

SUFFICIENT REASON & THE AXIOM OF CHOICE,

**AN ONTOLOGICAL PROOF FOR ONE
UNIQUE TRANSCENDENTAL GOD FOR
EVERY POSSIBLE WORLD.**

Chains of causes appear when the existence of **God** is discussed. It is claimed by some that these chains must be finite and terminated by God. But these chains seem endless through our knowledge search. This endlessness for the physical reasons for any world event expresses the greatness and complexity of God's creation and so the transcendence of God. So, *only we can put our hands on physical reasons in an endless forage for knowledge*. Yet, the endlessness of the physical causes chains confirms the existence of God, and this is what our paper tries using Zorn's lemma, an equivalent for one of the most celebrated axioms of mathematics, the ***Axiom of Choice***.

Introduction. God exists this is a leap of intuition the author believes our human intelligence makes in an early moment of our life. This leap is put by a refutation of our ***common sense*** to accept the existence, that we are part of it, to be without ***one great knowledgeable free will*** as maker and sustainer; Descartes' brief elegant phrasing for this leap is well known.

No man or woman goes alone without an embracing society, and no society goes without law, and no law works without morals, and the whole edifice of morals crumbles into dust if not founded on the belief in One Unique Transcendent Beneficent Just and Merciful God. This may be shown deductively but also inductively. A quick look in history for the last three thousand years and we see that it is monotheism and its morals that steadily gain ground

over time. Idolatry or denying the One Transcendent God led in a final stage, even in the societies that allowed democracy, to the monopoly of power and the god emperor rule. But then we saw the submission of Pax Romana and other god emperor societies to the monotheistic morals as milestones on history. A snapshot for the world after World War II and we see that; it is the morals based on monotheism that prevailed, even in those societies that denied God. So, believing in God is necessary at least in the eyes of some; it is the power for morals which without humanity perishes.

God has created us with this insurmountable sense of free will and a high capability to make our choices, even the choice of believing or not believing in Him. By common sense, God is Beneficent, thus for our good, God guides us to believe in Him, but *without coercion since God is Just*. Why has God made all this? Nobody can know the perfect answer, but at least the free will feeling is good and makes one feel happy, and the more of it the happier we feel. Anyhow, for those who do not believe God exists; no proof for God exists. As for those who are only skeptic; here is the perspective of this paper: ***A miracle or a tight snug evidence make a coercion towards believing in God, moreover, they put the trialed in guilt without any way out, if he saw them, thus, due the Justice and Mercy of God, whatever is the proof anyone tries about the existence of God, this proof will fail to be coercive.*** Meaning, any such argument never its common sense premises will be absolutely shielded against sophistry, and never will be purely analytic of sheer snug deductive steps in the mathematical style from whatever premises to the conclusion "God exists". From another perspective, the conclusion "God exists" always must be greater from whatever premises, where ***some*** leap of intuition must ***be added*** to the argument to complete the way to the conclusion. So, the argument is never purely analytic where the conclusion is implied or equivalent to

the premise. Thus, we may say, it is always a free style deductive proof, and the tact is to have the common sense premises to be impermeable for sophistry, the intuition leaps at minimum, short, obvious and their contrary absurd to common sense. *Analytic, but there somewhere in the argument, there must be a short leap for the **intelligence** to jump, a clear intuition or common sense.* Such a leap, as mentioned above, is clear, short and easy even for the mind of a child, thus any such proof may be seen as just a detour around the obvious and clear, **pedantry or sophistry maybe, its only excuse is that it is sophistry against sophistry.**

The instinct of causation, causality, sufficient reason, or causation is an intrinsic belief of our constitution. What exactly is causation? no one can decide, so it is undefinable, and so it is tolerant for more than one interpretation. Anyhow, all people believe they act according to it, and if not all of them then almost all of them. A newborn in the animal kingdom from his day one instinctively moves according to it, at least through its teleological way. We cannot proceed in our lives without acting according to this causation instinct, any everyday argument, court defense or scientific proof any one of us introduces tacitly nark it. ***All philosophers discuss it, Al Mutazila Muslim scholars and the philosophers of the Common Sense school adopted it, and a lot of them, such as Gottfried Wilhelm Leibniz and Thomas Reid, explicitly adhered to it and were fanatic advocates.*** Almost every philosopher had given his explicit formulation or statement for this belief as a principle of thought. Maybe, the most famous one is this:

“For every event there must exist a sufficient reason”.

Without rhetoric, the proof starts by taking the principle of sufficient reason with its implication “existence is consistent”, and the Axiom of Choice with its two equivalent results, **Zorn’s**

Lemma and the **Well Ordering Theorem**, as premises or axioms. Then, we introduce our ontological argument that exposes the infinite chains of causation and shows how their endless guise is a proof for the existence, transcendence, oneness, and uniqueness of God. Our proof contains a few intuitive steps. The justification for these steps, we think, is their invincibility as intuitions, and the absurdity of their contrary for common sense. By “intuition” we mean: *an immediate idea that evolves uniquely and exactly the same way in at least almost all minds if not all minds*. Also, some other steps are justified by expressions like, our mind refuses or accepts not the contrary, These steps could be put as axioms in the beginning of the proof, since they are common sense and their contraries are absurd par excellence, put precisely after a form of **Russel's paradox** that shows the contradictory aspect of the converse of our common sense **mind precedes matter**, which is the wedge for almost all these steps, but we preferred to let them appear as common sense through the proof.

