Adjoint Exactness

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Abstract

Plato's ideas and Aristotle's real types from the classical age, Nominalism and Realism of the mediaeval period and Whitehead's modern view of the world as process all come together in the formal representation by category theory of exactness in adjointness ($\dashv$). Concepts of exactness and co-exactness arise naturally from adjointness and are needed in current global problems of science. If a right co-exact valued left-adjoint functor (Σ) in a cartesian closed category has a right-adjoint left-exact functor (Δ), then physical stability is satisfied if Δ itself is also a right co-exact left-adjoint functor for the right-adjoint left exact functor (Π): $\Sigma \dashv \Delta \dashv \Pi$. These concepts are discussed here with examples in nuclear fusion, in database interrogation and in the cosmological fine structure constant by the Frederick construction.

1 Exactness

The principles of exactness and adjointness have appeared over the centuries in many guises in a multitude of phenomena and applications but it is only since the development of category theory that their interrelation has become transparent and their character as universal recognised [13, 12]. Exactness is that property of boundary at the closure at the top of any system, with its dual property of co-exactness corresponding to the origin or bottom of any system. A distinction between top-down and bottom-up methods is often made but it is not always appreciated that there is a fundamental type change between the two approaches: whichever is treated as covariant then the other is contravariant. In a world of 'process' (rather than of fixed sets) the respective vocabulary of source and sink for bottom and top might be more appropriate [1].

Aristotle was probably the first to make a serious study of the nature of categories in the *Organon* [1]. However, there was little formal work on categories for well over 2,000 years until (initially independent of but then) building on the work of Frege [11], Whitehead and Russell introduced the topic of typing at the beginning of the *Principia Mathematica* written in the first decade of the 20th century ([40] chapter II pp.39-68). Russell first developed a more advanced theory of types to deal with his eponymous paradox - the anomalous set of all sets that cannot be a member of itself. This was in his *Principles of Mathematics* [2].

At the time Russell thought that the doctrine of types in his Appendix B proved the existence of mathematical objects but in the second edition, he changed his view:

> What is said on existence-theorems in the last paragraph of the last chapter of the "Principles" (pp. 497-8) no longer appears to be valid: such existence-theorems, with certain exceptions, are, I should now say, examples of propositions which can be *enunciated* in logical terms, that can only be disproved or disproved with empirical evidence (at p. viii [36] 2nd edition).

This disenchantment with the theory of types is further confirmed by remarks of Spencer Brown:

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[1] In Rossiter, Heather & Sisiaridis *Process as a World Transaction* [33] the nature of banking database transactions provides an example where the zero balance might be implemented as a process when there is an arrangement with the bank automatically to top up an account from a second account to prevent the first from going into overdraft.

[2] Russell's *Principles of Mathematics* of 1903 [35] needs to be distinguished from the more formal mathematics of Whitehead & Russell's *Principia Mathematica* [40] although the latter was planned as a second volume to the former. However the Principia was found to be a much greater undertaking than originally anticipated. The outcome was that the *Principia* was published as a self-standing work in two volumes with a third volume planned but never published although much of it was written. Apparently most of the formal mathematical content was penned by Whitehead with Russell making policy decisions [Ivor Grattan-Guinness private communication 2007]. Whitehead himself says in a footnote in *Process and Reality* [41] that Russell was responsible for most of (and in the 2nd edition the whole of) the philosophical content.

> Recalling Russell's connection with the theory of types, it was with some trepidation that I approached him in 1967 with the proof that it was unnecessary. To my relief he was delighted. The Theory was, he said, the most arbitrary he and Whitehead had ever had to do, not really a theory but a stopgap, and he was glad to have lived long enough to see the matter resolved (pp. xiii-xiv in ([37]).

Whether this did resolve the matter is doubtful. Spencer Brown's theory of standard forms only took the theory further to a limited extent. In his words:

> Put as simply as I can make it, the resolution is as follows. All we have to show is that the self-referential paradoxes, discarded with the Theory of Types, are no worse than similar self-referential paradoxes, which are considered quite acceptable in the ordinary theory of equations.

Spencer Brown seems only therefore to seek to make the paradoxes respectable, not to remedy them. It is certainly a valid point he mentions not generally appreciated that Russell's paradox is not just a special case but is present everywhere in algebra. However he certainly made no attempt to deal with the much more far-reaching knock-out blow dealt to set theory by Gödel's theorems of undecidability ([10] at p.49) which we have discussed elsewhere [17].

