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WHAT DO PARACONSISTENT LOGICS REJECT, A DEFENSE OF THE LAW OF CONTRADICTION

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Aristotle discovered the law of contradiction more than 2000 years ago. Since then, this law has been regarded as one of the basic principles of logic. Aristotle considered this principle to be 'the most indisputable of all beliefs,' but nearly half a century ago, it began to be criticized. The voice of criticism came from a philosophical logic - paraconsistent logic. This study analyses in depth the specific properties of the positive logic plus approach, non-adjunctive approach, and relevant approach of paraconsistent logic and presents three definitions of the law of contradiction. It also shows that there are two types of the law of contradiction in C-logics and that the law of contradiction with classical negation is valid in them. Furthermore, this study proves that the law of contradiction with classical negation is also valid in a non-adjunctive approach and explains the reason this law cannot be deduced in some relevant logic systems. Based on these, we can clarify what type of the law of contradiction is questioned by paraconsistent logics and thoroughly reveal the exact meaning of 'the law of contradiction is invalid' in paraconsistent logics.

Keywords: C-logic, classical logic, law of contradiction, paraconsistent logics

INTRODUCTION

It is common knowledge that the law of contradiction, the law of excluded middle, and the law of identity are the three basic laws of logic. In particular, Aristotle stated that the law of contradiction is 'the most indisputable of all beliefs' in his philosophical work *The Metaphysics* (Aristotle 1991, 1011b13). Few logicians critically discussed the law for more than 2000 years after the time of Aristotle, and logicians believed it to be so clear that there was no need to study it. Leibniz affirmed the role of the law of contradiction in judging the truth value of a proposition, stating in *The Monadology* that, by means of this principle, one can judge that any statement containing contradictions is false and any proposition that is contrary or contradictory to the false one is true (Leibniz 1898, § 31).

However, that 'undisputed' situation began to change in the middle of the last century. In 1948, the Russian logician Jaśkowski proposed a paraconsistent logic (discussive/discursive logic), which can contain contradiction but does not lead to overcomplete results (Jaśkowski 1969, 145). Subsequently, the Brazilian logician Da Costa and his collaborators conducted axiomatic studies of this logic. (Da Costa and Dubikajtis 1977, Kotas and Da Costa 1979). Shortly after the birth of discussive logic, Da Costa presented a series of paraconsistent logic systems C_n, C_n^{*}, and $C_n^{\#}$ ($1 \leq n < \omega$) that limited the validity of the law of contradiction. Therefore, Da Costa also called them *C*-logics. He also claimed that the law of contradiction was universally invalid in *C*-logics (Da Costa 1974, 361–71). Avron subsequently presented a system combining relevance logic, paraconsistent logic, and classical logic (Avron 2005). Priest suggested a Da Costa type of propositional logic based on intuitive world semantics before extending it to first-order logic (Priest 2009, 2011b). Results such as those, create the impression that paraconsistent logics are correct and the law of contradiction should be doubted.

Priest and Routley (1984, 3) defined a logic as paraconsistent iff its logical consequence relation is not explosive. (A logical consequence relation is explosive iff formula A, and its negation can deduce formula B.) Da Costa, Krause, and Bueno (2007, 791) opined that paraconsistent logic is the logic of inconsistent but nontrivial theories. Because formula A and its negation imply inconsistency, and the explosive—A and its negation can deduce formula B—implies the nontrivial, these two definitions are not very different in nature. According to Priest and Routley (1984, 4–10), there are three approaches to studying paraconsistent logics. The first is the non-adjunctive approach, which is characterized by the rejection of adjunction: from formula A and B, 'A $\wedge$ B' can be obtained. The second is the relevant approach, which requires that a propositional variable be shared between the conclusion and a premise, and rejects such pure implicational formulas as A ⊃ (B ⊃ A). The third is the positive logic plus approach, which starts from the assumption that positive logic (sometimes classical and sometimes intuitionistic) is correct and adds a suitable negation to it.

Among these three approaches, Da Costa's *C*-logics is perhaps the one that has been studied most. It gives rise to the following questions: if the law of contradiction is invalid as *C*-logics claimed, does it mean that the logical belief we have held for two thousand years might be facing an existential crisis? If our logical belief is not wrong, how do we explain the viewpoint of paraconsistent logics represented by *C*-logics that invalidates the law of contradiction?