In more detail with little rhetoric, we preface in the first part of the paper by some talk about the axiom of choice, and we show how in *a subtle touch* it tacitly implies, by its very constitution, the existence of a mind higher than ours capable with an absolute power for free choice. Then, we delve into the axiom's equivalent almost counter intuitive implication, the Well Ordering Theorem, which confirms that *in some way* everything must have a beginning. After that we go into a brief word about a mathematical implication of the axiom of choice, namely **Banach/Tarsky paradox**, which hints to the plausibility of the idea of the creation and recreation of the world from almost nothing. The first part of the paper is an overture that shows how the Axiom of Choice touches on metaphysics, then in our principal argument in the second part, this touch will be explicitly exposed. There, after setting the stage with a suitable back scene, we let the axiom's vague equivalent implication, though the most applied in

mathematics proofs, Zorn's Lemma, go on to the stage to perform the principal act of our ontological proof, and so terminate this paper.

With some more rhetoric, in a spectacle by common sense, sufficient reason leads us to a spot on the physical side of the stream of thought. There, an easy leap of intelligence puts us on the side of the believers in One Unique Transcendent God. Some people do not see this spot for this short leap, and some others are not convinced it even exists. Here, the Axiom of Choice with its deputy Zorn's Lemma (explained in any introductory set theory book), which mathematicians rely on in critical moments of their proofs' growth, comes as extra guides. They take those who do not see the short jump spot and those who are not convinced this place exists to walk an uphill detour. Three guides with them, they reach some summit near the stream where the interlacing branches get less dense. There, a vantage vista opens and through it the place for the short jump is seen better and its existence and shortness are confirmed and even appears smaller. Thus, the skeptic intelligence gets encouragement to make its leap. This is the paper's hope and its outline, praying when scrutinized to be of least discrepancies.

Choice is something we face almost in every moment of our life, here we deal with a mathematical aspect for this notion. Consider a set , about which *we only know that it is not empty*, meaning it contains some elements. This means that no information is known about its content, the context it is in, or the things which it was formed from. When we describe a set of things by only saying that it is not empty, then this set in a sense has not even space or time

as a context, and what we say is only that there exist elements in this set.

Then, can we choose (or say, define/separate/pick/distinguish) any one of its elements? Meaning, does it exist a mechanism to define any one element of it? *No, we cannot, since picking or choosing needs some "additional information", sufficient to devise a way, or say, some guidance for choosing.* If the reader wants some approximative example about that, he can imagine a small aquarium containing tiny fish and covered by an opaque sheet. Can he then pick one of the fish (notice that picking by the guidance of sight is not allowed now)? Surely not, unless he uses some additional information, like for instance that he considers the aquarium is formed of say a thousand small cubes, then he says: I pick the fish that passes by cube 8 at 9 p.m. Obviously it is this additional information, namely that the aquarium has a volume partitioned into small cubes in some space time frame, that allowed the choice, in other words, this was the guidance that enabled making a choice. But suppose that no fish passed in the cube we picked by the coordinates (8,9) and no fish passed by any cube's coordinates we choose, then we must modify our mechanism of choice or find a new one. If we are confident that we can find such new mechanism, depending on some illuminating information, this means that we believe, or rely on, this claim taken without proof: *there exists a way or mechanism for picking or choosing from this aquarium. **Note that if the aquarium contains only one fish, we will not be able to define this one fish as our choice unless we knew this additional information, namely, that it exists only one fish there in the aquarium.***

Now consider a family of non empty sets $S_1, S_2, S_3, S_4, \dots$, meaning this family of sets is of one set, or a finite or infinite sequence of sets (infinite sequence means infinite, but ordered

like the sequence of natural numbers). If no information is told about these sets except, they are not empty, meaning we are dealing with sets in their utmost abstract situation, like in set theory, so no *additional* information even about the context where these sets are formed in, then as told above, we cannot choose any element from any one of these sets. But suppose that we are not in the utmost abstract situation, meaning we are in some *less abstract situation or context*. Then additional information about each one of these sets over that of their non emptiness can be tacitly assumed. We may assume that this information is the one that allowed us previously to separate, choose, and form these sets as a finite or even infinite sequence of sets from a known greater collection of things. Then with a grain of salt, we can swallow that we can choose from this family of sets.

Consider now the family where its sets are all non-empty subsets of the interval $[0.....1]$. Though we are in a less abstract context, the real numbers, this additional information turns out to be not enough to make our choice. Since, this family always has some sets such that, “*we*” cannot know forever any information about them except the information that they are not empty, containing some unknown real numbers. In fact, *almost all* these subsets impossible we can know forever anything about them; thus, we cannot choose any element from these sets. It is worthy to note that, in the first place, if we want to form these subsets, then *we need to know and choose their elements!* The reader can notice that yes, we can choose and form an infinite sequence of *some* of these subsets but never all of them.

The previous talk means that, in general, “*we*” alone cannot make a choice from an arbitrary family of non-empty sets, but this does not deny the possibility that there exists a way or mechanism for choosing from whatever non-empty set and so from an arbitrary

family of non-empty sets, and if this is the case, then this way is only in the capability of a mind higher than ours. So, unless we assume a higher mind who is knower and so capable to separate and choose, no choice from an arbitrary non empty set can be guaranteed. This impossibility in some cases for “us” to make a choice; in mathematical jargon can be stated as an axiom claiming:

There does not exist a choice (function) for every non empty family of non-empty sets.

This axiom is called “the negation of the axiom of choice”, so “the axiom of choice” is:

There exists a choice (function) for every non empty family of non-empty sets.

*Now since for a choice (function) to exist, it is necessary to know some additional information beside the information of the non-emptiness of the set, then the Axiom of Choice cannot be accepted, unless **we tacitly assume** there exists at least one mind who knows enough information about every existing set to make a choice, or say has a transcendental power to discriminate and choose. Note the repetition of the word “exist” and its derivations in the previous sentences.*

Assuming the **Zermelo Frankel system of axioms** for set theory to be consistent, then almost all what we can call beautiful mathematics can be figured inside a consistent realm of mathematics that is named: **Z F, choice**, which adopts the axiom of choice with those of Zermelo Frankel and has richer results. The other also consistent realm with the title: **Z F, negation of choice**, does not contain many beautiful results that are in the first realm.

