

# *Carnap and Newton: Two Approaches to the Method of Theory Construction (Part I)*

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**Abstract:** The paper compares Carnap's and Hempel's Standard Conception of Scientific Theories with Newton's method of theory construction as applied in his *Principia*. It is shown that the latter is built, contrary to Carnap's and Hempel's views, by a cyclical method.

**Keywords:** Carnap, Hempel, Standard Conception of Scientific Theories, Newton, *Principia*, cyclical method of theory construction.

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The aim of this paper is to compare the view on the method of theory-construction as given in the so-called "Standard Conception" of scientific theories with the method of theory-construction employed by Newton in Books I and III of his *Principia*. Because of its length we have split it up into two parts.

In Part I, we start with an analysis of the so-called "Standard Conception" of scientific theories developed in the framework of logical empiricism and analyze its main deficits. We then provide a reconstruction of the method of theory-construction as given in Book I and Book III of Newton's *Principia*,<sup>1</sup> which we regard as a paradigmatic case of a cyclical method of theory construction and show how it deviates from the views on the methods of theory construction stated in

<sup>1</sup> We reconstruct here, from the point of view of applied methods, neither the relation of Book II of the *Principia* to its two remaining books, nor the method of its internal construction. Books I and III deal with movements of bodies in nonresistant spaces (media), while Book II with their movement in resistant spaces.

the “Standard Conception”. Finally, in Part II, we deal with the meanings of the term “harmonic law of planetary motion” as used in the *Principia* and address the issue of meaning-change of this term within Book I and Book III of the *Principia*.

## 1. The “Standard Conception” of Scientific Theories

### 1.1 What is the “Standard Conception” About?

The “Standard Conception”, sometimes also dubbed as “the received view of scientific theories”, was, as Carnap confesses, “influenced by two different factors: the explicit development of the axiomatic method by Hilbert and his collaborators, and the emphasis on the importance and function of hypotheses in science, especially in physics, by men like Poincaré and Duhem” (1963a, 77).<sup>2</sup> At the same time, it should provide a logico-linguistic framework for the reconstruction of the structure of scientific theories and an explication of what these theories are “about.” The views stated in the standard conception take as their starting point Carnap’s *Testability and Meaning* of 1936/37, where he already clearly differentiates between observable and non-observable predicates of a language as follows:

A predicate ‘P’ of a language L is called *observable* for an organism (e.g. a person) N, if for suitable arguments, e.g. ‘b’, N is able under suitable circumstances to come to a decision with the help of few observations about a full sentence, say ‘P(b)’, i.e. to a confirmation of either ‘P(b)’ or ‘~P(b)’ of such a high degree that he will either accept or reject ‘P(b)’.

(Carnap 1936/37, 454 – 455)

In a next step Carnap propounds in his *Foundations of Logic and Mathematics* (FLM for short) the view that a scientific theory can be characterized by “two fundamentally different components, a factual and a logical” (Carnap 1939, 37). While the latter can be characterized as a set of axioms plus its logical consequences, the former comes only via a factual/observational interpretation of the logical consequences of axioms.

<sup>2</sup> On the Hilbertian roots of the “Standard Conception” see also Carnap (1958, 79) and Hempel (1973, 369).

Finally, in his *Methodological Character of Theoretical Concepts* of 1956, as well in his papers (1958), (1959/2000) and (1961), he divides the total language of science into an observational language  $L_O$  – as a language completely interpreted by means of its observational vocabulary  $V_O$  – and a theoretical language  $L_T$ , whose descriptive vocabulary  $V_T$  consists of theoretical terms, i.e., terms speaking about directly non-observable entities. For  $L_O$ , but *not* for  $L_T$ , Carnap accepts the following six requirements (Carnap 1956, 41– 42):

- (1) The requirement of *observability* for the primitive terms.
- (2) The requirement of explicit *definability* for the non-primitive descriptive terms.
- (3) The requirement of *nominalism*: the values of the variables must be concrete, observable entities (e.g. observable events, things, or thing-moments).
- (4) The requirement of *finitism*: the rules of the language  $L_O$  do not state or imply that the basic domain (the range of values of the individual variables) is infinite (i.e.,  $L_O$  has at least one finite model).
- (5) The requirement of *constructivism*: every value of any variable of  $L_O$  is expressed by an expression in  $L_O$ .
- (6) The requirement of *extensionality*: the language  $L_O$  contains only truth-functional connectives.

For  $L_T$  it holds:

that it contains a type-theoretic logic with an infinite sequence of domains  $D^0, D^1, D^2$ , etc. [...] The elements of  $D^0$  are the members of the infinite sequence  $0, 0', 0''$ , etc. Officially, no meaning is given to these logical expressions. Their use follows from the rules of the language. We shall, however, in order to relate these expressions to familiar concepts, unofficially regard  $D^0$  as the domain of natural numbers with '0' denoting the number 0, '0'' the number 1, etc. (1975, 76; 1958, 237)

In addition, a theory is built in the extra-logical part of  $L_T$  on the basis of theoretical postulates, i.e., on extra-logical axioms, and contains also other theoretical statements derived as theorems from these axioms. What also comes in – according to Carnap – are the so-called “correspondence-rules/postulates” (“C-rules” for short), i.e., statements containing both theoretical and observational terms. These

rules/postulates give to the theoretical postulates and terms from  $V_T$  a certain *interpretation*.<sup>3</sup>

A succinct characterization of the “Standard Conception” is given by Hempel as follows:

A scientific theory might be linked to a complex spatial network. Its terms are represented by the knots, while the threads connecting the latter correspond, in part, to the definitions and, in part, to the fundamental and derivative hypotheses included in the theory. The whole system floats, as it were, above the plane of observation and is anchored to it by the rules of correspondence. These might be viewed as strings which link certain points [of the network] with specific places in the plane of observation.

(Hempel 1952, 36)

This approach to scientific theories can be schematically represented as follows (here  $\alpha, \beta, \chi, \dots$  stand for undefined terms while the lines connecting them stand for axioms;  $a, b, c, \dots$  stand for defined terms and the lines connecting them stand for theorems;  $o_1, o_2, \dots$  stand for observational terms):<sup>4</sup>

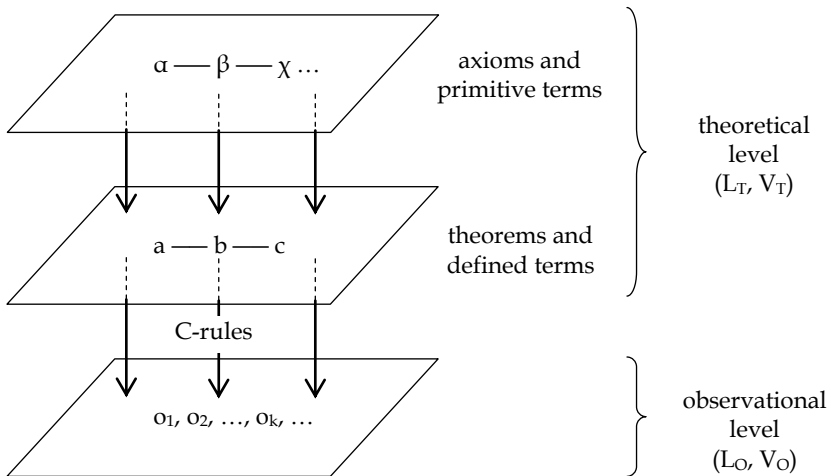


Fig. 1: A spatial representation of the “Standard Conception”

<sup>3</sup> On this see Carnap (1956, 46).