In order to solve these two problems, this study first takes the *C*-logics constructed by Da Costa and his collaborators as the starting point before showing why *C*-logic claims that the law of contradiction is invalid and clearly analyses the invalidating mechanisms. Thus, it clarifies in what sense the law of contradiction is invalid in *C*-logics. Based on these analyses, the study presents three strict definitions of the law of contradiction, and investigates the other types of typical, paraconsistent logic (including the non-adjunctive and relevant approaches), finally clarifies why paraconsistent logic claims that 'the law of contradiction is invalid.' Specifically, Section 2 describes and explains the special nature of *C*-logics, that is, the invalidity of the law of contradiction and the principle of explosion. Section 3 offers an analysis of the reason those two important principles are invalid and discloses that there are two different types of the laws of contradiction in *C*-logics. Section 4 discusses the situation of the law in the non-adjunctive, and relevant approaches of paraconsistent logic, proves that this law is valid in the non-adjunctive approach, and shows that there is no relation with this law in the relevant approach of paraconsistent logic. Section 5 presents three strict definitions of the law of contradiction and clarifies what types of the law of contradiction *C*-logics reject.

DISTINCTIVE FEATURES OF *C*-LOGICS: TWO PRINCIPLES ARE INVALID

The *C*-logics constructed by Da Costa retains all the inference modes of classical logic that have nothing to do with negative words, which is the reason why it is called the positive logic plus approach of paraconsistent logic. The most important characteristics of the logic are that (Da Costa, Krause, and Buéno 2007, 797):

> I. The principle of non-contradiction should not generally be valid.
> II. From two contradictory propositions, that is, one being the negation of the other, it should not be possible to deduce any proposition whatsoever.

These two important characteristics are closely related, as below.

First, in paraconsistent logic C_1, according to the definition of valuation, if $v(\neg A) = 0$, then $v(A) = 1$ (Da Costa, Krause and Buéno 2007, 821); based on this definition, paraconsistent logicians have presented quasi-matrices (or quasi-truth tables) on the law of contradiction $\neg(A \wedge \neg A)$ (Da Costa, Krause and Buéno 2007, 826).[1] It can be proved that this formula is not always true, so it is no longer valid. In simple terms, according to this negation definition, when $\neg A$ is 0, A is 1; but when $\neg A$ is 1, A can be 1 or 0. This means that $(A \wedge \neg A)$ is not always false, and $\neg(A \wedge \neg A)$ is not always true. Thus, the law of contradiction is invalid. Because any system of C_n $(1 \leq n < \omega)$ is a sub-system of C_1, $\neg(A \wedge \neg A)$ is invalid in any of C_i $(1 \leq i \leq n)$. Hence, it is not valid in *C*-logics. Because *C*-logics are sound (Da Costa, Krause, and Buéno 2007, 821), $\neg(A \wedge \neg A)$ is not a theorem of *C*-logics.

Second, the reason paraconsistent logic systems can tolerate contradictions is that the formula $(A \wedge \neg A) \rightarrow B$ is invalid,[2] although it is a theorem in classical logic. Its intuitive meaning is that if a contradiction holds, then any proposition holds – in other words, contradictions can deduce anything. As previously mentioned, $(A \wedge \neg A)$ is not always false, that is, it can be true or false. Thus, there is a possibility that $(A \wedge \neg A)$ is true and B is false. Hence, the formula $(A \wedge \neg A) \rightarrow B$ is not always true. Since this formula is invalid, according to the soundness of *C*-logics (Da Costa, Krause, and Buéno 2007, 821), it is not a theorem of *C*-logics. Thus, 'contradiction deduces anything' will never occur in paraconsistent logic systems. Based on the invalidity of the law of contradiction, the principle of explosion $(A \wedge \neg A) \rightarrow B$ is also invalid. Therefore, paraconsistent logic can tolerate 'contradictions' in its systems without leading to the explosive consequences of 'any proposition holds.'

