Idealization in Quantum Field Theory

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Abstract

This paper explores various functions of idealizations in quantum field theory. To this end it is important to first distinguish between different kinds of theories and models of or inspired by quantum field theory. Idealizations have *pragmatic* and *cognitive* functions. Analyzing a case-study from hadron physics, I demonstrate the virtues of studying highly idealized models for exploring the features of theories with an extremely rich structure such as quantum field theory and for gaining some understanding of the physical processes in the system under consideration.

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1 Introduction

In 1988, H. Brown and R. Harré edited a book that aimed at bringing quantum field theory (QFT) to the attention of philosophers of science. In the introductory section, the editors complain that QFT does not receive the attention in the philosophy of science community it deserves. And indeed, before 1988 only a few relevant publications appeared, including articles by J. Cushing (1982) and M. Redhead (1982). In the meantime, QFT has been gaining more interest; there are international symposia on the foundation of QFT, such as the Boston Colloquium in March 1996, and even a textbook is devoted to philosophical aspects of QFT (Teller 1995).[1]
Most relevant authors deal with *foundational problems* of QFT, such as the status of virtual particles and the niceties of renormalization. Especially the last topic has been discussed widely recently, mainly in the context of effective field theories (Cao & Schweber 1993).
Concerning *methodological problems* of QFT, much less work has been done so far. These are the problems J. Cushing, being skeptical that there are really new foundational problems in QFT, suggested to investigate (Cushing 1988). In this line of thought, Cushing himself studied, *inter alia*, the construction of a specific scientific theory (the S-Matrix program) and confronted his reconstruction with 'global' methodological models such as the ones by T. S. Kuhn and I. Lakatos (Cushing 1990).
Whereas Cushing focused on the development of scientific theories, I shall concentrate on the application of theories. More specifically, I shall deal with the role of idealizations in the various models and theories of or inspired by QFT. For a long time, philosophers ignored the investigation of idealizations from a philosophy of science point of view. One of the reasons for this neglect is, I understand, that idealizations were assumed to be motivated by purely pragmatic considerations only. In this paper I wish to show that idealizations have many more functions in science. I will especially focus on cognitive functions of idealizations by showing in a case-study how idealized models are used to explore the features of a theory.

The remainder of this article is organized as follows. Sec. 2 aimes at clarifying the terminology ('model', 'theory') and setting the stage for the case-study, that is presented in Sec. 3. In that section, three different models of hadron structure are discussed and related to the current philosophical debate of models and idealizations. Sec. 4 points out some more general metatheoretical implications of the case-study. Finally, Sec. 5 summarizes our main results.

2 Theories, Models and Idealizations

In this section I shall first define the terms 'theory' and 'model'. More specifically, I shall distinguish two different types of theories and two different typs of models. I will use this classification to give a survey of various kinds of idealizations performed in QFT.
It may be worth asking in parenthesis why it is necessary at all to define the terms 'theory' and 'model'. In fact, it turns out that this is a very difficult task, let alone the observation that there is a whole continuum of theoretical constructs (Suppes 1962, Balzer et al. 1987)

[1]See also (Auyang 1995) and (Clifton 1996).

so that a definition proper of 'theory' or 'model' is somewhat artificial. The very existence of this continuum is, however, one reason for the confusion in the philosophical literature on models. In order to avoid confusion I shall therefore start with a characterization of these terms in the way I use them in this paper.
Here, as well as in the whole paper, I restrict myself to the theories and models in the framework of QFT. But even within QFT it turns out to be difficult to exactly define what a model is and how it ought to be distinguished from a theory. Quite often, the terms are used interchangably by scientists. The term 'model' is, however, frequently preferred. Scientists tend to be reluctant to name their constructs a theory. Some constructs, however, reach this blessed status.

I propose the following characterization that is, I maintain, in accordance with the scientists' use of these terms. The term 'theory' is reserved only for the following two theoretical constructs.

Type A Theory: General Background Theories

> A Type A Theory is a theoretical construct that needs to be specified by additional assumptions (a 'model object' – in M. Bunge's terminology[2]) in order to be applicable to a concrete object or system.[3]
>
> The general formailism of QFT is a Type A Theory. The Wightman axioms, for instance, provide the framework for all concrete realizations of QFT such as quantum electrodynamics (QED) or quantum chromodynamics (QCD).[4]

Type B Theory: Fundamental Model-Theoretical Models

> A Type B theory is a model-theoretical model of a Type A Theory, that is a concrete realization of the general formalism of QFT. The attribute 'fundamental' is added since *at the moment* everything that can be said about the respective domain of applicability can be captured by these theories.
>
> QED and QCD are typical examples for Type B Theories.

All other theoretical constructs are called 'models' of which I shall also distinguish two types. These are:

Type A Model: Non-Fundamental Model-Theoretical Models

> A Type A Model is a model-theoretical model of a Type A Theory that is not a Type B Theory. Some of these models have, so far, no application at all ('toy models'[5]), others serve as an input for more fundamental constructs, such as models that provide certain symmetry breaking mechanisms.
>
> The frequently used φ^4–theory is a typical example for a Type A Model.

[2]See (Bunge 1973, 100 f).
[3]This is also the kind of construct N. Cartwright has in mind when talking about theories, see her (1983).
[4]See (Haag 1992) and (Straeter 1988).
[5]See my discussion in (Hartmann 1995a, 57f).

Type B Model: Phenomenological Models

A Type B Model is a set of assumptions about some object or system (Redhead 1980). Some of these assumptions may be *inspired* by a Type B Theory, others may even contradict the relevant theory (in case there is any). Theory (of Type A or B) here serves as *one* tool for the construction of models (Cartwright et al. 1995).