<sup>4</sup> For a different scheme see Feigl (1970, 6).

The “Standard Conception” is at the same time a vehicle for certain specific philosophical views on the nature of scientific theories and scientific knowledge in general, namely, that *only observational terms* from  $V_O$  have a *direct reference*; they are about *something*, while the terms from  $V_T$  have *no direct reference*, but at the same time are not just part of a purely computational device. Carnap claims that this approach should enable one to escape both the instrumentalist view of scientific theories (i.e., theories serve us just as instruments for computation) and the anti-nominalist (realist) view on the nature of theoretical terms, namely, that they refer to entities in the extra-linguistic realm. So, already in his *FLM* Carnap claims that:<sup>5</sup>

philosophers [...] content that [...] modern theories [...] are not at all theories about nature but “mere formalistic constructions”, “mere calculi”. But this is a fundamental misunderstanding of the function of a physical theory. It is true that a theory must not be a “mere calculus” but possess an interpretation, on the basis of which it can be applied to facts of nature. But it is sufficient [...] to make this interpretation explicit for elementary terms; the interpretation of the other terms is then indirectly determined by the formulas of the calculus, either definitions or laws, connecting them with elementary terms. (Carnap 1939, 68)

And in his papers Carnap (1956) and Carnap (1958) he underscores his nominalism by claiming that the variables of  $L_T$  do *not* range over non-observable entities but over *mathematical* entities. The example he gives for this is as follows: “Let the constant ‘ $n_p$ ’ be defined as ‘the cardinal number of planets’. This constant is descriptive, to be sure, but the thing described by it is a natural number which belongs to the domain of  $D^0$ . The number  $n_p$  is identical with 9, but the identity ‘ $n_p = 9$ ’ is synthetic” (1975, 80 – 81; 1958, 243). In general this means that according to Carnap we have two designators “ $f$ ” and “ $g$ ”, where the former is a *descriptive* constant and the latter a *mathematical* constant, but it holds that they have the *same extensions* for the same arguments, i.e., they are *extensionally identical*. And in order to escape any possible charge of realism with respect to “ $g$ ”’s ranging over natural numbers he states also that this “should not be taken literally but merely as didactic help by attaching familiar labels to certain kind of entities or, to

<sup>5</sup> On this see also Carnap (1966, 254 – 256).

say it in a still more cautious way, to certain kinds of entities of expressions in  $L_T$ " (Carnap 1956, 45 – 46).

## 1.2 Negative consequences of the nominalism of "Standard Conception"

The negative consequences of the nominalism of the "Standard Conception" are readily seen when one analyzes Carnap's views in *FLM* on the possible relations between the elementary (i.e., observational) terms and the abstract (i.e., theoretical) terms in the process of theory construction. As examples of the former he mentions terms like "yellow", "bright", "dark", etc. and of the latter he points to terms like "electric field", "frequency of oscillations", "wave function", etc. and then goes on as follows:

Suppose we intend to construct an interpreted system of physics – or of the whole science. We shall first lay down a calculus. Then we have to state the semantical rules [...] for the specific signs, i.e., for the physical terms. [...] Since the physical terms form a system, i.e. are connected with one another, obviously we need not state a semantical rule for each of them. For which terms, then, must we give rules, for the elementary or for the abstract ones? [...] Either procedure is [...] possible [...]. The *first* method consists in taking elementary terms as primitive and then introducing on their basis further terms step by step, up to those of highest abstraction. [...] The first method has the advantage of exhibiting clearly the connection between the system and observation [...]. However, when we shift our attention from the terms and the methods of empirical confirmation to the laws, i.e., universal theorems of the system, we get a different picture. Would it be possible to formulate all laws of physics in elementary terms, admitting more abstract terms only as abbreviations? [...] But it turns out [...] that it is not possible to arrive in this way at a powerful and efficacious system of laws. To be sure, historically, science started with laws formulated in terms of a low level of abstractness. But for any law of this kind, one nearly always later found some exceptions and thus had to confine it to a narrower realm of validity. The higher the physicist went in the scale of terms, the better did they succeed in formulating laws applying to a wide range of phenomena. Hence we understand that they are inclined to choose the *second method*. This method begins at the top of the system, so to speak, and then goes down to lower and lower levels. It consists in taking a few abstract terms as primitive signs and a few fundamen-

tal laws of great generality as axioms. Then further terms, less and less abstract, and finally elementary ones, are to be introduced by definitions [...]. At least this is the direction in which physicist have been striving with remarkable success, especially in the past few decades. [...] Now let us examine the result of the interpretation if the first or the second method for the construction of the calculus is chosen. In both cases the semantical rules concern the elementary signs. In the first method these signs are taken as primitive. Hence the semantical rules give a complete interpretation for these signs and those explicitly defined on their bases. [...] If, on the other hand, abstract terms are taken as primitive – according to the second method, the one used in scientific physics – then the semantical rules have no direct relation to the primitive terms of the system but refer to terms introduced by long chains of definitions. The calculus is first constructed floating in the air, so to speak; the construction begins at the top and then adds lower and lower levels. Finally, by the semantical rules, the lowest level is anchored at the solid ground of observable facts.

(Carnap 1939, 62 – 66)

Carnap can thus provide the following schematic representation of these two methods which he views as *alternative* methods of theory construction (1939, 63):

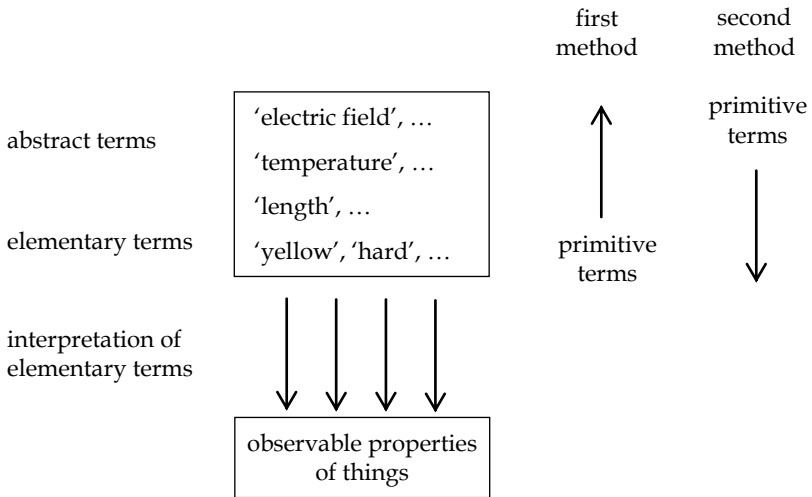


Fig. 2: Carnap on the alternative methods of theory construction

What has to be emphasized here are two important points. First, that already in his *FLM*, like in his later papers (1956), (1958), (1959/2000) and (1961), Carnap holds to the view that abstract terms like “electric field”, “wave function”, i.e., “ $\Psi$ ”, lack any reference. So, e.g., he claims:

If we demand from the modern physicist an answer to the question what he means by the symbol ‘ $\Psi$ ’ of his calculus, and are astonished that he cannot give an answer, we ought to realize that the situation was already the same in classical physics. There the physicist could not tell us what he meant by the symbol ‘ $E$ ’ in Maxwell’s equations. Perhaps, in order not to refuse an answer, he would tell us that ‘ $E$ ’ designates the electric field vector. To be sure, this statement has the form of a semantic rule, but it would not help us a bit to understand the theory. It simply refers from a symbol in a symbolic calculus to a corresponding word expression in a calculus of words. (Carnap 1939, 68)

Second, Carnap in *FLM*, contrary to his papers (1956), (1958), (1959/2000) and (1961), understands under semantical interpretation of a term *only* the assignment of its *designatum*; the meaning/intension of terms is in the *FLM* as yet *not* taken into account here because in 1939 Carnap in his semantics<sup>6</sup> did not as yet arrive at the distinction between the intension and extension of language expressions.

