Welfare, Voting and the Constitution of a Federal Assembly

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Abstract. Equal and proportional representation are two poles of a continuum of models of representation for the assembly of a federation of states. The choice of a model has repercussions on the welfare distribution in the federation. We determine, first by means of Monte Carlo simulations, what welfare distributions result after assemblies that were constituted on the basis of different models of representation have considered a large number of motions. We assess what model of representation is favored by a Rawlsian maximin measure and by the utilitarian measure and present matching analytical results for the utilitarian measure for a slightly idealized case. Our results show that degressive proportionality can be justified as a compromise between maximin and utilitarian considerations. There is little surprise in this result. What is more surprising, however, is that, within certain contexts of evaluation, degressive proportionality can also be justified on strictly utilitarian grounds.

Introduction. A federal assembly consists of a number of representatives for each of the nations (states, *Länder*, cantons...) that make up the federation. In determining the constitution of such an assembly there is the following tension. On the one hand, we may think of the federation as a federation of nation states. This would support a model of equal representation, i.e. a model in which each nation has the same number of representatives. On the other hand, we may think of the federation as a federation of people. This would support a model of proportional representation, i.e. a model in which the number of representatives for a nation is proportional to the number of its inhabitants. But presumably that is not the only motivation. What makes this issue worth quibbling about is that the model of representation that is instituted will have an impact on the welfare distribution over the nations in the federation that will ensue over due course. Welfare distributions can be evaluated on various measures that correspond to conceptions of justice. We will investigate what models of representation yield welfare distributions that score higher on the Rawlsian maximin measure and on the utilitarian measure. First, we construct a continuum of models of representation ranging from equal to proportional representation. In between these extremes are models of *degressive proportionality*. On such models, the larger nations receive more representatives, but less than would be warranted by proportionality, whereas the smaller nations receive less representatives, but more than would be warranted by proportionality. We take the European Union as a paradigm case of a federation. We run a Monte-Carlo simulation in which a large number of motions are voted up or down within varying contexts of evaluation and investigate how well the resulting welfare distributions score on the Rawlsian maximin and on the utilitarian measures. Simulation results give us more leeway in specifying values for the parameters in the model, but they do not provide complete insight in the functional dependences of the measures on these parameters. We will provide analytical results for the utilitarian measure for a slightly idealized case.

Our study deals with a question in voting theory and theories of justice that has direct relevance to today's political world. With the increase in autonomous nation states across the world comes the need to design institutions for transnational political structures that are responsive to certain conceptions of justice. With the projected extension of the European Union, there has been much discussion about how the various nations should be represented in the Council of the European Union. The Swedish proposal was for the number of representatives to be proportional to the square root of the population. This is a model of degressive proportionality. The French president responded that he failed to see *what was politically significant about the square root*. The question is of course what justification can be advanced for models of degressive proportionality. We will show that degressive proportionality can be justified as a compromise between maximin and utilitarian considerations. There is little surprise in this result. What is more surprising, however, is that, within certain contexts of evaluation, it can also be justified on strictly utilitarian grounds. We end with some suggestions about how our model could be made operational for empirical work.

1. The Federation, its Constituent Nations and Models of Representation.

Let there be a federation that has a total population of S people. It is divided into N nations, some of them larger, some of them smaller. Each nation i has a population size of s_i. The federal assembly is the decision-making organ for the federation. Our model can be readily generalized, but just to have some definite numbers, we will run our simulations with the actual population sizes of the European Union (see Table 1) and with the actual number of representatives in the Council of the European Union.

To represent the continuum between equal representation and proportional representation in the assembly, we construct the following measure, which determines the proportion of representatives of nation i in the assembly:

(1) $$r_i(\alpha) = \frac{x_i^{\alpha}}{\sum_{i=1}^{N} x_i^{\alpha}} \quad \text{for } x_i = s_i / S \text{ and } \alpha \in [0, 1]$$

α is a measure of the degree of proportionality. $r_i(0) = 1/N$ and there is equal representation in the assembly. $r_i(1) = x_i$ and there is proportional representation in the assembly.[1] Intermediate values of α represent models of representation of degressive proportionality that are located on the continuum between both extremes. For example, $\alpha = ½$ corresponds to the Swedish proposal. Obviously, $\sum_{i=1}^{N} x_i = 1$ and $\sum_{i=1}^{N} r_i(\alpha)$ = 1 for any value of α.

How can we turn ratios of representatives into actual whole numbers of representatives for assemblies of particular sizes? This is a complex question in voting theory, but for our purposes the following simple system suffices. There are currently $N = 15$ nations in the European Union and $T = 87$ representatives in the Council of the European Union. To determine the number of representative for each nation i we multiply T with the ratio $r_i(\alpha)$. We assign, in a first step, $[r_i(\alpha)T]$—i.e. the whole number smaller than or equal to $r_i(\alpha)T$—representatives to each nation i. The number of remaining seats is $k(\alpha) = T - \sum_{i=1}^{N}[r_i(\alpha)T]$. Clearly $k(\alpha) < N$. These $k(\alpha)$ seats are distributed as follows. We order the nations according to the relative sizes of the decimal parts $r_i(\alpha)T - [r_i(\alpha)T]$, going from larger to smaller. We now assign to each of the first $k(\alpha)$ nations in this ordering precisely one additional seat. Let $R_i(\alpha)$ be the number of seats that each nation i receives in the assembly on the proportionality measure α.