Anyhow, what concerns us from the previous are the following points:

Z F, choice which is the mainstream of mathematics, and the treasury of “existence results” and their elegant proofs, necessarily assumes, though tacitly, the existence of someone with greater knowledge than us. And we do not go away from truth when we say, this One is of transfinite knowledge, since these problems of mathematics that make explicit use of the Axiom of Choice, are problems that deals with high infinities problems. An important result in Z F, choice, (this result is equivalent to the axiom of choice) is the result which says, that any set can be well ordered, meaning, in this well ordering, any subset of this set has a first element. This particularly means that the set itself in this well ordering has a first element. The reader can see that this seems, though hardly, as a logical proof that can apply to the set of all events of the world relative to some *vague* frame. In short, the world can be well ordered and so it has a least or first event. To be precise, the physical world, in light of set theory, has some ranking and a first event, and so in “some way” it cannot be always existing. Obviously, this reinforces the common sense intuition that the world is created at some instant, backs the physical “**Big Bang theory**” of *Lemaître*, and assures the statements of the Monotheistic Scriptures about the creation of the world at some instant from nothing.

But well ordering for the real numbers though implied by the axiom of choice, and so it exists by a necessary entailment of sound rational reasoning, is something impossible to imagine. *This of course adds an example to the list of things accepted existing through reason but never can be figured out.*

Another important result which is called: the Banach-Tarski paradox, in fact it is not a paradox at all, it has this name because

it is in some sense, counter intuitive, for more precision it is not counter intuitive, it is just counter experience. The result says, it is possible to decompose a ball, say of volume 1, into a finite number of pieces, then reassemble these pieces into two balls where each one of these two balls is also of volume 1, like the original ball. A direct corollary of this, is that from one single ball an infinite number of balls of the same size can be created according to the geometrical sequence: 1, 2, 4, 8, 16, 32, 64, 128, ...*This implies at once, the existence of someone with unusual knowledge or discrimination power that can recreate some destroyed body once again even better. This also gives an approximate figuration for the creation from nothing.*

The sum of the above is that knowledge and choice are interlaced. If we looked deeply at this talk, we find it is about the **ability** to discriminate and choose, to form a set in the first place we need to discriminate and choose. Intelligence, discrimination and choice are necessary for forming sets, or are necessary to classification taxonomy, and so for knowledge. So, the higher one's mind distinguishing ability, the higher his ability to choose, classify and know. Obviously, the converse is also true. We close this section by remarking that the implication of the axiom of choice about the existence of a higher mind backs in some way our causation instinct about the existence of a sufficient reason for anything in existence.

Sufficient Reason, a word about it was given above, is the principal premise we use for our argument in this final section. Before starting it is important to notice that the principle of Sufficient Reason confirms the existence of a sufficient cause for any event, but it does not tell whether a given thing is the cause

for another, this depends on other human powers. Now, we begin by showing that **Baruch Spinoza** suggestion about the world, which reduces to accepting matter as eternal with some sort of web mind or laws, is some form of Russel's paradox. This will give reinforcement or justification for our common sense conviction; *mind precedes matter*.

We have three kinds of minds:

An object is a mind that does not cause anything.

A mind that cannot cause itself, obviously, an object satisfies this definition.

A mind that can cause itself.

Contain, in the below argument means understand, and understanding something implies the ability to cause this thing, but not necessarily that who understands a thing is he the one who caused it.

Now assume there exists a mind W that contains only those minds which cannot cause themselves.

If we consider W to be a mind that can cause itself, then by the definition of W , W does not contain its own mind, but then absolutely, W cannot cause itself, and we get a contradiction.

If we consider W to be a mind that cannot cause itself, then by the definition of W , W contains its own mind, but then for sure W can cause itself, and we get a contradiction.

So, anyway to accept W as existing leads to a contradiction, thus the notion of a mind W that contains only those minds that cannot cause themselves is just a delusion.

Now Spinoza's suggestion, as the author understands it, assumes the world always exists as ***one whole***, and this one whole is the cause for the individual things in this world, and nothing is outside of this world, and that this world as one whole is as one mind N that ties or knits or contains everything in the world through, say, some web of eternal laws. By the second part of this suggestion, this assumed mind N does not contain or knit anything except those things that are contained in the world, where each one of these things in the world, whether with intellect or not, as we know and Spinoza with us, cannot cause itself. *Obviously, N satisfies the definition of W above, so N as W cannot be but a delusion.* This argument has shown that assuming any kind of adornment for matter with some elusive guise of mind to swallow that matter could be eternal, meaning self-caused, is self-contradictory.

Thus, either we accept our common sense conviction that mind precedes matter, or we go in some nonsense sophistry about the alternative of something or some mind that "causes itself partially" (which amounts to the contradiction "a set contains and contains not itself", and the contradiction "self-causing and being subject to time & space"), or we accept matter without mind as eternal, which in no way is a common sense.

Now we sketch a scene for the events in our world. According to the principle of sufficient reason, each event has causes, where each one of these causes has causes, ... and so on. Moreover, this same event is a cause for other events, where each one of these events is again a cause for others, ... and so on. The previous sketch is a line drawing and does not pencil some nuances and

shades that render an expressive image. Thus, we pencil now some more shades for a more expressive scene, beginning by an elucidation for a difference between the notion of a **“sufficient reason/cause” and that of a “sufficient condition”**.

The reader is familiar with these *equivalent* statements available in mathematics and logic references,

A is a *sufficient condition* for B.

