Logic, the *A Priori*, and the Empirical

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ABSTRACT: The time-honored view that logic is a non-empirical enterprise is still widely accepted, but it is not always recognized that there are (at least) two distinct ways in which this view can be made precise. One way focuses on the knowledge we can have of logical matters, the other on the nature of the logical consequence relation itself. More specifically, the first way embodies the claim that knowledge of whether the logical consequence relation holds in a particular case is knowledge that can be had a priori (if at all). The second way presupposes a distinction between structural and non-structural properties and relations, and it holds that logical consequence is to be defined exclusively in terms of the former. It is shown that the two ways are not coextensive by giving an example of a logic that is non-empirical in the second way but not in the first.

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In "Logical Consequence and Logical Expressions" (2003) Mario Gómez-Torrente discusses many central issues in the philosophy of logic in an elegant, knowledgeable, and sensible manner. At the beginning of his paper Gómez-Torrente notes the importance of form and modality for the notion of logical consequence, and he asserts that a necessary condition for the conclusion of an argument to follow from the premises is that it "*follows by logical necessity from those premises*". He further asserts that this notion of following by logical necessity is "associated (in ways which are themselves unclear and vague) to unclear and vague notions like analytical implication, *a priori* implication and implication by necessity (*tout court*)". I agree that these notions and the connections between them, while important for logic, are nevertheless unclear and vague. Yet I think it is possible to make some headway in understanding them and the ways in which they are interrelated. Elsewhere I have discussed the first and the third of the notions Gómez-Torrente mentions. In the present paper I concentrate largely on issues related to the second[1].

The suggestion that something called '*a priori* implication' is part of the notion of following by logical necessity, and hence part of the notion of logical consequence itself, carries with it the suggestion that logic is fundamentally a non-empirical enterprise. I agree with Gómez-Torrente that this is indeed the case and that the competing view, the view that logic is in some sense empirical, is mistaken[2]. But the mere claim that logic is not empirical is itself unclear and vague. I suggest that a good way to begin improving our understanding of this claim is to reflect on a familiar passage from Tarski's classic paper on logical consequence. In (1936, 414-15) Tarski says:

> Certain considerations of an intuitive nature will form our starting-point. Consider any class *K* of sentences and a sentence *X* which follows from the sentences of this class. From an intuitive standpoint it can never

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[1] See my (2002b) for a discussion of the connections between analytic implication, necessity *tout court*, and logical consequence. I have previously discussed *a priori* implication in (1997) and (2002a). Some of the points made in the present paper are also made in (2002a) in the course of clarifying the position I took in (1997) and rebutting criticisms of it made by Sher (2001).

[2] Such a view was held by John Stuart Mill. A more recent, and much discussed, version of this view can be found in Putnam (1968).

happen that both the class *K* consists only of true sentences and the sentence *X* is false. Moreover, since we are concerned here with the concept of logical, i.e., *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects to which the sentence *X* or the sentences of the class *K* refer. The consequence relation cannot be affected by replacing the designations of the objects referred to in these sentences by the designations of any other objects.

The last part of the penultimate sentence is particularly germane to the present investigation: "[the logical consequence] relation cannot be influenced in any way by empirical *knowledge*, and in particular by *knowledge* of the objects to which the sentence *X* or the sentences of the class *K* refer" (emphasis added). Although this is somewhat obscure, one reading of it, for which I have argued elsewhere (1997, 2002a), is the following:

Logic is Not Empirical, Sense 1 (LNE1): Knowledge of whether the logical consequence relation holds in any particular case is knowledge that can be had *a priori*, if at all.

LNE1 is not just a criterion that can with some plausibility be extracted from Tarski's paper on logical consequence; it also reflects a view that has been widely held for a long time. Logic has long been held to be free, in some fundamental way, of all things empirical, and I believe many logicians have thought that logic achieves this freedom by satisfying LNE1 (or a similar standard). Thus I suggest that LNE1 embodies a plausible way of understanding Gómez-Torrente's *a priori* implication.