The first of these two features of Carnap’s *FLM* leads to the following negative consequences for the very reconstruction of the structure of scientific theories. Since the abstract terms are in the sequence Carnap labels as the “first method” viewed as the *end-product*, and at the same time are viewed as the *point of departure* in the sequence he labels as the “second method”, but at the same time are viewed by him as mere *syntactical entities lacking any extra-linguistic designata*, and thus their reference cannot have any causal determinations related to the causal determinations of the designata of the less-abstract terms, *the whole sense and importance of the movement from the elementary terms as primitive terms “back” to the elementary terms as derived terms gets lost*. Hempel expressed that sense and importance as follows:

Theories are normally constructed only when prior research in a given field has yielded a body of knowledge that includes empirical generalizations or putative laws concerning phenomena under study. A theory then aims at

<sup>6</sup> On this semantics see Carnap (1942).



providing a deeper understanding by construing those phenomena as manifestations of certain underlying processes governed by laws which account for uniformities previously studied [...]. (Hempel 1970, 142)

In the remarks that follow we will see how Newton employs the quantitative determinations of certain phenomena-effects in order to determine the quantitative determination of their cause and how he then derives the quantitative determinations of other phenomena-effects from the quantitative determinations of their cause. The first of the above-mentioned features of *FLM* leads also to the inability to realize that scientific theories are in fact *not* built by *either* the first *or* the second method, but via a *unity* of them, i.e., by a *cyclical method*. In Part II of this paper we will reconstruct the cyclical method of theory construction as it is given in Newton's *Principia*.

The second of the above-mentioned features of *FLM* enables us to understand the other negative consequences for the very reconstruction of the structure of scientific theories. Thus, even if Carnap (starting from 1942/43 on) substantially changed his semantics by bringing in the differentiation between the intension and extension of language expressions, and even if he explicitly provided the semantics for them in his *Meaning and Necessity* (*M&N* for short) of 1947, at the level of the very philosophy of science one finds a *surprising absence of the application of these semantics for a detailed analysis of the intension/meaning of the terms of scientific theories*. And even if he views the meaning of the terms of the observational language as fixed by analytic/meaning postulates and even if he deals with the *latter* in a separate paper (Carnap 1952), *neither this paper nor any other published work of his contains any analyses of the meaning of those terms which he labels as "elementary/observational terms."*

Let us now turn to the terms which Carnap labels as "abstract/theoretical terms." Worth quoting here is Carnap's letter to H. Feigl of August 4, 1958, where he refers to above mentioned example "The cardinal number of planets is 9" which appears in his paper (1958) in square brackets:<sup>7</sup>

<sup>7</sup> The symbol "RC" stands for the catalogue number of Carnap's manuscripts in the archives at the University of Pittsburgh. We would like to thank the archives at the University of Pittsburgh for the permission to quote from this letter of Carnap to Feigl.

[T]he entities to which the variables in the Ramsey-sentence refer are characterized not purely logically, but in a descriptive way; and this is the essential point. These entities are identical with mathematical entities only in the customary extensional way of speaking; see my example in square brackets [...] In an intensional language (in my own thinking I use mostly one of this kind) there is an important difference between the intension  $9$  and the intension  $n_p$ . The former is *L-determinate* [...] the latter is not. Thus, if by 'logical' or 'mathematical' we mean '*L-determinate*', then the entities to which the variables in the Ramsey-sentence refer are not logical.

(Carnap, RC 102-07-05)

From the point of view of the semantics of *M&N*<sup>8</sup> this means that while the extension of the expression " $9$ " can be found out by using only the rules of language of mathematics determining its intension, i.e., without turning to extra-linguistic facts, the finding of the extension of the expression " $n_p$ " requires, in addition to the use of rules fixing its intension, some investigation into the extra-linguistic facts. In addition, according to Carnap's views stated in this letter, for " $n_p = 9$ " holds that while the *extensions* of the expressions flanking the sign of identity are *identical*, their *intensions* are *different*. But if one turns to Carnap's papers (1956), (1958), (1959/2000) and (1961) written *after* he has already accomplished the shift to the semantics of extension and intension, one finds out that what Carnap investigates into are *not the intensions of theoretical terms of the language of science*, but the ways how to differentiate between the analytic and the synthetic components of the language  $L_T$  and how (via the Ramsey-sentences) to eliminate theoretical terms in favor of variables bound by existential quantifiers.<sup>9</sup>

Such a lack of any investigation into the intensions of the terms of the language of science leads to yet another negative consequence. Even if, as shown above in Figure 2 above, Carnap takes into account that there exist *terms of the language of science which are both the point of departure and the "end"-point* in the process of theory construction, he does not seem to even consider the following crucial question. *Does*

<sup>8</sup> "[...] an L-determinate intension is such that it conveys to us its extension" (Carnap 1947, §22, 88).

<sup>9</sup> Carnap's aim in his paper (1959/2000) is to define explicitly theoretical terms in a language that contains both Hilbert's  $\varepsilon$ -operator and the whole logic and mathematics.

*the intension of these terms change once they are transformed from the point of departure to the "end"-point of theory construction?*

What is behind such a possible change of meaning of terms was spelled out, at least at general level, by G. Schlesinger in his paper (1964). Let us suppose that  $M$  is a term which we should grant admission into our language of science. In order to do so, we should be able to construct a proposition  $S_M$  containing  $M$ , so that  $S_M$  entails an observation sentence  $S_O$  which has to be tested. If  $S_K$  stands for some other propositions,  $T$  for a theoretical system and  $C$  for correspondence rules, we should require no more than the following two requirements (Schlesinger 1964, 395 – 396):

- (A)  $S_O$  is logically implied by the conjunction of  $S_M$ ,  $S_K$ ,  $T$  and  $C$ .
- (B)  $S_O$  is not logically implied by the conjunction of  $S_K$ ,  $T$  and  $C$  alone.