2. Voting on Motions.

A motion affects the people of the respective nations in different ways. A motion to improve the defense of the federation may benefit each nation to the same extent. But a motion to improve the highway system in some nation on the periphery of the federation does little more than benefit the nation in question, while it constitutes a cost to the other nations. A motion can be thought off as a utility vector $\langle v_1, ..., v_i, ..., v_N \rangle$ in which each v_i represents the expected utility that the motion will bring to an arbitrary person of nation i if the motion were adopted.

There is a certain threshold value of utility so that all the representatives of a nation will vote in favor of the motion if the utility that this motion will bring to the nation in question exceeds the threshold value. They vote against the motion if the utility drops below the threshold value. They will abstain if the utility equals the threshold value. Let us say that the threshold value is the point at which the costs balance out against the benefits for the nation of question. Costs and benefits should be understood broadly. They may also reflect feelings of altruism between the nations in question.

David Hume (1888 [1739]: Book III, Part II, Section II) notoriously believed that questions of justice only arise if we can expect moderate selfishness (and not benevolence or extreme selfishness) and in times of relative scarcity (and not in times of extreme scarcity or abundance). We will not follow Hume's contention as to when questions of justice arise, but his taxonomy comes in handy in distinguishing between alternative *contexts of evaluation*:

(i) *Benevolence and Abundance*. In times of economic prosperity, or amongst nations that genuinely care about the well being of the other nations, the benefits that nations receive when a motion is adopted tend to outweigh the costs more often than not. There is money enough to go around so that costs matter minimally and there is a positive disposition towards political initiatives in general so that each nation's utility from a motion receives an added bonus. To model this situation, we let the utility values in the vector that represents a motion be random numbers generated under a uniform distribution over the range [-.5, 1] and we set the threshold value for acceptance at $v_t = 0$. Hence, the chance that an arbitrary nation will vote for a motion is 2/3. As a mnemonic aid, let us name this the context of *generous* voters.

(ii) *Extreme Scarcity and Extreme Selfishness*. In times of economic recession, or amongst nations that are strictly concerned with their own welfare, the costs of a motion tend to outweigh the benefits more often than not. The nations are wary of expenditures and they only benefit from the implementation of motions

[1] When $\alpha > 1$, then the larger nations will get a disproportionately larger representation and the smaller nations a disproportionately smaller representation in the federal assembly. We will restrict α to the closed interval [0,1], but our model can be readily extended to such models of representation.

in support of projects that directly affect their own welfare. We let the utility values be random numbers generated under a uniform distribution over the range [-1, .5] with $v_t = 0$. Hence, the chance that an arbitrary nation will vote for some motion is 1/3. This is the context of *stingy* voters.

(iii) *Moderate Selfishness and Relative Scarcity*. This context is intermediate between the previous two poles of the continuum. We let the utility values be random numbers generated under a uniform distribution over the range [-1,1] with $v_t = 0$. The chance that an arbitrary nation will support a motion is 1/2. This is the context of *balanced* voters.

Some observations are in order. First, in our model, the utility values are independent of each other. Alternatively, one could model, say, these contexts by stipulating different degrees of dependency between the utility values in a motion. Positive dependencies hold between nations that are in a close bond with one another. Or it may also be the case that the nations are sensitive to the common good of the federation. An extreme case hereof would be that all nations receive the same utility value from all motions. We will address the challenge of modeling dependencies in Section 6. Second, we chose uniform distributions. But the distributions do not need to be uniform. Our analytical results will show that the utilitarian measure remains invariant under an alternative choice of distribution as long as we keep a limited set of parameters that characterize the distribution fixed. Third, we vary the range of the utility values and keep the threshold values fixed. Alternatively, one could keep the range fixed and change the threshold value, but this does not make any difference to our results.

3. Evaluating Models of Representation.

Models of representation in a federal assembly are social arrangements. Each value of α constitutes an alternative social arrangement. We start with the model of equal representation ($\alpha = 0$) and increase the value of α with increments of $\Delta\alpha$ (which we set in our computer simulation at .01) until we reach the model of proportional representation ($\alpha = 1$). We consider m motions for some sufficiently[2] large m, say $m = 10{,}000$, in our calculations. That is, we generate m n-dimensional vectors $\langle v_1^k, ..., v_i^k, ..., v_N^k \rangle$ for $k = 1, ..., m$ with random numbers in the ranges that correspond to the respective contexts of evaluation. For each motion, a vote is taken by an assembly whose constitution is based on a particular value of α. The representatives of a nation i will all vote for a motion if v_i^k exceeds the threshold value v_t that we take to be same for all nations; they will all vote against the motion if v_i^k is lower than v_t; and they will abstain if v_i^k equals v_t. If the motion k is accepted, each nation i is assigned a utility value v_i^k. If the motion is not accepted, then it is discarded and each nation remains unaffected by the motion. After the m motions have all been considered, we divide the sum of the utilities that each nation has accrued by m: the resulting vector $\boldsymbol{u}(\alpha) = \langle u_1(\alpha), ..., u_i(\alpha), ..., u_N(\alpha) \rangle$ contains the utilities $u_i(\alpha)$ that a person in nation i can expect from a motion, given a particular model of representation represented by a specific value of the parameter α. At the end of this process we have a vector of utility distributions associated with the values of α, viz. $\langle \boldsymbol{u}(0), \boldsymbol{u}(\alpha), \boldsymbol{u}(2\alpha), ..., \boldsymbol{u}(1) \rangle$, or, more specifically, in our computer simulation, $\langle \boldsymbol{u}(0), \boldsymbol{u}(.01), \boldsymbol{u}(.02), ..., \boldsymbol{u}(1) \rangle$.

