5).

But is this rational, *contra* premise 2? It sounds like strict atheism: complete certainty in God's *non*-existence. This, in turn, may seem dogmatic. After all, it seems that our evidence against God's existence could not be *that* decisive. And even if God's existence is either necessary or impossible, arguably we should not assign it an extreme probability, given our cognitive limitations. Compare: we should not have a credence of 0 or 1 in Goldbach's Conjecture, even though it's either necessarily true or false. And whether God exists or not is plausibly not a *trivial* falsehood (like $0 = 1$) or truth (like $2 + 2 = 4$), which may warrant extreme credences.

Moreover, this probability assignment apparently violates a much-touted putative rationality constraint called *regularity*. A probability function is said to be *regular* if it assigns 0 only to impossibilities. (*Open-minded* or *undogmatic* would be more evocative names for this intuitive constraint.) Assuming, then, that God's existence is *possible*, regularity requires one's credence in it to be positive.

This brings us to a cottage industry in recent formal epistemology: arguing for and against regularity as a rationality constraint. (See Easwaran 2014, who argues against it, for references.) Folk thinking about probability plausibly upholds regularity. But it is well known that the standard Kolmogorov probability calculus, with its real-valued probabilities, cannot maintain regularity in uncountable probability spaces. (See Hájek 2003b for a proof.) For example, the probability that a radium atom decays at a particular instant of time—say, noon exactly—is 0. (The probability distribution for the decay time is continuous, and the probability of any particular point is 0.) And someone could assign probability 0 to God's existence without being completely certain of God's non-existence. (To be sure, that may still be dogmatic, unwarranted by one's evidence.) A notquite-strict atheist who regards God's existence as possible but nonetheless assigns it probability 0 may be unmoved by the expected utility calculations in Pascal's Wager. (They should, however, be moved by a dominance argument that Pascal also gives—see Hacking (1972) and Hájek (2012).)

4.2 Infinitesimal Probabilities

A number of authors seek to save regularity by invoking *infinitesimal* probabilities, positive probabilities that are smaller than every positive real number. We have seen that Hacking credits Pascal's Wager for being the first well-understood contribution to decision theory. We submit that in it Pascal also offers the first reference to infinitesimal probability, another remarkable innovation (1662):

> if there were an infinity of chances, of which one only would be for you, you would still be right in wagering one to win two, and you would act stupidly, being obliged to play, by refusing to stake one life against three at a game in which out of an infinity of chances there is one for you, if there were an infinity of an infinitely happy life to gain.

This anticipates contemporary appeals to infinitesimal probabilities by authors such as Bernstein & Wattenberg (1969), Lewis (1980), Skyrms (1980), Benci et al (2006), and Wenmackers (2016). There are various number systems that include infinitesimals, including hyperreals, surreals (see Easwaran et al. 2023 for a survey), and non-Archimedean probabilities (Benci et al 2006). Infinitesimal probabilities have ramifications for Pascal's Wager—see Oppy (1990, 2018), Hájek (2003a), Jackson (2016), Bartha (2016), and Wenmackers (2018). In particular, some have suggested that invoking infinitesimal probabilities is a way to "cancel out" the infinite utilities traditionally invoked in the Wager without assigning God's existence a probability of zero.

Wenmackers (2018) notes that Pascal explicitly excludes infinitesimal probabilities from the Wager; she examines whether this move is necessary for Pascal—that is, whether assigning God's existence an infinitesimal probability could neutralize the Wager. She concludes that, generally, "the larger the infinite utility is compared to [the infinitesimal probability], the easier it is for the Wager to be successful" (2018: 308). However, if a skeptic were to have a credence that is the reciprocal of the value of salvation, they may not be required to take Pascal's Wager; in general, whether one should take the Wager depends on what the infinite and infinitesimal values are (as Oppy 1991 notes). Finally, Wenmackers considers whether (a) it is psychologically possible for humans to have infinitesimal credences and whether (b) the fact that there are an infinite range of possible deities could ground assigning a infinitesimal probability to Pascal's God (Wenmackers 2018: 312–4).