The contradictions that are tolerated in logical systems and without leading to explosive consequences are referred to as dialetheias (or true contradictions) by logician Graham Priest (Priest and Routley 1984, 4). As the principle of explosion is invalid in the paraconsistent logic, true contradictions make it impossible for any proposition to hold, which helps avoid explosive consequences. Thus, paraconsistent logic can isolate these true contradictions and avoid 'contaminating' other propositions. This is the gauntlet thrown down at the law of contradiction by paraconsistent logic: a logical result can be reached via paraconsistent logic by limiting the validity of the law of contradiction.

TWO LAWS OF CONTRADICTION IN *C*-LOGICS: THIS IS THE RESULT OF TWO TYPES OF NEGATIONS

For more than 2000 years after the law of contradiction was discovered and summarized by Aristotle, philosophers rarely questioned it, believing it to be so clear that any discussion thereon would be a waste of precious time. However, paraconsistent logicians started casting doubt on the law of contradiction. They claimed that this law did not work in the paraconsistent systems and provided clear steps to support that claim. Does this constitute a threat to the law of contradiction? Are we really going to discard the logical beliefs we have held for over two thousand years?

In traditional or propositional logic, a truth table is often used to show whether a formula is valid; in *C*-logics, quasi-matrices are employed to make decisions. Through this simple method, similar to a truth table, it can be clearly shown that the law of contradiction, whose symbolic expression is $\neg(A \wedge \neg A)$, is not always true, so 'this law' is no longer valid in these systems (Da Costa, Krause and Bueno 2007, 823–26). Although the quasi-matrix method is different from the truth table approach, it also stems from the definition of value assignment of connectives. By examining the definition of value assignment, it can be found that similar to classical logic, a proposition has two different values in *C*-logics: true and false (i.e., 1 and 0) (Da Costa, Krause, and Bueno, 2007, 821). The method by which *C*-logics invalidates the formula $\neg(A \wedge \neg A)$ – the symbolic expression of the law of contradiction – in the condition of two truth values is indeed ingenious. An examination of how *C*-logics did it is presented below.

To invalidate the formula $\neg(A \wedge \neg A)$, the basic measure of *C*-logics is to define a negation with special logical semantics as follows: $v(A) = 0 \Rightarrow v(\neg A) = 1$ (Da Costa, Krause and Bueno 2007, 821). As usual, 1 means true, and 0 means false. This definition indicates that when A is false, $\neg A$ is true. However, when A is not false, that is, when A is true, what is the truth value of $\neg A$? At this point, there is no specific requirement. Therefore, when A is true, $\neg A$ can be true or false. Thus, the logical characteristic of this type of negation is that A and $\neg A$ cannot be false at the same time, but they can be true simultaneously. For the convenience of discussion, '$\neg$' is referred to as 'paraconsistent negation' in this paper.

Some people think that a paraconsistent negation is not a genuine negation because the relationship between A and $\neg A$ is subcontrary, rather than contradictory (Priest and Routley 1989, 164–65). While we will not contest the reason behind this view, we believe it is insufficient to conclude that paraconsistent negation is not a genuine negation. Indeed, the connective expressing a contradictory relation must be a negation, but conversely, must a negation express contradictory relation? Not necessarily. Some logic can stipulate that the negation used in its system expresses contradictory relations, but it cannot generally be thought that 'a negation can only express contradictory relation.' In fact, it is well known that there are other types of negations, such as intuitionistic negation, negation as falsity, negation as orthogonality, and perfect negation (Wansing 2001, 415–36). Even though the intuitionistic negation does not express the contradictory relation which Priest mentioned, but modern logic still regards it as a negation.

In fact, it is unnecessary to show that paraconsistent negation is not a genuine negation, but rather only to show that it is not the negation used by classical logic. It is easy to reach this conclusion by using the mapping relation and truth function concept of mathematical logic. According to the definition of value assignment of paraconsistent negation (i.e., $v(A) = 0 \Rightarrow v(\neg A) = 1$), the formula $\neg(A \wedge \neg A)$ – the symbolic expression of the law of contradiction – is indeed invalid as mentioned above.[3] A detailed analysis follows below to show that paraconsistent negation is not the negation used in classical logic, and what *C*-logics meant by their claim that 'the law of contradiction is invalid.'

