A Dynamic Approach to De Broglie's Theory

Nizar Hamdan
Department of Physics, University of Aleppo
P.O. Box 12083, Aleppo, SYRIA
e-mail:nhamdan59@hotmail.com

Einstein's relativity (SRT) [1], raised an important question: Do its laws apply to matter/energy or to the frame of reference containing it. In other words, is SRT a property characterizing matter/energy or is it an imposed dynamics as expressed by the Lorentz Transformation (LT). This article examines the subtle difference between the two. It proves that matter/energy is the fundamental reality, while the reference frame, although it exists, exerts no force. Therefore, we can start from Newton's second law (NSL) to rebuild SRT [2]. This will allow us to derive the de Broglie relations from NSL while circumventing the well-known contradictions between SRT and de Broglie's quantum wave theory.

Keywords: Newton second law, de Broglie matter-wave; special relativity theory.

1. Introduction

Einstein obviously subscribed to the hypothesis that physical aspects of the space-time continuum are the principle forces in LT and obey the relativity principle through space contraction and time dilation.

SRT is viewed historically as a brilliant achievement. However, there are still a few physicists who are skeptical about its fundamental logic. The theory liberated us from the ether myth, only to throw us into the darkness of space-time. Matter is no longer free as its properties are now determined by the continuum.

SRT has been presented as a unique solution, yet tens of alternative theories are put forward to replace it. These are largely ignored since it is believed that they have nothing new to offer. The theories lack the glamour and fame of SRT for two main reasons:

1- There is only a subtle difference between the concept of the primacy of matter and that of its frame of reference.
2- SRT is a unifying theory of space and time, matter and energy, and has become the basis of other unifying theories.

To be successful, alternative theories should demonstrate the shortcomings of SRT and show it does not in fact unify.

SRT has removed the barrier between matter and energy, but it created a new one between the domains of non-relativistic and relativistic phenomena. The physical laws of classical physics can not transcend this barrier. Relativistic physics is able to represent classical physics only through approximation. The logical approach would be to start with the laws of classical physics and make them applicable to all particle velocities, i.e. to expand the appropriateness of these laws to deal with the relativistic domain. This can not be achieved unless we revert to the invariance of physical laws among inertial frames regardless of the coordinate transformations. In this way, SRT could be formulated from a mechanical, rather than electromagnetic base. In doing this, we must extend NSL by describing the moving particle as a wave and circumvent the contradictions between de Broglie relations and SRT.

2. Energy (Mass), Momentum, Velocity and Force Transformation Relations

Let us consider two moving inertial system S and S' with a relative velocity u_{ox} between them. The initial laws in classical mechanics are NSL and its expression for force:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}, \quad \frac{d\varepsilon_t}{dt} = \mathbf{F}\mathbf{v} \tag{1a,b}$$

Where Eq.(1b) can be derived on Newtonian mechanical grounds[2], but in classical mechanics the kinetic energy is $\varepsilon_t = T = \frac{mv^2}{2}$ with $m = m_0$. In his second paper regarding SRT, Einstein [3] proposed the famous equation, $\varepsilon_t = mc^2$. However, many text books often devote great effort to discussing the process of an elastic collision between two particles in deriving $\varepsilon_t = mc^2$ and the relativistic mass $m = \gamma m_0$. Let us try to establish a law for total energy and relativistic mass without using LT or SRT precepts. As demonstrated in [2] and according to relativity principle NSL implies that "mass is variable." The total energy ε_t is dependent on this.

One understands relativistic mechanics as a modification (correction) of classical mechanics. The same modification can be obtained if we revert to NSL and use change in mass rather than space-time as in SRT. We will demonstrate that Eqs.(1) and the relativity principle are more natural for describing the physics of relativistic mechanics. We can then go further and derive relativistic mass as well as all of SRT's relations by this approach.

The Cartesian components of Eq. (1) in frame S are:

$$\frac{dp_x}{dt} = F_x\,,\ \frac{dp_y}{dt} = F_y\,,\ \frac{dp_z}{dt} = F_z \tag{2a,b,c}$$

And

$$\frac{d\varepsilon_t}{dt} = F_x v_x + F_y v_y + F_z v_z \tag{2d}$$

Applying the relativity principle to Eq. (2), we have

$$\frac{dp'_x}{dt'} = F'_x\,,\ \frac{dp'_y}{dt'} = F'_y\,,\ \frac{dp'_z}{dt'} = F'_z \tag{3a,b,c}$$