We have pointed out a parallel between infinite utility and zero probability—their swamping effect in expected utilities—and moves made regarding zero probability prompt us to look for parallel moves regarding infinite utility. Once we countenance infinitesimal probabilities drawn from some number system, it is natural to countenance infinite utilities drawn from the same system. For example, Hájek (2003a) introduces surreal infinitesimal probabilities and surreal infinite utilities into the presentation of Pascal's Wager. Chen and Rubio (2020) also apply surreals to the Wager, and they argue that while this both respects dominance reasoning and has the result that pure strategies are preferable to mixed strategies, this application cannot preserve Pascal's desideratum that one ought to wager, regardless of one's (positive) credence in God's existence.

4.3 Extremely Low Probabilities

Another route to zero probability for God's existence is to start with an *extremely low* probability and to round it down to 0. This initial probability could be infinitesimal, which may be regarded as extremely low (perhaps by definition), or perhaps finite but still regarded as such. Then we might appeal to some principle that allows us to treat sufficiently small probabilities *as if* they are 0. This is in the spirit of Cournot's Principle, which states that any event with small probability *will not happen*, and more generally, any proposition with small probability *is false* (Cournot 1843; Buffon 1777; Kolmogorov 1933; Borel 1963; Shafer & Vovk 2006; Shafer 2007; Wilhelm forthcoming; Barrett & Chen MS; Rubio MS). Closer to home, Nicholas J.J. Smith (2014: 472) explicitly endorses a principle that he calls "rationally negligible probabilities: "For any lottery featuring in any decision problem faced by any agent, there is an $e > 0$ such that the agent need not consider outcomes of that lottery of probability less than e in coming to a fully rational decision." Regarding Pascal's Wager as a lottery, this principle implies that the agent may treat a sufficiently small probability for God's existence *as if* it is 0. See Hájek (2014), Isaacs (2016), and Cibinel (2023) for objections to this principle.

One motivation to treat extremely low probabilities as if they were 0 involves Bostrom's (2009) thought experiment known as "Pascal's mugging."[7] Imagine Pascal walking down a dark alley late at night; a mugger jumps out and demands Pascal's wallet. The mugger tells Pascal that "If you hand me your wallet, I will perform magic that will give you an extra 1,000 quadrillion happy days of life" (Bostrom 2009: 445). The amount of money in the wallet is worth one happy day for Pascal. Despite his having a very, very low credence in the mugger's claim, he admits it is positive: it is one in 10 quadrillion. He does the expected utility calculation for handing over his wallet and finds that doing so is rationally required—and so he does.

We can take this even further. Never mind the mugger in Bostrom's story. Pick a random person on the street, or in a crowd in a stadium, who has not interacted with you at all. Still, there is *some* probability that they are like what the mugger claims in Bostrom's story: if you give them your wallet, they will give you a *much* greater amount back, so much so that the expected utility calculation favors your giving them your wallet. So don't wait to be mugged—go up to this person and voluntarily give them your wallet! And so it goes for any other random person. You should keep giving away your wallet, time and time again! But we are still not done. Perhaps you also give some credence to the mugger, or other random people, *harming you greatly* with many days of misery if you hand over your wallet. The expected utility calculations become yet more complicated, and it is hard to see how they will turn out if you dignify all of these scenarios with positive probability. It may be tempting, then, to round down the sufficiently low probabilities to 0, and to hang on to your wallet![8]

Pascal's mugging points to a more general view known as *fanaticism*, which can be applied to both practical and moral goods:

> **Fanaticism**: For any finite good x and for any non-zero probability p, there is some finite good y such that it is better to have y with probability p than to have x for sure.

For example, let x be certainly saving 1000 actual lives (by making a donation that guarantees preventing 1000 people from dying from malaria). Let p be 1/10,000. Then fanaticism implies that there is some finite good y that is *so* good that a 1/10,000 chance of it is better than saving the 1000

[7] This story and the name "Pascal's Mugging" originally appear in Yudkowsky (2007).