As the law of contradiction needs to be defined with negation, it is necessary to have a clear understanding of negation first, which can be found in any book on traditional logic or classical logic. It is defined as: if A is true, then $\sim A$ is false; if A is false, then $\sim A$ is true ('~' can be read as 'not'). Aristotle's *Metaphysics*, which describes the law of contradiction, also uses negation in this sense (Aristotle 1991, 1008b2). For the convenience of discussion, '~' can be called 'classical negation' (i.e., the negation used by Aristotle, traditional logic and classical logic are called classical negation).

The definition of classical logic clearly shows the typical logical characteristics of classical negation; namely, A and $\sim A$ cannot be both true and cannot be both false simultaneously. Therefore, the logical semantics of classical negation determines a truth value mapping from A to $\sim A$: from true to false and from false to true. This one-to-one mapping is obviously a typical unary function relationship. In contrast, the definition of paraconsistent negation also makes a truth value mapping from A to $\neg A$: from false to true and from true to true or false. It is obvious that 'from true to true or false' is not a one-to-one mapping, which means that this mapping is not a functional relationship. As classical negation is truth-functional while paraconsistent negation is not, there is a fundamental difference between the natures of these two negations.

After establishing a distinction between these two negations, the law of contradiction is further examined. It is clear that the law of contradiction is closely and directly related to negation, which is necessary to express this law. Thus, according to the above definition of classical negation, as the judgment that A and $\sim A$ are both true (i.e., $A \wedge \sim A$) must always be false, then obviously $\sim(A \wedge \sim A)$ – the symbolic expression of the law of contradiction – must always be true. The question then arises: is *C*-logics opposed to this? The answer is that $\sim(A \wedge \sim A)$ is still valid in *C*-logics.

Why? Consider what occurs in *C*-logics. There is a strong negation denoted by the symbol $\neg^\star$ (Da Costa, Krause and Bueno 2007, 804), which satisfies all schemas and inference rules of classical propositional logic in C_1 (Da Costa, Krause & Bueno 2007: 806). Hence, the so-called strong negation $\neg^\star$ is simply classical negation $\sim$. It is easy to show that $\sim(A \wedge \sim A)$ is valid in *C*-logics: $A \vee \neg^\star A$ is valid in C_1 (Da Costa, Krause and Bueno 2007, 806), because $\neg^\star$ satisfies all schemas and inference rules of classical propositional logic, and thus, according to De Morgan's Law, $\neg^\star(A \wedge \neg^\star A)$ is valid; and as $\neg^\star$ is $\sim$, $\sim(A \wedge \sim A)$ is also valid in C_1. Because strong negation can be introduced in any system of *C*-logics by giving a definition, $\sim(A \wedge \sim A)$ is valid in any system of *C*-logics. Therefore, the law of contradiction expressed by classical negation is still valid in *C*-logics.

INSPECTION OF NON-ADJUNCTIVE APPROACH AND RELEVANT APPROACH: THE LAW OF CONTRADICTION STILL HOLDS

Discussive logic was first proposed by Jaśkowski, which is referred to by Priest and Routley as the non-adjunctive approach (Priest and Routley 1984, 7). The basic idea of Jaśkowski is to take 'true' to be 'true according to the position of some person (e.g., in a discussion).' This concept can be represented logically as 'true in some possible world (the world of the position of that person).' In fact, anyone who understands modern modal logic will not be surprised about the view of discussive logic and will consider it consistent with the basic view of modern modal logic. For instance, in the basic modal logic system K, D, T, B, S1–S5, and so on, when a proposition is true in one possible world, its negation can be true or false in another. It does not present a problem as long as the proposition and its negation are not both true in the same possible world. The main reason that discussive logic is a type of paraconsistent logic is that the explosive principle is invalid in it (Priest and Routley 1984, 1), but the law of contradiction is still valid if $\neg(A \wedge \neg A)$ is still considered as a representative of this law in discussive logic. The proof can be briefly described as follows:

