Euler Contra Du Châtelet (and Wolff) on the Composition of Extension

Stephen Harrop

Introduction

"There was a time," wrote Leonhard Euler in a letter dated 5 May 1761 to Princess Friederike Charlotte of Brandenburg-Schwedt,

> when the dispute respecting monads employed such general attention, and was conducted with so much warmth, that it forced its way into company of every description, that of a guard-room not excepted. There was scarcely a lady at court who did not take a decided part in favor of monads or against them. In a word, all conversation was engrossed by monads – no other subject could find admission.[1]

In this paper, I'll carry on with this philosophical fad. Specifically, I'll examine some arguments for and against the thesis that composite, extended beings are ultimately composed of monads – simple, unextended substances. The arguments in its favor are

1. All English quotations from this correspondence will be from Euler 1833 unless otherwise indicated. The French edition I will use is the three volumes of Euler 1775. Going forward I will cite passages from the *Letters* as Letter [letter number in Euler 1775], [volume number in Euler 1775] [page number in Euler 1775] / [volume number in Euler 1833] [page number in Euler 1833]. Thus for instance this quoted passage is cited as Letter CXXV, II 195/ II 39-40

those given by Christian Wolff and Emilie Du Châtelet. The arguments against it will be the one given by Euler. I'll conclude with a possible way forward: The difference between Euler on the one hand and Du Châtelet and Wolff on the other does *not* lie in their acceptance or denial of a general explicability principle (in other words, of the principle of sufficient reason). Instead, it lies in their endorsement of subsidiary principles. Du Châtelet relies on an explicability principle in which an object's having a property is not fully explained by citing another object with that same property. Euler rejects this in favor of his own explicability principle: An object's being extended must always have, as its explanation, an object which is extended. I'll conclude by arguing that the difference between the two also turns on epistemological differences. First, Euler seems to accord the imagination higher epistemic value than Du Châtelet; and second, he seems to demand explanations not only state *that* something is the case but *how* something is the case.

Du Châtelet on The Composition of Matter

Chapter VII of Du Châtelet's *Institutions Physiques* is titled "The Elements of Matter." After taking us through a brief tour of ancient element theory, together with that of Descartes and Gassendi, she arrives at an explication of Leibniz's (and, she says, Wolff's) account of the elements. She says that she is "going to try and make you understand the ideas of these two great philosophers on the origin of matter."[2]

The next section of IP[3] runs this way. It's acknowledged by everyone, says Du Châtelet, that all bodies are spatially extended. Thus, since nothing is without a sufficient reason, there must be an explanation for why it is that bodies are spatially extended. This reason must make it intelligible how and why this extension is possible.

2. All quotations from the *Institutions Physiques*, the second edition, are my own translations from Du Châtelet (1742). It is cited as IP [page number]. Thus this passage is cited as IP 137

3. IP 137ff

But saying that a body is extended because it is composed of extended parts will not do, according to Du Châtelet. This is because the same question will just arise again with respect to each of the parts: What is the reason that it is extended? For

> [S]ince the sufficient reason must invoke [*oblige d'alléguer*] something which is not the same as that of which one demands the reason, because without it one would give no sufficient reason, and the question always stays in the same state, if one wishes to satisfy this principle on the *origin of extension* [*l'origine de l'etendue*, emphasis mine], it is necessary to come at last to something un-extended [*quelque chose de non-étendu*], and which has no parts, to give reason to that which is extended, and which has parts, since a non-extended and partless [*sans parties*] is a simple being. Therefore composites, extended beings exist, because there are simple beings. (IP 138)

Before going on to reconstruct this argument, let's remind ourselves of what Du Châtelet is trying to explain. It is that "[a]ll bodies extended in length, breadth, and depth." (IP 137) She further writes that "it is necessary that [*il faut que*] this extension has its sufficient reason." (IP 138) In other words, what she's out to argue for is that simple beings (or monads) must be the ultimate explanation of why extended beings are *extended*. There must be an explanation for where this extension comes from – it can't simply emerge from thin air.