In formal terms, $u_i(\alpha)$ is constructed as follows. Let the function[3] $g(y)$ equal 1 if $y > 0$ and 0 if $y \leq 0$ and let $sign(y)$ be the standard function in mathematics which equals 1 if $y > 0$, 0 if $y = 0$, and –1 if $y < 0$. The *decision function* $D_k(\alpha)$ yields 1 if the majority supports motion k and 0 if the majority does not support this motion:

$$D_k(\alpha) = g\left(\sum_{i=1}^{N} R_i(\alpha)\ sign\ (v_i^k)\right) \tag{2}$$

We can now express $u_i(\alpha)$, i.e. the expected utility of a motion for nation i, as the sum of the utilities that a nation i receives from accepted motions over the numbers of motions considered:

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[2] 'Sufficiently' means that higher values of m do not change our results.

[3] g is similar to the Heaviside function, except that the Heaviside function is undefined for 0.

(3) $u_i(\alpha) = \frac{1}{m}\sum_{k=1}^{m} v_i^k D_k(\alpha) \quad i = 1, ..., N.$

Following Harsanyi (1976), the model of representation α that is supported by utilitarianism is the model that maximizes expected utility. Hence, the measure that is to be maximized is the sum of the component utility values $u_i(\alpha)$ in the utility vector $\boldsymbol{u}(\alpha)$ weighted by the respective population proportions x_i:

(4) $M^{util}[\boldsymbol{u}(\alpha)] = \sum_{i=1}^{N} x_i u_i(\alpha)$

The model of representation α that maximizes this measure is the social arrangement that is supported by the utilitarian conception of justice.

Following Rawls's difference principle (or rather, the difference principle substituting utilities for primary goods) the distribution which maximizes the minimum expected utility $u_i(\alpha)$ of a person in nation i is the fairer distribution (1971: 125f.). The measure to be maximized is the minimum utility value in $\boldsymbol{u}(\alpha)$:

(5) $M^{min}[\boldsymbol{u}(\alpha)] = Min(u_i(\alpha))$ with $i \in \{1,\dots,N\}$.

The model of representation α that maximizes this measure is the social arrangement that is supported by the Rawlsian conception of justice.

4. A Justification for Degressive Proportionality.

Figure 1 presents the graph for the Rawlsian measure for the context of *balanced* voters. This measure supports a model of representation in the neighborhood of equal representation. It behaves in a similar manner for *generous* and *stingy* voters as for *balanced* voters except that the values of the measure are lower for *stingy* voters and higher for *generous* voters (graphs omitted). Figure 2 presents the graph for the utilitarian measure for the context of *balanced* voters. This measure supports a model of proportional representation within a context of *balanced* voters. But surprisingly, Figures 3 and 4 show that, in contexts of *stingy* and of *generous* voters, the utilitarian measure supports a degressively proportional model of representation.

There are two good reasons why one might favor some degressively proportional model on welfarist grounds. One might defend a welfare distribution that combines utilitarian with Rawlsian considerations. Independently of the context of voting, striking a balance between these considerations favors a degressively proportional model of representation. This is not unsurprising. The Rawlsian measure is motivated by certain egalitarian concerns. One can only move away from an egalitarian welfare distribution if it is the case that introducing some inequality does not make anyone worse off. If all the nations have an equal input in the vote, then the inhabitants of all nations can be expected to end up with equal utility. But if larger nations have a greater input in the vote than smaller nations, then larger nations will outvote smaller nations. The welfare levels of larger nations will gain as a consequence, whereas smaller nations will be outvoted and their welfare levels will suffer as a consequence. So the inequality that is introduced by allotting larger nations a greater input in the vote does make inhabitants of smaller nations worse off, which is not tolerated by a Rawlsian conception of justice. On the other hand, this inequality will be less of a concern for the utilitarian, since the chance of being an inhabitant of a larger nation is greater than the chance of being an inhabitant of a smaller nation. So the inequality may actually increase the expected utility and so there is nothing objectionable for the utilitarian in introducing at least some proportionality into the constitution of the assembly. So, in conclusion, if we want to strike a balance between Rawlsian and utilitarian considerations, one may reasonable expect that some model of degressive proportionality will be favored.

But how much proportionality can we introduce into the constitution of the assembly and still expect the expected utility to rise? What *is* surprising is that the answer to this question depends on the context of evaluation. For *generous* and *stingy* voters, the expected utility maximizes for some degree of degressive proportionality. For *balanced* voters the expected utility maximizes for full proportionality. Assuming that a context of *balanced* voters is not the norm, it turns out that a strict utilitarian should support some model of degressive proportionality, rather than full proportionality.

5. An Analytical Account.

We will derive analytical results for the more striking results of our simulation, viz. for the behavior of the utilitarian measure.[4] The measure M^{util} is an expectation, viz. the expected utility $E[U]$ from an arbitrary motion. We will compute this expectation by conditioning on the propositional variables $\boldsymbol{A}$ and $\boldsymbol{C}$. The variable $\boldsymbol{A}$ equals A when the motion is accepted and $\neg A$ when the motion is not accepted. To define the variable $\boldsymbol{C}$, construct all the combinations of i nations voting for the motion and $N - i$ nations voting against the motion. From combinatorial analysis, we know that there are $\sum_{i=0}^{N} \binom{N}{i} = 2^N$ such combinations. The variable $\boldsymbol{C}$ equals C_1 when all the nations vote for the motion, C_2 when all nations except for nation N vote for the motion,..., and C_{2^N} when all nations vote against the motions. By the probability calculus,

$$(6) \qquad E[U] = \sum_{\boldsymbol{A}=A,\neg A} \; \sum_{\boldsymbol{C}=C_1,\ldots,C_{2^N}} E[U|\boldsymbol{A}, \boldsymbol{C}]P(\boldsymbol{A}, \boldsymbol{C}).$$