[8] Hiller and Hansen (forthcoming) suggest a different response to Pascal's mugging: for each mugger, there's a symmetric mugger who will offer you a similar reward if you decline the former's offer. So you shouldn't give your wallet to anyone. This response is consistent with classical decision theory. On Pascal's mugging, see also Balfour (2021) and Kaczmarek (MS).

lives. Perhaps *y* is saving a billion lives (by donating to a charity that does speculative research on how to solve unlikely but possible catastrophes that could destroy humanity). In any case, fanaticism implies that *something* finite is sufficiently good to play the role of *y.* (Wilkinson 2022 gives a similar example.)

Many have the intuition that you ought not give the mugger your wallet, you ought not give random (possible) muggers your wallet, and it's morally better to save 1000 lives than to take a very small chance at saving a billion lives. In general, to many, fanaticism seems absurd. However, it is *sometimes* rational to pay a small cost for a low-probability, high-payoff outcome. Kaczmarek (MS: 1) gives another case: suppose a warlord has a medicine that could save millions of lives. If I give her my wallet, she will release the medicine if it snows in LA sometime next week. It seems reasonable (or at least more reasonable than in the mugger case) to hand over my wallet, even though the chance it snows in LA next week is astronomically small. But what's the difference? It's not clear that sorting these cases correctly is simply a matter of setting the "rounding to zero" threshold low enough.

People have attempted to provide refined iterations of the rationally negligible probabilities principle (or Cournot's Principle) in order to tell us when we can ignore small probabilities and when we should pay attention to them. For example, some argue that fanaticism is false because you can ignore "outlandish" or "far-fetched" possibilities (see Schwitzgebel 2017; Monton 2019; Smith forthcoming; Kaczmarek MS). However, this project is notoriously difficult; most solutions lead to their own problems and paradoxes.

Some authors, such as Kosonen (forthcoming), and Wilkinson (2022) conclude that ultimately we should not discount small probabilities. Kosonen argues that doing so leads to one's being susceptible to money pumps (i.e. guaranteed loss of money in a series of bets); Wilkinson (2022) explicitly defends fanaticism, arguing that fanaticism's counterintuitive consequences are much more palatable than those one must accept if one rejects fanaticism. Becksted and Thomas (forthcoming) argue that we must either accept fanaticism, be extraordinarily (i.e. implausibly) risk averse, or have intransitive preferences. Russell (forthcoming) draws on their argument and Wilkinson's, and argues that the general situation is "unstable"; there are plausible principles that support fanaticism and others that count against it. Thus, both accepting and rejecting fanaticism leads to problems.

The verdict is still out on fanaticism; as counterintuitive as it might seem, it's possible that rejecting it is worse than accepting it. Then, it's not clear that assigning God's existence a low probability then rounding it to 0 is the right response to the Wager. Furthermore, even if we permit rounding to 0 in some cases, this move won't work for everyone. For those whose probability that God exists is on the low end, but who don't consider God's existence to be a completely "far-fetched" possibility, even the rationally negligible probabilities principle (or similar) may not be enough for them to escape the Wager—*their* probability is *not* negligible.

4.4 Imprecise Probabilities

We have so far assumed that the relevant probabilities are precise—that is, one can give a precise probability to the proposition *God exists*. However, this assumption may be unrealistic. Even if we sometimes have precise credences—e.g. when facing a fair coin flip or in response to a precise statistic—we won't assign precise probabilities to many things that we consider. For example, what is the probability that your neighbor has a can of tuna in their pantry? Most of us won't reply with a precise number such as 0.6432; in fact, it might even be difficult to say if this probability is greater or lower than 0.5.[9]

On some views, even ideally rational agents can assign imprecise probabilities in the face of unspecific evidence (as Joyce 2005 argues). And recall that Pascal suggests that God's existence is a matter where "reason can decide nothing", which suggests this may be a matter where our evidence is impoverished—indeed, maximally so. From this, one might conclude that it's permissible to assign an imprecise probability to God's existence. This has a number of noteworthy implications.