I make this point because one might think Du Châtelet is trying to answer a different question: why is it that extended things are *things* worthy of the name rather than mere aggregates? If that was what she is after, expressing herself in the way she does is rather strange. She emphasizes that it is “this extension [*cette étendue*]” (IP 138) which needs a sufficient reason, not the being-ness of the extended being.

So how should the argument be reconstructed? A natural way to start runs as follows:

(P1) There are extended beings.

(P2) If there are extended beings, then there must be something which explains why those beings are extended.

So: (C1) There must be something which explains why extended beings are extended.

There are now (at least) two ways that the argument could go. The first is to read Du Châtelet as making a mere infinite regress argument. She does say, after all, that the answer that “there is extension, because there are small extended parts” is insufficient. (IP 138) The argument would then run something like the following going on:

(P3.1) If the explanation of the extension of an extended thing is always an extended thing, there would be an infinite explanatory regress.

(P4.1) There cannot be an infinite explanatory regress.

So: (C2.1) The explanation of the extension of an extended thing is not always an extended thing.

The difficulty with this is threefold. First, there is no mention of an infinite regress in this passage at all. Neither the French word for infinity nor any of its cognates does not appear in this passage. If the argument Du Châtelet intended to make was one of infinite regress, we should, all else being equal, expect her to make it explicitly.

Second, Du Châtelet is quite willing to concede for the sake of argument an infinite explanatory regress elsewhere in the *Institutions*. For example, in her version of the cosmological argument for the existence of God (found in chapter 2), she allows for an infinite chain of contingent beings, each one of which has a cause. (We'll examine this explicitly in the next section.) There, such an infinite explanatory chain is *not* ruled out *ab initio*. So if we want to give a uniform treatment of the *Institutions*, we should try only to attribute opposition to infinite explanatory chains to her where we find it explicitly. And we don't find it here.

Third, the argument that Du Châtelet *in fact* makes relies not on the infinity of the explanatory chain, but on the features that are being explained. She writes, recall, that the thing doing the explaining needs "is not the same as that of which one demands the reason." (IP 138) It seems, then, that the problem with extended parts explaining the extension of the whole is that there needs to be a difference of state between two things for the one to explain the state of the other. (Here I am being deliberately vague about the details of what this "state" is.)

What these considerations suggest, then, is something like the following continuation of the argument:

(P3.2) To explain fully the fact that a thing has some property, one must invoke something that does not have that property

(P4.2) If the full explanation of the extension of an extended thing is other extended things, one would not invoke something that is unextended as part of that thing's full explanation.

So: (C2.2) The full explanation of an extended thing is not an extended thing.

And now the argument is basically done. Since (C1) tells us something must explain the extension of extended things, and (C2.2) tells us that it cannot itself be extended. we get the following conclusion:

So: (C3) The extension of extended things is explained by something unextended.

The turning point of the argument, as I've read it, is precisely (P3.2). Du Châtelet expresses it by writing that "[a] sufficient reason must invoke something which is not the same as that of which one demands the reason." (IP 138) I read the "not the same" language as essentially referring to an unextended object's lack of extension. If we were simply to invoke the extension of another object, the question would simply recur.

What I want to draw attention to is that it's not *just* the principle of sufficient reason that's driving the argument. There is a subsidiary explanatory principle at play. This is important because, as we'll see, Euler, in both his arguments against extended objects being ultimately composed of monads and his other works, explicitly endorses the principle of sufficient reason and makes heavy use of it.