How does a phenomenological model relate to theory? Two cases have to be distinguished: If there is only a Type A Theory (and no relevant theory of Type B), the model exhibits additional structure that cannot be deduced from that theory. If the considered Type B Model is a model of a Type B Theory (and in this sense a *model of a theory*) then the relation is very delicate; I shall come back to this in the next section. Exact deduction is often not possible and even if it has been achieved does not provide much insight.[6] Certainly, deduction from theory was not the way the model in question has been developed.

Among the many examples of Type B Models I shall mention only the various models of nuclear and hadron structure. It is interesting to note that these models were studied before *and* after the development of QCD, the supposedly relevant Type B Theory. Why?

This category is a residual category. All theoretical constructs that do not fit in any other category described above are Type B Models.

When applying theories or models, theoretical physicists make use of various approximations and idealizations. Furthermore, idealizations are involved in the construction process as well. Following E. McMullin, I shall take the term ‘idealization’ to “signify a deliberate simplifying of something complicated (a situation, a concept, etc.) with a view to achieving at least a partial understanding of that thing.” (McMullin 1985, 248) It is not an easy task to clearly distinguish between approximations and idealizations[7]. One way to distinguish both is by *function*. I suggest that the one and only motive to perform an approximation is that it allows one to treat problems that cannot be treated otherwise. Besides this *pragmatic* function, idealizations have additional *cognitive* functions that I will focus on in the reminder.
Before considering a detailed case-study from hadron physics I shall survey different kinds of idealizations performed in QFT. In doing so, I follow the classification of theories and models given above.

1. Idealization in Type A Theories

   Here it is often not known which (if any) of the assumptions of the theory are in fact idealizations.[8] There are, nevertheless, some candidates:

   (a) Fundamental Entities

   QFT assumes that the fundamental entities (electrons, quarks, etc.) are point particles. However, in case some variant of string theory will replace QFT as

[6]See (Cartwright 1983) and (Hartmann 1995b, chap. 2.).
[7]See (Laymon 1995), (Moulines 1996) and (Redhead 1980).
[8]For a strategy to detect such idealizations see Laymon (1985).

the fundamental background theory, this assumption will be merely a – albeit good – idealization, valid presumably only at low energies.[9]

(b) Spacetime Dimension

Furthermore, current superstring theories are defined on a spacetime manifold with more than 4 dimensions. Then, of course, working with only 4 spacetime dimensions (as QFT does) will turn out to be an idealization.

2. Idealization in Type B Theories

Since Type B Theories add structure to a given Type A Theory, all idealizations concerning the corresponding Type A Theory are also idealizations of the Type B Theory. Furthermore, there are supplementary idealizations in Type B Theories. Here I shall discuss only Type B Theories. There are several ways to detect idealizations in a Type B Theory in practice. Just like in the case of a Type A Theory, comparison with the successive theory will help. Also, searching for internal or external inconsistencies may serve as a litmus test for possible idealizations.

(a) The Role of Spacetime

It is a fundamental assumption of QFT that the underlying space-time is flat. Furthermore, it is assumed that the spacetime structure itself is not affected by the presence of matter fields. From the perspective of general relativity, however, this is false (external inconsistency). There *is* a back-reaction of matter on space-time. Taking this into account leads, however, as research in recent years has demonstrated, to various diffculties (Birell & Davies 1982). A future quantum theory of gravity is expected to solve these problems. In any case, the assumption of a flat space-time is an idealization.

(b) Influence of Other Interactions

When applying a Type B Theory (such as QCD) one often neglects that there are, strictly speaking, also contributions from other theories (such as QED) to the processes under consideration. When studying, for example, proton–anti-proton scattering at very high energies, physicists usually only consider strong interactions. Due to their electrical charge these particles do, however, also interact electromagnetically and weakly. This neglect is reasonable for it is known that the corresponding corrections are negligibly small.

(c) Fundamental Entities

Even if strings won't constitute the fundamental ontology, current leptons and quarks may still not be pointlike. They may be extended objects just like protons and neutrons. For a long time, these particles were also assumed to be pointlike. In this case, leptons and quarks had a sub-structure that (supposedly) does not affect the physics at sufficiently low energies. Then, current leptons and quarks had the status of *effective degrees of freedom*.

(d) Spacetime Dimension

When applying a Type B Theory, physicists sometimes reduce the number of space-time dimensions. This is done for calculational purposes only. It is

[9] The relevant energy scale will be set by the new theory.

hoped, though, that such an analysis suggests features that are present also in four dimensions. G. t'Hooft, for example, demonstrated confinement in QCD_2, that is QCD in 1 (space) +1 (time) dimensions. This has not been achieved in 4 dimensions, although the relevant sub-community seems to be convinced that confinement is a feature of QCD in 4 dimensions.[10]

3. Idealization in Type A Models

   Idealizations are much more obvious in the case of models.

   Here are some of them:

   (a) Spacetime Dimension

   Working in 1+1 dimensions also proves to be a helpful way to analyze features of a Type B Model. Furthermore, those models may occasionally be relevant for practical calculations such as 2 dimensional models in condensed-matter physics for the investigation of surface phenomena.

   (b) Entities

   Fields that show up in the Lagrangian density of a QFT model are, by definition, fundamental: they have no structure. For Type A Models this is either an idealization or the entities do not relate to anything that is supposed to belong to the physical reality at all ('toy model').