Hájek (2000) argues that if it is permissible to assign God's existence an imprecise probability over an interval that includes 0, then the Wager fails. And philosophers of religion commonly think that God either exists or fails to exist necessarily, so perhaps the probability interval for God's existence can rightfully include 0 and 1. However, recall that proponents of regularity for precise probabilities insist that rationality precludes assigning credence *0* to a contingent proposition. They may similarly insist that it precludes assigning a credence that's imprecise over an interval that *includes 0*. Rinard (2018) argues that if the (non)-existence of God

[9] See Bradley (2019) for more on imprecise probabilities.

is contingent, then we can resist Hájek's move in this way.[10] But regularity for imprecise credences can be upheld with an *infinitesimal* lower endpoint, so that the interval does not include 0. Then the problems associated with infinitesimal probabilities from section 4.2 still kick in.

4.5 Undefined Probabilities

Pascal's idea that our evidence regarding God's existence is impoverished might support assigning God's existence an imprecise probability. However, it might instead suggest that the probability of God's existence is *undefined*. To quote Pascal further: "[r]eason can decide nothing here. There is an infinite chaos which separated us. A game is being played at the extremity of this infinite distance where heads or tails will turn up…" (1662). If, as Morris (1986) suggests, we have no evidence either way regarding God's existence, then it may not be appropriate to assign any probability at all to God's existence. This idea may find support in the apophatic tradition, the view that God is completely beyond our descriptions, categories, and comprehension (see Michael & Gabriel 2016). On this view, God's nature and existence is not like the toss of a coin toss that we know something about—especially a coin that we know to be fair, and thus can easily assign probabilities to its outcomes, despite our ignorance of the outcomes themselves. Instead, it is like the toss of a coin about which we know *nothing*. God may be beyond us and thus largely unknowable to finite creatures with our cognitive and epistemic limitations.[11] Then perhaps we cannot rationally assign any probability—precise or imprecise—to God's existence, or at least it may be permissible for us not to assign any probability. This poses another challenge to premise 2.

If this is the case, Pascal's Wager (understood as an argument from expected utilities) would not even get off the ground. Recall that we can *null*ify the swamping effect of infinite utility by multiplying it by 0 probability. Similarly, we may also render the swamping effect of infinite utility null and void by multiplying it by a probabilistic void—that is, the *absence* of a probability, a probability gap. See Hájek (2000) for more on this response.

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[10] Admittedly, that God exists (or fails to exist) contingently is a minority view, but it is defended by some, such as Swinburne (1979).

[11] See also the literature on transformative experiences, and specifically religious conversion as a transformative experience, which is similarity taken to pose a problem for decision theory, e.g. Paul (2014), Pettigrew (2015), Chan (2016).

Now suppose that you have a positive probability for God's existence, as per premise 2. Undefined probabilities also figure in further twists to Hájek's argument that Pascal's Wager is invalid—see Hájek (2024). Suppose that you are *deliberating* about whether to wager for or against God, and you assign credences to your performing each option. By Pascal's lights, you are already doing something with infinite expectation. So why wager for God?—Keep deliberating! However, there is an interesting response. A number of authors (e.g. Spohn 1977; Levi 2007; Liu & Price 2019, 2020) hold the thesis that *while deliberating about what to do, you cannot rationally have credences for what you will do.* Then while deliberating, you are *not* enjoying infinite expectation; rather, your expectation is *undefined*.

Hájek rejects this thesis, so he rejects this reply. But even granting the thesis, it *still* shows that the Wager is invalid. Then we must compare your infinite expectation of wagering for God with your *undefined* expectation of continuing to deliberate. Then wagering for God does *not* maximize your expectation. Infinite expectation cannot be compared to undefined expectation, and in particular the former does not exceed the latter. An inequality fails to hold when one side of it is undefined.