Wolff on Composition in General

While Du Châtelet focuses on the composition of extension, her predecessor and intellectual influence Christian Wolff targets composition generally in §75 of his *Vernünftige Gedanken von Gott, der Welt und der Seele des Menschen, auch allen Dingen überhaupt* ("Rational Thoughts on God, the World and the Soul of Man, and on All Things in General").[4] Here is the passage in question:

4. This is most likely the work that Du Châtelet had access to in writing the *Institutions*; see Barber (1967, 205).

> §75 *That there are simple things.* If there are composite things, there must also be simple beings. For if no simple beings were present, then all parts – they can be taken to be as small as you might ever like, even inconceivably small parts – would have to consist of other parts. But then, since one could provide no reason where the composite parts would ultimately come from, just as little as one could comprehend where a composite number would arise from if it contained no unities in itself, and yet nothing can be without a sufficient ground (§30), one must ultimately admit simple things from which the composites arise. Whoever has proper insight into the principle of sufficient reason comprehends that one does not arrive at such a ground until one has no more questions and does not receive the same answer, as happens when one admits parts to infinity.[5]

This in large part mirrors the argument he gives in §686 of his *Ontologia*:

> §686 *If a composite being is given, simple [beings] also necessarily must be given*, or [*seu*] *Without simple beings composite beings are unable to exist.*
>
> For composites are composed out of parts distinct from one another [*a se invicem distinctis*]. (§531) But if these parts were again composed out of parts distinct from one another, they will in the same way be composite beings. (§531) Thus, as long as other smaller parts

5. Watkins (2009, 17)

> are admitted, out of which the larger parts are composed, the question of why they are composed continuously arises. Consequently it will not yet be understood whence the smallest composites [*composita minima*], which enter into the composition of the other composites, come about. Indeed, since [*Cum adeo*], the sufficient reason why something[6] is a composite is not contained in the notion of a composite (§56), just the same without a sufficient reason why something[7] is able to composite rather than incomposite, something[8] will not be able to be composite (§70). [Hence] the sufficient reason of a composite is to be sought outside of a composite, and thus in a simple being. Therefore if composite beings exist, simple [beings] must also exist, or [*seu*] without simple beings composition is neither able to be conceived nor to be given.[9]

We may plausibly reconstruct the argument given in both these works thus:

(W1) Everything has a sufficient reason why it is rather than is not.

So: (W2) If composites exist, there is a sufficient reason why composites exist. (from (W1))

(W3) If every composite is made up of composite parts, there would be an infinite regress of composites.

6. The Latin here is *cur quid*, which I have read as *cur aliquid*, since that gives a subject for which *compositum* is a complement. I thank my friend Lily Hart for the suggestion here.

7. The Latin here is also *cur quid*, and I have read it the same way.

8. Here again the Latin is *quid*, and I have made the same emendation as twice previously.

9. I cite from Wolff's *Gessamelte Werke* as WW [division number] [volume number] [page number] from Wolff (1962-). Thus this passage is cited as WW II 3 517-8; the translation is my own.

(W4) If there is an infinite regress of composites, then the original composite would have no sufficient reason for why it is composite rather than not.

So: (W5) If every composite is made up of composite parts, then the original composite would have no sufficient reason for why it is composite rather than not. (from (W3), (W4))

So: (W6) Not every composite is made up of composite parts. (from (W1), (W5))

Wolff's argument shares at least one important feature with Du Châtelet's: It invokes the principle of sufficient reason to explain some feature of a composite being. But the two arguments differ in two key respects.

First, in Du Châtelet's argument, the feature that is to be explained is the *extension* of an extended thing. By contrast, what is to be explained in Wolff's argument is the *composition* of a composite being. The explanatory target in both cases is importantly different. Du Châtelet's argument may be successful where Wolff's isn't if it can be shown that composites generally needn't have simple ultimate parts. In that case, extended composites may need unextended explainers of their extension, whereas composites generally need not have simple explainers of their composition.

