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Predicate-term negation and the indeterminacy of the future

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Abstract This essay introduces a formal structure to model the indeterminacy of the future in Einstein-Minkowski space-time. We consider a first-order language, supplemented with an operator for predicate-term negation, and defend the claim that such an operation provides an appropriate model for the indeterminacy of future contingents. In the final section, it is proved that given a language otherwise adequate to represent a physical theory, at least some of the predicates of that language are indeterminate when the future is not causally determined by the present and the past. The essay concludes with a brief discussion of some possible extensions of this model as well as some open questions.

Keywords Tense Logic · Future Contingents · Spacetime

1 Introduction

In this essay, I prove that there is a meaningful "match" or "consilience" between two independently plausible ways of representing the openness of the future in relativistic space-times. One, semantic, model for the indeterminacy of the future provides a plausible explication of the classical problem of future contingents, without the need to postulate either additional space-time structure, not required by relativistic physics, or non-standard first order logic. This theory consists of a language and a model theory for first-order logic, supplemented by an operator for predicate term negation. As such, it provides a formal explication of Adolf Grünbaum's notion of "attribute indefiniteness." That is, the formal theory, to be introduced in the next section allows us to provide a rigorous statement of what it means to claim that a given object *neither* has *nor* lacks a particular property. We will say that such an object is *semantically indeterminate.*

However, it is also the case that the relation of causal connection on Einstein-Minkowski space-time provides us with an independent characterization of what it means for the past, or the past and the present, to fail to "fix" the physical state of the future. In §4, this conception of indeterminacy, *relational indeterminacy*, is defined, and it is shown that for a certain class of plausible cases such relational indeterminacy implies semantic indeterminacy.

Thus, the argument of the paper proceeds as follows. The definition of semantic indeterminacy is introduced in the next section. Beginning with a standard language for first-order predicate calculus, we introduce the operator for predicate term negation and define semantic indeterminacy. In the following

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section, I provide philosophical reasons for finding the characterization of the open future in terms of semantic indeterminacy at least as plausible as the proposed alternatives to be found in the literature.

Finally, §4 defines relational indeterminacy, the conception of the openness of the future appropriate to relativistic space-times and makes the connection to semantic indeterminacy. The paper concludes with a brief discussion of open questions and potential generalizations of this machinery.

2 Extending the Language and the Model to Account for Future Contingents

Let us begin with a basic logical language, $\mathscr{L}_{\mathrm{T}}$, consisting of a list of singular terms $\mathrm{t}_1 \cdots \mathrm{t}_\mathrm{n}$ and n-ary predicates, $\mathrm{P}^\mathrm{n}_1 \cdots \mathrm{P}^\mathrm{n}_\mathrm{i}$ the universal quantifier, $\forall$, negation,$\sim$, conjunction, &. The formation rules and valuation for $\mathscr{L}_{\mathrm{T}}$ are completely standard. However, it has long been recognized that the sentential operators of such languages can be supplemented with a further set of operators, *predicate operators*(cf. Prior and Fine 1977). For our purposes, we only need one such operator, to generate the language $\mathscr{L}_{\mathrm{TN}}$, a predicate negation operator. This will allow us to clarify an important sense in which there is a distinction between *denying* that an entity possesses a particular property and *asserting* that the same entity has the contrary property. In Section 3, I argue that the representation of the joint falsehood of what would otherwise be contrary statements is the best available conception of future contingents.

Finally, and purely as a matter of notational convenience, we will help ourselves to a future tense operator, $\boldsymbol{F}$, as shorthand to "In the future, ...".

One of the most basic intuitions about the indeterminate future is that there is something "fuzzy" about entities to the future. Even if they exist, it is far from clear what properties they possess or what events they are involved in–that is, to claim that future entities are indefinite is to claim that they suffer from what Adolf Grünbaum called "attribute indefiniteness."(Grünbaum 1963)What then would it mean to claim that an object neither truly possesses nor truly lacks a particular attribute? Consider the following two predicates: "...is red." and "...is not red." In standard first-order logic we interpret a sentence involving the second predicate as the truth-functional negation of one involving the second sentence. But, we need not commit ourselves to that interpretation. Instead, we can make use of our predicate-negation operator, **N** which obeys the following two formation rules:

$$\mathbf{N}\mathrm{P}^n_i \text{is an n-ary predicate of } \mathscr{L}_{\mathrm{TN}} \text{ if and only if } \mathrm{P}^n_i \text{ is.} \tag{2.1}$$

$$\mathbf{NN}\mathrm{P}^n_i = \mathrm{P}^n_i \tag{2.2}$$

Next, let's consider the models of such a language. As usual, such model consists of a set $\mathbb{D}$, the domain of objects, and a valuation, v,