We have discussed objections to Pascal's Wager, formulated in terms of expected utilities, from various problematic probability assignments to God's existence: zero, infinitesimal, extremely low, imprecise over an interval that includes 0, and undefined. However, the *dominance* argument that Pascal also gives for wagering God, as we mentioned above, is untouched by these objections. (It faces other objections: see Hájek (2012).)

5. Alternatives to Expected Utility Theory

Recall premise 3: rationality requires you to maximize expected utility. This is the central doctrine of orthodox modern decision theory. While this premise is widely-agreed upon—some even claim that utility maximization is *constitutive* of rationality—it has also faced significant objections.

One might think that this requirement is overly demanding. Perhaps in some situations it is permitted to *satisfice*, performing an action that has *sufficiently high* expected utility, though not maximal (see Simon 1956). However, if one in fact wagers against God, and gets a finite reward, then this is not even *close* to the infinite expected utility that Pascal countenances. For more pressing challenges to premise 3, both Allais (1953) and Ellsberg (1961) argue that maximizing utility rationally requires intuitively *inferior* courses of option. And even more relevant to the Wager, the St. Petersburg game (Bernoulli 1713) is one that most would not pay more than \$20 to play, but playing the game has infinite expected value—that is, you should be willing to pay any finite amount to play according to decision theory. (See also Nover & Hájek 2004's Pasadena game.)

There are various suggestions on how we might modify expected utility theory in light of these problems. One possibility is to consider expected *differences* between various options; when, and only when, the expected payoff of one option is positive relative to another, we should prefer it (see Hájek & Nover 2006; Hájek 2006; Colyvan 2008; Colyvan & Hájek 2016). Paul Bartha (2007) proposes a related but different modification. Its basic quantity is the ratio of two differences in utilities when this is defined:

$$U(A, B; Z) = (U(A) - U(Z))/(U(B) - U(Z)).$$

The 'base point' Z is needed, because there is no such thing as the worst possible option, which could be taken as a natural zero point.[12] We might also utilize different decision rules in different contexts.

These problems for expected utility theory—and the various solutions—complicate things, and they suggest that premise 3 is false as it stands. However, these modified rules might rescue a version of the Wager (with a tweaked third premise). Bartha (2007) argues that his modified decision theory can solve many of the contemporary objections to the Wager. And Hájek (2018) utilizes Bartha's alternative decision theory as a way of saving the validity of Pascal's Wager from worries that arise from the use of infinite utility in the Wager.[13]

As a final note, you might wonder what kind of rationality is at issue in premise 3. It's widely held that decision theory is about *practical* rationality, a kind of means-ends rationality. However, most think that practical rationality is not the only source of normativity—there is

[12] One might think that *damnation* is the worst possible option, with a utility of $-\infty$. Mirroring what Pascal says about infinity, we might say that unity subtracted from negative infinity subtracts nothing from it! It is *reflexive under subtraction*: $-\infty - 1 = -\infty$. See footnote 2 for why we think this does not capture Pascal's view about damnation.

[13] However, not all modifications of expected utility theory are as friendly to the Wager. Buchak (2013) provides a risk-weighted expected utility theory in response to the Allais paradox, but she notes that the preferences required in the Wager are inconsistent with one of her axioms (p. 97). For more on Buchak on the Wager, see Buchak (2023: fn. 35).

also *epistemic* rationality (rationality concerned with getting at truth and avoiding error), *moral* normativity, and perhaps *all-things-considered* rationality (i.e. what you should do, given all relevant sources of normativity). One may concede that, given Pascal's reasoning, it is practically rational to take the Wager, but this leaves open whether wagering is epistemically rational, morally permissible, or all-things-considered rational. If taking the Wager is not rational in these other senses (because it would require or involve false believing, moral wrongdoing, etc.) then perhaps we ought not wager, all things considered (see Oppy 2006: ch. 5 and Jackson 2023c for more on this issue).