Notice that $E[U|\neg \boldsymbol{A}, \boldsymbol{C}]$ equals 0 for any values of $\boldsymbol{C}$, since the expected utility of a rejected motion is 0. Furthermore, by the chain rule, $P(\boldsymbol{A}, \boldsymbol{C}) = P(\boldsymbol{A}|\boldsymbol{C})\mathrm{P}(\boldsymbol{C})$. Hence,

$$(7) \qquad E[U] = \sum_{\boldsymbol{C}=C_1,\ldots,C_{2^N}} E[U|A, \boldsymbol{C}]P(A|\boldsymbol{C})P(\boldsymbol{C}).$$

In Table 2 we illustrate this calculation for a federation of two nations named ‘1’ and ‘2’. Each row lists the factors within each term of the sum in (7). First, let u^+ be the utility that a nation derives from an accepted motion assuming that they voted for the motion. u^+ equals 1/2 for *generous* and *balanced* voters and 1/4 for *stingy* voters. Let u^- be the utility that a nation derives from an accepted motion assuming that they voted against the motion. u^- equals –1/2 for *generous* and *balanced* voters, and –1/4 for *stingy* voters. On row 2 of the table, nation 1 voted for and nation 2 voted against. Hence, assuming that the motion is accepted, the expected utility from this motion is the sum of the u^+ and u^-, weighted by the population proportions of the respective nations. Second, the chance that the motion will be accepted depends on the proportion of the representatives in the assembly. The function $g(y)$ is defined as before, i.e. it equals 1 if $y > 0$ and 0 if $y \leq 0$. The chance that a motion is accepted equals 1 if a majority supports the motion, i.e. if $R_1 - R_2 > 0$, and equals 0 if the majority does not support the motion, if i.e. $R_1 - R_2 \leq 0$. Note that the values of R_i are a function of x_i and α for $i = 1, 2$. Third, let p be the chance that an arbitrary nation will vote for a motion. We have seen before that p equals 1/2 for *balanced* voters, 2/3 for *generous* voters and 1/3 for *stingy* voters. On row 2, the chance that the particular combination of nation 1 voting for and nation 2 voting against the motion equals $p(1 - p)$. In the last column we construct the product of these factors on each row and on the last row we construct the sum of these products.

The computational time in constructing a plot for $\alpha \in [0, 1]$ can be substantially reduced by assuming that the assembly has an infinite number of members, so that we can actually conduct a vote by means of the ratios $r_i(\alpha)$. This may seem like an unrealistic idealization, but the fact of the matter is that this idealization makes very little difference as long as the assembly is sufficiently large. We calculate

[4] The analytical work in this section can be directly extended to the Rawlsian measure as well.

$E[U]$ for *stingy*, *generous* and *balanced* voters in the European Union for α ε [0, 1] and have plotted these functions in Figures 5, 6 and 7.[5] Note that the function is smoother, i.e. less of a sequence of step functions, than the simulation results in Figures 2, 3 and 4 suggest. The steps come about due to the relatively small size of the Council of the European Union. In Figure 8 we have simulated the utilitarian measure for the European Parliament with 626 representatives. Notice how this function virtually coincides with the function that is represented in Figure 5. This function is calculated with proportions of representatives, which is tantamount to calculating the function for an assembly with an infinite number of representatives.

It is worth noting that the function $E[U]$ is fully determined by the parameters u^+, u^- and p for a particular federation. In our simulation we specified a uniform distribution for v_i for i = 1,..., N. But the only features of this distribution that are relevant are the probability p that an arbitrary nation will accept a motion, the expected utility u^- of an accepted motion for a nation that voted against the motion and the expected utility u^+ of an accepted motion for a nation that voted against the motion. As long as we keep these parameters fixed, the particular shape of the distribution is of no consequence for the quantities of interest in this paper.

We can also explain why the function $M^{util}[\boldsymbol{u}(\alpha)]$ remains constant for low values of α within each context. The value of $M^{util}[\boldsymbol{u}(\alpha)]$ is determined by the values of x_i and the values of R_i for i = 1,..., N nations assuming a particular context of evaluation. For a given federation the values of x_i are fixed. The values of R_i are determined by the values of x_i and α. Set the value of α at 0. As we increase the value of α nothing will happen to $U(\alpha)$ unless there is some change in one of the g functions. Let's return to the European Union with $N = 15$ as an example. When α equals 0 then the 8 smallest nations can outvote the 7 largest nations, since they have a majority of representatives. The lowest value of α for which a change occurs in one of the g functions is when there is sufficient degressive proportionality so that the 8 smallest nations can no longer outvote the 7 largest nations. Order the nations according to size so that $\pi(1)$ refers to the smallest nation and $\pi(15)$ to the largest nation in the European Union. When $\alpha = 0$, then $g(R_{\pi(1)} + ... + R_{\pi(8)} - R_{\pi(9)} - ... - R_{\pi(15)}) = 1$. But as we allow for more proportionality in the system, i.e. as we increase the value of α, then $g(R_{\pi(1)} + ... + R_{\pi(8)} - R_{\pi(9)} - ... - R_{\pi(15)})$ will eventually flip to 0. For what value of α does this change occur? The change in the g function occurs precisely when $R_{\pi(1)} + ... + R_{\pi(8)}$ no longer exceeds $R_{\pi(9)} + ... + R_{\pi(15)}$. If the number of representatives is infinite, then R_i equals r_i and this condition is equivalent to the condition that $x^{\alpha}_{\pi(1)} + ... + x^{\alpha}_{\pi(8)}$ no longer exceeds $x^{\alpha}_{\pi(9)} + ... + x^{\alpha}_{\pi(15)}$. To find this value we solve the following equation for α:

(7) $$x^{\alpha}_{\pi(1)} + ... + x^{\alpha}_{\pi(8)} = x^{\alpha}_{\pi(9)} + ... + x^{\alpha}_{\pi(15)} .$$

With the proportional population sizes for the European Union this yields $\alpha \approx .065$. And indeed the function $M^{util}[\boldsymbol{u}(\alpha)]$ is constant roughly over the interval [0, .065] in Figures 1 through 8. (For finite numbers of representatives, the value .065 is only an approximation and this approximation is the more accurate, the greater the number of representatives.) There are also other less perspicuous plateaus in the curve representing the function $M^{util}[\boldsymbol{u}(\alpha)]$ which are due to various combinations of possible coalitions.