Definition 1 **v** *is a valuation function that assigns to each* t_i *in* $\mathscr{L}_T$ *an element* $\delta_i \in \mathbb{D}$ *and to each* P^n_i *an element* $\mathscr{P}(\mathbb{D}^n)$*, with the additional constraint that* $v(\boldsymbol{N}P^n_i) \bigcap v(P^n_i) = \emptyset$.

Thus, the operator '**N**' operates as, in the language of neo-Aristotelian term logic, a variety of *predicate-term* negation.[1] Now consider the difference between '$\sim$Pt' and '**N**Pt' for some one place predicate. These are logically equivalent if and only if '**N**P' designates the logical contrary to a predicate. Let us say that a predicate such that this is the case, according to a particular model, is *determinate in* $\mathscr{M}$. That is:

Definition 2

$$P^n_i \textit{ is } \textbf{\textit{determinate in }} \mathscr{M} \textit{ if and only if } \boldsymbol{v}[\mathscr{M}, P^n_i] \cup \boldsymbol{v}[\mathscr{M}, \boldsymbol{N}P^n_i] = \mathbb{D}^n$$

However, nothing in the concept of predicate negation seems to require that all of the predicates of our language be determinate in all of our models in this way. In addition, there are good reasons to weaken this restriction. First, because most predicates , *e.g.* colors, do possess a range of non-logical contraries. Second, given a collection of predicates representing what we take to be intrinsic or "projectible" properties, there does seem to be a "natural" contrary to each such predicate read as "definitely does not possess that property or relation."

Finally, when we weaken this restriction, we gain a useful framework for making sense of indeterminacy. The claim that a particular entity is indeterminate with respect to a particular property or relation results from the indeterminacy of the relevant predicate as to the particular entity–the entity falls into the "extension gap" of the predicate. In such a case, both the assertion that the predicate holds of a particular entity and its predicate-denial are false, and the sentential-negation of both the assertion and the predicate-denial are true.The basic intuition about "attribute-indefiniteness" seems to be captured precisely by the idea that an object neither definitely possesses nor definitely lacks a property (stands or fails to stand in a particular relation). Thus, on this account, if, for simplicity, P_i^n is a monadic predicate and is not determinate then there is some $\delta \in \mathbb{D}$ such that both $P_i(\mathrm{t})$ and $\mathbf{N}\mathrm{P}_i(\mathrm{t})$ are false. In the next section, I will argue that this is the most plausible way to understand the status of future contingents.

I argue in the next section that this is the correct interpretation of future contingents. When the predicate becomes determinate, then one of either the assertion or the predicate-denial becomes true and the other remains false.

3 Are Future Contingents Actually False?

The claim at the end of the previous section that future contingency reveals itself at the semantic level as the joint falseness of the assertion and of the predicate-denial that an object instantiates a particular property or relation runs counter to the standard attempts to deal with the semantics of future contingents. The standard accounts of the semantics of future contingents seem to be variations or combinations of three distinct positions–three, or more, valued logics(See, e.g. Prior 1953); truth-value gaps(See, e.g. Thomason 1970); or branching space-times(See, e.g., Belnap 1992; McCall 1976, 1994). The argument that the "predicate-denial theory of future contingency" best captures our usual conception proceeds in four phases. First, I characterize the four possibilities in terms of their account of Aristotle's classic sea-battle example. Next, I argue that this theory better accounts for a significant grammatical distinction between two modes of denying that the sea-battle will occur tomorrow. Third, that it best accounts for our ordinary intuitions about what it means for the sea-battle, or any other event to occur. And, finally that it has the significant advantage of doing so without "messing about" with the semantic values of our language, the structure of our logic, or the structure of space-time.

Consider the classical problem of future contingents from Aristotle's *de Interpretatione*. Assume that it is not now determined whether a sea-battle will occur tomorrow. It still seems to be the case that:

Either a sea-battle will occur tomorrow or a sea-battle will not occur tomorrow. (3.1)

The problem arises because (3.1) seems to be a tautology, and thus true at all times, including the present. However, this seems to require that one of its two disjuncts must also be true *now*. But, that seems to imply that it is, in fact, *now* determined whether a sea-battle will happen tomorrow, contrary to the initial assumption.