> First, discussive logic J is sound and complete (Da Costa and Doria 1995, 46); hence, we have $\vdash_J A \Leftrightarrow \vDash_J A$, that is, formula A is a theorem in system J iff A is valid in J. As it is well known that the modal logic system S5 is sound and complete (i.e., $\vdash_{S5} A \Leftrightarrow \vDash_{S5} A$), that is to say, formula A is a theorem in system S5 iff A is valid in S5. According to Proposition 3.3 of J, $\vDash_J A \Leftrightarrow \vDash_{S5} \Diamond A$ (Da Costa and Doria 1995, 44), namely, formula A is valid in system J iff A is valid in S5 (this is also a general principle of discussive logic); hence, we can deduce that $\vdash_J A \Leftrightarrow \vdash_{S5} \Diamond A$ (corollary 1). This conclusion means that if formula $\Diamond A$ is an S5-theorem, then A is a J-theorem.
>
> Second, because $\neg(A \wedge \neg A)$ is a theorem of propositional logic, it is

also an S5-theorem because S5 proper contains propositional logic; then, according to the Necessitation Rule, $\Box\neg(A \wedge \neg A)$ is an S5-theorem. As $\Box\neg(A \wedge \neg A) \rightarrow \Diamond\neg(A \wedge \neg A)$ is an S5-theorem, according to Modus Ponens, $\Diamond\neg(A \wedge \neg A)$ is also an S5-theorem.

Thus, according to corollary 1, $\neg(A \wedge \neg A)$ is a theorem of discussive logic J. Therefore, no matter what type of negations are used in the discussion logic system, the formula $\neg(A \wedge \neg A)$ that represents the law of contradiction in the discussion logic is always valid. Thus, the discussive logic does not pose a direct threat to the law of contradiction.

In addition, according to Priest and Routley (1984, 10), paraconsistent logic also has a relevant approach, which will now be examined. The typical axiomatic systems in the relevant approach are P and P^*, which were constructed by Arruda and Da Costa (1984, 33–49) and which have no logical semantics. Subsequently, Routley and Loparić presented the semantics of P-systems, and proved their soundness (Routley and Loparić 1978, 301–320). However, the reason that P-systems are classified as paraconsistent logics is also that the explosive principle does not hold in these systems. It is well known that the principle of relevant logic is that inferences must actually use the premises; that is, the conclusion of an inference must actually be relevant to the premises. For this reason, in relevant logic, it is not the law of contradiction but rather the relevant principle that is necessarily related to the explosive principle. Because there is no relevance between the premises and conclusion in the inference mode of the explosive principle $A \wedge \neg A \vdash B$, this mode does not hold in all types of relevant logic systems. In fact, any relevant logic can be regarded as a type of paraconsistent logic according to the definition of Priest and Routley (Priest and Routley 1984, 1), because the inference mode $A \wedge \neg A \vdash B$ does not obey this special principle and does not work in all relevant logics.

System P^* has one inference rule more than P has, that is $A \rightarrow B, A \rightarrow \neg B \vdash \neg A$ (Arruda and Da Costa 1984, 35), so P is a proper sub-system of P^*; there is also a five-valued mathematical interpretation that shows that $\neg(A \wedge \neg A)$ is deducible in P^* but not in P. This situation does not occur in any proper sub-systems of classical logic because the law of contradiction (i.e., formula $\neg(A \wedge \neg A)$) holds for classical logic and all its proper sub-systems. However, relevant logic tells us that this is not surprising. Relevant logic systems vary in inference ability because of the special requirements regarding the relations between the premises and the conclusion. In ascending order, from weak to strong, the common relevant systems are Min, B, Dw, Tw, T, R, and so on. System Min is the smallest and weakest relevant system. Compared with other relevant systems, Min has fewer axioms and more inference rules. When axioms are added, and inference rules corresponding to the added axioms are removed, stronger relevant logic systems can be obtained. Starting with system Min, the system can be reinforced until system T is obtained, and $\neg(A \wedge \neg A)$ becomes the theorem of the system T. Any relevant logic system weaker than T has no theorem that states $\neg(A \wedge \neg A)$ (Priest 2011a, 188–206). No relevant logicians claim that their logics reject the law of contradiction since they know that it is only because these relevant systems are so weak that the theorems that could be deduced are extremely limited. Therefore, whether the law of contradiction (i.e., $\neg(A \wedge \neg A)$) holds in a relevant logic system depends on whether the system is strong.