And

$$\frac{d\varepsilon'_t}{dt'} = F'_x v'_x + F'_y v'_y + F'_z v'_z \tag{3d}$$

Now multiplying Eq. (2d) by $\frac{u}{c^2}$ and then subtracting the result from Eq. (2a), dividing the result by $\left(1-\frac{uv_x}{c^2}\right)$, we obtain

$$\frac{d\left(p_x - \frac{u}{c^2}\varepsilon_t\right)}{dt\left(1-\frac{uv_x}{c^2}\right)} = F_x - \frac{u/c^2\left(F_y v_y + F_z v_z\right)}{\left(1-\frac{uv_x}{c^2}\right)}$$

Multiplying and dividing the link – hand side with the scalar factor γ, then comparing with Eq. (3a), we have

$$p'_x = \gamma\left(p_x - \frac{u}{c^2}\varepsilon_t\right),\ dt' = \gamma dt\left(1-\frac{uv_x}{c^2}\right) \tag{4a,b}$$

And

$$F'_x = F_x - \frac{\frac{u}{c^2} F_y v_y}{\left(1 - \frac{uv_x}{c^2}\right)} - \frac{\frac{u}{c^2} F_z v_z}{\left(1 - \frac{uv_x}{c^2}\right)} \tag{5a}$$

Dividing Eq. (2b) by $\gamma\left(1 - \frac{uv_x}{c^2}\right)$ then comparing with Eq. (5b), we have,

$$p'_y = p_y\,,\; F'_y = \frac{F_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \tag{4b, 5b}$$

The same result can be derived from the z and z' forces and momentum

$$p'_z = p_z\,,\; F'_z = \frac{F_z}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \tag{4c, 5c }$$

Starting from Eq. (2a) multiplying by u and then subtracting the result from Eq.(2d), dividing the result by $\left(1 - \frac{uv_x}{c^2}\right)$, we obtain

$$\frac{d\left(\varepsilon_t - up_x\right)}{dt\left(1 - \frac{uv_x}{c^2}\right)} = \frac{\left(v_x - u\right)}{\left(1 - \frac{uv_x}{c^2}\right)} F_x - \frac{\left(F_y v_y + F_z v_z\right)}{\left(1 - \frac{uv_x}{c^2}\right)}$$

Adding and subtracting $\frac{u^2 / c^2\left(F_y v_y + F_z v_z\right)}{\left(1 - \frac{uv_x}{c^2}\right)^2}$, we have

$$\frac{d\left(\varepsilon_t - up_x\right)}{dt\left(1-\frac{uv_x}{c^2}\right)} = \frac{\left(v_x - u\right)F_x\left(1-\frac{uv_x}{c^2}\right)+\left(F_y v_y + F_z v_z\right)\left(1-\frac{uv_x}{c^2}\right)}{\left(1-\frac{uv_x}{c^2}\right)^2}$$

$$+\frac{u^2}{c^2}\frac{\left(F_y v_y + F_z v_z\right)}{\left(1-\frac{uv}{c}\right)^2} - \frac{u^2}{c^2}\frac{\left(F_y v_y + F_z v_z\right)}{\left(1-\frac{uv}{c}\right)^2}$$

Or

$$\frac{d\left(\varepsilon_t - up_x\right)}{dt\left(1-\frac{uv_x}{c^2}\right)} = \frac{\left(v_x - u\right)}{\left(1-\frac{uv_x}{c^2}\right)}\left(\frac{F_x - \frac{u}{c^2}\mathbf{Fv}}{\left(1-\frac{uv_x}{c^2}\right)}\right) + \left(1-\frac{u^2}{c^2}\right)\frac{\left(F_y v_y + F_z v_z\right)}{\left(1-\frac{uv}{c}\right)^2}$$

Multiplying and dividing the left – hand side with the scalar factor γ , we obtain

$$\frac{d\gamma\left(\varepsilon_t - up_x\right)}{\gamma dt\left(1-\frac{uv_x}{c^2}\right)} = \frac{\left(v_x - u\right)}{\left(1-\frac{uv_x}{c^2}\right)}\left(F_x - \frac{u}{c^2}\frac{(F_y v_y + F_z v_z}{(1-\frac{uv_x}{c^2})}\right) + \frac{F_y\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{uv_x}{c^2}\right)}\frac{v_y\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{uv_x}{c^2}\right)}$$

$$+\frac{F_z\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{uv_x}{c^2}\right)}\frac{v_z\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{uv_x}{c^2}\right)}$$