Once  $S_O$  is confirmed then  $M$  acquires its meaning relative to  $T$  and  $C$  and becomes a member of the class of terms in our language. From this he draws the conclusion:<sup>10</sup>

that a word which is endowed with significance and is made to stand for a given concept may lose this significance upon the change of context and cease to stand for that concept. After it has lost its original significance it may assume some new signification and stand for another concept or it may not do so. In the latter case, it becomes an empty combination of letters. [...] On each occasion [... a term] is applied anew – in the context of the same theory but in different setting – one has to re-examine that collection of propositions which originally bestowed meaning on it and see whether it is still relevant to this situation. If it is not, one has to find out whether some other set of  $S_O$  is logically implied by the conjunction of  $T$ ,  $C$ ,  $S_K$  and  $S_O$  may not lend significance to it. If such a set is found then we have to probe into the question whether we ought to regard the concepts determined by the two sets as being essentially identical concepts and hence should employ the same term in both settings. We have to compare the two groups of  $T$ ,  $C$ ,  $S_K$  and  $S_O$  and use our judgment to decide whether there are sufficient connecting elements between them to render the use of the same term in both cases a reasonable and useful practice. [...] a con-

<sup>10</sup> We would like to thank Oxford University for the permission to quote from the paper of G. Schlesinger.

cept which belongs to a highly developed science is moulded by a very complex set consisting of an enormous number of T, C, S<sub>K</sub> and S<sub>O</sub> sentences. When a new sentence joins this set or an old leaves it the character of the concept is altered to some degree. Its complete loss of identity comes about by a gradual process. (Schlesinger 1964, 402, 404, 405)

In Part II of this paper we will analyze the change of meanings of the harmonic law of planetary motion inside Book I and Book III of the *Principia*.

The approach of the so-called “Standard Conception” to the method of theory construction can thus be characterized by the following three catch-phrases: *no meaning-change; one-directionality of thought-movement; movement from the non-observable to the observable.*

## 2. The Cyclical Method of Theory Construction

### 2.1 Definitions and Laws in the Principia

Newton begins the construction of the *Principia* by relying on notions like “time, space, place and motion,” which he notes “are very familiar to everyone” (1999, 408)<sup>11</sup> and then tries “to explain the senses in which less familiar words are to be taken in this treatise” (1999, 408). From the point of view of *order* in the process of theory construction, the sequence of concepts at the very beginning of the *Principia* is as follows: *space* and *time* (explicitly discussed in the scholium after the definitions), *velocity* and *acceleration* (not explicitly discussed), *inherent force* corresponding to inertia (Definition 3), *impressed force* (Definition 4), *centripetal force* (Definition 5), *accelerative measure of the centripetal force* (Definition 7).

Let us now compare definitions 4 and 8 with the respective laws (axioms)

<sup>11</sup> But even with respect to the former he makes the remark that “these quantities are popularly conceived solely with reference to the objects of sense perception. And this is the source of certain preconceptions; to eliminate them it is useful to distinguish these quantities into absolute and relative, true and apparent, mathematical and common” (1999, 408).

Definition 4	Law 1
<i>Impressed force is the action exerted on a body to change its state either of resting or of moving uniformly straight forward (1999, 405).</i>	<i>Every body preserves in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed (1999, 416)</i>
Definition 8	Law 2
<i>The motive quantity of force is the measure of this force that is proportional to the motion which it generates in a given time (1999, 407).</i>	<i>A change of motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed (1999, 416).</i>

What readily can be seen is the fact that each of these two laws is a converse of the respective definition, or, to be more precise, a special type of “converse.” In the definitions we identify (*discover*) the cause (impressed force on a body; centripetal force) by means of their effects (change of state of this body; the motive quantity generated proportional to the motion generated in a given time), that is to say, *we proceed in thought from the effects of forces to the very forces*. In both laws *we proceed from the forces* (their non-action on a body; motive force) *to their respective effects* (the preservation of the state of the body; change of motion in time). We view those two pairs of definition – law as a case of a *cyclical thought-movement* about which W. Harper states “Newton’s use of inferences from phenomena is a part of a process of theory construction that is like [...] an information feedback process” (1993, 156) and which Newton himself characterized in general physical terms in the 1687 preface of the *Principia* as follows: “the basic problem of philosophy seems to be to discover the forces of nature from the phenomena of motion and then to demonstrate the other phenomena from these forces” (1999, 382).

In Parts 2.2 and 2.3 we will use the phenomena of motion – forces of nature – phenomena of motion-type of theory construction as key to the analysis of Books I and III.<sup>12</sup> In such a way that we will try to

<sup>12</sup> It remains the task of a future investigation to find out if such a cyclical form of relation is given also between Book II of the *Principia* and its two other books, and if

find in it, *first*, thought-movements from certain types of phenomena-effects to their causes; *second*, thought-movements from those causes to the *same* types of phenomena, that is to say, from which the causes were initially derived; and, *third*, the thought-movements from causes to *other* phenomena, that is to say, to phenomena-effects *different* from those from which the causes were initially derived. Exempted from our reconstruction will be sections 4 through 6 of Book I because they represent an exercise in pure geometry.<sup>13</sup>

## 2.2 Book I of the Principia

### A. *From the Phenomena of Motion to the Forces of Nature*

Section 2<sup>14</sup> as a whole can be viewed as that part of the *Principia* where Newton accomplishes thought-movements from phenomena-effects to their causes; the whole section being labeled “*To find centripetal forces*” (1999, 444). In proposition 2 Newton states:

Every body that moves in some curved line described in a plane and, by a radius drawn to a point, either unmoving or moving uniformly forward with a rectilinear motion, describes areas proportional to the times, is urged by a centripetal force tending toward the same point.

(Newton 1999, 446)

By proving it he, thus, demonstrates that the effect-phenomenon of motion of a body according to the law of areas has as its cause the force acting in a centripetal manner on that body. Proposition 3 then generalizes proposition 2 to the case when the body *L* (the Moon) orbits a second body *T* (the Earth) under the impact of a centripetal force to that second body, so that both are subject to yet another force acting on them along parallel lines (e.g., from the Sun), proving that “the difference of the forces tends [...] toward the second body as a center” (1999, 448).

But, even if section 2 as a whole deals with finding centripetal forces from their effects, proposition 1 in this section is of a reversed or-

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such a cyclical form of relation can be discerned also inside Book II.

<sup>13</sup> This view on sections 4 through 6 of Book I was communicated to us in written correspondence by the late Professor J. B. Brackenridge.

<sup>14</sup> Section 1 provides with its “first and ultimate ratios” the mathematical apparatus for the *Principia*.

der. It states that “[t]he areas which bodies made to move in orbits describe by radii drawn to an unmoving center of forces lie in unmoving planes and are proportional to the times” (1999, 444), that is to say, that it moves from the centripetal force, acting on a body, to one of its effects: the character of areas described by the body moved by that force in orbits. This shows what we already have seen above in relation of Laws 1 and 2 to Definitions 4 and 8, namely, that Newton permanently accomplishes a *bi-directional thought-movement*: from the effects to their cause and the other way round.

Proposition 4 states: “*The centripetal forces of bodies that describe different circles with uniform motion tend toward the centers of those circles and are to one another as the squares of the arcs described in the same time divided by the radii of the circles*” (1999, 449). It is readily seen here that Newton determines here the ratio of forces acting on bodies moving uniformly in circular orbits via the characteristics of these orbits (arc and radius) or, as in corollaries 1 and 2 of the proposition 4, via the characteristics of the movement of these bodies (speed and period).