4. Idealization in Type B Models

   Here are some additional idealizations that are typical for Type B Models:

   (a) Treat Some Fields Classical

   It is often very helpful to treat some fields showing up in the model classical. For example, in ordinary laser theory, the radiation field is treated as a classical (that is un-quantized) field while the atomic degrees of freedom are quantized. It should be noted that this move can be motivated physically. Nevertheless, procedures like this are conceptionally somewhat delicate.

   (b) Non-Renormalizable Models

   It has been maintained for a long time that non-renormalizable QFT models are inconsitent. The non-curable infinities in higher order perturbation theory seem to demonstrate that the model is at its best provisional. On the other hand, cutting the 'responsible' integrals off at some finite cut-off Λ appeared to be arbitrary.

   In the meantime the program of effective field theories[11] (EFT) provided some interesting new insights concerning the nature of non-renormalizable theories. First, the cut-off can be physically interpreted: It reflects the energy scale of the domain of applicability of the EFT in question. Secondly, it has been shown that at a given energy scale E a suitably choosen EFT can be fundamental in the sense that (practically) nothing more can be said about the ongoing physics. The error is proportional to $(E/\Lambda)^2$ (Cao & Schweber 1994).

[10]We shall come back to this in Sec. 3.

[11]See (Weinberg 1995 & 1996) for a textbook exposition.

This list demonstrates a popular view on idealizations, namly the statement that idealizations are a vice. Physicists have to idealize in order to perform calculations that would be impossible to do otherwise.

I wish to challenge this view by pointing out other functions of idealizations. The following case-study serves as a basis for my subsequent analysis. The case-study is taken from a part of contemporary phyiscs that did not receive much attention in the philosophy of science literature so far: hadron physics.[12]

3 Case Study: Hadron Physics

Hadrons are strongly interacting particles such as protons, neutrons and pions. *Hadron physics* is the branch of particle physics that aims at understanding their structure and (strong) interactions. Here is a short sketch of the history of hadron physics.[13]

In 1932 Cavendish physicist J. Chadwick produced in a series of experiments electrically neutral particles with almost the same mass as the positively charged hydrogen nucleus (later called proton). This astonishing observation marked the begining of hadron physics. It soon turned out that atomic nuclei could be understood as composed systems of protons and neutrons. W. Heisenberg (1935) tried to take advantage of the similarities between protons and neutrons (now called nucleons) by introducing the isospin concept in analogy to the spin concept familiar from atomic physics and Japanese physicist H. Yukawa (1934) proposed a dynamical model for the short-ranged interaction of nucleons. Subsequently these theoretical works were extended, but a real boost did not occur before a wealth of new particles ('resonances', 'hadron zoo') were directly produced in the labs after the second World War. Besides, the analysis of cosmic-ray data revealed the existence of new particles.

These findings inspired the development of a variety of phenomenological models that attempted to organize and systematize these data. I shall here only mention the (more theoretical) investigations in the context of current algebra and, of course, the famous (more phenomenological) quark model, suggested independently by M. Gell-Mann and G. Zweig in 1964.[14]

Relying on analogies to quantum electrodynamics and with the (now somewhat dubious) requirement of renormalizability (*pace* EFT) in mind quarks proved to be an essential part (besides gluons) of the ontology of the then-developed non-abelian gauge QFT of strong interactions, QCD, in 1971. This theory is currently supposed to be *the* fundamental theory of strong interactions.

QCD has three characteristic features (Weinberg 1996, 152 f) that I shall explain now in some detail for it is needed for our subsequent discussion of the various models of QCD.

1. Asymptotic Freedom

   At very high energies (compared to the rest mass of the proton) quarks move quasi-free. This has been demonstrated (though indirectly) in accelerator experiments at facilities such as CERN near Geneva, Fermilab near Chicago and SLAC at Stanford.

[12] See, however, (Giere 1988, 179 f) and (Hartmann 1996 and forthcoming).

[13] For details see (Pais 1986). The history of QFT is masterly covered in (Schweber 1994).

[14] See also (Cushing 1990) and references cited therein.

Cosmological models tell us that these energies were in fact realized in the early universe. Here, the interaction between quarks, characterized by an effective ('running') coupling constant $\alpha_s(q^2)$ ($-q^2$ is the 4-momentum transfer), monotonously approaches zero. This feature, called asymptotic freedom, allows it to successfully apply perturbation theoretical tools, well-known from QED, in this regime (Field 1989).

At low energies, on the other hand, the opposite effect occurs. For decreasing momentum transfer $\alpha_s(q^2)$ increases and soon exceeds 1, making a perturbation theoretical treatment dubious and practically impossible.

This is the regime, where confinment and chiral symmetry are important.

2. Quark Confinement

   Quark confinement ('confinement' for short) has been suggested to account for the fact that so far no single free quark has been observed in experiments or cosmic-ray data. Quarks seem to be always clumped together in baryons or mesons[15]. It should be noted that so far it has not been demonstrated analytically that confinement is a consequence of QCD. It even does not seem to be exactly clear what confinement really is. I shall come back to this.

3. Chiral Symmetry

   Chiral symmetry and its dynamical breaking is the second important low-energy feature of QCD. Unlike confinement we know much better what it means.

   Chirality is a well-known property of many physical, chemical and biological systems. Some sugars, for example, only show up in a right-handed form. If there were a left-handed version with the same frequency as well, the system would be chirally symmetric. In this case the interaction would not distinguish between the left- and the right-handed version.