Second, in Du Châtelet's argument, no use is made of the impossibility of an infinite regress. In Wolff's argument, however, this is an important feature. This, interestingly, mirrors their corresponding cosmological arguments. Wolff's argument in *Rational Thoughts* (which Du Châtelet would most likely have been familiar with) goes as follows:

> We exist (§1). Everything that exists has its sufficient ground why it exists rather than does not exist (§30) and, therefore, we must have a sufficient ground why we exist. If we have a sufficient ground why

> we exist, that ground must be found either within us or external to us. If it is to be found within us, then we exist necessarily (§32), but if it is to be found in something else, then that something else must have in itself its ground why it exists and thus exists necessarily. Accordingly, there is a necessary being. Whoever might object that the ground for our existence could be found in something that does not have in itself the ground for its existence does not understand what a sufficient ground is. For one must in turn ask further of such a thing what has the ground for its existence, and one must ultimately arrive at something that needs no external ground for its existence.[10]

Again, important use is made of the impossibility of an infinite regress of sufficient reasons of a specific sort – here, of contingent sufficient reasons. Wolff explicitly states that "one must ultimately arrive," in the course of asking for sufficient reasons, at "something that needs no external ground for its existence." The parallel with the argument for the existence of simples is obvious.

On the other hand, Du Châtelet explicitly admits that an infinite regress of contingent beings is possible, at least for the sake of argument, and then proceeds to argue by disjunctive syllogism:

> [I]f [the being that has existed from all eternity] were to have received its existence from another Being, it would be necessary that that other Being existed through itself, and then either it is of that being that I speak, and it is God, or else it would again have had its existence from another. One sees easily that in thus going back to infinity, one

10. Wolff ((1720) 2009, §928, 51)

> must either arrive at a necessary Being who exists through itself, or else admit an infinite chain of beings, which taken all together will have no external cause of their existence (since all beings enter into that infinite chain), and which, each in particular, will have no internal cause, since each does not exist through itself, and that they have their existence the one from the other in a gradation to infinity. (IP 41)

Note that the infinite chain is present in both of the prongs of the argument. Du Châtelet speaks of "going back to infinity" as pertaining to both options she considers. In the one case, one will arrive at the existence of God, and in the other, one arrives ultimately at a contradiction. So there's nothing about an infinite regress that she intrinsically rejects. Instead, what she appeals to (as we saw in the previous section) is a specific kind of explicability principle.[11]

The Foundation of Euler's Mechanics

In this section, I'll be going into some detail about Leonhard Euler's mechanics. I do this not just because they're interesting (though they are) but because they illustrate the key role that the principle of sufficient reason plays in his thought.[12] This will be especially important when I turn to an examination of his arguments against monadological metaphysics.

Euler is probably best known to history for his mathematical and physical work. For example, his landmark two-volume work *Mechanica, sive motus scientia analytice exposita* (henceforth *Mechanica*) systematically formulates Newton's particle dynamics

11. For closer and more detailed examinations of Du Châtelet's cosmological arguments see Lascano 2011 and Harrop, n.d.

12. Somewhat surprisingly, he is sometimes seen as a foe of the principle of sufficient reason. See for instance Lin and Melamed 2023, §6.

using the newly-developing differential and integral calculus rather than geometry, as had Newton's *Principia Mathematica Philosophiae Naturalis*.[13] But while he gave an exposition of Newtonian mechanics, he also subtly varies Newton's methodology while putting it in analytical form.

This comes out clearly in his treatment of Newton's first law. Here's how Newton states it in the *Principia*:

> Law 1: Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.[14]

Newton gives no argument for the law, only illustrations. In volume 1 of *Mechanica*, Euler quotes this law almost verbatim.[15] But immediately before doing so, he writes that "authors have embraced these laws of absolute motion and rest into one."[16] The laws of absolute motion and rest that Euler sets out are:

> Proposition 7. A body absolutely at rest must persevere in rest perpetually, unless it be disturbed [*sollicitetur*] to motion by an external cause. (I §56)

13. He was not the first to do something like this. As early as 1716, Jakob Hermann formulated Newton's second law in something like its contemporary form. See Hermann 1716, §131.