With the advent of modern digital computers, typing soon became a very practical issue. This is one aspect of exactness. The simple sum of two entities has to be carefully specified if the entities are of different types. The human brain can adapt according to context. A computer (currently) needs specification because it lacks an awareness of context. These must be suggested either explicitly, by default or be determinable by a specified procedure.

The topic of types had not totally disappeared between Aristotle and Russell for there was a very active debate in mediaeval philosophy between the Nominalists and the Realists, which arose from the translation of the *Organon* into Latin. Aristotle's teaching on categories does not make plain, which is more fundamental, the intension or extension. Is there an absolute concept that enables us to identify a tree when we see one or do the examples of trees we see around enable us to construct an archetypal concept of a perfect tree? The Nominalists argued strongly for the former, the Realists for the latter.

The precise distinction between intension and extension was not really recognised until the treatment of these concepts in the Port-Royal logic, for instance in *Ideas* 1662-1683 ([2], Comprehension and Extension at pp. 39-40):

> Now in these universal ideas there are two things which it is most important to distinguish clearly, *the comprehension* and *the extension*. I call the *comprehension* of an idea the attributes that it contains in itself, and that cannot be removed without destroying the idea. For example, the comprehension of the idea of a triangle contains extension, shape, three lines, three angles, and the equality of these three angles to two right angles, etc. I call the *extension* of an idea the subjects to which this idea applies. These are also called the inferiors of a general term, which is superior with respect to them. For example, the idea of a triangle in general extends to all the different species of triangles.

2 Exactness in Category Theory

The formal development of a 'Gödel free' notion of types can now be found in category theory. We find that it is not a question of which came first, the intension or extension. It is neither. It is a matter of adjointness and not a question of one or the other. Nominalists and Realists are really addressing two sides of the same coin [3]. It is the context that makes typing so important. A local system can be treated as roughly homogeneous and therefore the methods of classical physics have given rise to some astonishingly exact theories derived from simple models based on number and set theory. Such simplicity however is not maintained across problems of biology and medicine or in many examples of global systems important today. There set theoretic methods can be very inexact.

In applied mathematics we are concerned *directly* only with cartesian closed categories [4]. Cartesian closed categories possess both limits and exponentials as well as possibly their duals. The property of co-exactness is existence. For it is the property of being 'spot-on' i.e. relevant. It seems hardly a chance coincidence that the vocabulary used by both lawyers and physicists for this concept of relevancy is 'material'. It seems to arise from the way natural language is constructed as a reflection of the world as it is to be found. Co-exactness is what makes Aristotelian reality, that is matter [1].

The fundamental particles (the spots in 'spot-on') of any category system constitute its initial object. Exactness on the other hand is Plato's sublime reality. It is the idea or ideal condition of perfection [5]. It is not the everyday reality we experience here. To

[3] It is probably the same explanation for the old conundrum 'which came first, the chicken or the egg? That is a general feature of the theory of causation'.

[4] Other categories from pure mathematics, for example n-categories, may still be used as models - but they are not *reality* in an Aristotelian sense.

[5] As in the Myth of a Cave and the Tripartite Soul [38].

the Platonist any system we are concerned with in this life is only a pale shadow of the true potential concept. Left exactness has a sense of coherence in classical and quantum physics. Coherence in a process view is not a fixed state but one of dynamic equilibrium and structural entropy which gives rise to the presently recognised phenomenon of emergence [32] ([31], at p.124) which was drawn to the attention of ANPA by Frederick Young [42].

Philip Clayton in his book *Mind & Emergence* ([9] at p.vi in preface) [6] defines emergence as:

> Emergence is the view that new and unpredictable phenomena are naturally produced by interactions in nature; that these new structures, organisms, and ideas are not reducible to the subsystems on which they depend; and that the newly evolved realities in turn exercise a causal influence on the parts of which they arose.

The first occurrence of the term emergence seems to be in [23], but has been traced back to Aristotle's *Entelechies* which connects to the Leibniz monad. The modern pioneer of the concept although he did not use the term was John Stuart Mill [7] in [26] where Mill was one of the first to see a connection between levels in his treatment of induction (Book III) [8].