6. Further Questions.

We have made the simplifying assumption that the utility levels of the various nations in a motion are independent and identically distributed variables, that the distribution in question is uniform, and that the threshold level for voting for a motion is kept constant across all nations. To give empirical content to this study in institutional design, many of these assumptions will need to be relaxed. The assumption of independence may need to be relaxed. It may well be the case that a number of nations have common interests – e.g. motions dealing with agriculture will often elicit strongly correlated utility levels for Mediterranean nations in the EU. The assumption of identical distribution may have to be relaxed. It may well be the case that some nations tend to benefit more while others benefit less from motions that affect the federation.

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[5] The reader will notice that the function $E[U]$ for *generous* voters equals the function $E[U]$ for *stingy* voters plus 1/4. This can be proven to be the case by replacing the function $g(y)$ by the Heaviside function and by appealing to the integral representation of the Heaviside function, which is a common technique in mathematical physics. (Proof omitted.) However, unlike the g function used in (2), the Heaviside function is not defined for 0, i.e. when the numbers of votes for the motion equals the numbers of votes against the motion. The theorem does not hold for instance when $\alpha = 0$ and there is an equal number of nations, since in this case it may be the case that the number of representatives voting for the motion equals the number of representatives voting against the motion.

The assumption of uniformity may need to be relaxed. It may well be the case that benefits and cost are less weighty for some nations so that a normal distribution with mean at the threshold value and a fairly low standard deviation is more characteristic. And finally, the assumption of a constant threshold level may need to be relaxed. Certain nations may, for example, be more picky and only vote for a motion if they stand to benefit substantially. It is easy to build any of these adjustments into the code of the simulation. The easiest way to do so is to generate random numbers for the utility values of the nations under a multivariate normal distribution. We can set different bounds and different means for each nation, increase or decrease the variance as we want the distribution to be more or less uniform for some nation, and specify covariance measures that express a commonality of interests between the nations. Of course, there is a serious empirical challenge to construct a multivariate normal that characterizes the impact of motions on the welfare of the nations in the federation. But to substitute this multivariate normal in our simulation is straightforward.

An alternative to degressive proportionality is to demand that the motion be approved both by an assembly with proportional representation and by an assembly with equal representation. In the United States, the House of Representatives and the Senate approximates this model. Or one may obtain similar results by demanding from a single assembly that the votes of the representatives reflect the majority of the population in the federation as well as the majority of the nations. It is an open question whether there exists a model of regressive proportionality that yields values on the Rawlsian maximin measure and the utilitarian measure with a single majority vote that are Pareto optimal to the values on a double majority vote. Is the answer to this question stable across different contexts of evaluations? Is it stable when we relax the assumptions in various ways? These are some of the questions for future research.

Figures

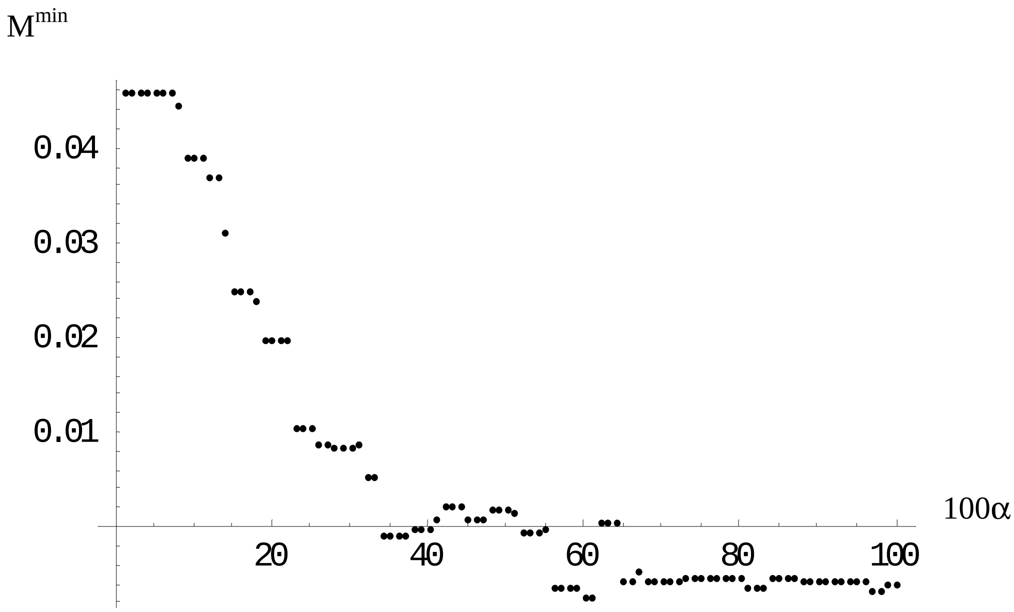

Fig. 1: The Rawlsian Measure for *Balanced* Voters for Europe and 87 Representatives