To get a better understanding of the method of determining the cause via its effects, let us go through the proof not of proposition 4 of the *Principia*, but of its corresponding to it second theorem in the manuscript *De Motu Corporum in Gyrum*,<sup>15</sup> which runs as follows: “The centripetal forces of bodies revolving uniformly in the circumferences of circles are as the squares of the arc described in the same time divided by the radii of the circles” (1965a, 278). The problem Newton faced was how to *measure* the centripetal force. The solution, upon which his thought-movement from an effect of a cause to the very cause is based, is as follows. Centripetal forces causes the deviation of bodies from their rectilinear motion and, thus, the former are proportional to the latter, or, “It is the centripetal forces that perpetually draw the bodies back from the tangent to the circumferences, and hence they are to each other as the distances [...] gained by their bodies” (1965a, 278). Based on this idea of how to measure the forces, the

<sup>15</sup> The correspondence of Theorem 2 to proposition 4 is, however, not complete. In the former Newton implicitly presupposes that the forces are centripetal; in the latter he provides a proof of that. A detailed comparison of *De Motu* with the *Principia* is given in Chapter 7 of Brackenridge (1995).

proof can be accomplished. We have two circles which can be represented as follows ( $S$  is the center of the smaller circle and  $s$  the center of the larger circle):

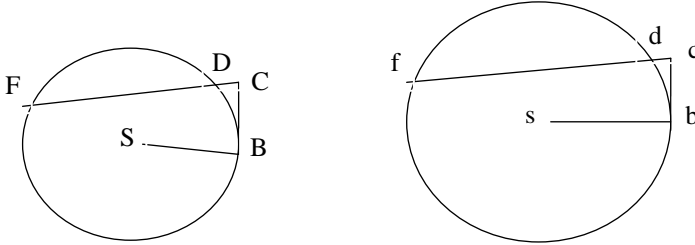


Fig. 3: Newton's measurement of forces by means of the deflection they cause

On each of them a body circulates, so that  $SB \neq sb$  and  $bd \neq BD$  holds. It can be proved<sup>16</sup> that ("×" stands for multiplication)  $cd \times cf = cb^2$  and  $CD \times CF = CB^2$  and, thus,  $CD = CB^2/CF$  and  $cd = cb^2/cf$  holds. So as the traversed arcs are very small,  $D$  is close to  $B$ ,  $d$  is close to  $b$ ,  $cf$  and  $CF$  are close to be the diameter of the respective circle, that is,  $2BS = CF$  and  $2bs = cf$  holds and therefore  $CD = DB^2/2BS$  and  $cd = db^2/2bs$  holds as well. Finally, one obtains the proportion between the forces  $f$  and  $F$  to their respective deflection  $cd$  and  $CD$  as  $f \propto db^2/b_s$  and  $F \propto DB^2/BS$  ("∝" stands here for "is directly proportional to"). From this result it is then possible to prove the respective corollaries in *De Motu* and the *Principia*. Because  $db = v \times t_1$  and  $DB = V \times t_2$  holds, where  $v, V$  are the respective speeds;  $t_1, t_2$  the time (and  $t_1 = t_2$ ), from the ratio  $F : f = DB^2/BS : db^2/b_s$  it follows  $F : f = V^2/BS : v^2/b_s$ , what is stated by corollary 1. Because for the periods  $t$  and  $T$  of the movement of the bodies along the whole circle it holds  $v \times t = 2\pi \times bs$  and  $V \times T = 2\pi \times BS$ , one obtains  $F : f = BS/T^2 : bs/t^2$  as stated by corollary 2. If  $T = t$ , then  $F : f = BS : bs$  as stated in corollary 3 of proposition 4, Book I, in the *Principia*. If the ratio on the right side of this equation is equal to one, then  $F = f$  as stated in corollary 3 of the *De Motu* and corollary 4 of the *Principia*. If the periods  $T$  and  $t$  are as the radii the ratio is  $F : f = bs : BS$  as stated by corollary 4 of *De Motu* and corollary 5 in the *Principia*.

<sup>16</sup> By means of proposition 36 (Book 3) of Euclid's *Elements*.



Corollary 6 (corollary 5 of *De Motu*) has a special place in proposition 4 of the *Principia* (theorem 2 of *De Motu*). Newton brings in here the relation between the period and the radius of the orbit typical for celestial bodies, namely, that the square of the periodic time is as the cube of the radius (i.e., what we today label as “Kepler’s third law”). He then proves that  $F : f = bs^2 : BS^2$ , that is to say, the centripetal force diminishes with the square of the distance.

Proposition 6 states the following:

If in a nonresisting space a body revolves in any orbit about an immobile center and describes any just-nascent arc in a minimally small time, and if the sagitta of the arc is understood to be drawn so as to bisect the chord and, when produced, to pass through the center of forces, the centripetal force in the middle of the arc will be as the sagitta directly and as the time twice inversely. (Newton 1999, 453 - 454)

In order to explain Newton’s proof, we partially draw upon the figure drawn not by Newton but by I. B. Cohen (1999, 319)<sup>17</sup> which is as follows

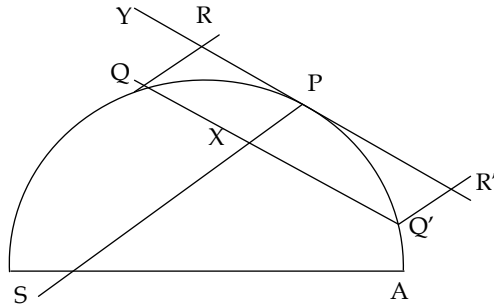


Fig. 4: Diagram for proposition 6 in Book I

Here  $XP$  is the sagitta (i.e., the line which when drawn bisects the chord and passes through the center  $S$  of force),  $P$  approaches  $Q$  and Newton proves<sup>18</sup> that  $F \propto PX / \Delta t^2$ . The “problematic” component in this expression is the magnitude of time. We are in need of an “Er-

<sup>17</sup> The figure used by Newton (1999, 454) contains neither the sagitta nor the chord.  
<sup>18</sup> The proof is based on corollary 4 of proposition 1 and corollaries 2 and 3 of Lemma 11.

satz"-measure for it. Newton, drawing upon proposition 1, uses what we today call Kepler's area law, and substitutes for  $\Delta t$  the area of the sector swept out by the line  $SP$ . The surface of the triangle  $SPQ$  can serve as the measure of  $\Delta t$ . If  $QT$  stands for the perpendicular dropped from  $Q$  on  $SP$  and  $SPR$  can, under the "nascent" considerations, be viewed as a triangle whose surface is given by  $\frac{1}{2}(SP \times QT)$ , then we end up with the proportion<sup>19</sup>  $F \propto QR/(SP^2 \times QT^2)$ .