For example, even though classical logic is generally accepted as obeying the law of contradiction, classical logic has a similar phenomenon. For the famous classical logic system PM, which has four axioms, if the other three axioms of this system are deleted, then only the first axiom A1, namely $(A \vee A) \rightarrow A$, is left. Due to the absence of three axioms, the new system is obviously less than the theorem of PM (with four axioms). This new system with only one axiom can be denoted as PM_1. Now, formula $\neg(A \wedge \neg A)$ is not a theorem in PM_1, because $\neg(A \wedge \neg A)$ cannot be deduced from $(A \vee A) \rightarrow A$. The method showing that $(A \vee A) \rightarrow A$ cannot deduce $\neg(A \wedge \neg A)$ is similar to the method showing the independence of an axiom as below.

Initially, we give an arithmetical interpretation. Proposition A and B can be 0, 1 or 2. For $\neg$: $\neg 0 = 1$, $\neg 1 = 0$, $\neg 2 = 2$. For $\vee$: $0 \vee 0 = 0 \vee 1 = 0 \vee 2 = 1 \vee 0 = 1 \vee 1 = 2 \vee 0 = 0$, $1 \vee 2 = 2 \vee 1 = 2$, $2 \vee 2 = 1$. Under this interpretation, axiom A1, that is $(A \vee A) \rightarrow A$, is always 0, and Modus Ponens has the property of preserving-0. However, we can find that $\neg(A \wedge \neg A)$ is not always 0. Therefore, $(A \vee A) \rightarrow A$ cannot deduce $\neg(A \wedge \neg A)$. At this time, just as paraconsistent logicians claim that 'there is a paraconsistent logic within relevant logic,' should we think that 'there is a paraconsistent logic within the sub-systems of classical logic'? Apparently not. The reason is similar to our explanation of relevant logic above: PM_1, which only has one axiom, is so weak that it can only derive a few formulas as theorems that do not include formulas $\neg(A \wedge \neg A)$. This is only a matter of system strength and size and does not qualify as a special phenomenon, let alone a threat to the law of contradiction.

WHAT DO PARACONSISTENT LOGICS REJECT? BOTH SIDES ARE CORRECT BASED ON ADEQUATE EXPLANATIONS

Based on the above analysis, some conclusions can be made. The non-adjunctive approach and relevant approach of paraconsistent logic do not constitute a substantial threat to the law of contradiction. However, the formula $\neg(A \wedge \neg A)$ indeed loses validity in *C*-logics (positive logic plus approach of paraconsistent logic). The details are as follows: first, two types of negation, that is, '$\sim$ (classical negation)' and '$\neg$ (paraconsistent negation),' are actually used in *C*-logics; the former is functional, while the latter does not express functional relations and plays the role of mapping. Second, based on those two negations, two types of the law of contradiction are actually described in *C*-logics, namely $\sim(A \wedge \sim A)$ and $\neg(A \wedge \neg A)$, where the former is based on classical negation, and the latter is based on the paraconsistent negation. Third, according to the definition of valuation, the law of contradiction based on classical negation $\sim(A \wedge \sim A)$ is still valid in *C*-logics, while the law of contradiction based on the paraconsistent negation $\neg(A \wedge \neg A)$ is invalid in *C*-logics.

Therefore, regarding the two questions explored in this study, all one needs to do is to revert to *C*-logics for the answer. That is, does the logic beliefs we have held for two thousand years face a crisis from *C*-logics, and how does one explain the viewpoint of *C*-logics that 'the law of contradiction is invalid'? To give a more accurate answer to this question, let us first define more clearly what the law of contradiction is and how it should be understood, and then go further to explore whether it has or does not have a certain property (e.g., validity).

The thought of the law of contradiction is clearly expressed in modern logic, and its formulation is given in theorems of classical logic. For example, the propositional calculus uses the formula $\sim(A \wedge \sim A)$, and the first-order predicate calculus uses the formula $(\forall x)\sim(F(x) \wedge \sim F(x))$ to express the law of contradiction.[4] They are valid formulas in the corresponding logical calculus. Their intuitional meaning is, respectively: proposition A and its negation $\sim A$ cannot hold simultaneously; for any object x, x has property F and x does not have property F cannot hold simultaneously. Because the law of contradiction, as a theorem, is closely related to the negation, it can be distinguished more carefully according to the different types of negation used: (1) If the negation used in the formula for the law of contradiction is the classical one, it is called the law of *classical negation definition*. (2) If a special negation with a logical meaning different from the classical negation is used, it is called the law of *special negation definition*. (3) If the negation in the so-called law of contradiction can be any type of negation, it is called the law of *general definition of meaning*.