Still LNE1 is not the only criterion that can be extracted, with some plausibility, from the passage recently quoted from Tarski. Consider again a (slightly larger) part of that passage, but this time with different emphasis: "[the logical consequence] relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the *objects* to which the sentence *X* or the sentences of the class *K* refer. The consequence relation cannot be affected by replacing the designations of the *objects* referred to in these sentences by the designations of any other *objects*" (emphasis added). This reading suggests a different way for logic to be free of matters empirical, one that has been championed by a number of philosophers of logic, including Gila Sher (1991, 1996, 2001). Sher holds that "objects and systems of objects have, in addition to physical, biological, sociological, and many other kinds of properties, also formal or structural properties", and that it is "the task of logic … to construct a theory of the transmission (preservation) of truth based on formal or structural grounds" (2001, 245-46). This formal-structural approach, as Sher calls it, thus requires "the indifference of logical consequence to empirical properties of individuals in a given universe" (2001, 258)[3].

I think it can be plausibly argued that taking indifference to empirical properties as a *desideratum* for logic is supported by the last two sentences in the passage recently quoted from Tarski. In (2002a) I expressed this view of logic as follows:

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[3] For details on the formal-structural approach see Sher (1991, 1996). Sher's account is based on work by Lindstrom (1969, 1974) that extends Mostowski's (1957) idea of cardinality quantifiers. It is similar to the view advocated by Tarski in (1986).

Logic is Not Empirical, Sense 2 (LNE2): The logical consequence relation is defined exclusively in terms of the formal properties of (and formal relations among) objects. Logic is blind to empirical, nonstructural properties and relations[4].

LNE2 is a way for logic to be free of the empirical that is very different than LNE1, but the latter is more plausible as an explication of Gómez-Torrente's *a priori* implication than the former. This is because LNE1 deals directly with the knowledge we can have of the logical consequence relation, and it sets a specific criterion of apriority for that knowledge. LNE2, on the other hand, deals with two different kinds of properties and relations of things, those that are empirical (e.g., physical, biological, sociological), and those that are formal or structural, and it has no obvious connection with *a priori* knowledge.

I suppose it might be suggested that these latter kinds of properties and relations, being in some sense the farthest removed from direct experience, could be labeled '*a priori*', and that an account of logical consequence satisfying LNE2 could thus be said to exhibit *a priori* implication. But this seems like an unacceptable distortion of the meaning of the widely used, though vague, philosophical term '*a priori*'. So I prefer to understand *a priori* implication as a kind of implication that satisfies LNE1.

There are thus (at least) two distinct ways in which it might be claimed that the logical consequence relation is free of the empirical. One claim would be that logical consequence, properly understood, incorporates *a priori* implication by satisfying LNE1, the other that it satisfies LNE2 and thus embodies Sher's formal-structural approach. It is significant, moreover, that the difference between LNE1 and LNE2 is not just one of sense, it is also one of reference. For it can be proved that LNE1 and LNE2 are not coextensive. The proof is contained in the following paragraphs, which begin with an account of the formal-structural approach.

In (1991, 1996) Sher develops the formal-structural view of logical consequence (hereafter FS) as a generalization of standard first-order logic that specifies precisely what it is for a term to be logical. The core idea is that a logical term is either a sentential connective that refers (rigidly, on Sher's preferred account) to a truth-functional operator or a term that refers (rigidly) to an operator that is *formal* in the sense of being "indifferent to all 1-1 replacements of individuals, both within and across universes", as she puts it in (2001, 247)[5]. Call this criterion of logicality the *FS criterion*. Examples of terms that satisfy the FS criterion are the usual logical constants of first-order logic (e.g., 'and', 'or', 'not', 'if-then', in their truth-functional senses, the binary

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4 Sher's contrastive use the terms 'empirical' and 'structural' strongly suggests that for her the terms 'empirical property' and 'nonstructural property' are coextensive. My use of the latter two terms also assumes this coextensiveness. Of course this entire approach assumes that a sharp distinction can be drawn between structural and nonstructural properties, but we shall see that this can indeed be done.

5 For a complete presentation of Sher's account of formal operators see (1991, especially 54-56) or (2001, section 1). A similar, yet significantly different, account of logical terms (one based on invariance over homomorphic rather than isomorphic structures) is presented and discussed by Feferman (1999).

predicate 'is identical with', the quantifiers 'some', and 'all'), and the quantifiers 'finitely many', 'uncountably many', 'most', and 'as many as not'.

The FS criterion classifies as logical many terms that are not ordinarily so classified, and the FS approach declares that it is appropriate to treat any such term as a logical term in the formal development of a logic. Let's call a language and its accompanying semantics (including the definitions of such basic logical notions as logical truth and logical consequence) an *FS logic* if it treats only terms satisfying the FS criterion as logical. So standard first-order logic is an FS logic, but there are many other FS logics as well. The FS approach is thus applicable to many different languages, subject only to the constraint on logical terms imposed by the FS criterion.