   There are also left and right-handed states in QFT. It can be shown that a QFT with explicit mass terms in its Lagrangian cannot be chirally invariant. Chiral symmetry is (almost) realized in the low-energy domain of QCD, because the current quark masses (of the relevant quarks in this regime) are small (about 10 MeV) compared to the rest mass of the proton (about 1000 MeV). Therefore, every eigenstate of the interaction should have a chiral partner with the same mass but opposite parity. Experiments, however, do not support this hypothesis. There are no chiral partners with the same mass but opposite parity.

   A way out of this messy situation is to assume that the interaction itself breaks the symmetry *dynamically.* As a result of this supposed mechanism, an effective quark mass is generated.[16]

[15] Baryons have three valence quarks, mesons are quark-antiquark composites.

[16] In the mathematical framework, an explicit mass term will then show up in the corresponding effective Lagrangian. This mass is, by the way, easily identified with the (dressed) constituent quark mass used in non-relativistic Constituent Quark Models ($m_{CQM} \approx 300$ MeV). Here, a field theoretical mechanism provides, therefore, a deeper motivation and theoretical legitimation of the CQM used for a long time without such a motivation.

It is theoretically well-established that confinement and dynamical chiral symmetry breaking cannot be obtained in a perturbation-theoretical analysis of QCD. They are low-energy phenomena and perturbation theory breaks down in this regime. Therefore, an infinite number of Feynman diagrams has to be added up to obtain those phenomena.
Summing up an infinite number of diagrams is, however, not an easy task. There are two entirely different ways to proceed: Lattice-QCD and modeling.
In Lattice–QCD the complete QCD action is replaced by a discretized action on a space-time lattice (Rothe 1992). This is, of course, an approximation. It is hoped, though, to get exact results of QCD by extrapolating the numerically generated results to zero lattice spacings. This method is presently the only way to test QCD in the low-energy regime. And for this reason, Lattice–QCD is certainly of utmost importance.
Despite this advantage, Lattice–QCD faces some serious problems. Some are technical (e. g. fermion doubling) and I shall not discuss them here (Rothe 1992), others are more conceptual.
Physicist T. Cohen points out the following problem:

> [W]hile high-quality numerical simulations may allow us to test whether QCD can explain low-energy hadronic phenomena, they will not, by themselves, give much insight into how QCD works in the low-energy regime. Simple intuitive pictures are essential to obtain insight, and models provide such pictures. (Cohen 1996, 599)

Cohen explains:

> Condensed-matter physics provides a useful analogy: even if one were able to solve the electron-ion many-body Schrödinger equation by brute force on a computer and directly predict observables, to have any real understanding of what is happening, one needs to understand the effective degrees of freedom which dominate the physics, such as photons, Cooper pairs, quasiparticles, and so forth. To gain intuition about these effective degrees of freedom, modeling is required. In much the same way, models of the hadrons are essential in developing intuition into how QCD functions in the low-energy domain. (Cohen 1996, 599 f)

In the reminder of the paper I will elaborate on this important point.
Confronted with the complexity of QCD physicist S. Klevanski also argues for modeling:

> One is . . . strongly motivated to look for a simpler model Lagrangian density that displays one for more of the essential features of QCD, but that is mathematically tractable. *We would then be able to explore the consequences of the features we have isolated as relevant.* Such an approach is a common one to both nuclear and solid-state physics: in nuclear physics, for example, the many excited states available to a nuclear system via the continuum may be averaged out to provide an coptical-model potential. In doing this, the N-body problem is reduced to a one-body problem for the purpose of calculating reaction rates. Basic to this approach is the recognition that many-body systems hare only exactly soluble in exceptional or oversimplified situations.

> Thus it is sensible to attempt to *isolate the relevant physics* for a process and to study this either by making an approximation cto the exact theory, or as is commonly done for a theory as intractable as QCD, by creating models that serve to accentuate the main features of the theory. (Klevanski 1992, 650, my emphasis)

Isolating the relevant physics is, however, not an easy task. Here, physics comes closesed to art (Redhead 1980). But even when we know parts of the relevant physics it might be ambiguous to model it. Confinement is a point in case. There are at least four different types of it.
In the reminder of this section I shall investigate in some detail how confinement and dynamical chiral symmetry breaking are modeled and how the consequences of these features are explored. I shall also study how de-idealization proceeds and how the corresponding models relate to QCD itself.
The models I shall discuss are:

- The MIT-Bag Model (Sec. 3.1)

  This model explores the consequences of confinement only.

- The Nambu–Jona-Lasinio Model (Sec. 3.2)

  This model explores the consequences of chiral symmetry and dynamical chiral symmetry breaking only.

- The Chromodielectric Soliton Model (Sec. 3.3)

  This model is the most ambitious model. It exhibits confinement as well as a mechanism for dynamcal chiral symmetry breaking.

3.1 The MIT-Bag Model

The MIT-Bag Model is a very simple model.[17] Developed in 1974 at the Massachusetts Institute of Technology in Cambridge (USA), shortly after the development of QCD, it soon became a major tool for hadron theorists. It essentially models confinement.[18]
In the MIT-Bag Model, confinement means that quarks are forced by an external pressure to move only inside a given spatial region (the bag). Therein quarks occupy single-particle orbits similar to nucleons in the shell model.[19] In the model's simplest variant the bag-shape is spherical. This holds true exactly for quarks in the ground state. When considering higher excitations, also non-spherical shapes have to be considered. This raises, however, additional technical problems (similar to the problems in nuclear physics with non-spherical nuclei).

[17]For a clear exposition of the model and its application see (Bhaduri 1988) and (DeTar & Donogue 1983).
[18]It is, however, sometimes claimed that asymptotic freedom is also taken into account, because the quarks move freely inside the bag.
[19]This is no surprise. The MIT-Bag Model was developed by nuclear physicists.