Thus, the law of contradiction can actually be understood in three different ways. Accordingly, we can conclude more clearly and concretely whether Da Costa's *C*-logics rejected the law of contradiction or not. One issue that has been clearly discussed in the study is that the law of contradiction described by classical negation – the 'classical negative definition' of the law of contradiction – is valid in *C*-logics. Since the law of contradiction described by Aristotle, expressed by traditional logic and classical logic, are all based on classical negation (described above), the law of contradiction defined by classical negation does not fail, even in *C*-logics.

Therefore, the logic belief that we have held for millennia actually is still valid in *C*-logics. This is the answer to the first question. The second question is now addressed, namely: how do we explain the claim by *C*-logics that 'the law of contradiction is invalid'?

First, based on the analysis above, only the formula $\neg(A \wedge \neg A)$ is invalid in *C*-logics. If the formula is considered to be a representative of the law of contradiction in the meaning of the theorem, more precisely speaking, the invalid one is the law of contradiction described by 'some negation,' such as paraconsistent negation, and this type of law of contradiction loses its validity. In other words, what is invalid in *C*-logics is the 'special negation definition,' that is, the law of contradiction described by a special negation is invalid in it. Therefore, what *C*-logics is actually rejecting is the law of 'special negative definition.'

Second, we all know that in addition to the classical negation, there are other connectives that can be called negative, but they are quite different compared to classical negations (Wansing 2001, 415–436). Since the law of contradiction needs to be expressed with negations, the question now is: can the formula obtained by replacing the classical negation in formula $\sim(A \wedge \sim A)$ with other negations still be valid? *C*-logics would claim that it is not. For example, the law of contradiction described with paraconsistent negation $\neg(A \wedge \neg A)$ is not valid, which is indeed a fact. Thus, the law of 'general meaning definition' is invalid because we now know that the law of contradiction described by a paraconsistent negation is invalid. Consequently, the exact meaning of 'law of contradiction is invalid' that *C*-logics claimed should be that the law' special negation definition' and 'general meaning definition' are invalid. Therefore, in actual fact, there is nothing wrong with our logical belief about the law of contradiction based on the classical negation definition (including Aristotle's logical beliefs), and the viewpoint of *C*-logics that this law is invalid (in the sense of special negation definition and general meaning definition) is also not wrong.

CONCLUSION

According to the detailed analysis above, the logical belief, that is, the law of contradiction that we have held for millennia is still valid in *C*-logics. Because this type of the law of contradiction is based on classical negation, and because the formula $\sim(A \wedge \sim A)$ is a theorem and is valid in *C*-logics, the classical meaning of the law of contradiction still holds even from the viewpoint of *C*-logics. However, if replacing the classical negation in formula $\sim(A \wedge \sim A)$ with other negations, such as paraconsistent negation, the new type of the law of contradiction might not always be valid. The view of *C*-logics is not wrong; however, its description is not clear enough. If *C*-logics were to argue that the law of contradiction using non-classical negation is invalid or the 'general meaning' in the law of contradiction is invalid, that would not be controversial.

NOTES

1. Where the capital letter A means a proposition or a statement, the symbol $\neg$ means negation or 'not,' the symbol $\wedge$ means conjunction, or 'and.'

2. The symbol $\rightarrow$ means material implication, or 'if …, then…'.

3. Obviously, with such an assignment definition, the formula $A \vee \neg A$ (that is, A or not A), which represents the law of excluded middle, is valid. The formula $A \rightarrow A$ (that is, A is A), which represents the law of identity, is valid as in classical logic because it does not involve negation. Therefore, we only discuss the law of contradiction here.

4. The letter A stands for any proposition, the symbol $\sim$ corresponds to 'not,' the symbol $\wedge$ corresponds to 'and,' the symbol $\forall$ stands for the quantifier 'all' or 'any,' x stands for any individual object, and $F(x)$ stands for x has some property F.

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