Aside from treating as logical many terms that are not ordinarily so treated, the semantics of FS logics is otherwise quite standard. It is based on *models* in the sense now common in logic, a model consisting of a nonempty set D (the domain) and a function F (the assignment function) that assigns to each nonlogical term of a specified language an element of D or a set-theoretic construct of elements of D. Truth in a model for a given FS logic is defined by means of a recursive definition that contains a clause for each logical term of the language. Finally, the definitions of *validity* (logical truth), *consistency*, and *logical consequence* are the familiar ones of standard model-theoretic semantics.

It is not difficult to show that there are FS logics (i.e., logics that satisfy LNE2) that violate LNE1. This is because there are FS logics containing arguments we can know to be valid or invalid *a posteriori* but not *a priori*. As an example consider an FS logic that contains all the usual logical constants of first-order logic plus the quantifier 'Q*', which is defined as behaving exactly like 'all' in models with domains of cardinality $\geq n$, but like 'at least one' in models with domains of cardinality $< n$, where the value of n is an integer we can know a posteriori but not a priori[6]. To make matters more specific, we can take n (as I did in (1997)) to be the least number of whole seconds in which, up through the end of the twenty-first century, a human runs a mile. Now consider the following argument:

$(Q^*x)(Dog(x) \rightarrow Black(x))$

$(Q^*x)Dog(x)$

$\therefore (Q^*x)Black(x)$

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[6] I take quantifiers to be linguistic expressions, such as 'all', 'some', '∀', '∃' (the latter two being read 'all', and 'some') that designate certain operators. In order to satisfy Sher's criterion for being a logical constant, the quantifier 'Q*' must be understood as an expression that rigidly designates the operator described in the text. This can be accomplished by reading 'Q*' as something like 'All, if the number of individuals $\geq$ dthat(n), otherwise at least one', where 'dthat' is the rigidifying functor of Kaplan (1979). (Compare 'Q*' with Gómez-Torrente's '∃', (2003 section 5).)

Since we know that $n \geq 3$, we know the argument is invalid, but we can't know this a priori. Yet 'Q*' counts as a logical term according to FS, so FS violates LNE1[7],[8].

The foregoing example shows that LNE1 and LNE2 are not coextensive criteria, since there are logics that satisfy the latter but not the former. This should not be surprising, since LNE2 sets a standard for the logical consequence relation itself, while LNE1 sets a standard for our knowledge of that relation. Yet these standards have sometimes been conflated, perhaps because each can be described, in general terms, as requiring that logic be non-empirical[9]. It is thus important to realize that this general requirement can be made precise in (at least) two different and demonstrably non-coextensive ways. The argument containing 'Q*' given above is logically correct in one non-empirical sense of logical correctness (because it satisfies LNE2), but it is logically incorrect in another non-empirical sense of logical correctness (because it fails to satisfy LNE1). There is of course no reason why the logical consequence relation could not be required to satisfy both LNE1 and LNE2, and one who is strongly committed to the non-empirical nature of logic may want to adopt such a position[10].

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[7] To see that the operator expressed by 'Q*' satisfies Sher's criterion for formal operators it is sufficient to see how that criterion applies to unary first-order quantifiers. Under Sher's account such quantifiers are linguistic expressions that (rigidly) designate functions from models to subsets of the power set of a model's domain. (For example, the quantifier 'at least one' designates the function that takes each model to the set of all nonempty subsets of the domain of that model, the quantifier 'all' designates the function that takes a model to the unit set of its domain, the quantifier 'finitely many' designates the function that takes a model to the set of all finite subsets of its domain, etc.) A quantified sentence is true in a model just in case the extension of the open sentence that results from deleting the quantifier is an element of the set that is the value of the function designated by the quantifier when that model is taken as argument. Suppose now that M_1 and M_2 are (not necessarily distinct) models, A_1 and A_2 their domains, and B_1 and B_2 subsets of these domains, respectively. Consider the structures $<A_1, B_1>$ and $<A_2, B_2>$. These structures are isomorphic if there is a one-one mapping from A_1 onto A_2 that is also one-one from B_1 onto B_2. According to Sher's criterion, the function designated by a quantifier is a *formal* operator if, for any such pair of models and isomorphic structures, B_1 is an element of the value of the function (when M_1 is taken as its argument) if and only if B_2 is an element of the value of the function (when M_2 is taken as its argument).