14. Newton 1999, 62. Note that Cohen and Whitman are translating from the third edition.

15. There are two changes. First, he omits the relative pronoun "*illud*" referring to the body persevering in its sate of motion (this suggests he may not have been working from the third edition of the *Principia*; see 62nbb and the statement in that edition at Newton 1726, 13). Second, while Newton begins the law with "*corpus omne*," Euler inverts the word order and writes "*omne corpus*."

16. I cite from *Mechanica* by volume and number, from Euler 1736. Thus this passage is cited as I §68. Translations from *Mechanica* are my own throughout. Note that Euler accepts the existence of absolute place (I §4), and hence of both absolute motion and rest (I §5, I §7).

> Proposition 8. A body having absolute motion will be moved uniformly, and with the same velocity now as at whatever time it was moved before, unless an external cause either is acting or will act [*agat aut egerit*] on it. (I §63)

> Proposition 9. A body possessed of absolute motion will go on in a straight line, or equivalently [*seu*] the space which it describes will be a straight line. (I §65)

What's important for our purposes is that after stating each of these laws Euler goes on to demonstrate (or maybe quasi-demonstrate) them. In each case the demonstration relies importantly on the principle of sufficient reason.

The demonstration of Proposition 7 (at I §56) reasons thus. Supposing an absolutely resting body to exist in an empty, infinite space, there is no reason why it should move from one part of that space to another, Hence, the reason for such a movement cannot lie in the body itself. As a result, if the body is moved, it must be so moved by some external cause. The demonstration of Proposition 8 runs similarly almost identically, except the body is now absolutely moved rather than absolutely resting (I §63). He also derives Proposition 9 using the principle of sufficient reason (I §65), reasoning that there can be no reason why a body moving from one part of space to another should deviate from a straight line unless by an external force.

Immediately after the demonstration of Proposition 7, he states that it has a corresponding law of nature:

> Corollary 1 [to Proposition 7]. It is therefore a law founded in the very nature of things that every body at rest [*corpus quiescens*] ought

> to persevere in rest unless it be disturbed to motion by some external cause. (I §57)

And after the demonstration of Propositions and 9, he states that there is a universal law corresponding to their conjunction:

> Corollary 1 [to Proposition 9]. From these two propositions is produced [*conficitur*] this universal law: Every body endowed with motion proceeds uniformly in a straight line. (I §66)

Thus Euler says that "we have shown [Newton's first law] from the principle of sufficient reason." (I §75)[17] Whereas Newton took the laws of motion to be postulates or axioms, Euler treats them as demonstrable from the principle of sufficient reason. I won't go into detail about Euler's actual derivations here. What matters for our purposes is that the principle plays a crucial role in establishing the foundations of his physics in *Mechanica*.

This use of the principle continues throughout his life. In his *Theory of the Movement of Solid or Rigid Bodies* (published in 1765 as *Theoria motus corporum solidorum seu rigidorum*), he likewise enunciates laws of inertial motion. The rest law (axiom 2, chapter 2) is:

17. He employs it elsewhere as well. For instance, he invokes it in his solution to the problem of of finding the effect of an arbitrary force on a point particle given the antecedent forces that particle is subject to. (I §146)

> §82 A body which rests absolutely [*absolute quiescit*], if it be[18] subjected to no external action, will perpetually persevere in rest.[19]

While this axiom "appears so evident through itself that it may need no proof," Euler nonetheless gives one, or at least an argument for it, so that "its strength [*vis*] may be more clearly understood."[20] And this quasi-demonstration proceeds almost exactly as it did in *Mechanica*, use of the principle of sufficient reason and all. Indeed, Euler says that this truth "depends on the principle of sufficient reason [*Nititur…principio sufficientis rationis*]."[21]