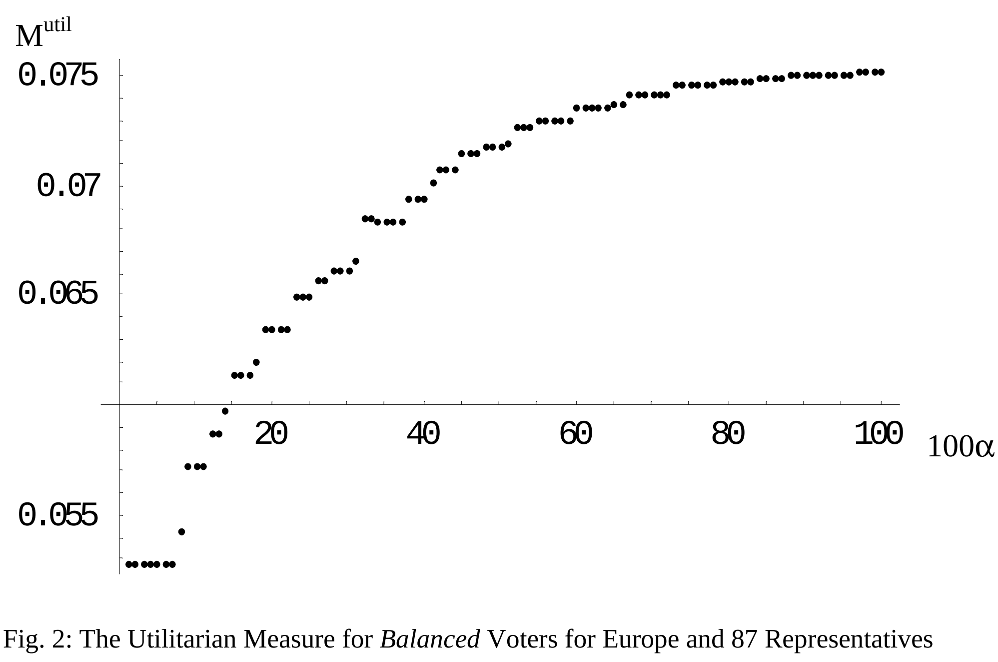

Fig. 2: The Utilitarian Measure for *Balanced* Voters for Europe and 87 Representatives

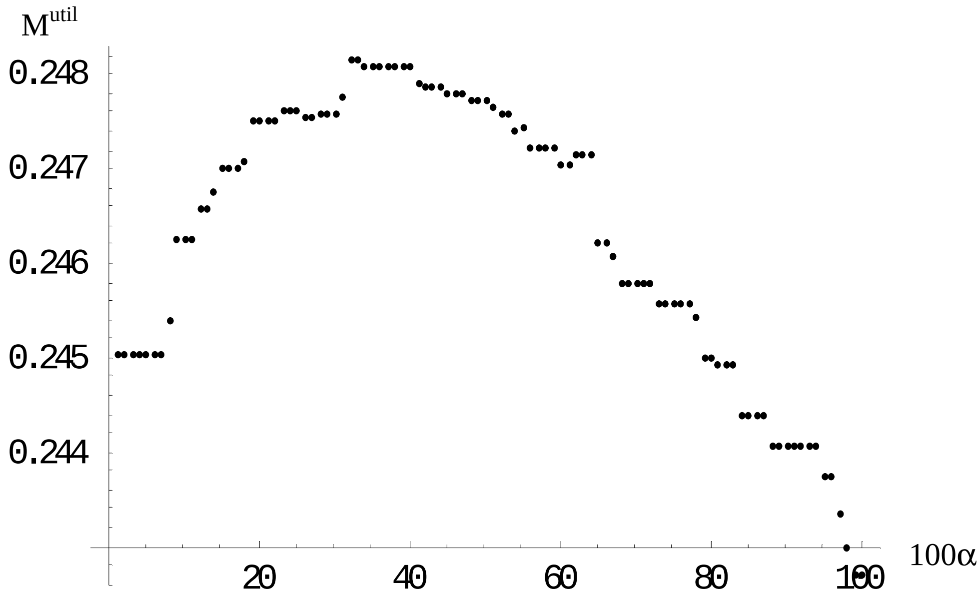

Fig. 3: The Utilitarian Measure for *Generous* Voters for Europe and 87 Representatives

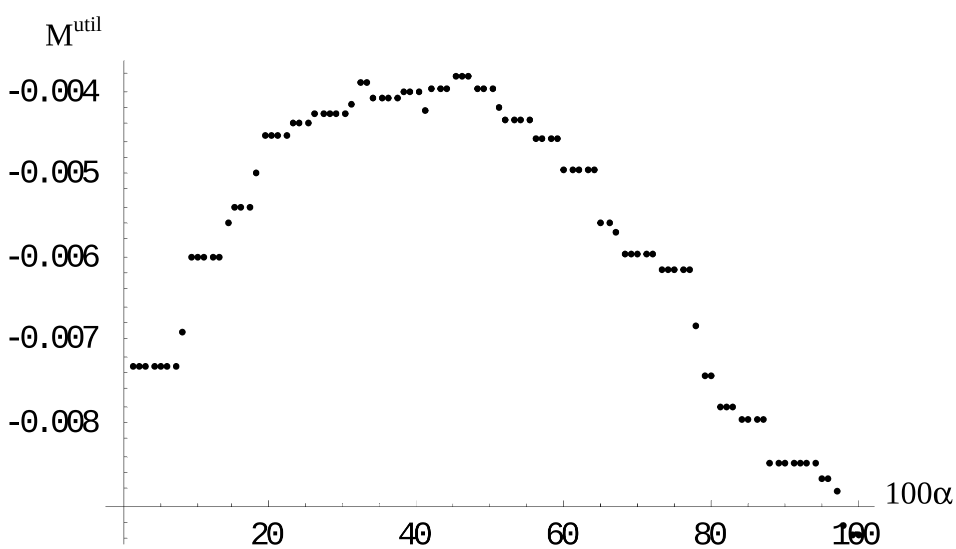

Fig. 4: The Utilitarian Measure for *Stingy* Voters for Europe and 87 Representatives