6. The Wager's Continuing Impact

The analogy between Pascal's reasoning in the Wager and reasoning about risky action more generally has been noted by various authors (Turner & Hartzell 2004; Manson 2002; Sandin 2009; Colyvan et al. 2010; Johnson 2012; Bykvist 2017; Munthe 2019). Wager-like reasoning has been especially influential in theoretical and applied ethics, and in fact, these arguments often explicitly mention Pascal's Wager.

In theoretical ethics, authors have considered how we should respond to moral uncertainty, including uncertainty about the correct view in normative ethics (see MacAskill 2019; MacAskill et al 2020; Hicks 2021). Colyvan, Cox, & Steele (2010) discuss Pascal's Wager-like problems for certain deontological moral theories, in which violations of duties are assigned negative infinite utility (see also Colyvan, Justus, & Regan 2011). Bartha and DesRoches (2017) argue that some of the difficulties that arise with uncertainty and infinite utilities in ethics can be addressed by relative utility theory.

In practical ethics, Pascalian reasoning has been applied to a number of issues, including nuclear policy, abortion, AI research, global warming, black holes, (anti)natalism, longtermism, DNA research, euthanasia, and antibiotic use. Several authors (Lockard 2000; Moller 2011) argue that considerations of moral risk, and not just evidential considerations, should play into our judgments about the overall moral permissibility of abortion. Stone (2007) applies Pascalian reasoning to euthanasia, arguing that the risks involved make euthanasia harder to justify (see Varelius 2013 for a response). Generally, a Pascalian-style argument is often applied to practical ethics and existential risks in a movement known as *effective altruism* (see Pummer & MacAskill 2020).

Some of these arguments appeal to infinite utilities; for example, Kenny (1985) argues that worldwide nuclear war has a negative infinite utility. On other views, humans have infinite value, so the loss of life would also have a negative infinite utility (this would also have ramifications for debates involving abortion and euthanasia). Others have suggested that DNA research could create a killer strain of bacteria that could wipe out humanity, and even if unlikely, we should ban DNA research because this outcome is infinitely bad. (However, see Stich 1978 for a dissenting view.)

More recently, developments in artificial intelligence (AI) technology have raised questions of AI safety, and some conclude that Pascalian precautionary reasoning applied to AI requires us to proceed with extreme caution (Bostrom 2014; see also Bales et al forthcoming and Gallow forthcoming). Derek Leben (2020) draws a clear analogy from the risks posed by AI and Pascal's Wager, and argues that, because there's a chance that pursuing AI technology could lead to an infinite loss, the risks of pursuing AI advancements outweigh the benefits.

7. Conclusion

After introducing expected utility theory and Pascal's Wager, we focused on utilities and probabilities, first discussing both infinite and finite utilities and implications of each for the Wager. Then, we discussed five types of probability: zero, infinitesimal, extremely low, imprecise, and undefined. Finally, we turned to some alternatives to expected utility theory and closed with implications of Pascalian reasoning, especially for questions involving ethics and risk.

Pascal's Wager competes with Anselm's Ontological Argument, Paley's Argument from Design, and the Problem of Evil for being the most famous argument in the philosophy of religion's Hall of Fame. But when it comes to their influence on cutting-edge contemporary philosophical developments, it's not even close: Pascal's Wager wins hands down.

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Biographies:

Alan Hájek is a Professor of Philosophy at Australian National University. His research interests include the philosophical foundations of probability and decision theory, formal epistemology, conditionals, and philosophy of religion. He studied statistics and mathematics at the University of Melbourne, received an M.A. in philosophy at the University of Western Ontario, and a Ph.D. in philosophy at Princeton University. He taught at the California Institute of Technology before moving to the ANU.

Elizabeth Jackson is an Associate Professor of Philosophy at Saint Louis University. Her research is in epistemology and philosophy of religion. In epistemology, she is interested in the belief-credence connection, epistemic permissivism, and pragmatic and moral encroachment. In philosophy of religion, she's interested in the rationality of religious commitment, and specifically Pascal's wager and the nature of faith. She completed her Ph.D. in Philosophy at the University of Notre Dame, and held previous positions at Australian National University and Toronto Metropolitan University.

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