Further, in a text published towards the end of his life, *An introduction to Natural Science* (published in 1775 as *Anleitung zur Natur-Lehre*), Euler makes the search after causes according to the principle of sufficient reason the cornerstone of natural science:

> *1. Natural science is a science that aims to explain the causes of change that occur on material bodies.*
>
> Wherever there is a change, there must be a cause for it, since it is certain that nothing can happen without a sufficient reason. Whoever can point to the reason why a change has occurred, has found its cause, and thus fulfils the ultimate aim of Natural Science. This ultimate aim is focussed only on changes, for as long as an object remains in the same state, the only conclusion that can be reached is that all causes that could produce a change are absent. But as soon

18. The Latin here is "*fuerit*," but in English the future perfect "will have been" sounds very strange here, so I have rendered it as "be." I don't think any substantive change in meaning results here.
19. Euler 1765, 32
20. 32
21. 32

> as a change occurs, one is entitled to ask for its cause, and Natural Science endeavors to determine the causes of all changes.[22]

The point here is that Euler makes copious and extremely important use of the principle of sufficient throughout his work on the foundations of mathematical physics and natural science generally. As a result, the difference between him and Du Châtelet (and Wolff) probably doesn't lie in his avowal of that principle. In the next section, I'll look at where it does in fact lie.

Euler *contra* Monadology

We have reason to believe that Euler thought highly of Du Châtelet. They had both submitted entries to the Paris Academy prize competition on the nature of fire, with Euler sharing the prize with two others (Louis-Antoine de Lozeran du Fesc and Jean-Antoine de Créquy) and De Châtelet receiving an honorable mention along with Voltaire.[23][24] In an undated letter to Du Châtelet, Euler writes that "in reading your *Institutions Physiques*, I have equally admired the clarity with which you have treated that science, as the facility with which you explain the most difficult things concerning motion."[25]

This admiration notwithstanding, Euler departs from Du Châtelet on the composition of extension. He gives many arguments, in the *Letters to a German Princess*,

22. Euler 1862, vol. 2, 449. I quote from the English translation by Ernest Hermann Hirsch, here. As one would expect, Euler uses the principle to argue for the laws of motion in this work as well; see §§26, 28 (vol. 2, 464, 466)

23. Calinger 2016, 148–9. For Du Châtelet's submission see Du Châtelet 2009, II

24. I cite from Euler's *Opera omnia*, still in compilation, as OO [series] [volume] [page] from Euler 1911–. Thus the citation to his entry to the Paris Academy competition, *Dissertatio de igne, in qua ejus natura et proprietates explicantur*, is cited as OO III 10 2-13. Zinnsner (Du Châtelet 2009, 54) refers to the winners as "three respected Cartesian authors," but it's not clear at this stage, with the recent publication of *Mechanica* and its avowal of Newtonian absolute space and motion, that Euler could correctly be called a Cartesian.

25. Euler 1963, 278. In the same letter he registers some disagreement with Wolff concerning the nature of force (279).

that extension does not ultimately reduce to unextended parts. But I want to focus on a specific objection he raises, because I believe it illustrates the primary difference between his views and those of both Wolff and the marquise, even when taking into account their basic agreement on the truth of the principle of sufficient reason.

First Euler asks: "[I]s it it possible for them to explain how bodies would be composed of monads?"[26] He proceeds as follows:

> Monads, having no extension, must be considered as points in geometry, or as we represent to ourselves spirits and souls. Now it is well known that many geometrical points, let the number be supposed ever so great, never can produce a line, and consequently still less a surface, or a body…[I]t is an incontestable truth, that take any number of points you will, they can never produce extension. I speak here of points such as we conceive in geometry, without any length, breadth, or thickness, and which in that respect are absolutely nothing.[27]

I think Euler's objection here goes something like this (with some charitable filling in of the dots). Since monads are supposed to explain the composition of bodies, they should be able to explain the features of the composite. One of these features is the *extension* of bodies. But monads are like points in having neither length nor breadth nor thickness – in short, no extension. So how can any number of monads explain the extension of an extended object? They would have to give something that they don't have. The extension of an extended body, then, would seem to arise from nothing. And this itself runs afoul of the principle of sufficient reason. The explanatory

26. Letter CXXIX, II 211 / II 52
27. Letter CXXIX, II 211 / II 52-3

demand which motivated the monadologist to introduce simple beings in the first place is unfulfilled even on their introduction.