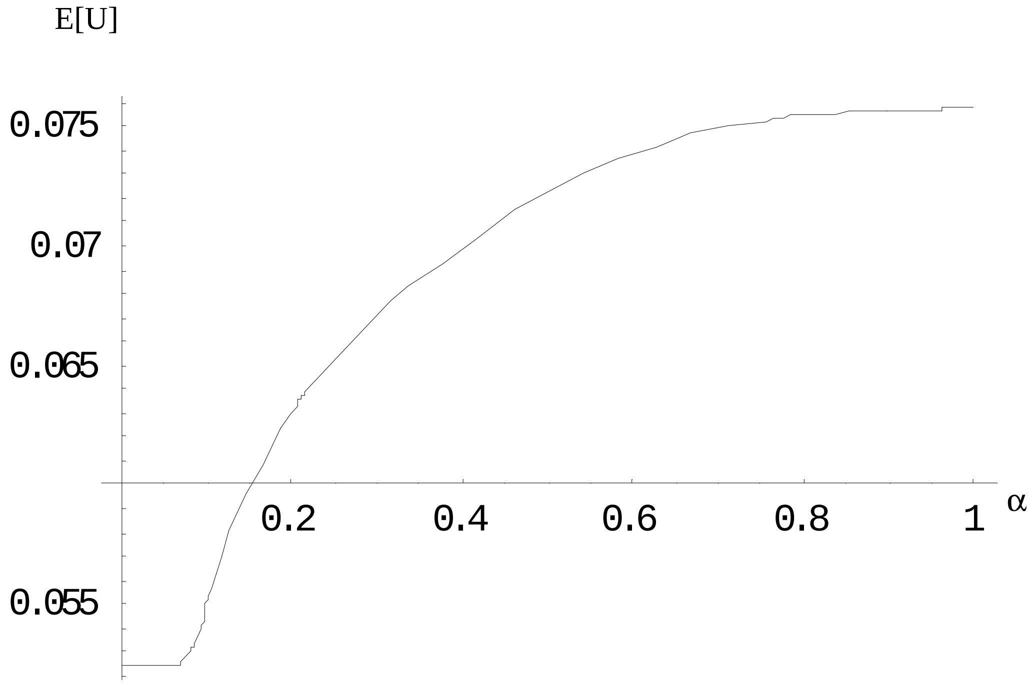

Fig. 5: The function E[U] for *Balanced* Voters for Europe and an Infinite Number of Representatives

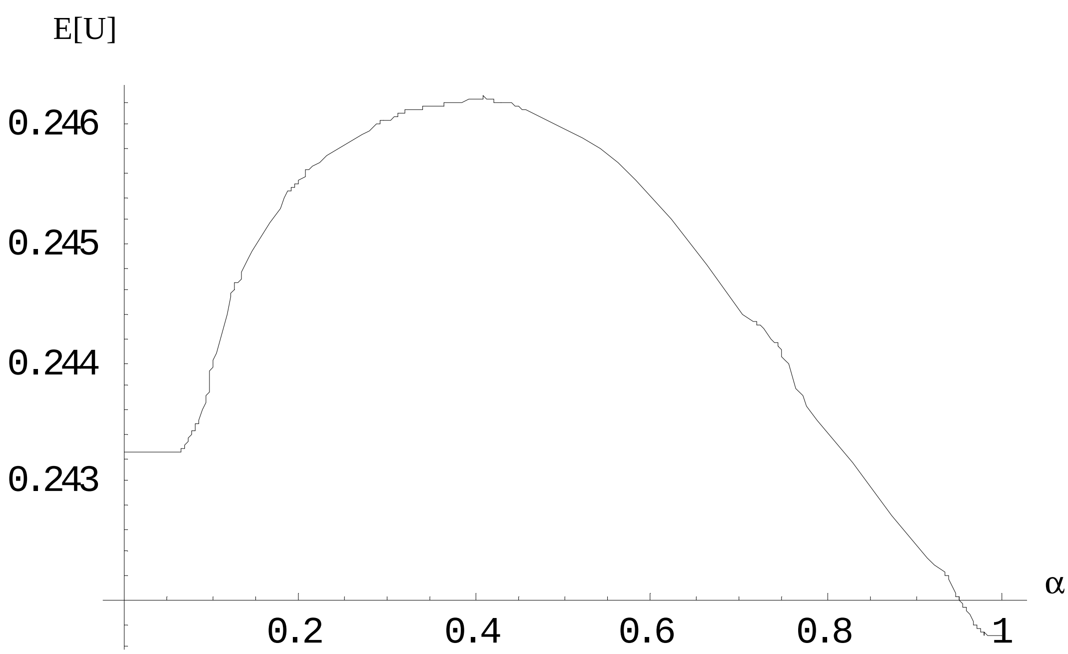

Fig. 6: The function E[U] for *Generous* Voters for Europe and an Infinite Number of Representatives