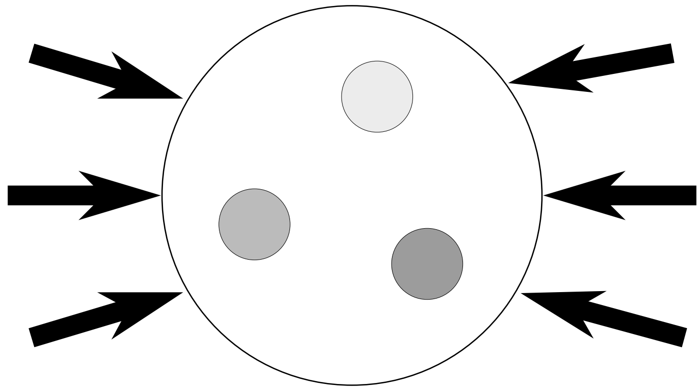

Figure 1: A baryon in the MIT-Bag Model

After presenting the general idea, let us now turn to the mathematical formulation. This is quite straightforward. The quark wave functions inside the bag are determined by solving the Dirac equation. The potential inside is supposed to be zero, while an appropriate boundary condition (the quark-flux through the surface must vanish) models an infinite outside–potential. This boundary condition leads – just like in ordinary quantum mechanics – to discrete energy eigenvalues of the bag. In the (easiest) case of massless quarks these energies scale for dimensional reasons with $1/R$, where R is the (yet undetermined) radius of the bag:

$$\epsilon_n = \frac{x_n}{R} \tag{1}$$

The x_n are solutions of a transcendental equation that can be easily solved numerically. The lowest value is $x_1 \approx 2.04$. Since we consider a collection of N_q ($N_q = 3$: baryons, $N_q = 2$: mesons) valence quarks, the total *kinetic* energy is

$$E_{kin}(R) = N_q \frac{x_n}{R} \quad . \tag{2}$$

Here it is assumed that all quarks are in the same (orbital) state.
The stabilizing potential energy of the bag results from the external pressure due to the boundary condition. It is given by

$$E_{pot}(R) = \frac{4}{3}\pi R^3 B \quad ; \tag{3}$$

B is the so-called bag-constant that reflects the bag pressure.
The total bag energy is the sum of both contributions:

$$E(R) = E_{kin}(R) + E_{pot}(R) \tag{4}$$

Minimizing $E(R)$ with respect to R yields the equilibrium radius of the system and, subsequently, the total bag energy:

$$R_n = \left(\frac{N_q x_n}{4\pi B}\right)^{1/4} \quad , \quad E_n = \frac{4}{3}\left(4\pi B N_q^3 x_n^3\right)^{1/4} \tag{5}$$

This is the model in its easiest version. The only parameter B is adjusted in order to get a best fit of hadronic observables (masses, charge radii etc.). Since the model is – at this

stage – pretty crude, the resulting phenomenology (= values for hadron masses and radii) is only fairly modest. When fixing the nucleon mass to its empirical value ($m_N = 938$ MeV), for example, the nucleon radius comes out to be $R_N = 1.7$ fm (compared to roughly 1 fm) and the pion mass comes out to be $m_\pi = 692$ MeV (compared to 138 MeV).
The MIT-Bag model received a lot of interest and has been improved in several ways. I shall here discuss only three of these improvements and pay special attention to the motivation of the corrections and additions as well as their relation to the theory to be modeled, QCD.

1. The One-Gluon-Exchange (OGE)

   So far the model neglects the mutual interaction of the quarks completely. There should be at least some residual interaction that is not effectively contained in the bag-pressure. There is also a phenomenological argument for quark interactions. So far, hadrons with quarks in the same overall orbital state should be degenerate. This is, however, empirically not the case. The nucleon and the Δ particle, for example, differ by roughly 300 MeV, that is one third of the mass of the nucleon.

   This can be fixed by adding an aditional term. It is supposed to reflect the perturbative one-gluon-exchange interaction between quarks. The corresponding energy conttribution is:

   $$E_X = \frac{\alpha_s \, \mathcal{M}_{qq'}}{R} \tag{6}$$

   with a constant matrix element $\mathcal{M}_{qq'}$ that depends on the single particle states (orbital, spin, etc.) of the quarks. Furthermore, α_s is the (constant!) strong coupling constant. It is treated as another adjustable parameter.

   The OGE term is phenomenologocally quite successful. Some problems remain, though:

   - The phenomenological value of α_s turns out to be larger than 1. This raises doubts concerning the legitimation of a perturbation-theoretical approach.
   - The motivation of the MIT-Bag Model and ist relation to QCD is unclear. Due to the static boundary condition it is very diffcult to relate it mathematically to QCD. This, of course, also holds for the OGE term. I conclude that the MIT-Bag Model itself is largely autonomous (Cartwright 1983).

2. The Casimir-Term

   This is an additional contribution to the energy of the form

   $$E_{Casimir} = \frac{Z}{R} \tag{7}$$

   with a parameter Z that can, in principle, be calculated. In practice, it is, however, usually treated as an additional adjustable parameter (for good reasons, as we will see below!).

Here is its physical motivation. The Casimir term is supposed to represent the zero-point energy of the quantum vacuum. It is a well known feature of QFT that this contribution cannot be turned off.

The term was first suggested in the context of quantum electrodynamics. Dutch physicist H. Casimir showed that two parallel conducting plates attract each other due to the presence of the quantum vacuum field (Milonni 1992). The case of a spherical cavity is, for theoretical reasons, much more complicated.