   It is easy to see that the operators expressed by 'all', 'at least one', and 'finitely many' are formal in this sense. It is also clear that the operator expressed by 'Q*' counts as a unary first-order quantifier and that it satisfies Sher's formality criterion. For, given any two models with domains of the same cardinality, that operator functions either as the operator expressed by 'all' in both models or as the operator expressed by 'at least one' in both. Hence the operator expressed by 'Q*' is formal for the same reasons these other two operators are. (Notice that not all operators designated by quantifiers count as formal operators. An example given by Sher (1991, 58) is the operator designated by the quantifier 'pebbles in the Red Sea' (i.e., the function that assigns to each model the set of nonempty subsets of its domain that contain only pebbles from the Red Sea)).

[8] I objected to Sher's FS approach in (1997, 2002a) because it violates LNE1. It is worth noting that Feferman objects to it for a different reason. He says: "No natural explanation is given by it of what constitutes the *same* logical operation over arbitrary basic domains" (1999, 37, emphasis in the original). Presumably the fact that FS classifies terms like 'Q*' as logical is behind this remark.

[9] Indeed Sher's conflation of LNE1 and LNE2 is responsible for much of her misunderstanding, expressed in (2001), of what I say in (1997) about FS.

[10] In connection with this point it is worth noting that in (2001) Sher withdraws her previous objection to LNE1 (and similar conditions), which she expressed forcefully in (1991, 64-65). Specifically, she

The foregoing discussion of FS and the difference between LNE1 and LNE2 is also relevant to Gómez-Torrente's criticism, in section 5 of (2003), of attempts to give a precise characterization of the notion of a logical expression. Indeed what I am calling 'FS' is essentially the same as the approach Gómez-Torrente there calls 'the Tarskian characterization'[11]. Gómez-Torrente criticizes the Tarskian characterization because it "doesn't satisfy the requirement of determining a set of expressions compatible with the use (vague as it is) of 'logical expression'." Similarly, I criticized FS in (1997, 2002a) for failing to satisfy LNE1, because I believe that LNE1 (or a very similar apriority criterion) has been and continues to be a widely accepted part of the intuitive notion of logical consequence. But Gómez-Torrente makes a more general criticism of attempts to draw a sharp distinction between logical and non-logical expressions:

> These attempts usually characterize the notion of a logical expression (or of the logical expressions to be found in a restricted set of expressions) in terms of alleged semantic, epistemic or mathematical peculiarities of the logical expressions. I conjecture that these attempts will not succeed, since it must be nearly impossible to model closely the vague and pragmatic notion of a logical expression in those terms.

I believe that Gómez-Torrente is basically correct about this. Indeed I have argued in (1997) that in spite of attempts, of the kind he considers, to provide precise criteria for distinguishing logical from non-logical expressions, in the end we are forced to rely on vague pragmatic principles to make this distinction[12].

In closing I'll add only that I agree with many other points that Gómez-Torrente makes in (2003), including much of what he says in section 4 about the logical consequence relation in higher-order languages. In particular I applaud his emphasis in section 4 on the importance, for higher-order languages, of the first implication in his (2). The first implication in (2), which he expresses as

$$\text{Val}_{\text{T}}(K,X) \Rightarrow \text{LC}(K,X),$$

is the claim that every argument that is valid in the precise model-theoretic sense that derives from Tarski exhibits the intuitive logical consequence relation. The fact that this implication can be proved for first-order languages but is open to question for higher-order languages (and indeed for any language, the logic of which lacks a sound and complete proof procedure) has not always received the attention it deserves in philosophical discussions of logical consequence[13].

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says in (2001, 256) that adding a requirement to FS that would have the effect of making it satisfy LNE1 is a matter of personal preference to which she is indifferent.

[11] For details on the relations between FS, the Tarskian characterization, and Mostowski's approach, see Gómez-Torrente (2002, especially 15, 18-19).

[12] Specifically, I argued in (1997, 375-379) that the choice of logical terms was a pragmatic one, subject only to the constraints that some terms be designated as logical, that some of these be ubiquitous (i.e., ones that appear in discourse on virtually every subject), and that the terms chosen, taken together, do not result in violation of LNE1.

[13] I have discussed this matter in (1997), (1999), and (2002a).

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