2.1 Adjoint Exactness

These two realities of Plato and Aristotle, exactness and co-exactness, are related formally by adjointness as shown in Figure 1.

[6] As a definition this raises concepts that need themselves to be more precisely defined like 'organisms', 'subsystems' and 'causal'.

[7] Not in Clayton's article (from [25] pp.37-39) which is fairly comprehensive with references to Morgan [27, 28], Beckermann *et al* [5] and Murphy [29].

[8] The usual edition is Philosophy of Scientific Method, Nagel, Ernest, editor, Hafner, New York (1950), which is an edited and repaginated version of the earlier one. In particular see in the 1950 edition: Book III (pp. 170-291) and Chapter X, Of Plurality of Causes and of the Intermixture of Effects (pp. 238-252).

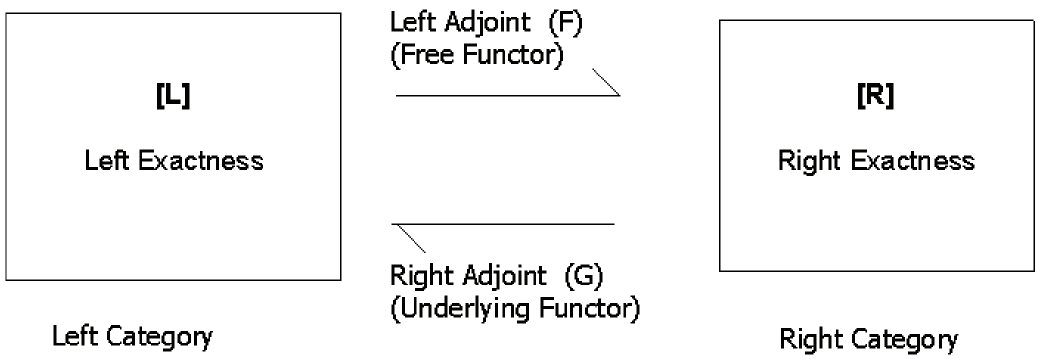

Figure 1: Left and Right Categories **L** and **R** respectively, related by the adjoint functors $F \dashv G$ where F is left adjoint to G and G is right adjoint to F

The formal representation in category theory of a left adjoint is that for a subobject process of L there is a subobject process that is its image in R. Its right adjoint means that there is a reverse image of that subobject process in any limit of L. This is the stability functor. In a free system where the only conditions are for possible existence of limits the left adjunct F is a free functor whose choice determines the existence of R. The right adjoint (G) then defines the axiom of choice and is known variously as the underlying functor or forgetful functor [9]. Because F therefore effectively creates R it is an existential functor to be identified with the existential qualifier in logic usually written as $\exists$ and read as *there exists* and its right adjoint (G) a stability functor identifiable as the diagonal functor (Δ). In the very simple case of the direct reverse, the functor (Δ) is simply an inverse image usually written as f^{-1}. Often it is a more complicated precompositional 'indexing functor' still written as f^* an old notation from functional analysis with the $*$ as the usual wild character representing the composition of all necessary functions. More precisely it is a natural transformation, sometimes represented by the symbol α^* [10].

[9]It enables L to be reinstated without leaving a trace of the choice of F so, that if unknown, R cannot be recovered by any form of reverse engineering.

[10]The asterisk wild character in α^* is really tautologous because a composition of natural transformation is just an ordinary natural transformation giving closure.

In the dual situation the same functor Δ plays the role of the left-adjoint free functor (F) and its right-adjoint is the universal quantifier usually written as $\forall$ and read as *for all*. As operators the existential Σ (sometimes written $\amalg$) generalises the sum of sorts and $\forall$ to the capital Π. More details are given by Paul Taylor ([39] Section 9.4). Given an intensional subobject there is a limit taken with all the extensional subobjects, where subobject is a monic equivalent class of arrows into an object.

$$\begin{tikzcd}
L \arrow[r, "\eta"] \arrow[d, "f"'] & GF(L) \arrow[dl, "G(g)"] \\
G(R) &
\end{tikzcd}$$

(a)

$$\begin{tikzcd}
& F(L) \arrow[dl, "F(f)"'] \arrow[d, "g"] \\
FG(R) \arrow[r, "\epsilon"] & R
\end{tikzcd}$$

(b)