Proposition 53 is also symptomatic for Newton's derivation of the cause by means of one of its effects. It states "*Granting the quadratures of curvilinear figures, it is required to find the forces by whose action bodies moving in given curved lines will make oscillations that are always isochronous*" (1999, 556). The pendulum with cycloidal cheeks can be represented by the following figure (the point  $T$  stands for the oscillating body,  $STRQ$  is the curved line in which the body oscillates,  $AR$  is the axis of  $STRQ$  which passes through the center of force  $C$ ):

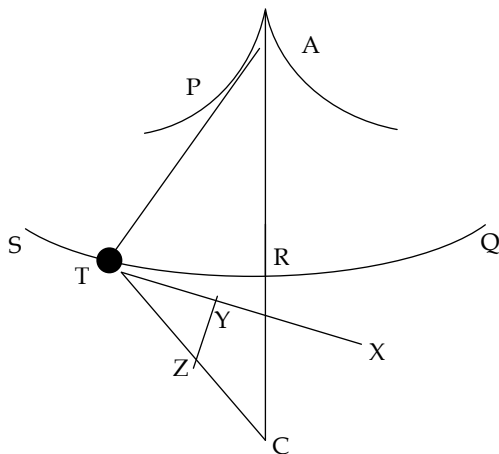


Fig. 5: Finding forces causing isochronous oscillations of bodies

Newton takes the line  $TX$  which is tangential to the path  $STRQ$  at  $T$  because this is the only direction, at the instantaneous position of the

<sup>19</sup>  $PX=QR$  and  $\frac{1}{2}$  can be omitted because we deal here not with equations but only with proportions.

body, in which the attracting force can *cause* a change of its speed, i.e., accelerate or decelerate it in the curve *STRQ*. He uses the path-effect *TC* of the action of the force of attraction on the bob and decomposes it into the components *TY* and *YZ*. The former is tangential to the path, and thus is *the* path-effect of the cause of the just mentioned possible changes of speed of the body, while the latter is the stretching-effect of the cord under the impact of the attracting force which, under the idealization that the length of the cord does not change under the impact of forces, can be neglected because it does not change the speed of the body. And he puts *TY* equal to the arc *TR* in order to claim that since force represented by *TR*:<sup>20</sup>

is as the projection *TR* to be described, the body's accelerations or retardations in describing proportional parts of two oscillations (a greater and a lesser oscillation) will always be as those parts and will therefore cause those parts to be described simultaneously. And bodies that in the same time describe parts always proportional to the wholes will describe the wholes simultaneously. (Newton 1999, 556 - 557)

### B. *From the Forces of Nature to the Same Phenomena of Motion*

It is worth noting that even in section 2, which as a whole deals with the discovery of forces from their respective effects, there are several places where Newton reverses the order of his thought-movements. The first instance is found in Newton's differentiation between proposition 1 and proposition 2 given above in 2.2.A. and the second instance appears in corollaries 3 through 6 of proposition 4 in the *Principia*, where he adds at the very end the phrase "and conversely," (1999, 451) which he utilizes to imply *once we are able to determine that the ratio of forces is such and such, then we can determine as well the specific ratio of the characteristics of movement of the bodies upon which these forces act*. The third instance of this reversal appears in the scholium to proposition 8, where he states: "a body will be found to move in an ellipse, or even in a hyperbola or a parabola, under the action of a cen-

<sup>20</sup> Professor Curtis A. Wilson *via* a written correspondence was very helpful in this reconstruction of Newton's ideas in proposition 35 of Book I.

tripetal force that is inversely as the cube of the ordinate tending toward an extremely distant center of forces" (1999, 458).

This scholium is the converse of proposition 8, where from the movement of a body in a semicircle and the "fact" that the center of force acting on it is very distant, he derives the distance-dependence of the force.

Finally, in corollary 1 of proposition 10 he states not only that "the force is as the distance of the body from the center of the ellipse," but also "and conversely, if the force is as the distance, the body will move in an ellipse having its center in the center of forces" (1999, 460).

A quite profound reversal, which we will discuss in all its details in Part II, is accomplished by Newton in Book I starting from corollary 1 of proposition 13 and ending in proposition 17. While in propositions 11 through 13 he moved *from the conic character of the trajectory of the body to the force producing it*, corollary 1 states:

From the last three propositions it follows that if any body P departs from the place P along any straight line PR with any velocity whatever and is at the same time acted upon by a centripetal force that is inversely proportional to the square of the distance of places from the center, this body will move in some one of the conics having a focus in the center of forces; and conversely. (Newton 1999, 467)

A similar movement from forces acting on bodies to the trajectory of these bodies is accomplished by Newton also in corollary 2 of proposition 13. Newton claims that<sup>21</sup>

If the velocity with which the body departs from its place P is such that the line element PR can be described by it in some minimally small particle of time, and if the centripetal force is able to move the same body through space QR in the same time, this body will move in some conic whose principal latus rectum is the quantity  $QT^2/QR$  [...] (Newton 1999, 467)

In proposition 14 Newton presupposes the existence of several (mutually non-interacting) bodies which orbit about the same center, and then goes on as follows: "If [...] the centripetal force is inversely as the

<sup>21</sup> The line element  $PR$  and the space  $QR$  are displayed in Figure 2. The latus rectum  $L$  of a conic is the chord perpendicular to the principal axes which passes through a focus of the conic, so that if  $AB$  is the major diameter and  $PD$  its minor diameter, it holds  $L=PD^2/AB$ .

square of the distance of places from the center, I say that the principal latera recta of the orbits are as the square of the areas which the bodies describe in the same time by radii drawn to the center" (1999, 467).

Proposition 15, presupposing the same like proposition 14, states that "the squares of the periodic times in ellipses are as the cubes of the major axis" (1999, 468). So while Kepler's third (or harmonic) law was in proposition 4 *presupposed* (for circular trajectories) and enabled to derive the character of the force, in proposition 15 it becomes a *consequence* (for elliptical orbits) by presupposing that  $F \propto 1/SP^2$ .

Proposition 17 concluding section 3 states:

Supposing that the centripetal force is inversely proportional to the square of the distance from the center and that the absolute quantity of this force is known, it is required to find the line which a body describes when going forth from a given place with a given velocity along a given straight line.

(Newton 1999, 470)

Newton then proves that the orbit can be either an ellipse, or a parabola, or a hyperbola (1999, 471).

Finally, in corollary 1 of proposition 53 Newton, drawing upon proposition 53 (reconstructed above in 2.2.A), proves that a pendulum under the action of a uniform force of gravity will have the period of all its oscillations equal. On the basis of this knowledge Newton explains in corollary 2 how to construct pendulum clocks with isochronous oscillations.

### C. From the Forces of Nature to New (Different) Phenomena

The movement from the forces of nature to types of phenomena, were the latter never before made their appearance in the *Principia*, can be identified for the first time in section 2, namely, in the scholium to proposition 10. It states: "If the center of the ellipse goes off to infinity, so that the ellipse turns into a parabola, the body will move in this parabola, and the force, now tending toward an infinitely distant center, will prove to be uniform. This is Galileo's theorem" (1999, 460).

One can view this scholium as a direct consequence of the "converse" section of corollary 1 of proposition 10, where Newton states that "if the force is as the distance, the body will move in an ellipse"

(1999, 460). So, if the center of force is, in respect to the body on which it the force acts, very distant, this body will be subject to a constant force and will move in a parabolic trajectory, e.g., like a projectile.

From section 3 we view proposition 16 as a representative of the force – other phenomena type of thought movement, because it explains how to find a previously unknown characteristic of the moving body – its velocity – and where its prove explicitly draws upon proposition 14 which starts/proceeds from the inverse-square character of the centripetal force.

Section 7 considers also a previously unanalyzed (i.e., a new) phenomenon, namely, the “*rectilinear descent and ascent of bodies*” (1999, 518). Starting (implicitly) from proposition 17, where the conic orbit is understood as an effect of the action of an inverse-square centripetal force, Newton states the problem to be solved in proposition 32 as follows: “*Given a centripetal force inversely proportional to the square of the distance of places from its center, to determine the spaces which a body in falling straight down describes in given times*” (1998, 518).

In the solution Newton, first, supposes that the body does not fall perpendicularly. By drawing, explicitly, on corollary 1 of proposition 13, he views its trajectory as a conic and, thus, considers three possible cases of the fall of a body along an orbit: ellipse (case 1); hyperbola (case 2); parabola (case 3).