Comparing with Eq.(3d), we have

$$F'_y = \frac{F_y\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{uv_x}{c^2}\right)},\ F'_z = \frac{F_z\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{uv_x}{c^2}\right)},$$

$$\varepsilon'_t = \gamma\left(\varepsilon_t - up_x\right) \tag{4d}$$

And

$$v'_x = \frac{v_x - u}{1-\frac{uv_x}{c^2}},\ v'_y = \frac{v_y\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{uv_x}{c^2}\right)},\ v'_z = \frac{v_z\sqrt{1-\frac{u^2}{c^2}}}{\left(1-\frac{uv_x}{c^2}\right)} \tag{6a,b,c}$$

The scalar factor can be fixed by applying the relativity principle on Eqs.(6a) and (5b). So Eqs.(6a) and (5b) could be written in frame S' as

$$v_x = \frac{v'_x + u}{1 + \frac{uv'_x}{c^2}}, \ F_y = \frac{F'_y}{\gamma\left(1 + \frac{uv'_x}{c^2}\right)} \tag{7a,b}$$

Hence we can substitute from Eq. (6a) into (7a) gives

$$\left(1 - \frac{uv_x}{c^2}\right)\left(1 + \frac{uv'_x}{c^2}\right) = 1 - \frac{u^2}{c^2} \tag{8}$$

Similarly, substituting Eq. (5b) into (7b) gives

$$\gamma^2\left(1 - \frac{uv_x}{c^2}\right)\left(1 + \frac{uv'_x}{c^2}\right) = 1 \tag{9}$$

Eqs. (8) and (9) lead to the determination of γ i.e.,

$$\gamma^2\left(1 - \frac{u^2}{c^2}\right) = 1, \text{ hence } \gamma = \frac{1}{\left(1 - \frac{u^2}{c^2}\right)^{\frac{1}{2}}} \tag{10}$$

We may write Eqs. (6) as

$$\frac{v'_x}{\sqrt{1 - \frac{v'_x}{c^2}}} = \frac{v_x - u}{\sqrt{1 - \frac{u^2}{c^2}}\sqrt{1 - \frac{v^2}{c^2}}} \tag{11}$$

I use Eq.(6b) into(4b) to get

$$m' = \gamma m\left(1 - \frac{uv_x}{c^2}\right) \tag{12}$$

Multiplying the Eq. (11) with m_0 , and comparing it with (12), we deduce

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},\ m' = \frac{m_0}{\sqrt{1-\frac{v'^2}{c^2}}} \tag{13a,b}$$

From Eq.(1b) , the total energy is given by

$$\begin{aligned} d\varepsilon_t &= Fvdt = d(mv)v \\ &= v^2 dm + mvdv \end{aligned} \tag{14}$$

And from Eq.(13a), we have

$$dm = \frac{mvdv}{c^2\left(1-\frac{v^2}{c^2}\right)} \qquad i.e. \qquad mvdv = \left(1-\frac{v^2}{c^2}\right)dm \tag{15}$$

Substituting Eq.(15) in Eq.(14), we get

$$d\varepsilon_t = c^2 dm \tag{16}$$

By integration, from v_1 to v_2, we get

$$\varepsilon_t \quad = mc^2\Big|_1^2 \tag{17}$$

In the particular case, if $v_1 = 0$ and $v_2 = v$, then ε_t should equal the kinetic energy ε_k , i.e.

$$\varepsilon_k = mc^2\Big|_1^2 = mc^2 - m_0c^2 \tag{18}$$

So the quantities mc^2 and $m'c^2$ are the total energy ε_t and ε_t' in frames S and S' respectively.

It is simple to prove, that Eqs. (6) and (11) lead to

$$\varepsilon'^2 - c^2\boldsymbol{P}'^2 = \varepsilon^2 - c^2\boldsymbol{P}^2 = m_0^2c^4 \tag{19}$$

Or

$$\varepsilon_t^{\,2} = c^2 \boldsymbol{P}^2 + m_0^{\,2} c^4 \,,\ \varepsilon_t'^2 = c^2 \boldsymbol{P}'^2 + m_0^{\,2} c^4 \tag{20a,b}$$

The dynamics of a moving particle are built into SRT to accommodate LT. Therefore, we introduced an alternative method which is not based on LT to derive all the mentioned relations in this section.

Depending on this formulation we continue to derive the de Broglie relations for particle-wave duality but without requiring SRT or any relativistic assumption such as LT.