Figure 2: Roles in Adjointness of a) η, the unit and b) ϵ, the counit of adjointness respectively

Mathematical results derived from the fundamental theorem of adjointness ([24], p. 121) show that left adjoints preserve colimits and right adjoints preserve limits. The physical analogue of this is that the free functor identifies potential right exact observables out of the left exact implicate order (as Bohm calls it [6]), or from the uncollapsed wave function in quantum mechanics. The right adjoint underlying functor provides the conditions (i.e. the laws of physics) for this solution. If the conditions are satisfied the result is physical existence. In terms of quantum theory it is the collapse of the wave function from quantum reality. In classical physics right exactness amounts only to a modelling of reality. Nevertheless this is rather a simplification. The fuller picture consists of a more elaborate left-right dichotomy. A very important example in practice of the process represented in Figure 1 is adjointness between syntax and semantics [22] that arises when the left and right categories are of opposite variance. In the applied mathematics of the real world there has to be added pragmatics to complete the trio with syntax and semantics. The finer detail is the diagram in Figure 2, that is a zoom into the contents of Figure 1 [11]. This is the effect of the stability functor that holds between the two categories. The potential existence identified by the qualificational existential functor Σ is not realisable in general by the quantificational universal functor Π. The diagonal stability functor doubles up as both a left and right adjoint so that it preserves both limits and co-limits. From the viewpoint of the stability functor both categories coincide as left and right categories.

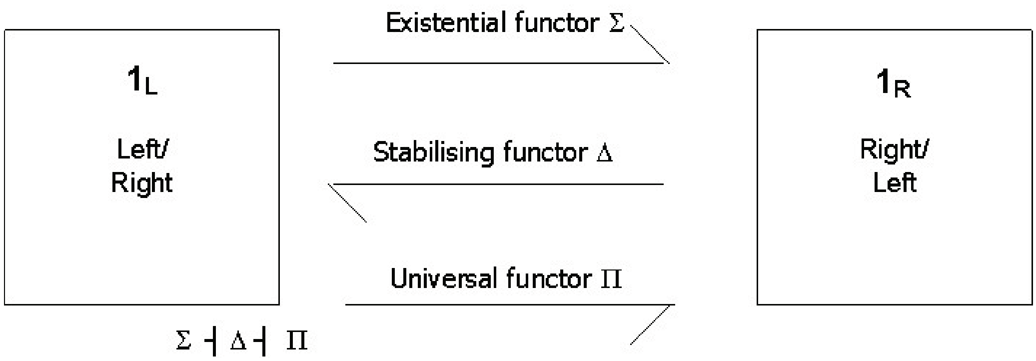

Figure 3: Left/Right and Right/Left Categories $\mathbf{1_L}$ and $\mathbf{1_R}$ respectively, related by the adjointness $\Sigma \dashv \Delta \dashv \Pi$

The exact adjointness $\Sigma \dashv \Delta \dashv \Pi$ formally defines the concept of *process* in the universe. It is the fundamental definition of the arrow in cartesian closed categories as a composition resolvable into three levels relating the left category $\mathbf{1_L}$ with the right category $\mathbf{1_R}$ in the diagram of Figure 3. Then Figure 4 shows that a process is a composition of $\Sigma \dashv \Delta \dashv \Pi$.

[11] The triangle (a) of Figure 2 is in the left category of Figure 1 and triangle (b) in its right category. The two triangles mutually establish the unique existence of the exactness and co-exactness relatively of the left- and right-categories respectively.