First, consider the interpretation of 3.1 suggested by the schema in the previous section . The basic strategy is to diagnose a previously unrecognized ambiguity in 3.1, specifically in the second disjunct. Consider two readings of the second disjunct of (3.1).

It is not the case that a sea-battle will occur tomorrow. (3.2)

$$\text{A sea-battle will not occur tomorrow.} \tag{3.3}$$

On this account (3.1) is only a tautology when we read the second disjunct as (3.2). In that case it is true because (3.2) is. But, if we read the second disjunct as (3.3), then (3.1) is not a tautology, and if the occurrence of the sea-battle is indeterminate, it is in fact false. Consider a name, t, and the predicate

$$\text{S='}\ldots\text{is a sea-battle in the Mediteranean.'} \tag{3.4}$$

Then, (3.2) would be translated into $\mathscr{L}_{\mathrm{TN}}$ as:

$$\sim \boldsymbol{F}\mathrm{St} \tag{3.5}$$

and (3.3) as:

$$\boldsymbol{F}\mathbf{N}\mathrm{St} \tag{3.6}$$

However, notice the suggestive difference between these two readings. (3.5) is quite plausibly read as the *present* denial that a sea-battle takes place in the future. But, since the sea-battle has not actually occurred, it does seem plausible that the denial that it has occurred is true. (3.6), on the other hand, *now* asserts, about the future, that it will not be occupied by a sea-battle. But, since nothing *now* determines whether that will be the case, it seems equally plausible that such an assertion be presently false. Such a grammatical distinction can never be anything but suggestive. However, I claim that the model introduced here has additional advantages.

Next consider the standard non-fatalist accounts of 3.1. On a 3-valued account of logic, the interpretation of Aristotle's sea-battle depends on the precise version of "disjunction" one chooses to use to interpret (3.1). Since the first disjunct is to be assigned the middle truth-value, so must the second. Then, if one chooses the "weak" definition, such that the value of the disjunction is the maximum of the disjuncts, (3.1) is also indeterminate. The "weak" definition, thus, "solves" the problem by diagnosing a previously unrecognized ambiguity in the logical connectives. On the "strong," or additive, definition, the disjunction is true. Thus, it solves the puzzle only by denying excluded middle. Alternatively, we might introduce truth-value gaps, an an account such that neither disjunct currently possesses a truth-value. The tautology is true because the truth of the disjunction holds on every possible consistent assignment of truth values to the disjuncts. Finally, on a branching space-time account the relevant disjunction is true because, while it is not now determined which branch will become actual, whatever branch becomes actual will have one disjunct true and one false. Thus, both of these "solutions" solve the problem by denying excluded middle. The second, where the branching allegedly takes place in physical space-time, also poses massive problems for the physics of space-time.

This last claim requires some further discussion. Recently, defenders of branching space-time theories have claimed that branching space-time theory requires at most minor amendments to our current space-time physics. Although I believe that my theory has significant interest independently of this question, I will argue, briefly, that branching space-time theories continue to suffer from this disadvantage.

Unfortunately, there are nearly as many theories of branching space-times as there are branching space-times theorists. Therefore, a complete survey would be beyond the scope of this discussion. However, a brief discussion of two aspects of the issue should be sufficient for our purposes. First, what is required for a given theory to be a theory of branching *space-times*? Minimally, if it is not simply to collapse into the claim that two possible worlds diverge, in the sense of David Lewis(1986), the branching must occur *at* a particular spatio-temporal location. But, this implies that the causal future of that event consists of two topologically disconnected regions. Even if one manages to avoid particular problems with topology change and the loss of the Hausdorff property, perhaps by restricting space-time physics to being the theory of the structure of each history rather than of the entire manifold of events, one faces problems. One would need to explain how to connect this "pre-physical" picture of the collection of possible point-events with existing physical theories of the structure of the manifold of locations of possible point-events. While some progress seems to have been made on the topological concerns raised by such branching structures, no general account connecting branching space-times with space-time physics seems to occur in the literature.