Propositions 33 through 38, drawing upon the results from proposition 32, then deal with various aspects of the descent and ascent of bodies (their velocity, fall along a parabola, time of descent from a certain point, time of ascent/descent of a projected body). In proposition 39, concluding section 7, Newton presupposes the action of a centripetal force of any kind (not only an inverse-square one) and then determines the time to reach a place and the speed of a body in any place it reaches when ascending/descending straight up/down.

Section 8 with its propositions 40 through 42 has as its aim “[t]o find orbits in which bodies revolve when acted upon by any centripetal force,” (1999, 528) that is to say, which is not an inverse-square one, but in general some unspecified function of distance. For example proposition 41 states: “*Supposing a centripetal force of any kind and granting the quadratures of curvilinear figures, it is required to find the trajecto-*

ries in which bodies will move and also the times of their motions in the trajectories so found" (1999, 529).<sup>22</sup>

From the point of view of the cyclical method of theory construction the structure of Book I of the *Principia* can be represented in a concise way by means of the following table listing the respective propositions in this book:

Type of thought-movement in the <i>Principia</i>	From the phenomena of motion to the forces of nature	From the forces of nature to the same phenomena of motion	From the forces of nature to new phenomena of motion
<b>Book I</b>	prop. 2 through 13, propos. 43, prop. 44, prop. 53	prop. 1, corollaries 3 through 6 of prop. 4, scholium to prop. 8, corollary 1 to prop. 10, corollary 1 to prop. 13, prop. 14, prop. 15, prop. 17, corollary 1 to prop. 53	scholium to prop. 10, prop. 16, prop. 32 through 42, prop. 54 through 69

Fig. 6: The structure of Book I of the *Principia* from the point of view of its method of construction

### 2.3 Book III of the Principia

In Book III of the *Principia* it is possible, like in Book I, to discern the phenomena of motion – forces of nature – phenomena of motion type of thought-movement. In fact Newton himself in the introduction to the 1687 edition of the *Principia*, after mentioning this/that type of thought-movement, states that “in Book 3 our explanation of the system of the world illustrates these propositions. For in Book 3 [...] we derive from celestial phenomena the gravitational forces by which bodies tend toward the sun and toward the individual planets. Then the motions of the planets, the comets, the moon, and the sea are deduced from these forces” (1999, 382).

<sup>22</sup> I.B. Cohen provides in (1999) a detailed proof of this proposition.

### A. *From the Phenomena of Motion to the Forces of Nature*

Newton starts Book III by stating four rules of reasoning and six phenomena, where the latter pertain to the “Keplerian”<sup>23</sup> characteristics of the primary planets circling the Sun and of the satellites of Jupiter and Saturn. The first rule states that “[n]o more causes of natural things should be admitted than are both true and necessary to explain their phenomena,” (1999, 794) while the second rule states that “the causes assigned to natural effects of the same kind must be, so far as possible, the same” (1999, 795).

From those/these phenomena and rules, together with the knowledge obtained/derived in Book I, Newton then starts his thought-movement to the forces that cause those “Keplerian” characteristics. Proposition 1 states:

The forces by which the circumjovial planets are continuously drawn away from rectilinear motions and are maintained in their respective orbits are directed to the center of Jupiter and are inversely as the squares of the distances of their places from that center. (Newton 1999, 802)

The first part of this proposition – that there is a force directed to the center of Jupiter – is proved on the basis of proposition 2 or 3 (Book I), because for the satellites of Jupiter holds, by phenomenon 1, the area law. The second part of the proposition, that that force is an inverse-square one, is proved by corollary 6 of proposition 4 (Book I) because, by the second part of phenomenon 1, Kepler’s third law holds for the satellites of Jupiter. Newton then adds, on the basis of both parts of phenomenon 2, that “[t]he same is to be understood for the planets that are Saturn’s companions” (1999, 803).

Proposition 2 states the same for the planets, while the proof follows the same pattern and draws upon the phenomenon 5 (the area law for the planets) and phenomenon 4 (the harmonic law for the planets). Newton proves its second part also by considering corollary 1 of proposition 45 (Book I), namely, that “the slightest departure from the ratio of the square would [...] necessary result in a noticeable motion of the apsides in a single revolution and an immense such mo-

<sup>23</sup> Newton mentions in the phenomena 1 through 6 what we today label as “Kepler’s second” and “Kepler’s third law”, but not what we today label as “Kepler’s first law.”



tion in many revolution" (1999, 802). But because the aphelia of the planets are at rest, the centripetal force tending toward the Sun is really an inverse-square one.

Proposition 3 states for the Moon what proposition 1 and 2 stated for satellites Jupiter, Saturn, and for the planets. Its first part is proved in the same way as are the proofs performed for the first parts of propositions 1 and 2 (by drawing upon the area law for the moon stated in phenomenon 6). But the proof of its second part follows another path, simply because the moon was in the time of Newton the only satellite of the Earth and, thus, Newton could not apply Kepler's area law to the system Earth – Moon. He draws, instead, again – as in proposition 2 – on corollary 1 of proposition 45 (Book I). Even if the apogee of the moon is moving, it is still "[...] very slow [...]" and "this motion of the apogee arises from the action of the sun [...] and accordingly is to be ignored here," so that "the remaining force by which the moon is maintained in its orbit will be inversely as  $D^2$ " (1999, 802 – 803), where  $D$  expresses the distance of the Moon from the center of the Earth, while the semidiameter of the Earth is put equal to 1.

Proposition 4 states that "[t]he moon gravitates toward the earth and by the force of gravity is always drawn back from the rectilinear motion and kept in its orbit" (1999, 803). Newton proves it by means of the well known first<sup>24</sup> moon-test,<sup>25</sup> and then claims that "that force by which the moon is kept in its orbit, in descending from the moon's orbit to the surface of the earth, comes out equal to the force of gravity here on earth, and so (by rules 1 and 2) is that very force which we generally call gravity" (1999, 804).

Proposition 5 then states that the satellites of Jupiter, Saturn, and the planets orbiting the sun "gravitate" to the respective celestial object because the revolutions

are phenomena of the same kind as the revolution of the moon about the earth, and therefore (by rule 2) depend on causes of the same kind, especially since it has been proved that the forces on which those revolutions

<sup>24</sup> The second moon-test is performed in corollary 7 of proposition 37.

<sup>25</sup> On the moon-test see S. Aoki's paper (1992). Newton performs in the scholium of proposition 4 a different proof by considering an imagined second satellite orbiting the earth (1999, 805).

depend are directed toward the centers of Jupiter, Saturn and the sun, and decrease according to the same ratio and law (in receding from Jupiter, Saturn, and the sun) as the force of gravity (in receding from earth).

(Newton 1999, 806)

In the scholium of this proposition Newton then performs an important shift from the *concept of centripetal force* to that of *force of gravity*, “[f]or the cause of the centripetal force by which the moon is kept in its orbit ought to be extended to all the planets [...]” (1999, 806).

Proposition 25 sets as its aim “[t]o find the forces of the sun that perturb the motions of the moon” (1999, 839). This aim is achieved (by drawing upon proposition 66 in Book I) via a decomposition of the effect of the force of gravity which perturbs the moon. He represents that force of sun by lines and views them as representing “accelerative gravity,” (1999, 840) that is to say, he is able to deal with the perturbing force of the sun by representing it via its effect.