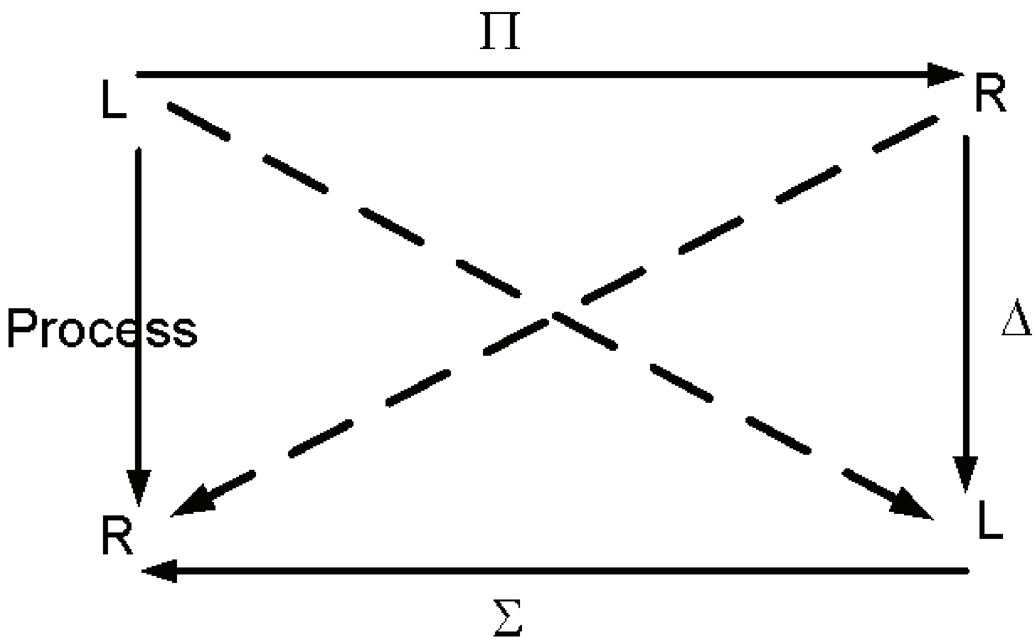

Figure 4: Commuting Diagram showing that Process $\equiv \Sigma \dashv \Delta \dashv \Pi$; dashed diagonal arrows give simultaneity

3 Examples of Exactness

A physical example of exactness is the discovery of unstable chemical elements e.g. Lawrencium Lr (atomic number 103) and Ununoctium Uuo (118). Lr was the first of the trans-uranium elements to be identified entirely by nuclear, rather than by chemical, means. Lr was discovered at the Heavy Ion Linear Accelerator (Hilac) by bombarding a target of californium (with 98 protons) with boron nuclei (with five protons) thus creating a new element with 103 protons [14]. Lr is very unstable having a half-life of only four hours, indicating that the balance between the nuclear reactions creating and decomposing it is very much biased towards the decomposition side.

The much more recent example is the discovery in 2006 of the heaviest element known of Uuo 118 which is so unstable that only three atoms of it have been detected, through collisions of californium-249 atoms and calcium-48 ions [43]. A half-life of 0.89

ms was observed, indicating the great instability of this element. Because of the very small probability that a fusion reaction occurs, more than 410^{19} calcium ions had to be shot at the californium to have only three fusion reactions.

The fusion reaction probability is greater for Lr than for Uuo but in both cases is very small. This probability is a balance between F the free functor, producing the heavy elements under collisions made at very high speeds and with great intensity, and G the underlying functor, decomposing the heavy elements back into smaller nuclei. The balance between F and G is indicated experimentally by the half-life of the element. Categorially the balance is represented by the relative values for F and G: F will have a very low probability as the fusion process is very difficult and G a very high one as the heavy ions rapidly decompose. The values of η and ϵ, the unit and counit respectively of adjunction shown in Figure 2, provide additional perspective. Because little reaction takes place from the calcium ions viewpoint then η will be small but as much reaction takes place from the heavy ions viewpoint then ϵ will be large. The unit η and counit ϵ will be smaller and larger respectively for Uuo than for Lr as Uuo is the more unstable of the two elements.

This unstable element resides in the preorder of exactness in the universe and may be observed briefly as co-exactness under these right adjoint conditions of certain partial orders. From the viewpoint of logic the functors expressed as F, G are the propositions $\exists$ and Δ in first-order predicate logic. For modal logic the pair are correspondingly $\diamond$, $\square$, the usual symbols for possibility and necessity. It is this ubiquity in mathematics and physics that suggests to us that adjointness is fundamental and the universal logic that regulates the world both physical and metaphysical [16, 15, 18, 34].

Exactness and co-exactness is a critical feature of very many problems in modern life. To illustrate the role of the interplay between left and right adjoint functors two further examples will be given. The first shows how computing procedures are an implementation of these functors in very common operations like searching databases. The development of algorithms by Newton and Leibniz for differential and integral calculus based on the concept of limits in mathematical analysis is "one of the great achievements of the human mind [12].