If A then B.

A implies B.

A proves B.

A gives proof for B.

B is a necessary result/consequence of A.

No B then No A.

No B proves No A.

Only If B then A.

B is a necessary condition for A.

B is necessary in proving A.

In these expressions we assume that both A and B are claims or objects tacitly interpreted as claims.

For instance, if A is divisibility by 8 and B is divisibility by 2. Then “divisibility by 8 is a sufficient condition for divisibility by 2”, and this is equivalent to “divisibility by 2 is a necessary result of divisibility by 8”, and also equivalent to “divisibility by 2 is a necessary condition for divisibility by 8”, and to “no divisibility by 2 then no divisibility by 8”. *Obviously, in this example both A and B are independent of will and time in the causation connecting them. All mathematics statements are in a two valued logic frame independent from time and will.*

While if we assume say that A is “man on earth” and B is “car”, and we consider,

“Man on earth is a *sufficient reason* for car”,

then this obviously is not equivalent to “no car proves no man on earth”, or equivalent to “car is a necessary reason for man on earth”, or equivalent to “car is a necessary result of man on earth”. These obviously are false statements. The reason for this is clear, there is a difference between these two expressions “sufficient reason/cause” and “sufficient condition” in our talk here. For the first expression A is sufficient=capable for B, and willed B, but has the choice for doing B or not. While also for the second, A is sufficient=capable for B, but A has no choice but to yield B in no time, or we can say that B is a side of A, this is clear from the necessity in the sixth formula. So, it is clear now that in the considered example this is not a sufficient condition but a sufficient reason that depends on the free will of man. Meaning the will is not considered inactive in the statement; the will is active involved in the statement’s scene. Also, the word “reason” has its guise “make”, and the causing is imagined through time where man on earth precedes car in time. Meaning we have in mind “man on earth is a sufficient cause for car later in time and man willed car to exist”. Obviously, no need to mention that when

will or time are involved, the “*reflected implication*” of the “necessary & sufficient condition” is not allowed in general for the “necessary & sufficient reason”. It will be an implication in one direction only, from the reason to the event.

We demonstrated the difference between a *sufficient reason* and a *sufficient condition*, and our coming discussion involves will and time, namely like the case of man on earth & car, and in this discussion we do not involve the contrapositive formula or anyone of its area, for these reasons and to avoid redundant talk, we back-bench the contrapositive players in our mind (the last sixth formulas) to avoid any conflict in the field. This back-benching in anyway will not harm our discussion. Although in mathematics “sufficient condition= implication” and its “contrapositive” are free players, due to the independence of the mathematical objects and statements from will and time; this same reason makes mathematics in general and in particular its results: the axiom of choice and its equivalents, to be also independent from will and time, and so neutral and resilient for an insertion into a scene that involves will and time. And this is confirmed by what we clearly see every time everywhere, mathematics is inserted in, or permeates, every aspect of our everyday life which goes with causation entangled with will and time.

Briefly, our argument context is the set of all things in Existence, where a binary relation called Causation acts between these things. Causation is “Transitive”, but in general it is “not symmetric” and “not reflexive”, thus as we shall see later, a strict order relation emerges. In our argument we test the things of existence in respect to the “causation relation”, where, of course, the usual logic of mathematics is with us including its indispensable notions of “sufficient, necessary, and necessary & sufficient conditions”, but we must not

confuse between these notions present in every argument (including ours here) and the broad notion of the “causation relation” (which goes in the variations “sufficient, necessary, and necessary & sufficient causes/reasons”) which we test the things of Existence in respect to it in our argument.

The logical formulas are just abstractions or simplifications for our perception of a very complex reality. Therefore, to maintain these logical formulas sensible, or more generally to maintain the whole logical team at our side without conflict in a larger discussion that includes objects with free will, we must be sensitive to the subtle differences of their meanings in a broader discussion. We confine ourselves by the above word about the semantics interlaced with our subject.

Now after the above elucidation. We go into our addition of shades and nuances. ***We have a present event C at hand.*** From the first glance we are convinced that it is not the cause of itself. And so, by the principle of sufficient reason, or say our causation instinct, this event ***needs*** a sufficient reason to be here. We searched, then we found a cause for the event C. We looked at it, and we get the conviction that the cause we have found is not a sufficient reason either for event C or for itself of course. *So, if we are sure that this is a cause for the event, and since any cause is one of two kinds, necessary or sufficient (“necessary & sufficient” can be seen as necessary or sufficient), then what we have found is only a pure necessary cause for the event.* So again, under a sting of our causation instinct this ***pure necessary cause*** also needs a sufficient reason, so we go into a search for a reason for this cause. We found one. We inspected it, but again we saw that it is not a sufficient reason for the pure necessary cause at hand, and not for event C and surely not for itself. Thus, again what we have got is ***a pure necessary cause for the pure necessary cause.*** And again, a sting of our causation instinct spurs us for a ***new search***

or research, a more accurate word, ... and so on. Obviously, we go into *a path that maybe never ends*.

At this point we need to say a word about the proof we sketch; our proof may be described as a scene for the present and past of the world but not its future. More precisely, ***our proof is a sketch for an event of the world within its past to the present moment, but not with the future***. We may say that imagining the future alters the scene completely such that then we will be looking at a new totally different scene, although exactly with the same structure or composition.

Now to an important remark about the exact meaning of “pure necessity”: if we assume there are two distinct pure necessary reasons for C such that each one of them its role for C is the same, then one of them is not actually necessary for C, which is a contradiction. Thus, we can say that each pure necessary reason of C is “unique” in its role for C. So, we have this result:

if we reached a sufficient reason for C, then it must be a sufficient reason for “all” the pure necessary reasons of C (all the pure necessary reasons for C must happen before C happens, and a sufficient reason for C must be enough for that). This also means that no pure necessary reason for C comes above a sufficient reason of C. It is also important to note that the pure necessary reasons for C may be on different paths going into C, but each sufficient reason of C must be on every one of these paths above all the pure necessary reasons of C on each path.