I think that Euler implicitly avows something like the following principle of explicability:

(EE) If x explains the extension of y, x must be extended.

The reasoning here seems to arrive pretty simply from the quoted passage. If some simple has no extension whatsoever, then putting more of these simples together, even infinitely many, cannot produce what was not there in the first place. The partisans of monads may say that these simples have qualities which "render them proper to produce the phenomenon of extension," but Euler rejects this as well.[28] He explicitly states it later on:

> [T]hese simple beings, which must enter into the composition of bodies, being monads which have no extension, neither can the compounds, that is bodes, have any extension.[29]

Further, her thinks that the partisans of monads hold it too:

> [H]aving hence deduced that bodies are compounded of simple beings, they are obliged to allow that simple beings are incapable of producing real extension, and consequently that the extension of bodies is

28. Letter CXXIX, II 212 / II 53
29. Letter CXXXI, II 219 / II 58

mere illusion.[30]

So this prong of Euler's criticism goes as follows. If extended beings are composed of parts, then the parts should be able to explain the extension of the compound. But if the ultimate, simple parts of an extended object are unextended, then the extension cannot be explained. So either, Euler says, "[w]e are under the necessity of acknowledging the divisibility of bodies in infinitum, or admitting the system of monads."[31]

Is a rapprochement possible?

So how are these conflicts to be solved? Euler, Wolff, and Du Châtelet alike claim to accept the principle of sufficient reason, and yet they reach opposite conclusions. Is there some way to reconcile these arguments?

Wolff's argument is, by my estimation, weaker than Du Châtelet's. As with his cosmological argument, it simply relies on the impossibility of an infinite regress without a good argument in favor of this impossibility, at least my my judgment. Dialectically, Euler is well within his rights simply to reject the impossibility.

And, indeed, there seem to be principled reasons to accept the infinite divisibility of extension, reasons that many early moderns were aware of. For example, a well-known argument for the infinite divisibility of extension was that if extension were *not* infinitely divisible, then many well-known theorems of geometry would simply turn out to be false. This is noted by multiple early modern authors. Isaac Barrow, for example, writes:

30. Letter CXXXI, II 220 / II 59
31. Letter CXXXI, II 220 / II 59

> How does not this [the finite divisibility of magnitude] destroy every Incongruity of Magnitudes, which is shewn by Geometricians in so many Examples, and supported with so many Demonstrations? Since a Point is the common Measure of all Magnitudes, and every one Magnitude is to another as a Number of Points to a Number of Points, if Lines consist of Points, Superficies of Lines, and Bodies of Superficies.[32]

Similarly, Antoine Arnauld and Pierre Nicole write in the *Port-Royal Logic* that:

> [Geometry] shows us that there are certain lines having no common measure, that for this reason are called incommensurable, such as the diagonal of the square and its sides. Now if the diagonal and the sides were made up of a certain number of indivisible parts, one of these indivisible parts would be a measure common to these two lines. Consequently it is impossible for these two lines to be made up of a certain number of indivisible parts.[33]

It's worth noting, however, that at least on this count Du Châtelet has something of a rejoinder. She believes that, while *abstract* extension is infinitely divisible, *actually existing* extension is not. Let's spell this out in more detail.