Having an additional parameter, the Casimir term surely improves the phenomenology of the MIT-Bag Model. But also this term has some problems. The main problem is that theory suggests that the term must be negative, while the best fits are achieved by taking a slightly positive value[20].

3. Center-of-Mass Corrections

   This is a really strange correction. It had to be introduced in order to remove known side effects of the basic assumptions of the model. Neglecting them right from the beginning would imply working with a completely different model.

   As a consequence of the cavity approximation the center-of-mass of the many-boby state is inevitably in motion and the corresponding kinetic energy is included in the total energy. This contribution should be removed from the bag energy to obtain the mass. Even in the non-relativistic case this is not an easy task; an exact relativistic treatment is not possible (Wilets 1989). Only approximate approaches are known.

   The simplest way is to evaluate the rest mass like this

$$m = \sqrt{E^2 - < p^2 >} \quad . \tag{8}$$

   Here $< p^2 >$ denotes the expectation value of the squared momentum of the composite system. There are better though more complicated procedures to remove the unwanted ("spurious") center-of-mass excitations (Wilets 1989).

Besides these modifications, the MIT-Bag Model inspired the construction of a variety of related models. I shall mention only two types of them:

1. The Cloudy–Bag Model

   This model reacted to the explicit breaking of chiral symmetry in the MIT-Bag Model – a serious draw-back of this model – by adding an aditional (vector-isovector) pion field that couples to the quarks at the bag surface in a chirally invariant manner.

   This and related models are much more difficult to treat mathematically. Nevertheless, they have been used quite a lot in the literature.[21]

[20] See (DeTar & Donogue 1983), (Plunien, Müller & Greiner 1987) and (Wilets 1989).

[21] See (Bhaduri 1988) for a survey.

2. Soliton Models

   This class of models was inspired by another well-known flaw of the MIT-Bag Model. It turns out that dynamical problems cannot be analysed within the MIT-Bag Model because of the static boundary condition. Soliton Models add a dynamical mechanism that generates confinement by making the bag-surface an explicit physical degree of freedom. This is achieved by introducing a scalar field σ that couples to the quarks (thereby, though, violating chiral symmetry) in the easiest version of the model.[22]

3.2 The Nambu–Jona-Lasinio Model

The Nambu–Jona-Lasinio (NJL) Model is a simple field theoretical model that provides a mechanism for dynamical chiral symmetry breaking.[23] According to this model, quarks are not confined. In its extended versions, the model serves currently as a model for low-energy QCD: When it was introduced in 1961, its purpose was somewhat different. (This, by the way, demonstrates the flexibility of these models.) However, the wanted mechanism was inspired by an analogy between the Dirac-theory and superconductivity, as is already noticed in the title of the original publication (Nambu and Jona-Lasinio 1961).

Similar to the MIT-Bag Model the only degrees of freedom are quarks. Gluons, the mediators of the interaction between the quarks, are only effectively taken into account. The short-ranged quark-quark interaction is approximated to be point-like (this is a good approximation for low momenta), gluons are the "frozen in".

Here is the Lagrangian density of the NJL Model:

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + \mathcal{L}_{int} \tag{9}$$

with the free (= non-interacting) Dirac part

$$\mathcal{L}_0 = \bar{q}(\mathrm{i}\gamma_\mu \partial^\mu - m)q \tag{10}$$

(q is the quark wave function) and the interaction part (in Flavour–$SU(2)$)

$$\mathcal{L}_{int} = G\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma_5 \tau q)^2\right] \tag{11}$$

G is a coupling constant with the dimension *length*^{2}. The model has three main features:

- The NJL Model is non-renormalizable

  This is a consequence of the fact that the coupling constant has dimension *length*^{2} (Weinberg 1996). The model is therefore not defined unless a suitably chosen (momentum) cutoff Λ is specified. This cutoff reflects the energy scale where the model is applicable.

[22] More on soliton models and chiral variants of it can be found in (Wilets 1989). See also Sec. 3.3.

[23] A recent review of the NJL Model is (Klevanski 1992).

- The NJL Model is chirally invariant

  For $m = 0$ the Lagrangian density is chirally invariant. For $m \neq 0$ this symmetry is explicitly broken. Since the relevant current quark masses are small compared to the rest mass of the proton, this does not change the features of the model considerably.

- The NJL Model provides a mechanism for dynamical chiral symmetry breaking

  This remarkable feature is already "visible" in the Mean-Field-Approximation (MFA). Here is the Lagrangian density in this approximation:

$$\mathcal{L}_{NJL}^{MFA} = \mathcal{L}_0 + \bar{q}Mq \tag{12}$$

  The "effective" quark mass M is a solution of the following self-consistent equation:

$$M = \frac{2GM}{\pi^2} \int_0^\Lambda \frac{p^2 dp}{\sqrt{p^2 + M^2}} \tag{13}$$

  This equation has a non-vanishing solution in case G exceeds a critical coupling strength G_{crit}. In this regime, chiral symmetry is dynamically broken.[24]