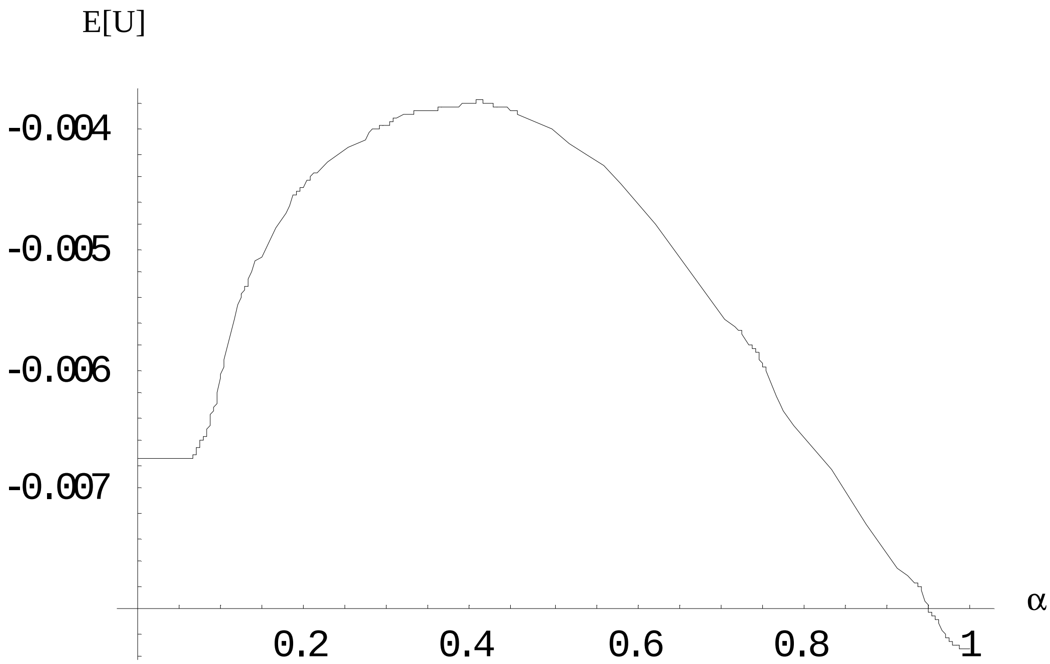

Fig. 7: The function E[U] for *Stingy* Voters for Europe and an Infinite Number of Representatives

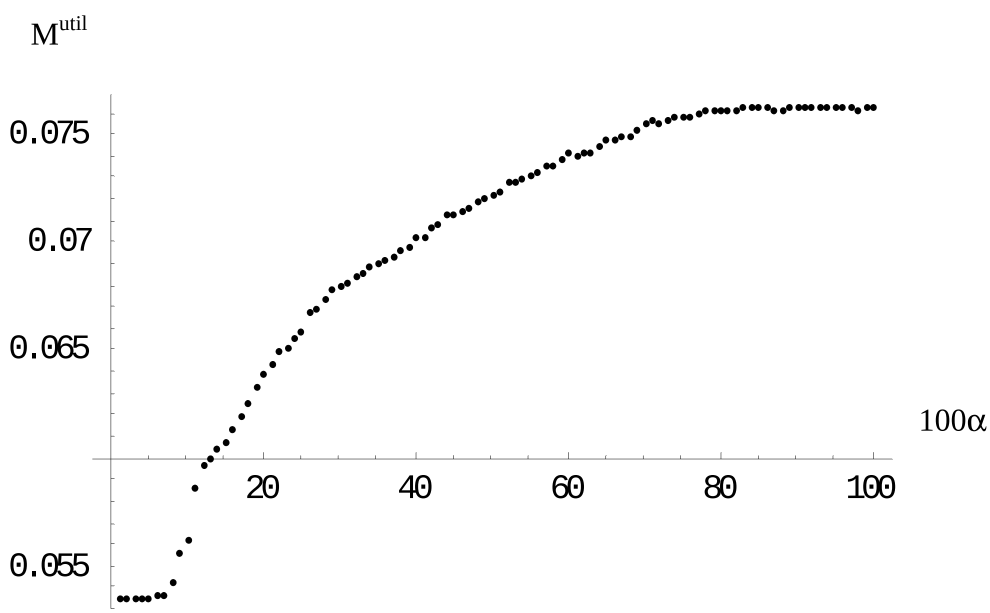

Fig. 8: The Utilitarian Measure for *Balanced* Voters for Europe and 626 Representatives

Tables

Austria	7.9	.0214
Belgium	10	.0271
Denmark	5.2	.0141
Finland	5	.0136
France	57.2	.1550
Germany	81.2	.2201
Greece	10.2	.0276
Ireland	3.5	.0095
Italy	57.8	.1566
Luxembourg	.3897	.0011
Netherlands	15.1	.0409
Portugal	9.8	.0266
Spain	39.1	.1060
Sweden	8.8	.0238
UK	57.6	.1561
Total	369	1

Table 1: Population sizes in Millions (Second Column) and Population Proportions (Third Column) of the Constituent Nations of the EU

	1	2	$E[U \mid A, C_i]$	$P(A \mid C_i)$	$P(C_i)$	$\prod$
C_1	+	+	$u^+x_1 + u^+x_2$	$g(R_1 + R_2)$	p^2	
C_2	+	–	$u^+x_1 + u^-x_2$	$g(R_1 – R_2)$	$p(1 – p)$	
C_3	–	+	$u^-x_1 + u^+x_2$	$g(–R_1 + R_2)$	$(1 – p)p$	
C_4	–	–	$u^-x_1 + u^-x_2$	$g(–R_1 – R_2)$	$(1 – p)^2$	
						$\sum$

Table 2: Construction of the Function $E[U]$ in Equation (7)

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