Now assume for a moment that we have found *a sufficient reason for C*, as said, this immediately will mean that this sufficient reason at hand is a sufficient reason for all the pure necessary reasons on every path of C under it to the event C we started with.

At this moment under a spur of our causation instinct we will ask about a reason for this sufficient reason at hand. *If we assume the reason for this sufficient reason to be a pure necessary reason (meaning not “necessary & sufficient”), this will mean that it will be a pure necessary reason for the sufficient reason at hand and for every one of the pure necessary reasons under it including C, which contradicts our assumption that we have reached a sufficient reason for the event C and no other pure necessary reasons for C come above this sufficient reason. Thus, the reason for the sufficient reason at hand must be also a sufficient reason or “sufficient & necessary”, so the sufficiency for this reason is guaranteed.*

Obviously, this last argument can be repeated again and again, so we get maybe an endless sequence of sufficient reasons where each one of them is a sufficient reason for all the causes under it to the present event C we started with. **Also, it is possible that we have more than one sequence of sufficient reasons.** It is important here to remind the reader by what was told earlier, since the discussion under consideration entangles will and time with causation, then to avoid any conflict with the formulas of the contrapositive area, we back-bench these formulas in our mind. And obviously the **will** for any reason of an event is assumed to be in consent.

Now the world may be depicted on a paper as some square or neighborhood dense with points, each point is a pure necessary cause/event where *each event* has maybe an infinite number of paths going into it. Each one of these paths is dense with **pure necessary causes, where each cause is a pure necessary cause for every event that comes under it.** So, we have paths dense with pure necessary causes leading to the present event. Then other events are imagined also on different paths that go out of our present event. Going up toward the paper top, you go to the past, going down you go to the future, the simultaneous events are the points on the same horizontal line.

We consider in our talk “one” path of the many paths that pass by our present event C, more precisely, we consider the part of this path starting by C at present and going up within past, we denote this path’s part or chain by B.

Here is an embracing conclusion for the previous talk with some different words. By asking who, how, why, where, ... , ***we express our deep desire for knowledge motivated by our instinct of causation.*** Sometimes we get an answer after some research, and with more questions and more answers knowledge accumulates. But though we get some answers we always feel that the answer has missed something and is not complete, the perpetual feeling that our knowledge is always incomplete, so we ask again and again in ***an endless knowledge forage...*** . So, we always feel that we only have known pure necessary causes and not sufficient ones. So, we have got paths or chains of causes that extend infinitely, or seem infinite, where the rings on every chain are ***pure necessary reasons.***

A shadow of determinism is cast now, but we are sure by our instinct that we have a free will, hence this shadow of determinism is a mere delusion. This instinct of ours that denies this shadow of determinism is also an implication of our causation instinct. ***This is justified once we show that the sufficient top of the chain has “a maximal that is the reason for itself”, and so we have at least one free will on the chain.***

Anyhow this is not the concern of this paper, so we get back to the track of our subject. ***As told above, if we assume we have reached a sufficient reason for C, then this precisely means that this is a sufficient reason for all the pure necessary causes under it to the event under consideration, the present event C. Meaning all the pure necessary causes chains of C are held by “every” sufficient reason of C.*** Obviously this immediately means that we have reached our

goal for finding a sufficient cause for our event, and so more search about extra sufficient causes is redundancy in **some sense**, unless justified. More precisely, this puts **in a sense** an end to the search on **every** chain of the collection of all chains of pure necessary causes that lead to the event C.

Also, it is suitable here to mention that if we assume one of the sufficient reasons on one path to be the “reason for itself”, then this par excellence will truncate one of the sufficient reasons extension sequences of a chain B of C, and this chain of C absolutely ends. Also, if something is the cause of itself, then an invincible common sense dictates at once that this self must be eternal, or say, always exists independent, complete, not subject to, or say, above time and space, meaning does not evolve or change over time and space.

Now as elucidated, a path or chain of causes is infinite or seems to us infinite, and its rings are pure necessary reasons, but this puts us in what seems as a dilemma, since it is the principle of sufficient reason that has led to this conclusion about this endlessness character of the chain of causes, but also it is by the very essence of this principle that we must have a sufficient reason meaningful in some sense for the considered event. Though these two opposite implications are **the two sides of the key for research and the accumulation of knowledge**, their opposition is perplexing. But this opposition is only on the face. Since, though the part of the pure necessary causes is endless, according to our causation instinct there must exist sufficient reasons that **bound from above** the endlessness of every pure necessary causes chain B of C (*obviously this will be explicitly confirmed once we know a first sufficient reason*). So, we are allowed to consider the sufficient causes come above every chain of pure necessary causes of C as a whole, something as 1, 2, 3, ... bound the endless sequence [0, 1/2, 3/4, 7/8, 15/16, ...] and as said above, we denote the pure necessary part of a chain of C by B.

Notice that in general, we cannot say that the sufficient reasons extension of chain B has a first sufficient reason. Meaning the structure of our endless chain B of pure necessary reasons topped by **one** of its sufficient extension sequences is like that:

[C < < ncs <) < suf < < suf < < suf <]

It is also good to remember what was mentioned above, that it is enough to know only one sufficient reason of C to feel our search has succeeded.

Due to the structure of the chains, especially the hazy border between the necessary and sufficient on the chains, a condition is needed to illuminate the exploration for some implications of the chain of causes.