Proposition 36, then, as a continuation of proposition 25, sets as its aim “[t]o find the force of the sun to move the sea,” (1999, 874) while proposition 37’s aim is “[t]o find the force of the moon to move the sea” (1999, 875). It is, in respect to the latter worth to be noted that Newton performs here his computation of the size/quantity of forces by means of the size/quantity of their effects. He states that “[t]he force of the moon to move the sea is to be reckoned from its proportion to the force of the sun, and this proportion is to be determined from the proportion of the motions of the sea that arises from these forces” (1999, 875).

### B. *From the Forces of Nature to the Same Phenomena of Motion*

The first and only proposition which is the result of the thought-movement forces of nature – phenomena of motion in Book III is proposition 13. It states “*The planets move in ellipses that have a focus in the center of the sun, and by radii drawn to that center they describe areas proportional to the times*” (1999, 817). This proposition is viewed by Newton himself as a result of such a type of movement. He claims

We have already discussed these motions from the phenomena. Now that the principles of motions have been found, we deduce the celestial motions from these principles a priori. Since the weights of the planets toward the sun are inversely as the squares of the distances from the center

of the sun, it follows (from book 1, prop. 1, and prop. 13, corol. 1) that if the sun were at rest and the remaining planets did not act upon one another, their orbits would be elliptical, having the sun in their common focus, and they would describe areas proportional to the times.

(Newton 1999, 817 – 818)

### C. *From the Forces of Nature to New (Different) Phenomena*

The first proposition belonging to the forces of nature – new (other) phenomena type of thought-movement is proposition 6. It states that “[a]ll bodies gravitate toward each of the planets, and at any given distance from the center of any one planet the weight of any body whatever toward the planet is proportional to the quantity of matter which the body contains” (1999, 806). Newton’s proof consists in fact of five different proofs. We will analyze only the first one to show that proposition 6 really belongs to that type of thought-movement, but that it differs from those analyzed in Part 2.2.C above as well as from the type of thought-movement in propositions 8 through 39 in Book III, which we will analyze below.

The first proof is *theoretically* based on proposition 24 of Book II which states that for a pair of pendulums “whose centers of oscillation are equally distant from the center of suspension, the quantities of matter are in a ratio compounded of the ratio of the weights and the squared ratio of the times of oscillation in a vacuum,” (1999, 700) from which he then derives corollary 1 stating “[a]nd thus if the times are equal, the quantities of matter in the bodies will be as their weights,” as well corollary 6 which runs as follows:

But in a nonresisting medium also, the quantity of matter in the bob of a simple pendulum is as the relative weight and the square of the time directly and the length of the pendulum inversely. For the relative weight is the motive force of a body in any heavy medium [...] and thus fulfills the same function in such a nonresisting medium as absolute weight does in a vacuum.

(Newton 1999, 701)

If one looks into the structure of proposition 24 of Book II, it is readily seen that it is based on the following chain of thoughts:

- i) velocity generated by a force in a given time (i.e., acceleration) in a given quantity of matter is proportional to the force and time and

- inversely as that quantity of matter; “this is manifest from the second law of motion” (1999, 700).
- ii) For two pendulums of the same length, the motive forces in places of deflections equally distant from the perpendicular are as the weights.
  - iii) If two oscillating bodies describe equal arcs and if the arcs are divided into equal parts, then the velocities in corresponding parts of oscillation will be to one another as the motive forces and the whole times of the oscillations directly and the quantities of matter inversely.
  - iv) Therefore, the quantities of matter will be as the forces and the times of the oscillations directly and the velocities inversely. But the velocities are inversely as the times, so, the quantities of matter are as the motive force and the square of the times, that is, “as the weights and the square of the times” (1999, 700).

From steps i) through iv) it is readily seen that Newton draws here upon the second law, which – as shown above – is just the converse of definition 7, and where the latter derives force from its effect. This is also apparent from the fact that Newton substantiates the concept of motive force by means of the concept of weight, which is the type of force theoretically assigned to acceleration. From this we can readily draw the conclusion that after Newton has accomplished his thought-movement from space and time, velocity and acceleration, accelerative measure of force-weight, he can theoretically deal with the motive quantity of force (definition 8), then move to the second law, and then, via proposition 24 and its corollaries in Book II, finally arrive at proposition 6.

What has to be emphasized here is, *first*, that proposition 6 belongs to that part of the *Principia*, where Newton, *after he initially moved from phenomena of motion to the forces causing them, moves from the forces to other (new) phenomena* and, *second*, that the thought-movement from the concept of force as weight of a body to the discovery of its proportionality to the mass of this body is *not* the discovery of a new phenomenon as an *effect* of this force. In the conceptual hierarchy of the *Principia* mass (quantity of matter) of a body on which the force of gravity acts is not an effect of the latter. We only discover, via that thought-movement from the phenomenon of motion to the forces causing them by acting on that body, what that force (also) depends

on from the “side” of that very body. Mass, as stated by I. B. Cohen, is in the *Principia* a *primary quantity* (1999, 92), but which we are able to discover as a magnitude in the framework of the *Principia*, only via the derived concept of force, so that the concept of mass is that framework also a derived concept.

Proposition 7 makes the universal claim that “gravity exists in all bodies universally and is proportional to the quantity of matter in each” (1999, 810). In contrast to proposition 6, which dealt with the force of gravity acting on a body, here the force of gravity is considered as having its source in a body and as being related to that body’s mass. Newton proves this proposition by drawing upon the claim from proposition 69 of Book I reconstructed above in 2.2.C which states that “*the absolute forces of attracting bodies A and B will be to each other in the same ratio as the bodies A and B themselves to which these forces belong*” (1999, 587). And it is precisely because the force labeled in Book I by Newton as the “force of attraction” is labeled in Book III as the “force of gravity,” that he can state proposition 7.

Thus, for proposition 7 holds what was stated already for proposition 6. Accordingly, after accomplishing the thought-movement from phenomena-effects of motion in definitions 7 and 8, Newton can, via the third law – which expresses the mutual ratio of the forces of two bodies acting on each other via the ratio of the effect each of them has on the other – derive the proposition stating the relation between the (absolute) force belonging to a body and the mass of this body.

Propositions 8 through 42 of Book III contain Newton’s reflections on various phenomena not previously discussed or used in the derivation of the forces. These phenomena are characteristics of planets (their masses, sizes, shapes, etc.), the motion of the moon, the motion of comets, and the phenomena of tides of the sea.

Proposition 8 states that

If two globes gravitate toward each other, and their matter is homogeneous on all sides in regions that are equally distant from their centers, then the weight of either globe toward the other will be inversely as the square of the distance between the centers. (Newton 1999, 811)

The fact that Book III of the *Principia* is built upon the point of view of the cyclical movement between the phenomena of motion and their corresponding forces can be expressed in a concise way as follows:

Type of thought-movement in the <i>Principia</i>	From the phenomena of motion to the forces of nature	From the forces of nature to the same phenomena of motion	From the forces of nature to new phenomena of motion
<b>Book III</b>	prop. 1 through 5, prop. 25, prop. 36	13	prop. 6 through 24, prop. 26 through 35, prop. 37 through 42

Fig. 7: The structure of Book III from the point of view of its method of construction

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