3- De Broglie Theory and SRT

After the creation of the electromagnetic theory of light, it become possible to formulate the laws of the corpuscular properties of radiation and the wave properties of the corpuscular as

$$\varepsilon_t = hf = \hbar w \,,\ p = \frac{h}{\lambda} = \hbar k \tag{21}$$

De Broglie [4], postulated the validity of relation (21) for a particle with rest mass m_0 through his hypothesis of the "periodic phenomenon," i.e.,

$$hf_0 = m_0 c^2 \tag{22}$$

When Eq. (22) is written with respect to the frame S, then Eq. (22) takes the form

$$hf = \frac{m_0 c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}} \tag{23}$$

According to Eq.(22),

$$f = \frac{f_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{24a}$$

However, as is well known in SRT, that if the clock has a frequency f_0 in the rest frame of the particle, its frequency at velocity v in frame S, (according to the so-called time dilation), is

$$f = f_0\sqrt{1-\frac{v^2}{c^2}} \tag{24b}$$

Evidently, Eq.(24b) is just the opposite of Eq.(24a). Indeed, accounting for time dilation leads to a slowing down in the frequency of the "moving clock," Eq.(24b), while accounting for the energy increase of a "moving particle" yields an increased frequency, Eq.(24a). Thus, it is clear that some additional assumption is needed to overcome such a fundamental contradiction. Namely the phase of the "periodic phenomenon," i.e., $2\pi f_0 t' = w_0 t'$ should be obtained using phase conservation with respect to LT, i.e.

$$w_0 t' = 2\pi\frac{m_0c^2}{h}t' = \frac{2\pi m_0 c^2\left(t-\frac{u}{c^2}x\right)}{h\sqrt{1-\frac{u^2}{c^2}}} \tag{25}$$

$$= wt - kx$$

From Eq. (25) we get Eq.(24a). On the other hand, since Eq.(25) must hold for every x and every t, it also becomes the well-known formula for connecting a particle's momentum with its wavelength .

$$k = \frac{2\pi m_0 v}{h\sqrt{1-\frac{u^2}{c^2}}} = \frac{p}{\hbar}, \text{ i.e. } \lambda = \frac{h}{mv}, u = v \tag{26a}$$

As well as

$$v_p v = c^2 \tag{26b}$$

As usual, the phase velocity of a wave is

$$v_p = \frac{\varepsilon_t}{p} = \frac{w}{k} \tag{26c}$$

Whereas the particle velocity equals to the group velocity of wave

$$v = v_g = \frac{dw}{dk} \tag{26d}$$

An obvious contradiction exists between de Broglie's theory and SRT (through which it was formulated). For instance, according to SRT, Eq. (26b) has the contradiction that the velocity v_p is different from the mechanical velocity v for the same particle, therefore the superluminal velocity v_p is said to be devoid of any physical meaning [5]. Although of this, the Lorentz transformation for wave vector and frequency, i.e.

$$k' = \gamma\left(k - \frac{uw}{c^2}\right) = \gamma k\left(1 - \frac{u}{c^2}\frac{w}{k}\right) \qquad , w' = \gamma\left(w - uk\right) = \gamma w\left(1 - \frac{uk}{w}\right)$$

are expressed by the phase velocity as:

$$k' = \gamma k\left(1 - \frac{uv_p}{c^2}\right), \; w' = \gamma w\left(1 - \frac{u}{v_p}\right) \tag{27a,b}$$

For a light wave $v = c$, Eq (27b) is a longitudinal Doppler shift formula, while for a matter wave, Eq (27b) is a non-longitudinal Doppler shift formula. So the union of SRT and de Broglie's wave formalism has always been precarious because of the different velocities for the same particle.

Due to the difference between phase velocity and group velocity of de Broglie waves, de Broglie through Eq. (26a), developed the concept of a wave associated with material particles and removed the point–particle phenomena to a matter–wave phenomen. This led to a scientific controversy that was subject to much discussion and attempts at resolution. The first of these attempts can be attributed to J. Wesely [6], who supposed a real wave function instead of the complex wave function in traditional quantum theory. He could prove that the phase velocity equals the particle velocity. Another attempt in this context was M. Wolff [7]. The wave–structure of moving electron is analyzed on the basis of spherical waves. He frees SRT from the usual contradiction and concludes SRT and de Broglie's theory are compatible.

4- Derivation of de Broglie Relations for the Moving Duality on a Dynamical Basis

Recently R. Ferber[8], showed that Eq. (26b) is a result of using LT, and not a result of de Broglie's hypothesis.