We call a chain of causes B of C as having an upper bound cause, if it has a "first sufficient cause". Obviously, this is not the well-known condition for upper boundedness, but we shall see later, that for our argument if B has a first sufficient cause, then the conventional upper boundedness condition will be satisfied.

As shown above no pure necessary causes are there above this first sufficient cause above the chain B, only sufficient causes may be there. So, the structure of this endless chain B of pure necessary reasons topped by **one** of its sufficient extension sequences is like that:

[C < < ncs <) |< **suf** < < suf < < suf <]

We may say that this definition demands a clear strong confirmation that there is a bound for chain B where no more *pure necessary causes* for the event C exist above this bound. If

this demand is confirmed and since this bound is a sufficient cause for C, then we have succeeded. So, we repeat what was said above, this in a sense, or say almost, puts an end for anymore search for causes for C, because any more possible reasons above this first sufficient reason will be all sufficient, and so redundant sufficient reasons, unless justified, for B and C. In some sense the search ends, “if we have known a trusted supplier for our needs, why we will search for who supplies him”.

Now to a new point. An implication for the principle of sufficient reason is:

“Existence is Consistent”.

We will not discuss this implication, but we give for it one very concise justification; without such conviction the whole enterprise of scientific research will be mere fancies of hope. We will use the above implication as an axiom or premise in the main proof of this paper.

Projecting the principle of causation as a premise with its above mentioned implication about existence, and the axiom of choice with its two equivalent results, Zorn’s Lemma and the Well Ordering Theorem, against a background of the impossible or the almost impossible for our mind, we show in what follows that every chain of causes has a first sufficient reason and so an upper bound. Moreover, every chain of causes will be shown to be meaningful, meaning it has a ***peculiar unique sufficient reason***. So, this will confirm the refusal of our mind to accept that any one chain of causation to be without a ***limit***, (note that we say limit and not last element, something maybe like 1 being a limit for the infinite sequence; 0, 1/2, 3/4, 7/8, 15/16, ...). Also, it will be shown that this peculiar limit is with mind or say of one free will.

In the next argument we justify these claims, and consequently this will provide our ontological proof.

Since:

The events in our world assumed over the aggregation of time, as shown above, can be “strictly ordered by causation as cause precedes event”. Obviously, this at once gives the image of different paths or chains spreading from the present event to its preceding causes, where the rings of the chain are causes/events. If we consider only one of these chains of pure necessary causes for an event C, say B, then we will find it **totally ordered**, meaning for any two rings on the chain one must precede the other.

Now an arbitrary chain B of the event C, according to our causation instinct, must be topped by some sufficient cause A. Also, we consider the set U of “all” the pure necessary causes for B, obviously, U contains “every” pure necessary causes chain going into any point on B. Thus, it is very important to note that U contains pure necessary causes for B not on B, meaning on other paths going into the points of B. Also, we must note that the event C lies on B, therefore “every” pure necessary causes chain of C is contained in U. Obviously U exists, or say U is an element in existence, and due to “all”, U is a sufficient cause for B and so also for C. But U evolves over time and space, so, it is not the cause of itself, thus, according to the principle of Sufficient Reason U must also have a sufficient reason, so, “above” U there exists sufficient reasons. Now if we consider A, meaning any sufficient cause for B, then by page 18 above, A must be a sufficient reason for all the pure necessary causes of B. Thus, A is a sufficient reason for U. Argued in the language of sets, A must hold all the pure necessary causes of B, meaning holds U. Thus, U is the first sufficient reason for “every” chain B of the event C, also “every”

sufficient extension sequence of B begins by U, and so “a” uniqueness of U is established.

Also, we consider the set of “all” sufficient reasons “above” U. This set, or the “sufficient top” above B and U, is an “element in existence” that bounds B and U (U and any element on B come under this sufficient top), so, the sufficient top satisfies the conventional condition of an upper bound for B and U (required to apply Zorn’s Lemma). It is very important to notice that: we do not know anything about the sufficient top which is above B and U except that it is not empty, meaning the sufficient top has at least one element.

Above we talked about an axiom of set theory, the “axiom of choice”, which has two equivalent implications, “Zorn’s Lemma” and “Well Ordering”.

Zorn’s Lemma states that: if we have a ***set S*** of elements that allows an order between ***some*** of its elements (partial order, strict or not strict), and we find that any ordered sequence (totally ordered subset) of elements has an upper bound which is an element of S, then S must have at least one maximal element, ***meaning an element of S that cannot be preceded or topped by any other element of S.***

Well Ordering states that: for any set of elements there is some way of ordering such that any subset of this set has a first element, including this set itself.

Thus, considering the suggestion ***S as all the things that exist***, including the collection of events of the world over time, with causation as the order, and since in respect to this order, every chain B has its first sufficient reason U ***an element in S, existence***,

and since above B and U there is an upper bound “the sufficient top”, *an element in S, existence*, then by Zorn’s lemma:

“There is at least one element of S, existence, which must be a maximal cause, meaning a cause that needs no other cause, or say that is not caused by any other thing in S. But then due to the maximality of this cause and our causation instinct, we are forced to accept this cause as a sufficient cause/reason for itself.

But self-causation is absolute independence, meaning no other thing in existence prevents or bounds or affects the existence of this self, so it is perfect or complete and never changes, meaning it is complete above time and space. In other words that involves time and space expressions, a maximal needs no time to exist perfect complete everywhere, or was always there everywhere and will be always there everywhere, or always exist complete anywhere invincible, or a maximal is above imagination in space/time, or a maximal is neither bounded nor changing by time or space. So, a maximal cannot be but assumed as one complete indivisible whole above time and space. Moreover, each maximal absolutely ends “one” sufficient reasons sequence extension of some chain B of some event C, no other cause comes above a maximal. How is this end or final connection of the maximal with this chain B followed by one of its sufficient extension sequences? This is impossible for imagination, since, as said: the maximals are above space/time. Thus, the imagination of the world events in space/time as a paper’s surface is impossible to be extended to include the maximals, it stops here”.