The great success of the use of the calculus in applied mathematics, theoretical physics and engineering must outweigh all other methods put together. Even alternative methods like statistical modelling often rely heavily on differentiation and integration. Yet no satisfactory explanation is involved for why these work. The adjoint exactness of limits and co-exactness in category theory is an obvious candidate for an explanation. But this remains to be shown. In the meantime other calculi have appeared where the connection with adjoint exactness is easier to see. In particular the rise of computing has brought on to the scene a new class of algorithm where the workings of these adjoint functors is more obvious. These workings will be shown here for an SQL exact search for the purpose of data mining in data warehouses.

The second example concerns whether the main ANPA interest in the use of the Frederick construction is a 3-level combinatorial hierarchy over an integral binary field of natural numbers. The program universe [30] was an early computational model to generate the cosmological fine structure constant. The categorial version seeks to eliminate any unnecessary assumptions. An important feature of the Frederick construction is to climb the level. The special interest of the Frederick construction form a categorial perspective is to provide the structure for dimensionless parameters

[12]Richard Courant in the preface to Boyer's *History of Calculus* [8].

[20]. There is some suggestion that these are not constants but have been weakening in time. There seems more attention paid to the variation in the gravitational constant than the fine structure constant. As a dimensionless parameter it is arguably a type free quantity.

A common view of data mining is that it enables rules between clusters of data to be derived [3]. To illustrate a very simple example of this type of problem, we show below the kinds of powerobjects which need to be constructed for complex queries. Standard query languages like SQL are designed to deal with questions such as: *Which students have at least grade B in at least three subjects?*.

Probabilistic methods using statistical models can provide very precise results but there are some application areas where precision is inadequate. For example a piece of knowledge such as 22.7% of the candidates obtained at least grade B in three subjects is precise information but it is not an example of exact knowledge discovery. This information can tell a particular student the probability of obtaining three Bs but it does not tell the student what grade that student actually obtained or would or, even, could obtain.

3.1 Example I. Requirements for Mining Data Warehouses ('Mashup')

Which students have at least grade B in at least three subjects?.

might appear in an undergraduate text as:

SELECT E1.Cand
FROM Exam E1, Exam E2, Exam E3
WHERE E1.Subj $! =$ E2.Subj AND E1.Subj $! =$ E3.Subj AND E2.Subj $! =$ E3.Subj
AND E1.Cand = E2.Cand AND E1.Cand = E3.Cand
AND E1.Grade $<=$ $'B'$ AND E2.Grade $<=$ $'B'$ AND

E3.Grade <= 'B';
where the symbol != means *not equal to* and the table *Exam* appears as follows:

Cand	Subj	Grade
1	Maths	B
1	Chems	A
1	Phys	B
2	Chem	C
3	Biol	B
3	Phys	B
3	Chem	C
4	Chem	A
5	Maths	B

This statement first takes the triple product of the table *Exam* by multiplying the table twice with itself. It then retrieves from the product those rows where there are three different course numbers for the same student and each course is associated with a grade of B or of lower lexical order. The table (logical silo) below shows an extract from the resulting triple-product table:

E1			E2			E3		
Cand	Subj	Grade	Cand	Subj	Grade	Cand	Subj	Grade
1	Maths	B	2	Chem	C	3	Biol	B
1	Maths	B	4	Chem	A	3	Phys	B
1	Maths	B	4	Chem	A	1	Phys	B
5	Maths	B	5	Maths	B	5	Biol	B
3	Chem	C	3	Phys	B	3	Biol	B
1	Maths	B	1	Chem	A	1	Phys	B

Only the last tuple satisfies the request. A practitioner would probably not follow this textbook style but would rather use grouping as:
SELECT Cand

FROM Exam
WHERE Grade <= 'B'
GROUP BY Cand
HAVING COUNT(*) >= 3;

The query on this table provides an example of exactness in set theory. The powerset on which the query is applied is a subset of the whole powerset for exams namely that part comprising three exam entries only but is still large containing $|Exam|^3$ entries where $|Exam|$ is the cardinality of the $Exam$ table.

Knowledge discovery is always looking for links and this query is almost archetypal as an example of data mining. However the SQL is not an archetypal method for dealing with it. For example this SQL query relies on inherent typing which may not be natural such as whether A is less or greater than B in the ordering system employed. This can only be resolved by resorting to a higher-level view. Any query can contain many possible problems of this nature. A SQL expression like the above example might appear somewhat contrived. In fact it is not *ad hoc* but systematic. Nevertheless this simple example illustrates the difficulties for the casual user in knowledge discovery in information systems. More fundamental problems like this with the relational data model in general can be found discussed in [19].