Second, in his “Pruning some branches from ‘branching spacetimes’ ”(2008), John Earman offers a classification of branching space-times theories into three classes: ensemble branching, individual branching and *Belnap* branching. Roughly, Earman argues that ensemble branching is relatively harmless, largely because it’s a divergence theory in Lewis’s sense; that individual branching falls afoul of various theorems regarding topology change from general relativity; and that Belnap branching is ambiguous. In the main text, I treated Belnap’s theory simply as a more sophisticated version of Storrs McCall’s individually branching theory. I want to suggest that the difficulty categorizing the Belnap program results, at least in part, from an equivocation on the part of its defenders, including Nuel Belnap. Consider Belnap’s “EPR-like ‘funny business’ in the theory of branching space-times” (2002). Apparently under pressure from the kinds of concerns raised here, Belnap changes his terminology so as to emphasize “the intention that the branching be between space-times.[Note 1]” However, it remains the case that this theory “place[es] a causal order on concrete ‘events,’ ” and that it is distinct from branching time largely in that, “in branching time, the terms of the causal order are giant Laplacean ‘simultaneity slices,’ not tiny little point events.” Thus, it seems that the first part of the note moves us towards a theory of branching in *modal* space, while the second moves us back toward a theory of individual branching.

However, in the remainder of that essay, he retains the basic structure of Belnap (1992) such that the primitives of the theory consist in “Our World,” the set of all possible point-events, and a causal order on “Our World.” The primary consequence of this, for our purposes, is that individual point-events in “Our World” can belong to more than one history. But, then, as I indicated in the main text, we have branching within space-time. Alternatively, we have numerically the same event occupying more than one space-time. Either way, fitting this “pre-physical” picture into our ordinary physics of space-time structure remains a significant, and apparently unmet, challenge.

To return to the main line of our discussion, consider what we ordinarily mean when we claim that an event occurs. As a general rule, this amounts to a claim that some entity comes to possess some property or to stand in some relation that it previously did not. On this reading, we can give a precise content to this notion. Consider some property, represented by the predicate P_i, that is indeterminate with respect to an object δ at time t_o. If the object later comes to possess the property, it is *literally* added to the extension of P_i. The availability of this straightforward account of occurrence confers a significant theoretical advantage on this approach.

Finally, it handles future contingents without any of the messiness of the other three accounts, although certainly with its own kind. We need not become involved in the messy debates over the plausibility of multiply valued logics and of the need to formulate plausible consequence relations for them. We need not deny the law of excluded middle. And, we can continue to use our ordinary physics on ordinary space-time. However, do we have any reason to believe that ordinary space-time contains such future contingents? In the next section, I argue that we do.

4 Does the World Contain Future Contingents?

In order to show that the actual world does contain future contingents, the argument proceeds in two stages. First, it is necessary to connect the *semantic indeterminacy* of the previous section with *causal indeterminacy.* That is, the standard reason for thinking that the sea-battle is not yet determinate is that nothing about the past and the present has yet determined whether the sea-battle will actually occur. Thus, the first, and trickier, step of the argument is to characterize the openness of the future in this sense and to demonstrate that a causally open future is semantically indeterminate in the sense above. The second, and fairly trivial, step is to show that Einstein-Minkowski space-time in fact possesses the relevant indeterminacy.

What does it mean to claim that the future is not determined by the past? In order to remain at the greatest level of generality, consider space-time theories. Let's say that, abstractly, such a theory consists of a space-time and the assignment of possible values to every point of space-time. In the standard cases, assignments of scalar, vector or tensor fields to the space-time. A specification of all of the available types of values to each point of the space-time, I will call the state of the space-time, relative to a particular theory.[2] Now, let us consider topologically open regions of space-time, down to points. Obviously, given the state of space-time, the relevant values are also determined for each such region. In the absence of a dynamics, there is a meaningful sense in which all possible assignments of such values are equally allowable. But, of course, this is not what we want to know. We do not deal in God's eye views of space-time, except at the most abstract levels. We deal with regions of space-time.

What we really want to know is: given the (full or partial) specification of the state of a region of space-time, how does that, given a relevant theory, constrain the states of other regions of space-time? Obviously, to do this we need a dynamics that connects the states of different regions of space-time and connects the various "elements" of the state. What we seem to require is a specification of how likely a region R is to be in a particular state, r, given that some other region, S, is in a particular state, s. The obvious option is to locate some natural probability measure and use the conditional probability, where we say that the state of S determines the state of R when the conditional probability equals 1.[3]

The natural assignment seems to be to an equal probability to all dynamically consistent states of the entire space-time. Now consider an arbitrary region, an open set in the usual topology, R of the space-time. Some of the dynamically possible states of the space-time assign the same state to that region, some of them assign different states. Thus, the states of regions inherit the probability that they will be in a given state from the initial probability assignment to states of the space-time. That is, for all possible states r of R, we can derive the probability that R is in state r, prob(r≡R). But, now consider another region, S. Via the usual definition of conditional probability, we can define for each state s of S, the probability that S is in s given that R is in r, prob(s≡S|r≡R) for each of possible state r of R.