Du Châtelet thinks that Geometry is founded on "abstractions of our mind."[34] Points, lines, and surfaces are abstractions, since "there are nothing but solids in

32. Barrow 1734, 155
33. Arnauld and Nicole 1996, 231
34. IP 189

Nature."[35] Further, "[t]he greater part of Philosophers [have] confused the abstractions of our mind with Physical Bodies."[36] And it is this confusion which engenders the demonstrations that bodies are infinitely divisible.[37] While geometrical extension is infinitely divisible, physical extension, which is distinct from it, is not. The number of parts of geometrical extension is "absolutely indeterminate"[38] whereas "everything which actually exists must be determined in every way."[39]

Du Châtelet appeals to this distinction to solve Zeno's paradox of Achilles and the tortoise. While the paradox may apply to extension abstractly considered, it does not apply to actually existing physical extension:

> But secondly, this ingenious Sophism being founded on the divisibility of extension to infinity, the principle of sufficient reason overthrows it [*le renverse*] easily. For you have seen that it is proved by that principle, that Physical extension is at the last composed of simple beings, and that by consequence its divisions, even the possible ones, have real and positive limits.[40]

Du Châtelet's response to the problem raised above should now be clear: The incommensurability theorems adverted to above are true of *abstract* or *geometrical* extension, but not of *physical* extension. Geometrical extension is infinitely divisible, but physical extension has what she calls "primitive corpuscles," "those which have

35. IP 189
36. IP 190
37. IP 190
38. IP 190
39. IP 191
40. IP 192

the immediate reason of their composition in the Elements."[41] These elements are, of course, simple beings or monads (see chapter VII of IP).

Still, this does not answer Euler's fundamental question: How is it that unextended things can give rise to extended things? Where does this extension come from? Du Châtelet has an answer to this as well, but it is less-than-satisfactory. It is, in essence, the answer supposedly given by Samuel Johnson: "I have found you an argument, but I am not obliged to find you an understanding." She writes:

> The difficulty which one has of conceiving how simple and unextended Beings may by their assemblage compose extended Beings, and the aversion one has to admit simple Beings, are not a reason to reject them. This revolt of the imagination against simple Beings comes from our habit of[42] representing to ourselves our ideas under sensible images, which cannot help us here.[43]

But that is fundamentally *not* Euler's complaint, it seems to me. Rather than not seeing *how* it is that unextended things can make extended things by their composition, he appears to think that he *can see* its impossibility. He writes:

> They are right in saying their monads are not nothings, but beings endowed with an excellent quality, on which the nature of the bodies which they compose is founded. Now, the only question here is

41. IP 195
42. The French here is "*l'habitude ou nos sommes*." I have here rendered it as "our habit of" because that's the best way I can put what seems like the sense of the French into good English.
43. IP 158

> respecting extension; and as they are under the necessity of admitting that the monads have none, several nothings, according to them, would always be something.[44]

Conclusion

What is clear from the above, I think, is that the difference between Euler and Du Châtelet (and indeed Wolff) is *not* one of varying degrees of commitment to the principle of sufficient reason. Rather, it seems to turn on a question of what sorts of explicability principles each holds to *in addition* to a general principle of sufficient reason. There are also two subsidiary epistemological points of difference.

First, there is the question of what epistemic weight we should give to our imagination or similar cognitive processes. In the passages we examined above, it seems that Du Châtelet seems to accord low epistemic status to these imaginings. Euler, on the other hand, appears to rely on them heavily.[45]

Second, there is the question of what we might call that-explanations and how-explanations. A that-explanation is, or may be, an argument or a demonstration that something is the case. A how-explanation, by contrast, is a display of how it is that something comes about. (Think of the difference between a constructive and non-constructive proof in mathematics.) Apparently, Du Châtelet seems to think that all that's needed to say that unextended simple beings can give rise to extension is a that-explanation. But Euler appears to want a why-explanation.

44. Letter CXXVII, II 204 / II 47

45. Given his reliance on thought experiments in his demonstrations of Newton's laws, this shouldn't be surprising.

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