The components of an SQL command are represented in terms of algebra and category theory in the table below:

SQL construct	algebraic operation	functor
SELECT attributes	projection	component of Σ
FROM tables	product/join	Δ
WHERE predicate	restrict	component of Σ
GROUP BY	quotient	Π

Figure 5, adapted from Figure 3, then shows how this structure of an SQL query maps onto the adjoint relationship. The adjointness $\Sigma \dashv \Delta \dashv \Pi$ in terms of SQL is: WHERE $\circ$ SELECT $\dashv$ FROM $\dashv$ GROUP BY.

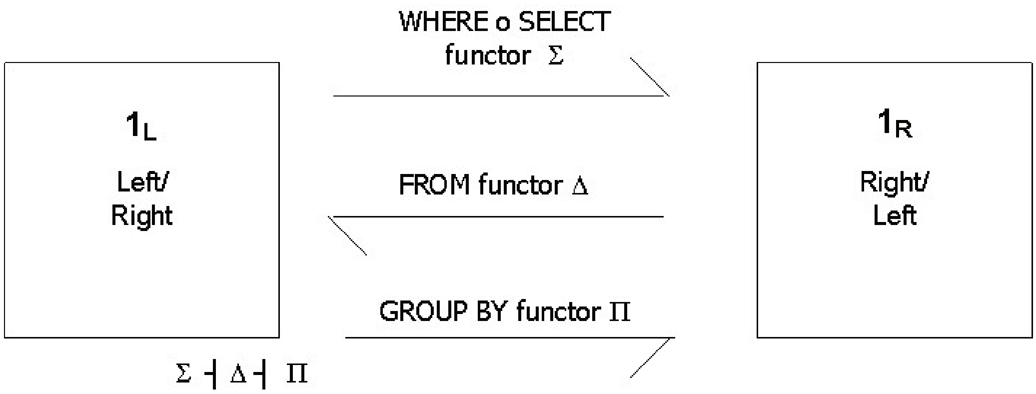

Figure 5: Left/Right and Right/Left Categories $\mathbf{1_L}$ and $\mathbf{1_R}$ respectively, related by the adjointness $\Sigma \dashv \Delta \dashv \Pi$ in terms of SQL constructs

This application of the SQL calculus illustrates well the tripartite structure of a process. Note the simultaneity of the tripartite structure $\Sigma \dashv \Delta \dashv \Pi$: composition is not time dependent and the expression can be evaluated in any way, giving associativity.

3.2 Example II. Frederick Construction of Combinatorial Hierarchy CH

The process view of the universe has been a main theme in ANPA strongly canvassed by Bastin [4]. It appears that Whitehead gradually came round to the view of process as reality [41]. Apparently he may have reached this view while writing *Principia* with Russell when they found they needed to abolish classes in order to understand even the proposition $1 + 1 = 2$. Russell later said (in his 2nd edition, at page xi):

> Dr. Whitehead, at this stage, persuaded me to abandon points of space, instance of time, and particles of matter, substituting for them logical constructions composed of events. In the end, it seemed to result that none of

> the raw material of the world has smooth logical properties, but that whatever appears to have such properties is constructed artificially in order to have them.

In the process view of the universe, mathematical objects are indistinguishable from physical objects. The structure of the universe is left exact while observables are right exact. Observations of the fine structure constant have been made with some very precise experimental results. These will therefore have an underlying functor (Δ) that relates it to the left exact reality of this fine structure. In classical terms this will relate mathematical objects, Frederick Parker Rhodes put forward his algorithm of a combinatorial hierarchy that gave a value in very close agreement with the experimental results. The most recent work shows very good agreement to seven significant decimal places. However it is clear that the Frederick construction [7] is only a model and not reality. The need to include the McGovern correction makes this very plain by adding probability theory. Nevertheless the method of Frederick Parker Rhodes is a fine example of the Δ functor that mediates between the mathematical and physical worlds of exactness.

The Frederick construction requires some explanation for the jump between levels. Parker-Rhodes himself relied on an implicit association in the numbers suggesting a natural recursive operation.

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