Finally, given that our goal is to investigate what regions of space-time are indeterminate relative to certain other regions in various space-times, the minimal constraints on the dynamics are the relations of causal connection appropriate to the space-time under consideration.

Definition 3 ***caus****(S,R) if and only if the state of S could causally influence the state of R.*

First, note that in these definitions regions can be replaced with points as the degenerate case of regions in the usual topology. Second, Definition 3 allows us to define a, not necessarily exhaustive, partition of the space-time, relative to any given region R, into the causal past, $\mathbb{C}$(R); the causal present, $\mathbb{P}$(R); and the causal future, $\mathbb{F}$(R), as follows:

Definition 4 $\forall q, R\ \exists p\{(q \in \mathbb{C}(R)) \Leftrightarrow (p \in R\ \&\ \boldsymbol{caus}qp\ \&\ \sim \boldsymbol{caus}pq)\}$

Definition 5 $\forall q, R\ \exists p\{(q \in \mathbb{P}(R)) \Leftrightarrow (p \in R\ \&\ \boldsymbol{caus}qp\ \&\ \boldsymbol{caus}pq)\}$

Definition 6 $\forall q, R\ \exists p\{(q \in \mathbb{F}(R)) \Leftrightarrow (p \in R\ \&\ \sim \boldsymbol{caus}qp\ \&\ \boldsymbol{caus}pq)\}$

It will also be useful to have a name for the entire region from which R is causally accessible, i.e. the union of $\mathbb{C}$(R) and $\mathbb{P}$(R): call it, $\mathbb{A}$(R). For clarity note that in relativistic space-times, assuming the standard connection between actual causal connectability and the geometry of the space-time, these regions consist of the future null-cone and its interior, the space-time location itself, and the past null-cone and its interior respectively.We are now ready to say what it is for the state of one region of space-time to determine the state of another region. As we should expect from the above:

Definition 7 *S **determines** R [**det**(S,R)] if and only if for each possible state s of S, there exists a state r of R, such that for some Q⊆S, Q⊆ $\mathbb{A}$(R) and for the state q of Q induced by s, prob(r≡R|q≡Q)=1.*

Now, we possess the necessary resources to define *relational indeterminacy*(RI). Given the above it must be that the future is open if it is not determined, in the sense of Def. 7, by the past. Which past? Given that the future is defined relative to the changing space-time location of entities along their world-lines, it must be relative to *their* past. Thus, define the following two concepts:

Definition 8 *A point q_o is **Relationally Indeterminate** (RI) relative to a point q_1 if and only if there is no R⊆ $\mathbb{A}(q_1)$ such that **det**(R,q_0)*

Definition 9 *A point q_o is **determinate** relative to a point q_1 if and only if there is an R⊆ $\mathbb{A}(q_1)$ such that **det**(R,q_0)*

Therefore, we can finally state that the future is open, in at least one significant sense, if and only if it is relationally indeterminate relative to our changing location. Or, alternatively, that an event happens at q_o relative to q_1 only when the state of q_o is determinate relative to q_1.

It is important to distinguish *relational indeterminacy* from two closely related concepts: *predictive indeterminacy* and *indeterminism.* Predictive indeterminacy is, as the name implies, a fundamentally epistemic notion. It is merely the claim that a particular epistemic agent, in a particular situation, lacks the resources to establish the state of a particular region of space-time. Karl Popper, for example, confuses predictive with relational indeterminacy in *The Open Universe*(Popper 1991). Note that while relational indeterminacy implies predictive indeterminacy, on the assumption that causally accessible information exhausts the information available to epistemic agents, the converse is not true.