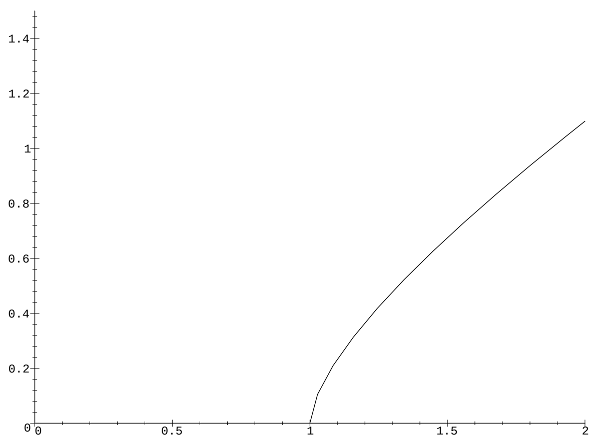

Figure 2: Dynamical chiral symmetry breaking in the NJL Model

Dynamical chiral symmetry breaking has several consequences, many of which can be demonstrated in the NJL Model. Firstly, according to Goldstone's Theorem, a massless boson (the 'Goldstone boson') is created. The corresponding Goldstone boson of chiral symmetry is the pion that comes out to be massless for vanishing current quark masses. Secondly, famous low-energy theorems, such as the Gell-Mann–Oakes–Renner relation

[24] This equation is called 'gap equation' in analogy to a similar effect in superconductors.

$$m_\pi^2 f_\pi^2 = -\frac{m_u + m_d}{2} < \bar{q}q > \tag{14}$$

can be derived from the model. This relation accounts for the finite pion mass due to an (additional) explicit breaking of chiral symmetry.[25]

In order to apply the NJL Model for phenomenologocal purposes it has been extended in several ways. Here are some of them:

1. More quark flavours

   Including strange quarks made it possible to calculate the properties of many other mesons (apart from the pion).

2. Include Other Chirally Invariant Interactions

   Including these interactions results in better fits, not only because of the additional parameters that can be adjusted. Besides mesons, some research groups also investigate the structure of baryons in this model. These calculations are extremely complicated from a mathematical point of view.

3. Finite temperature

   The NJL model has also been extended to finite temperature. This is a natural extension for the underlying Type A Theory accounts for this. In this context, physicists studied hadron properties as a function of temperature and the nature of a supposed phase transition to a phase where chiral symmetry is restored.

3.3 The Chromodielectric Soliton Model

The Chromodielectric Soliton (CDS) Model is the most ambitious of all the models discussed so far. It exhibits mechanisms for confinement *and* dynamical chiral symmetry breaking and may, therefore, serve as a tool to investigate the relation between both features of QCD.[26]

The CDS Model is a chiral version of the already mentioned class of soliton models (Sec. 3.1). By adding additional explicit gluon degrees of freedom it is made chirally invariant. The gluons couple to the quarks as well as to the σ-field, and thereby mediate the quark-σ-interaction.

Here is the Lagrangian density of the CDS Model:

$$\mathcal{L}_{CDS} = \bar{q}\ \mathrm{i}\gamma_\mu D^\mu q - \frac{\kappa(\sigma)}{4} F_{a\mu\nu} F^{a\mu\nu} + \frac{1}{2}(\partial\sigma)^2 - U(\sigma) \tag{15}$$

In this equation, q represents the quark field, D^μ is the covariant derivative that accounts for the quark-gluon interaction and $U(\sigma)$ is the potential of the σ field. Note that this Lagrangian density equals the QCD Lagrangian density in case the color-dielectric function $\kappa(\sigma) \to 1$.

[25] In this equation, f_π is the pion decay constant, m_π, m_u and m_d are the masses of the pion and of the u– and d–quark, respectively, and $< \bar{q}q > \approx (-250MeV)^3$ is the quark condensate.

[26] An exposition of the model and some of its applications can be found in (Krein et al. 1991) and (Wilets et al. forthcoming).

It is interesting to note that this model is, therefore, more complicated than the theory it is supposed to model, QCD. The CDS Model exhibits an additional field and its interactions. So why not solving QCD directly? It turns out the additional σ-field cotains contributions of the gluon field that would show up in a QCD treatment only in higher orders of perturbation theory.
The color dielectric function $\kappa(\sigma)$ is modeled in order to obtain color-confinement. Color-confinement means that only color neutral states are physically observable.
Here is the idea, developed by T.D. Lee (Wilets 1989): Consider the total energy W of a (color-) charge Q_c in a color-dielectric medium characterized by a position dependent color-dielectric function $\kappa(r)$. This energy is given by

$$W \sim \int \vec{E} \cdot \vec{D} d^3 r \sim Q_c^2 \int_0^\infty \frac{dr}{r^2 \kappa(r)} \tag{16}$$

For suitably chosen functions $\kappa(r)$ this integral diverges. Then W is finite only if the total (color-) charge vanishes: $Q_c = 0$. This is just color-confinement.

Choosing $\kappa(\sigma(r))$ appropriately therefore guarantees color-confinement. It has been demonstrated that the CDS Model exhibits spatial confinement and dynamical chiral symmetry breaking with this assumption (Krein et al. 1991).
(Wilets et al. forthcoming) used the CDS Model to determine hadronic observables, such as the mass of the nucleon and the mass of the pion. These calculations are extremely (computer-) time consuming. In order to perform them nevertheless, further mathematical truncations of the model were necessary. It is, therefore, very hard to interpret the resulting data and to decide, whether deviations from experimental data result from (wrong) model assumptions or from various approximations made in the process of solving the model.
Models, such as the CDS Model, that model more than one feature of some fundamental Type B Theory face an even more serious problem. They do not lead to reliable insights concerning the *relation* between the modeled features. Are we allowed to conclude from the CDS Model that dynamical chiral symmetry breaking is a consequence of the more fundamental assumption of confinement? I don't think so. Furthermore, since both features are modeled in a specific way, we cannot expect their combined effect in the model to resemble in any sense their combined effect in QCD. This observation stresses again the importance of studying simple models that allow physicists to explore the consequences of one single feature of a theory.