So maximality implies self-causation. Conversely, saying that something causes itself means that this thing is not preceded or topped by any other cause, so it is maximal. Thus

maximality = self-causation = eternity, completeness anywhere invincibility, oneness... . By this we notice that: it is awkward to assume many maximals. Anyhow if we consider the realm of all maximals of S, and in light of the Well Ordering statement, then in this realm of maximals **there exists some vague well order for the maximals (surely not the order of causation, since having the maximals subject to causation contradicts their maximality) such that any subset of maximals of S, including the set of all maximals, has a first maximal.** This last conclusion is not used in the rest of our argument, though it hints to something very interesting, it asserts the existence of a unique first maximal, and at the same time hints to the vagueness of this order that exists between the maximals. Thus, if there is only one maximal then this vagueness dissolves, and a clarity relief springs.

Here it is very important to notice that since every maximal is above space and time, then it cannot be an event of the world. Also, a maximal cannot be U, because U is an aggregation of things separated by time and space, U evolves and changes over time and space, meaning U is not one complete indivisible whole above time and space. So, as said earlier, U is not the cause of itself and not a maximal.

Self-causation implies a very peculiar existence property; saying that something “caused itself”, immediately means that this self is very aware, and knowledgeable of “itself perfectly and completely”. Since it is against common sense to say that a self causes itself without completely knowing itself and be of one free will, our minds do not accept that (see Russel’s paradox above).

In other words, it is invincible our belief that no maximal is without mind and one free will. Therefore, wisdom, knowledge, oneness and free will are guaranteed for any maximal.

Conversely if “a self completely knows itself” this immediately means that this self “knows itself wholly perfectly and needs no other thing to exist, or say always exist by itself anywhere, or eternal complete anywhere invincible”. Thus, a self that causes itself, or say a maximal always exist as knowledgeable one indivisible whole perfect complete neither bounded nor changing over time or space, or say above space/time, never a composition of parts, of complete knowledge of itself. The previous is a justification for the next belief of our mind.

Our mind always believed that a self with a ***mind& knowledge, consciousness, or free will***, must be ***one indivisible whole***, and sometimes ***fuses these notions***. Meaning ***a conscious self is never a composition of parts***, especially if this self is the cause of itself. This is demonstrated now by the above arguments at least for a “self that causes itself”.

So, there is nothing called a thing that causes itself partially. Therefore, anything is one of two; caused by other things, or caused completely and wholly as “one indivisible self” by itself. Now, a self that causes itself cannot be assumed but as one indivisible self without parts, thus a self that causes itself is not a physical body, meaning is not any form of matter, since matter is divisible, extended in parts in a space time frame (note that eternity is denied for matter by our physical science, it changes over time, and it has a beginning in its most accepted existence physical theory). So, a maximal transcends the physical world. ***So, Oneness, Transcendence, and absolute Knowledge are guaranteed to any maximal.***

Now, as mentioned, the maximals are above time and no maximal is above the other in time, they are eternal, anywhere invincible, meaning every maximal from eternity is self-sufficient alone without need for any other cause and cannot be affected by any

other cause. ***Succinctly, the maximals are not subject to causation in any way.***

Suppose now that a maximal causes an event C in existence, then it is a must that it causes this event “sufficiently alone”, since it is against common sense, or say, we cannot imagine causing an event in the world to be harder or even on the same par to “self-causing”. This exactly means that every maximal is eternal anywhere invincible alone without any other maximal, except itself, and it is “separately alone sufficient” for itself and for any event it causes/initiates into existence. Shortly, if a maximal A initiates an event (with its time/space) in existence, then it is a must that it initiates it separately and sufficiently alone from nothing, “ex nihilo”. So, any maximal is Eternal, Almighty. Also, this means that, if the sufficient reasons for this event are assumed on different sufficient extension sequences above U, then “all” these sufficient extension sequences must be held below this one unique maximal.

But the separation entailed by the self-sufficiency of the maximals mentioned above, means that inside existence each maximal with the events it causes must be “isolated” from every other maximal with the events it causes (though each maximal exists always and anywhere invincible !!!). So, it is harder now to assume more than one maximal.

Anyhow, due to this isolation no two maximals have common events, or say no event is initiated by more than one maximal, therefore, “uniqueness is guaranteed for the maximal of the collection of all pure necessary chains (topped by their sufficient extensions) of the event C. So, also maximality discards the assumption of more than one maximal clumped together above the collection of all pure necessary chains (topped by their sufficient extensions) of the event C.

And for this same isolation reason it is “almost impossible” that an event of one maximal is even consistent with an event of another maximal.

Existence is split into two non-intersecting sets; the physical world of events which are subject to causation and time, and the maximals, where each maximal is self-caused, indivisible one, eternal, transcendent, alone above time and space. ***So, “each” maximal is sufficient alone for itself and for any event it causes, unaffected by anything, isolated though everywhere!!!***

So far, we have established the following,

Maximals exist, and if an event C some has a maximal then it is one and only one maximal. And since maximals exist, then there exists an event C such that every chain B of it has all its sufficient extension sequences absolutely ending with one and only one maximal, no other sufficient reasons above it. But as shown earlier, the first sufficient reason U of B, is a composition of parts, so U cannot be a maximal, so only A, a sufficient reason above U, could be maximal. Therefore, there exists an event C such that every chain B of it has all its sufficient extension sequences absolutely ending with one unique maximal, no other sufficient reasons above it, and this maximal is not U, but it could only be a sufficient reason A above U. But as shown earlier, all events chains have the same structure with an A above its U, so no essential difference is between any two chains for two different events, so it is highly likely that for “every” event, its chains their sufficient extension sequences absolutely end with one unique maximal. And as clarified above reaching a first sufficient reason in a sense ends the search on the chain, since having “more” sufficient reasons following the first sufficient reason is redundancy, unless justified. And if this “more” is endless without limit, then redundancy becomes absurdity. An absurdity that

empties our causation instinct, which we absolutely trust, from its very meaning. Therefore, assuming an event's chains that does not have some sufficient reason **A as maximal** above its U will be redundant and absurd. Also, we always anticipate that a sufficient cause for an event must be of a higher rank than the event, so a maximal is a more convincing candidate as a sufficient reason for an arbitrary chain B than a common sufficient reason.