Alternatively, *indeterminism* implies relational indeterminacy, but not *vice versa.* That is, consider a point, p such that the causal past *of* p fails to determine its state: there is some causal indeterminism in the universe. Then, from any region not containing p, p is relationally indeterminate. But, again, there are other sources of relational indeterminacy in the universe. For example, even in a deterministic universe, the future light-cone of every point in Einstein-Minkowski space-time is relationally indeterminate to that point. It is this relationship between indeterminism and relational indeterminacy that seems to have led Hans Reichenbach to insist on the relationship between indeterminism and temporal becoming(cf. Reichenbach 1991, 1925)

To show that *relational indeterminacy*, plus plausible assumptions, yields semantic indeterminacy, begin by considering a covering, $\mathcal{C}$, of the space-time extracted from the usual topology.[4] This covering now constitutes a domain for the model structure of section **??**, with the identity function as the occupation function. Next, define what it is for the predicates of a language, $\mathscr{L}_\psi$, to provide an adequate representation of a space-time theory, **T**, where$\{\psi_i\}$ is the set of possible states of the space-time according to **T**, relative to $\mathcal{C}$. Let us say that:

Definition 10 *A set of predicates $\mathbb{P} = \{P_1 \cdots P_n\}$ is* adequate *to **T** and $\mathcal{C}$ if and only if for all ψ and all R∈ $\mathcal{C}$ $P_i R$ for exactly one $P_i \in \mathbb{P}$.*

Intuitively, $\mathbb{P}$ provides a partition of the possible states of the regions specified in $\mathcal{C}$ such that we can use $\mathbb{P}$ to specify the state of the space-time to a precision limited by the precision with which we specify $\mathbb{P}$ and the size of the regions in $\mathcal{C}$. Call a language with such an adequate set of predicates, $\mathscr{L}_\psi$. Next,

Definition 11 *A* local interpretation *of $\mathscr{L}_\psi$ relative to a region, R, consists of a partial interpretation, with valuation v_R, such that for all s∈ $\mathcal{C}$,*

1. *$s \in v_r(P_i)$ if and only if r* determines *s.*
2. *$s \in v_r(\boldsymbol{N}P_j)$ if and only if for some $i \neq j$ $s \in v_r(P_i)$*

Finally,

Definition 12 *A region* ***S*** *is* semantically indeterminate *relative to region* ***R****, if, under the local interpretation induced by* ***R****,* $\sim [P_i S \vee \mathbf{N} P_i S]$.

We can now prove the following theorem:

Theorem 1 *For regions R and S, if S is* relationally indeterminate *to R, then for all adequate* $\mathscr{L}_\psi$ *S is* semantically indeterminate *relative to R.*

Proof From definition 8, there is at least one state of s of S such that for all possible states of r of R, prob(s≡S|r≡R)≠1. Then, from definition 11, S is not in the extension of any predicate in $\mathbb{P}$ of the local interpretation induced by R, independently of the state of R. Nor is it in the extension of the predicate-negation of any of the predicates. Therefore, $\sim [P_i s \vee \mathbf{N} P_i s]$ for all P. Therefore, given that $\mathbb{P}$ is adequate and from definition 12 S is semantically indeterminate relative to R.

For a more intuitive presentation of the basic ideas here, consider a toy space-time theory in which the only values assigned to space-time are two scalar fields, A and B. Such a pair of scalar field assigns two real number values to each point of space-time. We can then define two sets of monadic predicates, A_i and B_i, as follows. Divide the possible values of each of the two fields into a collection of half-open sets, {[0,1), [1,2)... [n,+∞)}, where n is an arbitrary integer value. For simplicity assume that, for any particular application of the scheme, n is chosen so that the actual value of the relevant field is everywhere less than n. Associate a predicate with each element of the partition so that:

Definition 13 $A_n w$ *iff the value of the A-field is such that* $n \leq A(w) < (n+1)$

And, similarly for B. Clearly the set of predicates $\{A_n, B_n\}$ is adequate to this theory in the sense of Definition 10. Using the definitions from §§2 and 3 above, we can take the domain to be open sets on the space-time and the occupation function to be identity. Now, assume that we have a dynamics for A and B given by ordinary partial-differential equations with a well-formed initial value problem(IVP). Given the existence of a well-formed IVP, then when we specify the values of both fields on any single space-like hyper-surface (Cauchy surface) there is a single *dynamically acceptable* extension to the remainder of space-time. But, for any region less than a complete Cauchy surface we have the same only for some spatio-temporal volume, less than the entire space-time, determined by the volume of the initial region. However, outside of that determinate region, A and B can take pretty much any values depending on the state of the remainder of space-time.