4 Lessons

I shall now draw some more general ('philosophical') conclusions from the material presented in the last section.
Let me start by asking for the motives for performing idealizations. Ever since the times of Galilei it was very important that idealizations allow scientists to solve a problem that turned out to be mathematically too complex. This is the *pragmatic aspect* of idealization. Although this aspect is still of some importance, I shall argue that this is not the main reason to perform an idealization in contemporary physics, for current high-powered computers allow us to solve highly complicated equations exactly, and due

to the rapid development in computer science and technology that motive to idealize will become even less important in the future (Hartmann 1996).

But even when we can solve whatever equation we want on a computer, physicists will certainly not stop investigating idealized models. This is because of the *cognitive role* of idealizations. Idealized models give us a partial understanding of the relevant mechanisms for the processes in the system under study. They alllow us, for example, to get some insight into the highly complicated dynamics inside a hadron by describing the physics in terms of (simple) effective degrees of freedom. In practice, this is done by extracting a single feature of a theory (in our example QCD) and exploring their consequences, such as confinement in the MIT-Bag Model or chiral symmetry and dynamical chiral symmetry breaking in the NJL Model, in a *numerical experiment*.[27]

I have stressed the importance of simple models that explore only the consequences of one single feature. Improving on models such as the ones presented in the case-study in order to make them empirically more adequate leads to the following problems: Firstly, the model under consideration will never be empirically adequate provided that the underlying theory (QCD) is in fact fundamental and therefore assumed to be empirically adequate. Since the model is not the full theory, it will never agree with all relevant empirical data. Secondly, models that aim at becoming empirically more adequate than others (such as the CDS Model) do not give us much insight into the dynamics, for they are too complicated.

This brings me to a related point. If empirical adequacy were the only goal scientists aim at (van Fraassen 1980), the practice of science could not be understood. Why do physicists work with models such as the MIT-Bag Model or the NJL Model? Why don't they contribute to the program of Lattice-QCD? There are certainly pragmatic (and possibly social) motives for this. Models are easier to apply, they allow the calculation of observables that cannot (yet) be calculated in full QCD. But still, I hope to have shown that there is more to modeling than this.

Modeling may have many aims. I wish to argue – jointly with T. Cohen – that *gaining understanding of the processes involved by means of exploring the consequences of single features of a theory* is certainly very central to the very idea of modeling.

Having argued that the main purpose of these models is not empirical adequacy, how can we understand that scientists try to make their models more realistic anyway? A way to understand this is to realize that improving a model phenomenologically supports the overall aim to explore the consequences of the feature under consideration. Which observables can be adequatly numerically reproduced by assuming only that feature alone, which ones need additional assumptions and mechanisms?

Here is another argument why I maintain that models are not made 'more realistic' in the first place in order to improve on their empirical adequacy, but in order to learn something about isolated features. Why do scientists stop at some point to explore a given model? In principle they could 'de-idealize' the model ad infinitum. Instead, at some stage they consider a new model that allows them to clarify the physics of other features of the underlying theory.

As a consequence of this observation, I do not see much evidence (in hadron physics) for E. McMullin's thesis that the practice of de-idealizing a model supports the idea of

[27]See (Humphreys 1991) and (Rohrlich 1991).

scientific realism. McMullin writes

> Formal idealization is . . . a quite powerful epistemic technique, not only the original model already supports theoretical laws that account approximately for some of the regularities to be explained, but even more because the 'adding back', if it accounts for additional experimental data and especially if it leads to the discovery of new empirical laws, is a strong validation for the model and its accompanying theory, within the limits of the idealizations employed. Indeed, this becomes a strong (though not conclusive) argument for the existence of the structures postulated by the model. If the original model merely 'saved the phenomena' without in any way approximating to the structure of the object whose behaviour is under scrutiny, there would be no reason why this reversal of a simplifying assumption, motivated by the belief that the object *does* possess something like the structure attributed to it, would work as it does. Taking the model seriously as an approximately true account is what leads us to *expect* the correction to produce a verifyable prediction. The fact that formal idealization rather consistently *does* work in this way is a strong argument for a moderate version of scientific realism. (McMullin 1985, 261 f)

Having a fundamental theory like QCD allows us, however, also to argue against philosophers of science who wish to strengthen anti-realism by pointing to the various 'contradictory' models of nuclear and hadron physics. The relation between those models is certainly very subtile (Cohen 1996) but there seems to be an underlying theory that inspired them all. Modeling seems to be neutral concerning the realism-anti-realism issue. This, however, is an issue for another article.

5 Conclusions

Although QFT is one of the most fundamental theories we have, idealizations are used extensively in the context of working with this theory and constructing models of it. In order to get an understanding of the various uses of idealizations it proves useful to distinguish different types of theories and models. The distinctions I make are, I claim, in accordance with the physicists use of the terms. After reviewing idealizations in theories of Types A and B, as well as Models of Type A, this paper focuses on Models of Type B, that is phenomenological models. The case-study I presented analysed phenomenological models of hadron structure. Interestingly, there is a fundamental (Type B) theory for this part of physics that can be solved numerically by high-powered computers. How can we make sense of the observation that models are still used extensively? In tackling this question, I argued for two theses. Firstly, empirical adequacy cannot be the only aim of science and secondly, this practice can be understood by assuming that one important aim of science is to gain understanding of the physical processes involved. One way to gain understanding is to consider simplified models that allow the describtion of relevant aspects of the underlying theory in terms of effective degrees of freedom. Idealizations are, therefore, not only a vice, necessary for pragmatic reasons, but also a virtue: they have a cognitive function that is much more important than the pragmatic aspect.

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