Therefore, to deal with these contradictions, we must re-derive all relations in section 3 without LT. To remove the kinematical contradiction in the de Broglie formalism, we first derive Eq.(26a) on a dynamical basis starting with Eq. (20a), i.e.

$$\varepsilon_t^2 = c^2 p^2 + m_0^2 c^4$$

The last relation yields:

$$\varepsilon_t d\varepsilon_t = c^2 pdp \text{, i.e.; } d\varepsilon_t = vdp$$

Using the Plank–Einstein relation, that $\varepsilon_t = hv = \hbar w$, yields

$$dw = \frac{v}{\hbar} dp$$

Now using the definition of group velocity, i.e.; Eq.(26d), we have

$$dk = \frac{dp}{\hbar}$$

By integration, $v = 0$ i.e. $k = 0$, we get

$$k = \frac{p}{\hbar} \text{ i.e. } \lambda = \frac{h}{p} \tag{28}$$

We can now remove the contradiction in Eq.(27) if we take into consideration that Eq.(17) could be written as

$$\varepsilon_t = mv^2 + m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}} \tag{29}$$

The new form of total energy ε_t, Eq. (29), is very important, because it shows us a new hidden variable, which is the relative kinetic energy

$$\varepsilon_v = mv^2 \tag{30}$$

Eq. (30) takes the name of "the relative kinetic energy" because it relates to total energy ε_t as

$$\varepsilon_v = \frac{v^2}{c^2} mc^2 = \frac{v^2}{c^2} \varepsilon_t$$

Now in addition to the right relation $\varepsilon_t = hf = mc^2$, we can specify the following relation

$$\varepsilon_v = hf = mv^2 \tag{31}$$

Eq. (31) helps us prove that $v_p = v$, if we substitute both Eqs. (31) and (28) in Eq. (26c)

$$v_p = \frac{w}{k} = \frac{mv^2/\hbar}{mv/\hbar} = v \tag{32}$$

Now setting Eqs.(28) and (26c) in Eq.(4a), we have:

$$k' = \gamma k\left(1 - \frac{uv_p}{c^2}\right)$$

Using Eq.(32) in the last relation, we get

$$k' = \gamma k\left(1 - \frac{uv_x}{c^2}\right)$$

Or

$$kv'_x = \gamma k v_x\left(1 - \frac{u}{v_x}\right)$$

According to Eqs.(32) and (26c) the velocity of a moving duality in frames S and S' is $w = v_x k, \quad w' = v'_x k'$, so we have :

$$w' = \gamma w\left(1 - \frac{u}{v}\right) \tag{33}$$

Eq.(33) is now a longitudinal Doppler shift formula for a moving matter-wave duality, and reduces to a longitudinal Doppler shift formula for a photon-wave duality , $v = c$ [9].

Conclusion

The incompatibility between SRT and particle dynamics arise because the LT and its kinematical effects have primacy over the physical aspects in deriving the relativistic dynamical quantities and in the interpretation of relativistic phenomena [2,10a,10b].

De Broglie formulated his theory of quantum waves, through LT, and the hypothesis of “periodic phenomenon.” Therefore de Broglie's formalism was the best model to show that kinematical effects in SRT are not compatible with the dynamics of the moving particle. To deal with these contradictions, we reformulated de Broglie's theory on a dynamical basis through NSL and the relativity principle. Depending on this approach, we can also find the intrinsic energy of a particle, which allows us to construct the framework of de Broglie's wave theory and SRT without the usual contradictions.

References

[1] A. Einstein, “On the Electrodynamics of Moving Bodies” Ann. Phys. 17, 891 (1905).

[2] N. Hamdan, “Newton’s Second Law is a Relativistic law without Einstein’s Relativity,” Galilean Electrodynamics July/August (2005).

[3] A. Einstein, “Does the Inertia of a Body Depend upon its Energy-Content,” Ann. Phys. 18, p. 639-641 (1905).

[4] L. De Broglie, “ The current Interpretation of Wave Mechanics, A critical Study(Elsevier, Amsterdam,1964).

[5] P. Bergman, Introduction to the Theory of Relativity: Dover, New York, p.145, (1976).

[6] J.Wesley, “Classical Quantum Theory,” Apeiron, Vol. 2, Nr. 2, 27 – 32 (1995).

[7] M. Wolff, “Beyond the Point Particle – A wave structure for the electron,” GED, 6, 83 – 91 (1995).

[8] R. Ferber, "A Missing Link: What is behind de Broglie's periodic phenomenon"?, Foundation of physics Letters, Vol. 9, No. 6, 575 – 586 (1996).

[9] N. Hamdan, "Derivation of the Relativistic Doppler Effect from the Lorentz Force," Apeiron, Vol 12, No. 1, Jan.(2005).

[10a] N. Hamdan, "Abandoning the Ideas of Length Contraction and time dilation," GED, 14, 83 – 88 (2003).

[10b] N. Hamdan, "On the Invariance of Maxwell's Field Equations under Lorentz Transformations," to appear in Galilean Electrodynamics magazine.