Define a state description for a region of space-time,

Definition 14 Ω_R*, is the set* $\{\sim B_1 R \ldots \sim B_{(i-1)} R,\ B_i R, \sim B_{(i+1)} R \ldots \sim B_n R,$
$\sim A_1 R \ldots \sim A_{(j-1)} R,\ A_j R, \sim A_{(j+1)} R \ldots \sim A_n R\}$
where $i \leq B(R) < (i+1)$ *and* $j \leq A(R) < (j+1)$.

Clearly a specification of the state of space-time, as above, determines a state description in this sense for every region of space-time. Just as straightforwardly, from the existence and uniqueness theorem for ordinary differential equations, the state description, Ω_C of a Cauchy surface possesses a single dynamically acceptable extension to a state-description, $\Omega_{\mathbb{W}}$ for the entire space-time. However, just as clearly there will be more than one dynamically acceptable extension of any region less than a complete Cauchy surface. There will, therefore, be at least two such state descriptions of at least some region compatible with the present state, thus inducing *semantic indeterminacy.*

But, despite various puzzles regarding the first-order representation of modern physical theories, I do not see any principled bar to defining such a language for any physical theory defined as above.[5]

Now consider Einstein-Minkowski space-time. Given a standard and natural reading of special relativity, where $\mathbb{C}$(R) and $\mathbb{F}$(R), the causally past and causally future regions of space-time are the past and future null-cones and their interiors respectively, then all regions of space-time that are not in the past relative to a given region are, in fact, relationally indeterminate(Harrington 2008).

5 Conclusion

I have argued, first, that Einstein-Minkowski space-time with the standard topology and a time ordering provides a model structure for standard tense logic when we restrict the use of the present tense to local regions of space-time. In addition, when we consider *proper time* along a world-line within Einstein-Minkowski space-time, it constitutes a completely standard dense linear time flow. Given this we have a full conception of tensed becoming within Einstein-Minkowski space-time when we postulate that objects are sequentially "present" along their world-lines, whether we explain that sequential presence indexically or take it as a metaphysical primitive.

Second, I have argued that when we extend standard first-order tense logic to include a predicate-negation operator, we can formulate a precise definition of the status of future-contingents as the joint falsehood of the assertion and of the predicate-denial that an object within the model possesses a certain property or stands in a particular relation. This provides a rigorous account of the concept of the "attribute indefiniteness" of future entities. Finally, even in a deterministic universe, the causal structure of special relativity guarantees that at least some indeterminacy will occur to the future of any region of space-time that does not constitute a Cauchy surface of the dynamics.

Finally, let me indicate two additional features of the theory developed here. First, at least for worlds with a fixed background space-time, the model structure can easily be adapted to other possible worlds, and probably to the actual world, where space-time is at best locally Minkowskian. Second, it also allows us to formulate a rigorous notion of what it means to say that time is or is not real within a given world. That is, we should accept that time is real in a given possible world just in case the space-time of that world supports a model for tense logic. Even more so, we might have worlds where time is, in a sense, partly real. Thus, suppose that a given space-time is not globally orientable and, thus does not possess a global time orientation. In such a world, for any time-orientation, $<$, there are points p, q such that both $p<q$ and $q<p$. It might still be locally orientable and particular curves within the space-time might possess an orientation. Such curves, when viewed as the world-lines of objects in the space-time, constitute models of tense logic for objects occupying those world-lines. It would then be reasonable to claim that time is real for those objects, but not for objects in other regions of the space-time. And, this would be a perfectly objective fact about the entities and their histories within space-time. Not a subjective fact about time-perception.

Notes

[1]This possibility was first suggested to me a discussion of similar cases in (Prior and Fine 1977). The analogy with neo-Aristotelian term logic comes from Heinrich Wansing at Logica 2007. He also directed me to his useful essay, "Negation" (Wansing 2001)

[2]From here on out, the relationship of the state to some particular theory specifying the range of possible states will be left implicit.

[3]Technically, since the state space is likely to be continuous this would allow for for some measure zero set of states other than the one of interest to be compatible with S. I will follow the usual practice in physics and ignore this possibility. If one wished to significantly complicate the exposition at no substantive gain, one can replace all discussion of particular states with finite ranges of states.

[4]Technically a countable covering is required as long we retain the standard restriction on a countable number of terms in the corresponding language.

[5]Actually, I am eliding a whole class of problems in first-order representations of physical values, here. Including problems about values that vary along multiple dimensions; the nature of vector and tensor fields; and zero-value physical quantities, just to name a few. For an interesting discussion of these issues, focused on the last problem, see Balashov (1999)

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