But, for all that, an immediate leap of intuition our intelligence puts here; Not only some event's chains sufficient extension sequences absolutely end with one unique maximal but, for "every" event, its chains sufficient extension sequences must absolutely end with one unique maximal, no other sufficient reason above it, and this maximal is an A, a sufficient reason above its U.

In other words, "every" event C in the world has its unique maximal A, where A is above every sufficient extension sequence of every chain B of C. And due to A maximality no other sufficient reasons, in particular no other maximals, come above A, and due to this maximality A owns all the positive properties mentioned earlier.

So, by the above, every event in existence is made by no more than one maximal, and it is almost impossible that two events initiated by two different maximals to be consistent. But this "almost impossibility" contradicts if not fully contradicts our axiom about existence is consistent, and the world inside existence means the events of the world are consistent. Thus, this almost contradictory situation has only one way to get out of it, namely that there is one and only one maximal that initiates every chain and the whole world, so it is one unique maximal for the world. And obviously any extra talk assuming another world different and isolated from ours and initiated

by a different maximal, adopts a vision for existence as isolated worlds where each world has its unique maximal. But then, this complete isolation, or say peaceful existence of the maximals and their worlds, due to the principle of sufficient reason, must have a sufficient cause. So, there must exist a maximal that “causes” for the maximals this complete isolation. But this means that the maximals are affected by other maximals, or say, in some way are subject to causation, which as demonstrated above, contradicts their maximality. So, it is one and only one maximal for every possible world in existence. So, there is one and only one maximal and Uniqueness, or we may say “Absolute Uniqueness”, is its characteristic. Therefore,

There is one and only One God for all worlds, He is, One, Absolutely Unique, Transcendent, Wise, Knower, Eternal, Almighty, He is the First.

We close our argument by three remarks,

The **endlessness** of every chain of pure necessary causes, more accurately of temporal physical necessary causes, in more than one way, confirms the **Transcendence of God** above time and matter, or the physical space/time, or the world of physical events. How is the sufficient top above U of chain B? Of only one sufficient reason, the maximal; of a finite or infinite number of sufficient reasons on one or different sufficient extension sequences below the maximal; the answer does not affect the argument.

To use Zorn’s Lemma in our argument, maybe it was enough to apply the premise of the existence of a sufficient reason for the chain B of pure necessary causes as a whole, and this confirms B is bounded by **the existence of its sufficient top or maybe we**

can say past. But this in addition to be less illuminating and even with more awkward joints, it will also demand the leap of intuition mentioned above to be wider. Since the **juxtaposition** quality of the **first sufficient reason** as a boundary to the endless chain of pure necessary causes without doubt sheds light for a faster snap shot of intuition by our mind. Briefly, our consideration for the set U is auxiliary.

The last remark is, In the proof we demonstrated that the maximals are not subject to causation in any way. **This precisely means that who, why, when, where ... in their "exact sense" do not apply in any way to God.** Since such interrogations try to apply causation to a maximal, meaning try to explain/reduce/subject a maximal to other causes. So phrases, or interrogations such as; **the nature of God, or why God, ...?, which imply a subjection to causation for God are nonsense and futile.**

The previous argument was drawn over the impossible for our mind. Meaning using these points which we justified them by saying "it is impossible for our mind to accept, or our mind refuses, or it is against common sense ...". These points could be stated as axioms for our proof from the start, but we preferred to let them appear naturally in their due time through the argument as tacit common sense or implications for **mind precedes matter.** Also, we used the principle of sufficient reason, and its implication; existence is consistent, along with variations on the Axiom of choice, in addition, to few leaps of invincible intuition, immediate and natural.

The proof is closed, but in the following we have what we may describe as a "good end" that entails Optimism. Consider the following:

In one word we state that God is the Last, whether by intuition or as an implication for the above argument. But let us forget about direct intuition and the above argument, and interpret an “aim” as a cause in the future for **a future existence of the present event C**, then we have these definitions and statements for another long detour argument,

A necessary aim for an event C is something existing in the future that is necessary for a future existence of C.

A good aim for an event C is something existing in the future that is sufficient for a good future existence of C.

“For every event there must exist a good aim/result”, which is the teleological **optimistic** face of our causation instinct,

And the implication, “Existence is consistent”,

And the Axiom of choice with its two equivalent results, Zorn’s Lemma and the Well Ordering Theorem,

If we take the above statements as premises or axioms, and noting that aims are “necessary”, “good”, and “necessary & good” aims, also noting that the events of the world can be ordered in respect to “aiming”, **Then**, with a little modification for some expressions like, **good** instead of **sufficient**, **aim** instead of **cause**, so **aim of itself** instead of **cause of itself**, we can repeat exactly the previous proof and conclude that we have one and only one maximal **good aim, the Last** where no other aim comes after it. Summing this to the conclusion of our main argument, we have this last leap of intuition:

There is one and only One God for all worlds, He is, One, Absolutely Unique, Transcendent, Wise, Knower, Eternal, Almighty, He is the First and the Last.

Thank God.